4 Evaluation of prospective modelling methods: fuzzy logics and cellular automaton applied to deforestation in Venezuela

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Abstract

Spatial evolutions of anthropized ecosystems and the progressive transformation of spaces through the course of time emerge more and more as a special interest issue in research about the environment. This evolution constitutes one of the major concerns in the domain of environmental space management. The landscape evolution of a regional area and the perspectives for a future state raise particularly important issue. What will the state of the region be in 15, 30 or 50 years?

Time can produce transformations over a regional area such as emergence, disappearance or the union of spatial entities. These transformations are called temporal phenomena. We propose two different methods to predict the forestry development for the forthcoming years in the experimental area, which reveals these spatial transformations. The proposed methods are based on fuzzy logic and Cellular Automata (CA).

The methods are supported by the analysis of the landscape dynamics of a test site located in a tropical rain forest country: the oriental piedmont of the Andes Mountains in Venezuela. This large area, at the scale of a Spot satellite image, is typical of tropical deforestation in a pioneer front. The presented approaches allow the geographer interested in environmental prospective problems to acquire type cartographical documents showing future conditions of a landscape. The experimental tests have showed promising results.

Keywords: Spatial dynamic of environment, modelling, fuzzy logic, cellular automata, prospective maps, tropical pioneer front.

4.1 Introduction

The spatial evolution of anthropized ecosystems and the progressive transformation of spaces over time is a large preoccupation in space accommodation, environmental domains, and prospective studies. There is an underlying question that arises concerning the landscape development and the prospective of the state of a forest area in future: How will conditions of a regional area develop within the next 15, 30 or 50 years?

In fact, the time consists of hierarchical events and can produce transformations upon a terrain landscape such as emergence, disappearing, and the union of spatial entities. These transformations are called temporal phenomena (Claramunt 1994).

Simulation with digital images has become an important and an interesting topic for research related to environment monitoring (Centeno and Selleron 2001).

A sequence of digital maps of different dates allows the analysis of the landscape dynamics of a region. Images collected by satellite (SPOT and Landsat) from the forest of Ticoporo, a tropical rain country that is located in Venezuela (South America), were used to investigate different methods of spatio-temporal prediction: fuzzy logic and cellular automata.

These methods enable us to study the future evolution of the forest by analysing the forest's progression and regression zones from a sequence of n thematic maps through time. The evolution modelling of regions, for an established date, is obtained with help of the sequence of satellite images representing the terrain conditions for distinct years. Thus, sensitive factors on region evolution are considered for the prediction purpose. It allows the geographer interested in environmental prospective problems to acquire type cartographical documents showing future conditions of a landscape.

4.1.1 Fuzzy sets in spatial modelling

Many works have been developed based on fuzzy systems to solve problems related to geo-processing. According to Saint-Joan and Desachy (1995) fuzzy systems deal with imprecise and uncertain information in a more efficient way when compared with algebra maps systems based on Boolean logic. Many authors point out some advantages in the use of fuzzy inference systems to solve problems associated with the environment (Centeno and Gois 2005, Zadeh 1965, Schultz et al*.* 2006):

- − The integration of diverse and heterogeneous sources of information in different scales of magnitude allows a formal trade-off between favourable and unfavourable conditions.
- − The possibility of manipulating linguistic terms instead of mathematical formulas can facilitate the use of the systems by specialists unfamiliar with the mathematical terminology.
- − The definition of a fuzzy rule base allows the reasoning process to focus on specific regions of interest.
- − Smoother decision regions resulting from the fuzzy reasoning can reduce abrupt changes in the final decision-making.

4.1.1.1 Modelling imprecision

Geographical data has a number of properties, which present challenges to the modelling process. Sometimes in image analysis approaches, it is more appropriate to regard the geographical regions as fuzzy subsets of the image. This includes complex definitions of location, multidimensionality and the inherent fuzziness in many features of the regions and their relationships (Peuquet 1984). The resultant model should be able to represent a simplified approximation of reality and manage the imprecision or indistinctness, which characterizes a lot of geographical information.

