A Novel Chaotic Neural Network for Function Optimization

Ting Zhou, Zhenhong Jia, and Xiuling Liu

College of Information science & engineering, Xinjiang University, Urumqi 830046, P.R. China
Corresponding author: Zhenhong Jia
jzhh@xju.edu.cn

Abstract. Chaotic neural networks have been proved to be powerful tools to solve the optimization problems. And the chaotic neural networks whose activation function is non-monotonous will be more effective than Chen's chaotic neural network in solving optimization problems, especially in searching global minima of continuous function and traveling salesman problems. In this paper, a novel chaotic neural network for function optimization is introduced. In contrast to the Chen's chaotic neural network, the activation function of the novel chaotic neural network is wavelet function and the different-parameters annealing function are adopted in different period, so it performs extremely better when compared to the convergence speed and the accuracy of the results. And two elaborate examples of function optimization are given to show its superiority. This chaotic neural network can be a new powerful approach to solving a class of function optimization problems.

Keywords: Chaotic neural network, Wavelet function, Annealing function, Function optimization.

1 Introduction

Neural networks have been shown to be powerful tools for solving optimization problems. The Hopfield neural network is proposed by Hopfield and Tank [1] and [2], has been extensively applied to many fields in the past years. Unfortunately, it was shown that the simple HNN often yields infeasible solutions for complicated optimization problems, such as TSP [3]. The main reason of this inefficiency is the structure of energy function in HNN, which has many local minima in which the network get stuck in one of them due to its strictly energy reducing behavior [4].

To overcome this difficulty, chaotic neural networks exploiting the rich behaviors of nonlinear dynamics have been developed as a new approach to extend the problem solving ability of standard HNN [5]-[7]. There have been much research interests and efforts in theory and applications of chaotic neural networks [8]-[10].

However, since CNN base on the periodic oscillations property of chaotic dynamics to search the optimal solution, the search time must be spent more than the HNN. There is a new trend in using improved simulated annealing mechanics to accelerate the convergence speed of CNN [11]-[13].

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Actually, some researchers have pointed out that the single neural unit can easily behave chaotic behavior if its activation function is non-monotonous [14]. And the reference [15] has presented that the effective activation function may adopt kinds of different forms, and should embody non-monotonous behavior. In many CNN model the activation functions almost adopt sigmoid function, theoretically speaking, they are not the basic function, so the ability of solving optimization problems is less effective than whose activation functions are composed of kinds of basic functions in chaotic neural networks [16]-[18].

We benefit from these ideas in our architecture. In this paper, we introduced a novel chaotic neural network to solve function optimization problems.

The organization of this paper is as follows: The WSAN model is formulated in Section 2. Afterward, the simulations of function optimization problems that show the superiority of our method are described in Section 3. Finally the conclusion will be presented in Section 4.

2 The Novel Chaotic Neural Network

In order to take advantage of the chaotic dynamics, convergent speed, and the activation function being wavelet function, the novel chaotic neural networks are defined as:

$$x_i(t)=exp(-(u\cdot y_i(t)\cdot(1+\eta(t)))^2/2)\cdot cos(5\cdot u\cdot y(t)\cdot(1+\eta(t))) \qquad (1)$$

$$y_i(t+1)=ky_i(t)+\alpha[\sum_j W_{ij}x_j+I_i]-z_i(t)(x_i(t)-I_0)\ . \qquad (2)$$

$$z_i(t+1)=\begin{cases}(1-\beta_1)z_i(t), & if\ z_i(t)>z_i(0)/2\\ (1-\beta_2)z_i(t), & if\ z_i(t)\le z_i(0)/2\\ & and\ |x_i(t+1)-x_i(t)|>\delta\\ 0, & if\ z_i(t)\le z_i(0)/2\\ & and\ |x_i(t+1)-x_i(t)|\le\delta\end{cases}\ . \qquad (3)$$

