Robust Impedance Control of a Delayed Telemanipulator Considering Hysteresis Nonlinearity of the Piezo-actuated Slave Robot

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Abstract. The slave robot in this research is a 1-DOF piezo actuator which includes hysteresis nonlinearity. Nonlinear hysteresis behavior makes robot control a complex task. In this research, the nonlinear and uncertain dynamics of the slave robot has been considered through the teleoperation control loop. LuGre friction model is used as the estimator of the hysteresis loop. An impedance controller for the master side and a sliding-mode-based impedance controller for the slave side have been proposed. The latter is a sliding mode controller, because the plant is nonlinear and uncertain. Also, it is an impedance controller providing both high performances during contact and excellent tracking in free space motion. These controllers make teleoperator robustly stable against uncertainties and bounded constant time delay. Meanwhile, scaling factors, known as sources of instability, have no disturbing effect. After canceling the nonlinear term out of the teleoperator by the controllers, stability of the entire system will be guaranteed by Llewellyn's absolute stability criterion. Performance of the proposed controllers is investigated through simulation.

Keywords: macro-micro telemanipulation, Hysteresis, Nonlinear, piezo-actuator, LuGre model, sliding mode, impedance controller, scaling, time delay, robustness.

1 Introduction

Micromanipulation has been fascinating a growing interest of teleoperation researchers. There are many cases in which slave environment is in micrometer dimensions. The operation complexity urges human operator to be present in control loop. Operator interacts with a macro-scaled master robot, say a joystick, whereas slave robot interacts with a micro-scaled environment. Micro-assembly [1] and In Vitro Fertilization [2] are

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two common exemplary applications of such systems. Piezo-actuated slave robots have been interested in due to their ability in high precision positioning [3].

The most important drawback in using of piezo-actuators is their nonlinear hysteresis behavior, making their control complex [4], [5]. In this research, the nonlinear dynamics of the slave robot has been entered directly into the teleoperation control loop. Then, the system stability and transparency is discussed. There are two common approaches for dealing with nonlinearity problem: 1) application of non-model based methods (e.g. neural network (N.N)) [6] for controlling and compensating the slave/environment nonlinearity. 2) Use of a cascaded feedback linearization controller which delivers a linear slave dynamics for teleoperation system analyses. The former lacks of rigorous stability discussion, while the latter has insufficient robustness against uncertainties and noises [7].

LuGre friction model is used; because it is proved that the model estimates the hysteresis behavior of piezo-actuator precisely [8].

Due to phenomenal uncertainty existing within the estimated slave model, a sliding mode as the slave controller has been used. This approach was first used in teleoperation systems by Buttolo [9] for a 1-DOF linear system. It was shown that the approach would enhance transparency in comparison with the conventional controllers, since it takes model parameter uncertainty into consideration. Park and Cho [10], [11] showed that a sliding mode controller (as the slave controller) could stabilize teleoperator (i.e. master, slave, and communication channels) against time delay. The delay may be considered either constant or varying, but known. In this way, an alternative approach was established for the common problem in teleoperation research area, that is stabilization of teleoperator against time delay. Up to date, numerous approaches have been used for solving the problem. One uses wave variables to make teleoperator passive against delay. It was contributed by Niemeyer in 1991 based on scattering theory to deal with constant time delay [12]. Later, it was generalized to any type of delay until 1998 [13], [14]. Another one was a novel control framework proposed by Lee. It uses passivity concept, the Lyapanov-Krasovskii technique, and Parsval's identity to make the combination of the delayed communication and control block passive [15]. Besides these two, there are several other approaches not based on passivity concepts like H-infinity optimization, other P and PD controllers, event-based planning, and so on.

Using a sliding-mode based impedance controller (SMBIC) not only stabilizes teleoperator against time delay and uncertainties, but also poses desired impedance to the slave robot. Meanwhile, scaling factors, known as resources of instability [16], [17], would never threaten the system stability.

