How Does an Intelligent Learning Environment with Novel Design Affect the Students' Learning Results?

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Abstract. We present an intelligent learning environment, T-algebra, for step-by-step solving of algebra problems using a novel design of step dialogue, which combines two known approaches: conversion by rules and entering the result. Each solution step in T-algebra consists of three stages: selection of the transformation rule, marking the parts of expression, entering the result of the operation. The designed dialogue enables the student to make the same mistakes as on paper and to receive understandable feedback about mistakes. The evaluation demonstrated that even a brief use of T-algebra affects the results of learning. The students who used T-algebra did better on consecutive paper test than the students who did not use T-algebra. Furthermore, T-algebra tends to affect specific error types, i.e., after using T-algebra the students make fewer mistakes of certain type on paper as well.

1 Introduction

Learning environments for step-by-step solving of expression manipulation problems (inc. linear equations) have been designed for a long time. Even the earlier environments could be divided into two groups according to the type of dialogue they use: rule-based or command-based environments (EXPRESSIONS [21], ALGREBRALAND [4], DISSOLVE [14]) and input-based environments (BUGGY/DEBUGGY system [5, 6], LMS [19], EMMA [16], Algebra tutor [2]). This division is still applicable today.

Rule-based environments (such as MathXpert [3], AlgeBrain [1], Cognitive algebra tutor [7], E-tutor: An Equation Solving Tutor [17]) are based on the principle that the student selects the transformation rule and in some cases a part of the expression; the transformation itself is made by the computer. In such environments, the student learns and practices the solution algorithm, but the learning of performing algorithm steps is passive, because the computer performs more work than the user. In addition, the student is not given the possibility to make certain mistakes; many typical mistakes are simply impossible.

Input-based environments (like Aplusix [13], Treefrog [20]) use paper-and-pencil-like dialogue design where a transformation step consists mainly of entering the next line. The student has the possibility to perform whatever steps and as much as he/she wants in one step and to make arbitrary mistakes. Yet such programs usually do not handle the solution algorithms of different types of problems and do not provide a precise diagnosis of the errors made.

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This article presents the intelligent learning environment T-algebra with a novel design of step dialogue [12, 15]. The design is novel, because it combines two known approaches: rule-based and input-based environments (conversion by rules is supplemented by entering the result). By choosing the rule, the appropriate parts of the expression and entering the result of the operation, the student can learn the algorithms and their steps and make mistakes in the same way as on paper. The proposed dialogue enables the program to check the knowledge and skills of the student, to diagnose errors and to offer feedback.

Before distribution of T-algebra to all Estonian schools, we organized an experiment to clarify how the program affects the learning results. 126 students of 7th grade (about 13 years old) from four different Estonian schools participated in the experiment. Pretest-posttest control-group design [8] was used in research. Pre-test and post-test were solved on paper in both (experimental and control) groups. Between these tests the control group received traditional instruction using paper and pencil, while the experimental group received experimental instruction using T-algebra. This experiment compared T-algebra with the paper-and-pencil method, not with other learning environments.

The second part of this article presents an overview of the T-algebra environment and error diagnosis principles in T-algebra. The third section describes the conditions of the experiment. The fourth part summarizes the pre-test and post-test results of the experimental and control groups, compares them, presents interesting findings of this comparison and answers the question: How does an interactive learning environment with novel design affect the students' learning results? The article also examines the types of errors in post-test in experimental and control groups.

2 Description of T-algebra Environment

T-algebra is an interactive learning environment for step-by-step solving of school algebra problems, including linear equations. Each solution step in T-algebra consists of