The fuzziness of geographical information can be related to the representation of regions, whose location or boundaries are not known precisely and to the representation of the information, which is expressed in imprecise terms. For all these reasons, there is now considerable interest in issues of uncertainty and imprecision in geoscientific information (Altman 1994). Fuzzy set theory is an appropriate means of modelling imprecision or vagueness and there are many areas to which fuzzy sets are being applied.

4.1.2 Cellular Automata Models

The cellular automata theory was first introduced by John Von Neumann in the forties and it gained considerable popularity in the 1970's, through the work of John Conway, called "game of life"(Gardner 1970).

A cellular automaton is a discrete dynamic system whose behaviour is specified in terms of a local relation (Toffoli and Margolus 1998). According to White et al. (2000) a cellular automata model consists of:

- − a one or n-dimensional space divided into an array of identical cells;
- − a cell neighbourhood of a defined size and shape;
- − a set of discrete cell states;
- − a set of transition rules, which determine the state of a cell as a function of the states of cells in a neighborhood;
- − discrete time step with all cell state updated simultaneously.

At each time step, all cells in the array update their current state according to the transition rule (representing the dynamic nature of the system) (Wolfram 1994). The number of possible configurations for a cellular automata (considering the transition cell updated) with s states and n neighbourhood cells is s^{s^n} (Weimar 1998).

According to the classical Cellular Automata Theory, a rule is called totalistic if it only depends on the sum of the states of all cells in the neighbourhood. Another classification is to distinguish between deterministic or probabilistic rules. In the first case, the transition rule is a function which has exactly one result for each neighbourhood configuration. However, probabilistic rules provide one or more possible states with associated probabilities, whose sum must be one for each input configuration (Weimar 1998). Each cell must be in one state. A set of discrete cell states can be defined by some property linked with the simulation of the phenomenon to be modelled.

The size of the neighbourhood must be defined. The Fig. 4.1 shows three examples of neighbourhoods that can be defined in two dimensions. The choice of neighbourhood depends on the context and it influences the propagation velocity of the phenomenon to be modelled (Weimar 1998).

Cellular automata can also be implemented with rules of different range. A range of 1 means that only the nearest cells are considered as neighbour cells, and a higher range means that more nearby cells are considered neighbours, as shown in Fig. 4.2.

The characteristics of CA used in today's geographic cellular automata (GCA) models are a mixture of the original CA formalism (Wolfram 1984) and the multiple transformations required for the modelling of the geographic space (Couclelis 1997, Torrens and O'Sullivan 2001). However, GCA can be used in any context where one of the main drivers of land use change is the influence of spatial neighbors. Some studies listed above exemplify this situation since they have consistently shown that the GCA modelling framework is well suited to capture the highly decentralized, multi-criteria, and spatial dynamics of geographic space.

Fig. 4.1 (a) Von Neuman's neighbourhood (b) Moore's neighbourhood (c) Arbitrary neighbourhood

Fig. 4.2 Range of cellular automata

4.1.3 Spatio-temporal Prediction technique

Predictions are important methods of reasoning about the geographic space and they are based primarily on inferences, rather than observations (Chase and Chi 1981).

The aim of a prescriptive modelling is to represent facts, to simulate processes, to express judgements or to provide for effective descriptions of geographic phenomena, through sets of properties or constraints. The computer has to generate the potential answers to these descriptions and to present them to the users (Falcidieno et al. 1992). Prescriptive modelling attempts to answer questions such as "what should be" by simulating the effects of certain actions effecting spatial objects/phenomena/processes. Prescriptive modelling is often based on the assumption that the problem domain has been well understood it provides effective descriptions of geographical phenomena in order to help users to make in spatial decision (Centeno 1998).

The problems addressed by prescriptive models generally involve two different uses for them: exploration and generation. The first requires a selective exploration of the spatial data model using geometric, topological, geographical properties in order to satisfy the objectives. The second problem generates a simulation of geographical phenomena. The initial statement of an allocation problem is a descriptive task, which consists of an explicit specification of some geographic conditions necessary to achieve the stated objective. The set of conditions expressed by the user defines the conceptual model of the spatial phenomena; it depends on the user's requirements. The simulation of geographical phenomena can be used, for example, to foresee potential site modifications in time.