An SMBIC is used as the slave robot controller for the mentioned reasons. Nevertheless, the most important reason motivating us to use this control scheme is as follow: after replacing the nonlinear slave dynamics with a linear one (i.e. the desired slave impedance), it will be possible to use Llewellyn's absolute stability criterion to assure the stability of the entire system. In fact, using this control scheme, it has been benefited from a sliding mode controller (which is essential due to slave nonlinearity and slave dynamic parameter uncertainties) and an impedance controller (enabling us to use Llewellyn's criterion which can be only used for linear systems [18]), simultaneously.

Section 2 of this paper defines teleoperator dynamic model in the presence of time delay, considering scaling factors. In section 3, an impedance controller for the master and an SMBIC for the slave robot are proposed. Hence, teleoperator has been stabilized against time delay, uncertainties, and scaling factors. In section 4, stability of the entire system is established through Llewellyn's absolute stability criterion. Sections 5 and 6 include the designed parameters and simulation results, respectively. The paper ends with conclusions and some future work remarks, in section 7.

2 Teleoperator Modeling

2.1 Dynamic Modeling for the Master Robot

The master robot is a single degree of freedom mass-damper system.

$$m_m \ddot{x}_m(t) + b_m \dot{x}_m(t) = u_m(t) + f_h(t) \tag{1}$$

where x_m denotes master position, m_m and b_m denote the inertia and viscous damping coefficient of the master, f_h denotes the force applied at the master side by the operator and u_m is the master control signal.

2.2 Dynamic Modeling for the Slave Robot

The slave robot is a 1-DOF piezo-stage with hysteresis behavior. It is well known that there exist two difficulties in modeling of the hysteresis nonlinearity of piezo-positioning mechanism; they are non-local memory phenomenon and asymmetric loop between descending and ascending paths [8]. Therefore, the development of a dynamic model to describe the hysteresis behavior is very important for the improvement of the control performance of the piezo-positioning mechanism. The hysteresis friction model, called LuGre model, is used due to reasons explained in [8]. Meanwhile, there is a further important benefit leading us to use this model; hysteresis effect of piezo-positioning mechanism can be *separately* added to the slave linear dynamics. This feature will cause tremendous facility in the stage of designing of the SMBIC.

The slave robot is now modified to describe hysteresis effect of the piezo-actuator by separately adding a load term $F_H(\dot{x}_s)$. The slave dynamic is shown in Fig.1.

$$m_s \ddot{x}_s(t) + b_s \dot{x}_s(t) + F_H(\dot{x}_s) = u_s(t) - f_e(t) \tag{2}$$

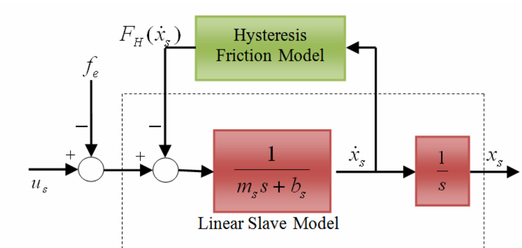

Fig. 1. Block diagram of the piezo-positioning based slave robot

where x_s is the slave position, m_s and b_s are inertia and viscous coefficient, u_s denotes the slave control law. f_e is the force exerted by the environment to the slave, and F_H is calculated from (3):

$$F_H = \sigma_0 \bar{z} - \frac{\sigma_1}{h(\dot{x}_S)} \bar{z} |\dot{x}_S| + (\sigma_1 + \sigma_2) \dot{x}_S \tag{3}$$

where

$$\sigma_0 h(\dot{x}_s) = f_C + (f_S - f_C).e^{-(\dot{x}_s / \dot{x}_{St})^2} \tag{4}$$

Here σ_0, σ_1, σ_2, f_C, f_S, and $\dot{x}_{St}$ are constants. $\bar{z}$, the contact force applied average bristle deflection, is achievable from (5):

$$\frac{d\bar{z}}{dt} = \dot{x}_s - \frac{|\dot{x}_s|}{h(\dot{x}_s)} \bar{z} \tag{5}$$

For the parameters definition and more details about LuGre model, reader is referred to [19] and [8].