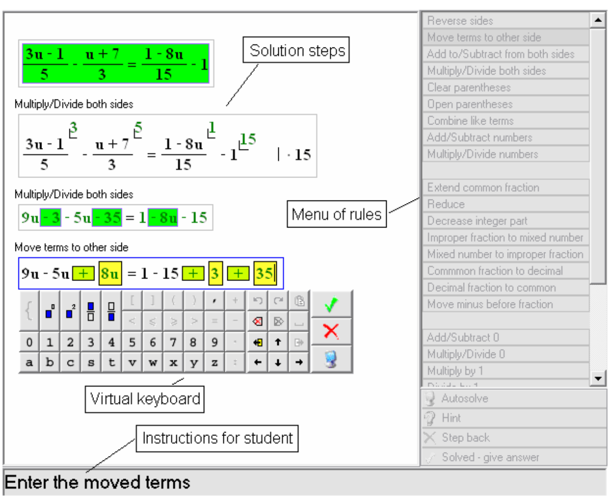

Fig. 1. The problem-solution window of the T-algebra program

three stages: selection of the transformation rule, marking the parts of expression, entering the result of the operation. The presented scheme improves the ability of the program to check the student's solutions, respond to the errors made by the student and give advice when the student is at loss. The program monitors whether the student works according to the algorithm, and supports it with the respective dialogue, diagnoses transformation errors, offers advice and, if necessary, performs the next stage of the step by itself.

The problem solution window of T-algebra is shown on Figure 1. The main part of the window contains solution steps and a virtual keyboard that can be used for active input. On the right side is the menu of possible actions. The lower part includes instructions for the student in this particular situation.

Figure 2 demonstrates performance of one step in the program (applying the rule *Move terms to other side*).

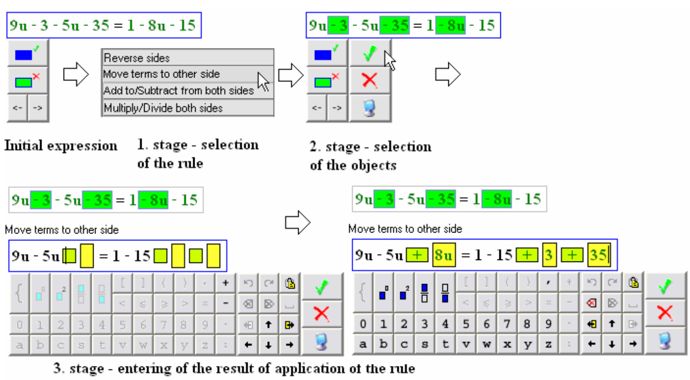

Fig. 2. Example of one step in T-algebra

In T-algebra, the student is left the possibility to make mistakes at all three stages of the step. If a mistake can be made then T-algebra can respond to it as well. First, the student could err in choosing the rule. If the application of the selected rule is impossible, the program does not immediately inform the student about the error, because the student will not find suitable objects for applying this rule or will make an error by choosing unsuitable objects. This gives the student a chance to correct the error without assistance.

Second, the student can make mistakes in marking the parts of expression. The program performs a number of different checks, like syntactical correctness, compatibility, position, etc. When wrong parts have been selected, the program does not permit to continue.

The input stage has the largest selection of potential mistakes, because the student must apply the rule for the marked parts and enter the result. The program tries to determine whether the student has made a standard error, which occurs often in student solutions (for example, not changing the sign of a moved term is a common mistake made by Estonian students). If the mistake is in the set of standard mistakes (some studies have been conducted to collect the students' mistakes made on paper [9, 11, 18]) then T-algebra is able to diagnose it and offer an appropriate error message (Fig. 3). Besides standard mistakes, T-algebra can also check the non-equivalence of equations.

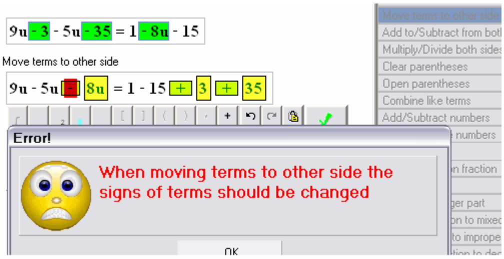

Fig. 3. Error message displayed when entering the result

3 Description of Experiment

The study was carried out in the winter 2007. Seven classes (126 students) of 7th grade (13 years old) from four different Estonian schools participated in the experiment. Classes from two schools, where there was more than one 7th grade class, were divided into experimental classes and control classes. The remaining two schools participated as experimental classes. After the division, we had 2 control classes and 5 experimental classes. Classes from the schools with more than one 7th grade class were taught by the same teacher.