$$\eta_i(t+1)=\frac{\eta_i(t)}{ln[e+\lambda(1-\eta_i(t))]}\ . \qquad (4)$$

where i is the index of neurons and n is the number of neurons, $x_i(t)$ the output of neuron i, $y_i(t)$ the internal state for neuron i, W_{ij} the connection weight from neuron j to neuron i, Ii the input bias of neuron i, α the positive scaling parameter for inputs, $k(0 \le k \le 1)$the damping factor of the nerve membrane, $z_i(t)$ the self-feedback connection weight, $\beta_1, \beta_2 (0 \le \beta_1 < \beta_2 \le 1)$ are the simulated annealing parameter of $z_i(t)$, δ is a given positive constant which magnitude order is 10^{-3}., λ the damping factors of $\eta_i(t)$, I_0 the positive parameter.

In this model, the equation (1) is different from the activation function of conventional CNN, which is a wavelet function other than sigmoid function, so it has a better ability in local approaching [18]. The variable $z_i(t)$ corresponds to the temperature in the usual stochastic annealing process and the equation (3) [13] is an exponential cooling schedule for the annealing. Obviously, if the value of $z_i(t)$ tends towards zero with time evolution in the form of $z_i(t) = z_i(0)e^{-\beta t}$, the novel CNN converts into HNN. In this paper, we adopt a smaller value of $\beta(\beta_1)$ before the chaotic dynamics reach the steady period-doubling bifurcated points. Then, a larger value of $\beta(\beta_2)$ is used after the chaotic dynamics tend toward steady bifurcated points. In order to banish disturbance of the self-feedback connection, we subjectively put $z_i(t) = 0$ when the difference of $|x_i(t+1) - x_i(t)|$ is less than a given positive constant (δ).

3 Application to Continuous Function Optimization

In this section, we use this novel chaotic neural network to solve continuous function optimization problems. And two examples are presented to demonstrate the superiority of our method to other methods. When HNN model is applied to solve complicated optimization problems, its energy function is defined as:

$$\mathcal{E}_{Hop}(t) = -\frac{1}{2}\sum_i \sum_{j \ne i} W_{ij} x_i(t) x_j(t) - \sum_i I_i x_i(t) + \frac{1}{\tau}\sum_i \int_0^{x_i(t)} f^{-1}(v)dv \tag{5}$$

Without going further, we know that the stable points of the very high-gain, continuous deterministic Hopfield model corresponds to the stable points of the discrete stochastic Hopfield model with the following Lyapunov energy function [19]:

$$E_{Hop}(t) = -\frac{1}{2}\sum_i \sum_{j \ne i} W_{ij} x_i(t) x_j(t) - \sum_i I_i x_i(t) \ . \tag{6}$$

Comparing (6) with the cost function of our method:

$$-\frac{\partial E}{\partial x_i} = -\frac{\partial f}{\partial x_i} = -\Big(\sum_{j=1, j\neq i} W_{ij} x_j + I_i \Big) \cdot \tag{7}$$

Where f is a function that needs to be calculated the global optimal solution.

Example 1: A classic nonlinear function optimization problem

$$\begin{aligned} &min\ f_1(x_1, x_2) = (x_1 - 0.7)^2((x_2 + 0.6)^2 + 0.1) \\ &+(x_2 - 0.5)^2((x_1 + 0.4)^2 + 0.15) \end{aligned} \tag{8}$$

The minimum value of this object function [equation (8)] is 0 and its responding point is (0.7, 0.5), and the total number of local optimal value is 3: (0.6, 0.4), (0.6, 0.5) and (0.7, 0.4).

The parameters are set as follows:

$$u = 2, k = 1, \alpha = 0.05, I_0 = 0.05, \beta_1 = 0.02, \beta_2 = 0.1, \lambda = 0.05, \delta = 0.001.$$