2.3 Delayed Signals and Scaling Factors

Delayed signals at both sides of the communication channel are as follow:

$$x_m^d(t) = x_m(t - T_1), \qquad f_h^d = f_h(t - T_1), \qquad f_e^d = f_e(t - T_2)$$

$x_m^d, \dot{x}_m^d$,and f_h^d represent position, velocity and the force applied to the master by operator. The force is transmitted from master to slave facing delay T_1. f_e^d is the force exerted to the slave by environment facing the delay T_2 .

These delayed signals out of the communication block are then scaled up or down by some factors depending on teleoperation tasks. Therefore:

$$x_s = k_p x_m^d, \qquad \dot{x}_s = k_p \dot{x}_m^d, \qquad f_h = k_f f_e^d$$

k_p and k_f are scaling factors for position/velocity and force, respectively.

3 Control Design

In teleoperation tasks, maximum performance is required. It means position tracking in free space and force tracking during contact. It is well known that impedance controller, which controls the relationship between the applied force and the position of the manipulator, is suitable for achieving this goal.

An impedance controller is used for the master robot. It is also desired to use this controller for the slave. But, piezo-positioning based slave robot has nonlinearity and uncertainty. Hence, sliding mode controller has been augmented with an impedance controller to establish a SMBIC.

3.1 Impedance Control for the Master

The target impedance for the master is supposed to be:

$$\bar{m}_m \ddot{x}_m(t) + \bar{b}_m \dot{x}_m(t) + \bar{k}_m x_m(t) = f_h(t) - k_f f_e(t) \quad (6)$$

where $\bar{m}_m, \bar{b}_m, and\ \bar{k}_m$ are the desired inertia, viscous damping coefficient, and stiffness, respectively. It is possible to replace the master dynamic (1) with the desired dynamic (6) using the following control law:

$$u_m(t) = (b_m - \frac{m_m}{\bar{m}_m}\bar{b}_m)\dot{x}_m(t) + (\frac{m_m}{\bar{m}_m} - 1)f_h(t) - \frac{m_m}{\bar{m}_m}\left\{k_f f_e^d(t) + \bar{k}_m x_m(t)\right\} \quad (7)$$

3.2 SMBIC for the Slave

The target impedance characteristic for the slave is specified such that:

$$\bar{m}_s \ddot{\tilde{x}}(t) + \bar{b}_s \dot{\tilde{x}}(t) + \bar{k}_s \tilde{x}(t) = f_e(t) \quad (8)$$

where $\bar{m}_s, \bar{b}_s, and\ \bar{k}_s$ are the desired inertia, viscous damping coefficient, and stiffness of slave, respectively. Also, $\tilde{x}(t) = x_s(t) - k_p x_m^d(t)$.

If f_e is set to be zero (as in free space motion occurs), and desired parameters are selected appropriately, $\tilde{x}$ will converge to zero as t increases, i.e. position tracking will be achieved. In the other words, the SMBIC guarantees position tracking in free space motion. If a hard contact happens, f_e will not to be zero. Consequently, $\tilde{x}$ will converge to a constant value, with respect to the value of f_e (see Fig.4)

The slave robot has remarkable uncertainty, especially within $F_H(\dot{x}_s)$ describing parameters. It is because of the fact that hysteresis loop (and therefore its describing constant parameters) fundamentally depends on the frequency and amplitude of the input signal [8]. That means, the exact slave dynamic parameters m_s, b_s, and those of $F_H(\dot{x}_s)$ are not exactly known. Therefore, their corresponding estimations are considered inside the slave control law (represented with hat):