The topic of linear equations was chosen for the experiment and the experiment began when the topic had been explained and practiced in the schools. The experiment consisted of four 45-minute sessions. In the first session, the students solved a pre-test on paper. In the next two sessions, the students practiced solving the problems of the same topic (linear equations). The experimental group practiced solving these problems with T-algebra, while the control group practiced solving exactly the same problems using traditional instruction technology – paper and pencil. In the last session, the students solved a post-test on paper. Teachers had exact instructions what, when and how to do and the same materials (pre-test, problems for practicing and post-test) were prepared for all teachers.

The pre-test was solved in both groups using paper and pencil. During the pre-test, the students could not use any assistance materials. The test contained 17 problems (6 types of problems) and it was possible to earn 39 points in total. Several examples of the problems (with maximum points) are listed below:

- Check if number *5* is solution of equation $10-2(3y-1)=2y-7(y-1)$ (3 p.);
- Reverse equation sides: $24=3y+7$ (1 p.);
- Divide equation sides by variable coefficient: $1{,}3n=-3{,}9$ (1 p.);
- Multiply both sides of the equation by common denominator: $\frac{x+3}{9}-\frac{3x+1}{12}=\frac{2x+5}{4}+\frac{x-5}{6}$ (2 p.);
- Move all variable terms to the left side and all constant terms to the right side and then combine like terms: $5-7x+4-x=3x-9+x+11$ (2 p.);
- Solve an equation: $5y-2(3y+4)=7-4y$ (5 p.).

The students were graded according to the following scale:

5: 35.5 – 39 points (91 - 100 %)
4: 27.5 – 35 points (71 - 90 %)
3: 19.5 – 27 points (51 - 70 %)
2: 11.5 – 19 points (31 - 50 %)
1: 0.0 – 11 points (0 - 30 %)

During the next two sessions (mathematics lessons), the experimental group practiced solving similar problems using T-algebra in computer class. The practice took place immediately after the pre-test in the next mathematics lesson and linear equations were not taught in the ordinary class between pre- and post-tests. The students had seen and tried T-algebra before when learning other topics, so the teachers did not have to explain the environment to the students. After the second lesson the students saved their solutions, the teachers collected them and sent to us for examination.

While the experimental group practiced solving in T-algebra, the control group solved exactly the same problems using paper and pencil. During the sessions the students solved the problems in their notebook and one of them wrote the solution on the blackboard. The teacher highlighted and corrected mistakes in the solutions on the blackboard, but did not explain anything new and did not correct solutions in the notebooks.

During the fourth consecutive session, both groups solved a post-test using paper and pencil. The arrangement of the post-test was the same as in pre-test and similar types of problems were used. Again, the students could not use any assistance materials, least of all the corrected pre-test.

After the experiment we collected the papers of the pre- and post-tests and the files with solutions in T-algebra for analysis.

4 Results of Experiment

After an analysis of papers and files, the students who had missed at least one session were excluded and the work of 115 students remained. The tests were analyzed further and the students whose pre-test result was less than 11 points were excluded, because in the preconditions of the experiment we assumed that the topic had been taught to the students, i.e., the students should be able to score at least 30 % of the points. We wanted to evaluate how T-algebra affects practicing after the topic has been explained by the teacher, not how it influences learning new material. While all other students had some basic knowledge about linear equations, these students (who scored under 30 %) did not. The work of 106 students remained after this step; 76 of them had participated in the experimental group and 30 in the control. Table 1 shows the results (average number of points) of pre- and post-test in both (experimental and control) groups. As we can see, the average number of points in pre-test is almost equal in experimental and control groups. The difference is not statistically significant (unpaired t-test t = 0.0368, p = 0.97) and the groups can be considered as equal.