The achievement of simulation of geographical phenomena through time consists of observing the changes of the spatial entities. The sequence of events must be considered in order to study the influence of spatial processes over the entities' transformation. The past states of an entity influence its current state, the current state in turn influences the future states of this entity. In this paper, the interest lies in the techniques of simulation.

4.1.4 Related works

Some approaches described in literature use a sequence of satellite images to generate a prediction for a specific region. In Centeno et al. (1996) the prediction method uses geographical data in vector representation and it is based on the position and form study of the spatial entities contained in each map. However, this method does not take into account relevant land area features such as valleys, rivers, slopes, roads, villages or indeed regions frequently destroyed by fire. Therefore the regions are constrained to uniform morphological transformations.

In St-Joan and Vidal (1996), the proposed approach applies mathematical morphology to zones of forestry progression and regression considering shape and surface of the regions, but the prediction task is occurs without regard of important factors related to forestry evolution.

The approach of Centeno and Selleron (2001) is founded on the principle that we must make use of regression and progression zones within the forest in order to discover the privileged directions of evolution that is the growth or decline in specific areas.

Schultz et al. (2008) have developed an approach based on the work of Centeno and Selleron (2001), but the method uses genetic algorithms and genetic programming to adjust coefficients that limit the process.

The discrete nature of cell states makes CA attractive for spatialtemporal modelling in a geographic information system (GIS) raster-based environment, which describes the world as a static representation based on a discrete array of cells. GIS and CA are complementary with regards to spatio-temporal modelling as the former provides the spatial framework for geographic data while the latter contributes the temporal dimension for describing change. Furthermore, the ability to develop realistic spatial models within a GIS environment has progressed due to the increasing availability of remote sensing (RS) data.

Cellular automata have already been used in some works related to prediction using geographic data. Rothermel (1972) has developed a model that simulates and predicts surface forest fire together with a GIS terrain data. In Vale et al. (1999), a process is described to simulate a viral epidemic through time. First it defines an initial state with some characteristics in a two-dimensional space and then the evolution is modelled by CA. Jants et al. (2003) described and tested a predictive modelling system to simulate the impacts of future policy scenarios on urban land use based on four different types of urban land use change. Sullivan and Knight (2004) provide a potential model for operational fire spread prediction.

Few studies focused on the land use dynamics of rural or more natural landscapes; examples are provided by the modelling of rural residential settlement patterns in the periphery of Toronto (Deadman et al. 1993) and in the Rocky Mountains (Tehobald and Hobbs 1998), and deforestation in the Brazilian Amazonian forest (Soares-Filho et al. 2002, 2004).

4.2 Test areas and data sets

4.2.1 The "Forest Reserve" of Ticoporo and the problematic

The material used to test the prediction modelling is from the forest of Ticoporo, on the oriental piedmont of the Andes in Venezuela (Fig. 4.3).

Fig. 4.3 Location of the test site of Ticoporo in Venezuela

The experimental site called *«Ticoporo Forest Reserve»,* lies on the eastern perimeter of the Venezuelan Andes, in the vast plain of Llanos crossed by the Orinoco river. This rainforest, covering an area of about 200,000 hectares, is very rich in tree species. It is very dense and has different physionomies.

It has acquired the protector status of "reserve" (Fig. 4.4), after it was already one of the last bits of the forest Llanos.

Indeed, a phenomenon of deforestation, which appeared in the early 60's, greatly increased during the 80's and continues to today. Its origin is the result of spontaneous movements of Andean peasants fleeing the land of the economically poor mountain region to conquer land in the plains. For them, these new "virgin" forest areas on a flat topography became a territory that allows the transformation of forest into extensive grazing pasture (land).

The phenomenon of deforestation is illegal and it has grown in an unbalanced manner in both time and space, due to several factors: the legal status of the land, the level of technology achieved and the social groups involved. Thus, at the end of the 70's, the shape of the massif was affected by human activity and then very quickly the heart of the "reserve" was reached. The regression of the forest was driven by two very distinct forces: on the one hand, a mechanized, industrial logging (a front of methodical and mechanized cutting), this occurs at the eastern and western edges of the forest; on the other hand a deforestation by fire (an ancestral culture of burn) or by cuts in the central area.