$$\begin{aligned} u_s = & -\frac{\hat{m}_s}{\bar{m}_s}\left\{\bar{b}_s \dot{\tilde{x}}(t) + \bar{k}_s \tilde{x}(t) + f_e(t)\right\} + \hat{b}_s \dot{x}_s(t) + \hat{F}_H(\dot{x}_s) + f_e(t) + \\ & k_p \frac{\hat{m}_s}{\bar{m}_s}\left\{-\bar{b}_m \dot{x}_m^d(t) - \bar{k}_m x_m^d(t) + f_h^d(t) - k_f f_e^{dd}\right\} - K_g.sat(\frac{s(t)}{\phi}) \end{aligned} \quad (9)$$

where $f_e^{dd} = f_e(t - T_1 - T_2)$, K_g is the nonlinear gain, ϕ is the boundary layer thickness reducing the chattering of the control input, and $s(t)$ is the sliding surface [20]. The last term in (9) is added to make sure the uncertain control law (9) satisfies the sliding condition $\dot{s}(t).s(t) \le -\eta.|s(t)|$. η is the minimum speed for states to reach on sliding surface $s(t)$=0.

For sliding condition $\dot{s}(t).s(t) \le -\eta.|s(t)|$ being satisfied, nonlinear gain K_g is given as:

$$K_g \ge \eta.m_s + |\alpha(t)| \quad (10)$$

where

$$\alpha(t) = \Delta m_s \left\{ \frac{\bar{b}_s \dot{\tilde{x}}(t) + \bar{k}_s \tilde{x}(t) + f_e(t)}{\bar{m}_s} + k_p \ddot{x}_m^d(t) \right\} + \Delta b_s \dot{x}_s(t) + \Delta F_H(\dot{x}_s) \qquad (11)$$

and

$$\Delta m_s = |m_s - \hat{m}_s|,\ \Delta b_s = |b_s - \hat{b}_s|,\ and\ \Delta F_H = |F_H - \hat{F}_H|.$$

The method in which ΔF_H is related to the hysteresis model parameters is provided in Appendix A.

4 Stability of the Entire System

The entire of the teleoperation system must be stabilized for every human operator and environment. Llewellyn's criterion providing necessary and sufficient conditions for absolute stability is introduced in [18]. A two-port system is absolutely stable if and only if:

i) $h_{11}$and h_{22}have no poles in the right half plane (RHP).
ii) Any pole of them on the imaginary axis is simple with real and positive residues.
iii) For all ω:

$$\begin{aligned} &\mathrm{Re}[h_{11}] \geq 0 \ \ , \ \ \mathrm{Re}[h_{22}] \geq 0 \\ &\eta(\omega) = -\frac{\mathrm{Re}[h_{12}h_{21}] + 2\,\mathrm{Re}[h_{11}]\mathrm{Re}[h_{22}]}{|h_{12}h_{21}|} \geq 1 \end{aligned} \qquad (12)$$

where h_{ij} are the elements of the hybrid matrix representation:

$$h_{11} = \bar{m}_m p + \bar{b}_m + \frac{\bar{k}_m}{p},\ \ h_{12} = -k_f e^{-T_2 p},\quad h_{21} = -k_f e^{-T_1 p},\ \ h_{22} = \frac{p}{\bar{m}_s p^2 + \bar{b}_s p + \bar{k}_s}$$

p is used in place of the familiar Laplace domain variable s to avoid confusion with sliding surface $s(t)$. Conditions (i), (ii) together with the first and second part of (iii) are satisfied by choosing positive impedance parameters. The third part of condition (iii) will be satisfied if:

$$0 < \bar{b}_s < \frac{\bar{b}_m}{k_p k_f} \qquad (13)$$

Equation (13) is derived in [21], assuming $k_p = k_f^{-1}$). It is also reminded that the system is stable regardless of the value of time delay if the condition (13) is satisfied.