We adopt the same initial values of network in Reference [13]:

$$y(0) = [-0.283, -0.283], \ z(0) = [0.065, 0.065], \ \eta(0) = [0.05, 0.08].$$

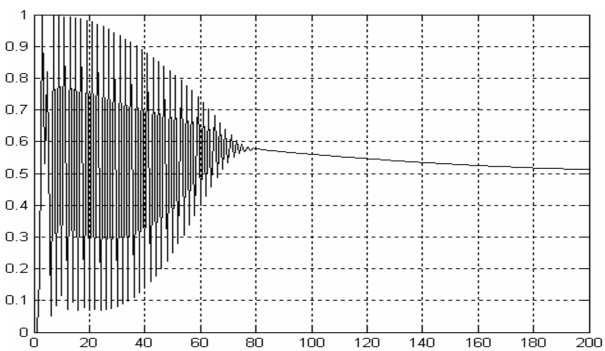

Fig. 1. The time evolution of $x_2(t)$ in simulation of Chen's chaotic neural network

The CNN in figure 1, the activation function of neural unit is sigmoid function and the value of the simulated annealing parameter is only put a single value in the whole optimization procedure, so we can see that $x_2(t)$ converges the global optimal value 0.5 more than 200 iterations. While in figure 2 $x_2(t)$ reaches the global optimal value 0.5 only iterations 45.

In order to make it be understood much clearer, we divide the whole optimization procedure into two processes: the first process is based on the chaotic dynamics and

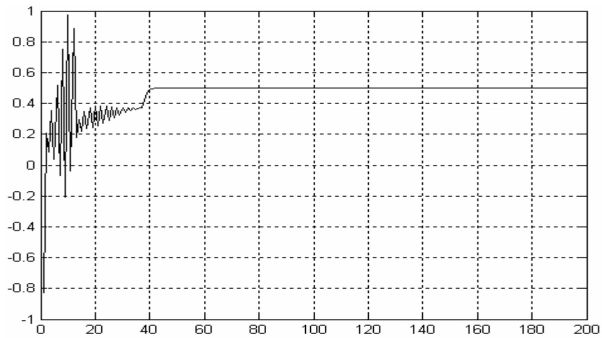

Fig. 2. The time evolution of $x_2(t)$ in simulation of the novel chaotic neural network

the second process is based on the gradient decent dynamics. By transferring sigmoid function to wavelet function in the novel chaotic neural network model, it can accomplish the ergodic chaotic dynamics more quickly in the first process and arrive at the global optimal value round. The main reason is the activation function of neural unit is non-monotonous wavelet function, so it has a better ability in local approaching.

In the second process in figure 1 when $x_2(t)$ tends toward to the global optimal value point 0.5, the value of the self-feedback connection weight remains very small. Moreover this small value continuously takes disturbance to the gradient convergent procedure. Therefore it leads to waste much more time to converge at the global optimal value. However, in this paper the different-parameters annealing function are adopted in different period which has been described in details in section 2, so it can overcome the above problems.

Compared figure 1 with figure 2, we can see that the CNN in this paper spends less time finding the global optimal value than Chen's CNN does. Furthermore it guarantees the accuracy of global optimal value to function optimization.

Example 2: Six-Hump Camel -Back Function [16]:

$$min\ f_2(x_1,x_2)=4x_1^2-2.1x_1^4+x_1^6/3+x_1x_2-4x_2^2+4x_2^4 \quad |x_i \le 1| \tag{9}$$

The minimal value of Equation (9) is –1.0316285, and its responding point is (0.08983, -0.7126) or (-0.08983, 0.7126).

We adopt our method to solve this function optimization problem, and we'll make a comparison with Reference [16] and [20] in Table 1. The parameters are set as follows:

$$u=0.05, k=1, \alpha=0.2, I_0=0.05, \lambda=0.3, \beta_1=0.015, \beta_2=0.1, \delta=0.001$$

The initial values of network are set as follows:

$$y(0)=[0.6,0.6], z(0)=[17.5,17.5], \eta(0)=[0.01,0.01]$$

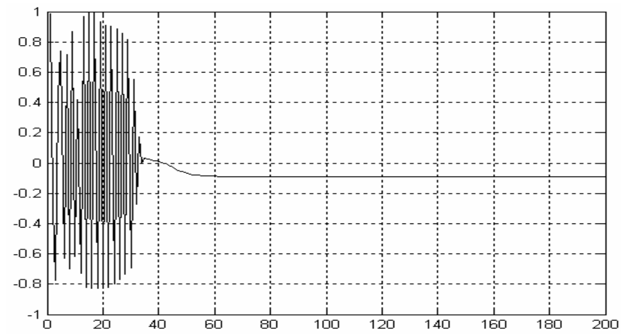

Fig. 3. The time evolution of $x_1(t)$ in simulation of function (9)