So there are two distinct phenomena that we will distinctly separate in order to consider the modelling. First, on the eastern and western edges there is the private concessions.

If the forest is exploited, it has survived only as a biogeographic entity, because the cut trees are systematically replaced by other tree species with rapid growth and quite often this regrowth is of a single species. This part of the reserve shows significant impoverishment. Both of the logging operations are even protected by private militias!

At the center, where the second distinct phenomena occurs, it is quite different. The forest has a hybrid status –Public and Private State–. It is the prime destination of the new usurping peasants, whose aim is the systematic destruction of the forest to create new pastures. These pastures will be redeemed by the major landowners of the surrounding area, and therefore this process will in gradually increase over time.

Both types of spaces, shown in the satellite images from 1989 (Fig. 4.4), are very different: the heart of the reserve is very sparse and surrounded by the two private industrial forest-covered properties. Together these two outlaying properties form a horse-shape around the barren central region. This central area is undergoing the phenomena, which are of concern for the modelling of this work.

4.2.2 Images of the forest "Reserve of Ticoporo"

For the experience, we have three Spot images (1987, 1989 and 1994) with 20 meters resolution and one LANDSAT-MSS image (1975) with 80 meters resolution, one of the oldest images (and without any cloud cover) acquired for this site.

After geometric correction and thematic classifications of all satellite images, we performed binary maps (512 x 512; resolution 70 m.) Each satellite image contains dynamic information about the forest area with its states. The Fig. 4.4 shows an example of the experimental space on the image from 1989.

Fig. 4.4 Channel red image from 1989 (left) and threshold image from 1989 (right)

The reader sees that the original satellite image (left) contains a very large variation in hues (here translated into gray-scale) that can not possibly all be taken into account by the model. As far as the variation of shades, they mostly deal with various grades of existing pastures. The problem is centered on the process of deforestation: the transformation of the forest into pasture over time. It begins with a simplification of the basic image data through a spectral binary segmentation of the image. So we use a simplified nomenclature: "forest - non-forest". This treatment is carried out on each image using the image processing *software Er-Mapper*. Fig. 4.4 shows the results of the binarization for the year 1989, which clearly differentiates between the forest (in black), and the rest of the open space (in white).

4.2.3 Creation of a geographical mask

A geographical treatment is added to this radiometric pre-treatment. Indeed, we have two very different phenomena of deforestation and the

model can not represent so many differences. Therefore, we chose to represent the space of the private industrial forest by a mask as shown in Fig. 4.5. The evolution in this area has a different behavior from the evolution of natural forest as we indicated in Sect. 4.2.1. For this reason, this region is not included in the analysis. Thus, the process of modelling, takes into account only those spaces left blank in this figure. Therefore, each image includes 128,450 hectares.

Thus, we have only one image with twelve waves (green, red and near infrared for each date). The most interesting dynamics take place in the center of the scene in quarter of 512 x 512 pixels.

Fig. 4.5 Geographic mask of two industrial forests in Ticoporo

The Table 4.1 gives an account of the gradual statistical evolution of the forest. The table begins with the treatment of the satellite images on the test site since 1975. The total assessment corresponds to a deforestation of 40,987 hectares in 19 years, that is to say 36% of the forest cover at the beginning but with an equivalent pastoral development.

	1975 -image	1987 -image	1989 -image	1994 -image
Forest	113, 123.85	78,193.71	70,914.27	72,136.33
Non Forest	15,326.71	50,256.85	57,536.29	56,314.23
Wood Rate	88.07	60.87	55.21	56.16
total Hectares	128,450.56	12,8450.56	12.8450.56	128,450.56

Table 4.1 Dynamical statistics about deforestation from 1975 to 1994 in the forest of Ticoporo

4.2.4 The forest Ticoporo Reserve: the known state and the estimation by space remote sensing

Fig. 4.6 and Table 4.1 thus account for the quantitative evolution of the forest and pastoral occupation from 1975 to 1994 by means of the binarization on the red channel of the Landsat and Spot satellites (1975) with a simplified nomenclature, "forest - non forest".