5 Parameter Design

Dynamic parameters of the master and slave robots, i.e. m_m, b_m and m_s , b_s and those of communication channels, i.e. k_p and k_f, are chosen with respect to our specific micromanipulation task. The constant characteristic parameters for estimating the hysteresis loop of the piezo-positioning mechanism of the slave robot, i.e. σ_0, σ_1, σ_2, f_C, f_S, and $\dot{x}_{St}$, are extracted from [8], which uses similar piezo-stage. Through the slave controller, estimated amount of slave parameters (i.e. $\hat{m}_s, \hat{b}_s, \hat{\sigma}_0, \hat{\sigma}_1, \hat{\sigma}_2, \hat{f}_C, \hat{f}_S$, and $\hat{x}_{St}$) are determined to be uncertain enough in order to enable us validate our claims about controller robustness. Both impedance parameters of the master and slave controllers (i.e. $\bar{m}_m, \bar{b}_m, \bar{k}_m$ and $\bar{m}_s, \bar{b}_s, \bar{k}_s$) are designed to induce high dexterity to operator plus satisfying absolute stability criterion (13). Value of K_g is determined to satisfy (10), when slave dynamic parameters are involved with uncertainty. Φ is the smallest possible positive number which eliminates the unwanted chattering. Maximum amount of time delay in round trip is assumed to be 1.5 second which seems to be a good estimation.

All parameters are listed in Table1.

Table 1. Designed parameters

Symbol	Quantiy SI	Symbol	QUANTIY SI	SYMBOL	Quantity SI
m_m	1	$\bar{m}_s$	0.02	f_S	1.5
b_m	4	$\bar{b}_s$	3.5	$\dot{x}_{St}$	0.001
m_s	1	$\bar{k}_s$	100	K_g	150
b_s	0.015	$\hat{m}_s$	0.1	Φ	0.5
σ_0	10^5	$\hat{b}_s$	10	$\bar{m}_m$	0.2
σ_1	$\sqrt{10^5}$	$\hat{\sigma}_0$	4×10^5	$\bar{b}_m$	4
σ_2	0.4	$\hat{\sigma}_1$	$4\times\sqrt{10^5}$	$\bar{k}_m$	0.4
f_C	1	$\hat{\sigma}_2$	6	$f\hat{}_C$	6
$f\hat{}_S$	6	k_p	0.1	T_1+T_2	≤ 1.5
$\hat{x}_{St}$	0.005	k_f	10		

6 Simulation Results

In this section, the simulation of the macro-micro teleoperation system is presented. The overall block diagram of this system including master, slave and proposed controllers is shown in Fig.2.

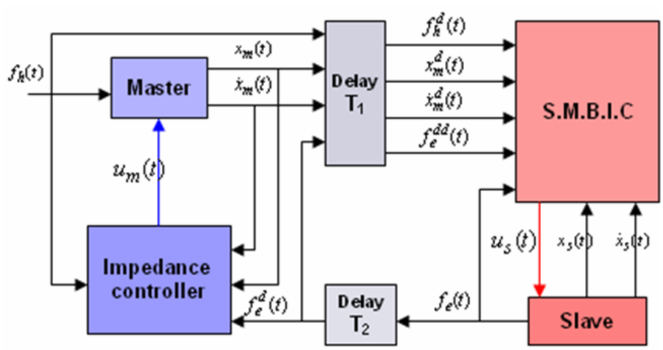

Fig. 2. The overall system block diagram

To demonstrate the performance of the controller the following scenario is organized. The human operator is modeled by a PD position tracking controller using spring and damping gains 70 N/m, and 50 N.s respectively. At the 0-8.5 second interval, the master robot is stabilized at the position 1.5 cm. Then, at 8.5-20 s, the master robot is pushed to a new position, i.e. 2.5 cm. While moving the robot to this target, the operator realizes existence of a hard wall, receiving step-like force feedback. Finally, at 20-27 s, the human operator retracts the master to the origin.

In Fig.3, the slave robot is involved with uncertainty, as introduced in table 1. However, K_g is set to be zero to highlight undesirable effect of uncertainty. The system is supposed to be free from delay to focus us only on uncertainty. As it is observed, the teleoperation system is oscillatory, caused by model dynamic uncertainty. Simulation is truncated at 16.1 s in order to avoid long time of simulation.