Table 1. Results (average number of points) of the tests

	Experimental	Control
Pre-test	29.4	29.3
Post-test	31.3	29.9

Table 1 indicates that the knowledge of students from experimental group is statistically significantly improved (paired *t*-test t = 3.571, p < 0.01), but no statistically significant difference (improvement) can be found in the points earned by the control group (paired *t*-test t = 1.2024, p > 0.05). Effect size (using Cohen's *d*) is 0.179. This implies that even a brief use (2 lessons) of T-algebra affects the results of learning.

Table 2 shows the percentages of students from the experimental group with different grades in pre- and post-tests. As we can see, the percentage of students with the highest grade has grown. The percentage of students with grade 4 remained the same while the percentage of students with low grades (3 and 2) has decreased.

Table 2. Division of students (from the experimental group) by pre- and post-test grades

	Students with grade 5	**Students with grade 4**	**Students with grade 3**	**Students with grade 2**
Pre-test	25 %	38 %	24 %	13 %
Post-test	39.5 %	38 %	14.5 %	8 %

Checking the post-test of the experimental group, we noticed that one experimental student emulated the writing style of T-algebra. The operation *Multiply/Divide both sides* has a slightly different appearance in Estonian textbooks and in students notebooks from that used in T-algebra. On paper, the students perform this operation in two rows and the result is a solution like the one on the left on Figure 4. However, the application of this rule in T-algebra is written in three rows (Fig. 1). The mentioned student was not able to solve problems of the type *Multiply both sides* in the pre-test. In the post-test he solved all five problems of this type successfully, but he always wrote the solution in three rows as illustrated on the right side on Figure 4.

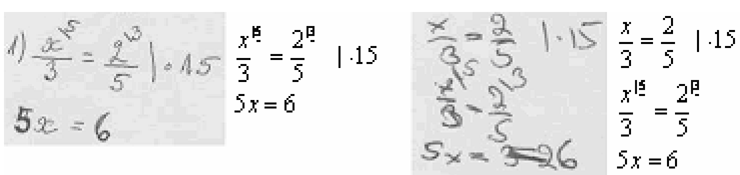

Fig. 4. Multiplication of both sides of equation in two rows (in textbooks, on paper) (left) and in three rows (emulating T-algebra) (right)

This picture was very unusual on paper, so we drew the conclusion that even two hours with T-algebra could affect the students' writing style. Naturally, this assumption still needs to be confirmed or refuted through future experiments.

Now we could look in more detail at the mistakes made in pre- and post-tests in the experimental and control groups. Table 3 shows the percentage of students in each group who made a specific mistake in the pre-test or in the post-test while the last columns show the percentage of the students who repeated the mistake they had made in the pre-test in the post-test as well. Many different kinds of mistakes were made, but Table 3 presents only the mistakes that were made in the pre-test by more than ten percent of the students in both groups (experimental and control). This restriction was introduced to enable comparison of mistakes in pre- and post-tests (it would be very hard to distinguish the influence of T-algebra in case of mistakes made only by a few students in one or another group). Table 3 does not reflect whether the student made this mistake more than once in one test.

Table 3. Mistakes made in pre- and post-tests in experimental and control groups

No	Nature of mistake	Example of mistake	Pre-test		Post-test		Recurrence in post-test	
			experimental	control	experimental	control	experimental	control
1	Minus sign before fraction is taken into account only at first term	$\frac{4y}{3}^{\lfloor 5} - \frac{2y+3}{5}^{\lfloor 3} = \frac{4}{15}^{\lfloor 1} \quad \vert \cdot 15$ $20y-6y\underline{+}9=4$	55	56	29	46	52	82
2	Arithmetic mistake in combining and in evaluating	$7s-9s=2-12$ $-2s=\underline{-24}$	46	50	21	33	45	67
3	In the problem *Check if number is a solution* the equation is solved		40	30	19	10	48	33
4	In the problem *Reverse sides* all variable terms are moved to the left side and all constant terms to the right side		32	23	8	10	24	43
5	Minus sign before parentheses is taken into account only at first term	$9-2(3y-1)=3-2y$ $9-6y\underline{-}2=3-2y$	22	46	10	40	47	85
6	Mistake in sign in dividing	$-4y=-8 \quad \vert:(-4)$ $y=\underline{-}2$	21	20	7	7	31	33
7	Arithmetic mistake in dividing	$-0.3y=-1.2 \quad \vert:(-0.3)$ $y=\underline{0.4}$	17	14	9	7	53	50
8	Sign is not changed when moving to other side	$8u-5u+15=4u+6$ $8u-5u-4u=\underline{15}+6$	16	20	8	10	50	50
9	Whole number is not multiplied	$\frac{x}{5}^{\lfloor 2}+3=\frac{7}{10}^{\lfloor 1} \quad \vert \cdot 10$ $2x+\underline{3}=7$	15	23	3	10	18	43