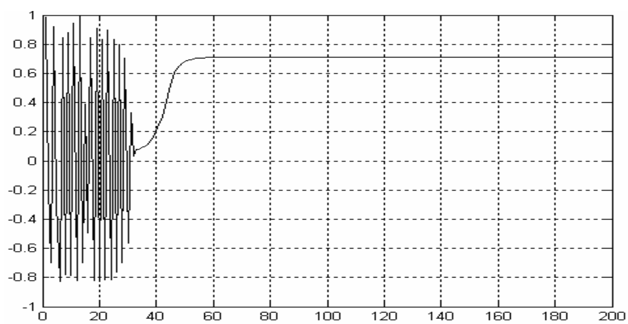

Fig. 4. The time evolution of $x_2(t)$ in simulation of function (9)

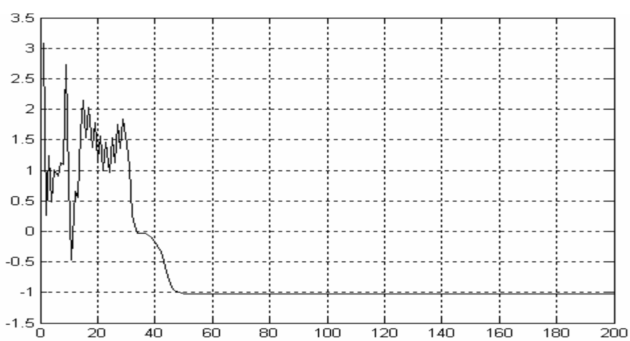

Fig. 5. The time evolution of energy function of (9)

The above figures suggest that a search of the global minima is through chaotic dynamics, the practical global minimal value of Equation (9) in Fig.5 is –1.0316 and its responding point of the simulation in Fig.5 is (-0.0898, 0.7127).

Analysis of the Simulation Results:

Table 1. Simulation results of equation (9) obtained from this paper, Reference [16] and Reference [20]

	f_2 (this paper)	f_2(Reference [16])	f_2 (Reference [20])
TGM	-1.0316285	-1.0316285	-1.0316285
PGM	-1.0316	-1	-1
ERR	-0.0000285	-0.0316285	-0.0316285

In Table 1, we compare the result of figure 5 obtained from this paper with the results obtained from others, such as the Reference [16] and Reference [20]. And the columns "TGM", "PGM" and "ERR" represent, respectively, theoretical global value, practical global value and error.

In figure 5, the energy function of Equation (9) in our paper reaches the global optimal value only with 60 iterations. It's still faster than Reference [16] and [20] which reached the practical global value with about 100 iterations under the same simulated parameters. Besides, In Table 1 the theoretical global value is –1.0316285, and the practice global value obtained from ours is –1.0316 while Reference [16] and Reference [20] are -1. It's obviously that the global value obtained from this paper is much closer to the theoretical global value.

And we also use this model to other function optimizations, such as the famous function called Rosenbrock function problem [21]. The overall data obtained proved this novel CNN to be effective in solving optimization problems.

4 Conclusion

In this paper, we introduced a novel chaotic neural network which activation function of neural unit is wavelet function and the different-parameters annealing function are adopted in the different period. In contrast to Chen's chaotic neural network, application of this model to continuous function optimization showed its superiority when compared to the convergence speed and the accuracy of the results. This model can be a new approach to solving a class of function optimization problems.

This paper has shown the potential of chaotic neural network model which activation function is composed of non-monotonic basic function for solving the optimization problems. From which has been shown that this neural techniques can find the global optimal value much faster and more accurate. And the model may also be well suited to solving the combinatorial optimization problems such as TSP and CAP, due to its inherently adaptive nature. Applications of the model for this purpose will be the subject of our future research.

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