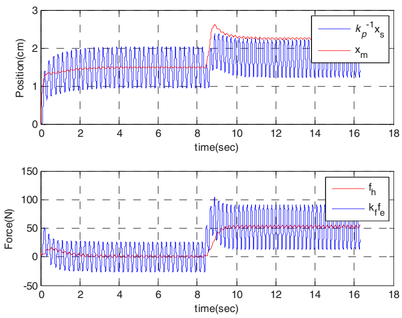

Fig. 3. Master/slave position and force signals when $T_1+T_2 = 0$ and K_g =0

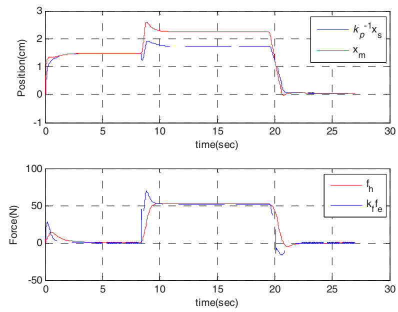

Fig. 4. Master/slave position and force signals when $T_1+T_2 = 0$ and K_g =150

In Fig.4, the slave robot is again involved with uncertainty. However, K_g is determined according to (10). The system is supposed to be free from delay, because we are focusing just on the instability caused by uncertainty. As shown in this figure, the teleoperation system is now stable. When the slave robot is pushing against the obstacle (8.5-20 s), the contact force is faithfully reflected to the human. Also, when the

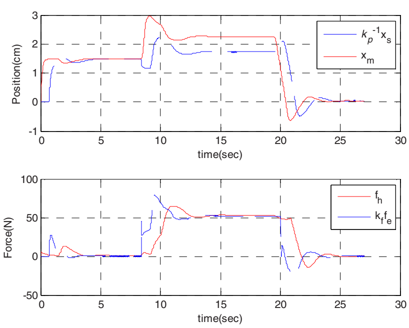

Fig. 5. Master/slave position and force signals when $T_1+T_2 = 1.5$ s and K_g =150

slave does not interact with the wall and the human force is negligible (3-8.5 s and 22-27 s), the master and slave coordination is achieved. These verify our desired objectives, i.e. position tracking during free motion and force tracking in hard contact.

In Fig.5, the uncertain teleoperation system is involved with a time delay (T_1+T_2 = 1.5 s). As depicted in this figure, the teleoperation system is stable even in presence of remarkable time delay. Although the delay has degraded force/position tracking (especially force) during the transition periods, force and position coordination are achieved at the steady state situations.

7 Conclusions

In this research the nonlinear and uncertain dynamics of a 1-DOF piezo-actuator based slave robot has been entered directly into the teleoperation control loop. LuGre friction model is used as the estimator of the hysteresis loop. An impedance controller for the master and a sliding-mode-based impedance controller for the slave robot have been proposed. The proposed controllers make teleoperator robustly stable against uncertainties and bounded constant time delays. After canceling out of the nonlinear term of teleoperator through the controllers, stability of the entire system is guaranteed by Llewellyn's absolute stability criterion.

Despite its outstanding performance, there is still a limitation during application of LuGre friction model. That is, the model is not dependant on the frequency and amplitude of the input signal. Obviously, this limitation would increase model uncertainty and consequently, degrades force/position tracking. Therefore, establishing an amplitude/rate dependant loop estimator is an open problem for the future.

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Appendix A

$$F_H = F_H(\sigma_0, \sigma_1, \sigma_2, f_C, f_S, \dot{x}_{St}) \Rightarrow \Delta F_H = \left| F_H - \hat{F}_H \right| \Rightarrow$$

$$\Delta F_H = \sum_{1}^{6} \left(\left(\frac{\partial F_H}{\partial *} \Delta * \right)^2 \right)^{\frac{1}{2}}, \ * \in \{\sigma_0, \sigma_1, \sigma_2, f_C, f_S, \dot{x}_{St}\}$$