As we assumed, T-algebra affects some error types. We can see that the students from the experimental group made fewer mistakes in the sign of second term (mistakes number 1 and 5) in the post-test. The same was observed in the earlier experiments [10] and we believe that showing the error message immediately and directing attention to the mistake and its location (box) is the cause of that. However, T-algebra does not affect mistakes in sign, which are not connected to the second term, such as mistakes in sign in dividing and moving to other side (mistakes 6 and 8).

A decrease in the number of students from the experimental group who made the mistake number 9 was also predictable. The same was noticed in the earlier experiment [10]. The students working on paper often forget to multiply a number, which is not fraction, but T-algebra does not allow proceeding with such solution and notifies that all terms should be multiplied. This causes the reduction of this mistake in the post-test.

T-algebra can also affect learning of algorithms. The students from the experimental group made the mistake number 4 (in the problem *Reverse sides*) less frequently than the students from the control group. Experimental students made the mistake number 3 (in the problem *Check if number is a solution*) in the post-test more often than the control students, because teachers request checking the solution of linear equations when solving on paper (sixth type of problems – solve an equation). Therefore, the students solving equation on paper also practice checking the solution. This checking stage is omitted in T-algebra, because the program does not permit incorrect solutions. Consequently, the percentage of the students from the control group who made this mistake decreased, because they had more practice with this type than the experimental students.

It is hard to say whether T-algebra affects arithmetic mistakes or not. The students from the experimental group made the mistake number 2 less frequently than the students from the control group, but the frequency of the mistake number 7 was equal (and even slightly higher in the experimental group). We assumed that T-algebra does not affect arithmetic mistakes and we hope future experiments will explain the decrease in the frequency of the mistake number 2.

5 Conclusions

We have combined two known approaches for step-by-step solving of algebra problems and have designed a three-stage dialogue in T-algebra intelligent learning environment. We have succeeded in creating such rule dialogue in T-algebra that gives the student the possibility to learn both the solution algorithms and their steps, to make the same mistakes as on paper, and enables the program to check the knowledge and skills of the student, understand the student's mistakes, offer feedback and give advice. We have conducted the experiment to answer the question: How do T-algebra environment and its novel design affect the students' learning results? The experiment comprised pre- and post-tests on paper and practice with or without T-algebra.

As we saw in the post-test, the students from the experimental group did better than the students from the control group (the average pre-test score was almost equal in experimental and control groups). This shows that even a brief use (2 lessons) of T-algebra affects the results of learning. We also saw a progress of the students from the experimental group to a higher score in the post-test.

The experiment showed that even two hours with T-algebra could affect the students' writing style on paper. However, this observation needs further experiments for confirmation or refutation.

Finally, we saw that T-algebra could affect some error types, i.e., the students from experimental group made fewer mistakes of certain types (like mistakes in the sign of second term). However, this short period of use of T-algebra gave strange results for arithmetic mistakes; we hope to find an explanation for the change in these mistakes from a long-term experiment.

Obviously our decisions and ideas need some years of practical classroom trials before they can be finally confirmed. Starting from the school year 2006-2007, tens of teachers in Estonian schools use T-algebra for practice. The results of the school trials and teacher experiences will contribute to and support further development of T-algebra.

The conducted experiment examined the influence of the T-algebra environment on students and did not compare T-algebra with other environments. It would be very interesting to organize an experiment to compare whether the novel design of T-algebra produces better results than, for example, Aplusix [13] or MathXpert [3] environment.

Acknowledgements

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