On the basis of a rate of timbering of 88% in 1975, this corresponds to more than 113,000 hectares of tropical forest. The threshold of 56% was reached in 1994. Binary cartographic projections "forest - not forest" of Fig. 4.6 translate the changes in space dynamics from one date to another. It is immediately noted that the metamorphosis of the landscape is not homogeneous in all places, since it affects mainly the heart of the forest reserve while, simultaneously, the "two arms" of the private and protected forest fields appear relatively unchanged. Thus, from 1975 to 1994, 36% of the territory permuted into pasture, which represents an average of 2,157 hectares devastated per year, i.e., 2% of the initial capital forest.

It is important to note that if these data cover a period of 19 years, they are not very "recent". Indeed, it was not possible to acquire new images due to cloud cover, which is almost always present in intertropical areas. Therefore, in the face of this observation, we chose to calibrate the model from the first three dates (1975-1987-1989) to make projections of space in 1994, 2000, 2005 and 2010. Thus, we reserved the last acquired image, – the real image of 1994– to validate the model to this date.

4.3 Methodology and practical application to the data sets

Through a sequence of Satellite Remote Sensing Images for n instants t_1, \ldots, t_n so that $t_1 < ... < t_n$, this work proposes to predict the evolution of a temporal phenomenon for the time $t_n + I$.

The method proposed in this paper is founded on the principle described in the latter approaches (see Sect. 4.1.1 and Sect. 4.1.2) provided that we make use of regression and progression zones of the forest to find the direction of evolution (appearing or disappearing). However, we achieve the prediction by coupling fuzzy set theory and out-image data. Thus, the approach adopted here seeks to yield improved results, since the prediction takes into account the zones that are more or less favourable to the evolution. Considering the following facts may aid the prediction approach:

- − studied temporal events are continuous.
- − geographical data are in raster representation.

Fig. 4.6 Geographic binarization maps for each date from the forest of Ticoporo

Alpha-level sets

In fact, the resulting evolution image represents a fuzzy set that will be analyzed to determine the final shapes and positions of the regions for the predicted map. Fuzzy sets can also be defined by means of their families of α-level sets (Klir and Yuan 1995), according to the resolution identity theorem (Zadeh 1995). Given a fuzzy set *S*, its α-level sets S_α from *U* associated to $S \in f(u)$ are given by the following equation:

$$
S_{\alpha} = \{x \in U \mid \mu_s(x) \ge \alpha\} \tag{4.1}
$$

4.3.1 Problem description

We wish to predict the evolution of a temporal phenomenon for time t_{n+1} from a sequence of n thematic maps characterizing this cartographic continuous temporal phenomenon for *n* instants t_1, \ldots, t_n so that $t_1 \leq \ldots \leq t_n$.

The first step of the approach proceeds by first predicting the overall surface of the region area. What makes it possible to obtain a quantity of space permutation on the basis of fixed nomenclature: forest not - forest.

This stage contains the prediction of the overall surface value applying the analytical data in an adaptive linear adjust method. This value is calculated for the instant t_{n+1} .

4.3.2 Progression and regression maps

So, after predicting the overall surface of the region area, the next step is to obtain maps representing the progression and regression zones. Each map representing a progression or a regression zone is obtained from two consecutive images (the geographical maps taken at t_i and t_{i+1}). The regression map corresponds to the subtraction of the images taken at instants t_i and t_{i+1} , and the progression map corresponds to the subtraction of images taken at t_{i+1} and t_i . Thus, for *n* instants of time, there will be *n*-*l* progression maps and *n-1* regression maps. The method proposed in this work is the use of regression and progression maps of the forest to acquire the privileged directions of evolution (appearing or disappearing). Thus, the approach adopted here seeks to yield improved results, since the prediction takes into account the zones that are more or less favorable to the forest or pasture evolution.

4.3.2.1 Stages of the predictive modelling

From thematic mapping (satellite images in raster format), the basis of the proposed methods can be divided into five basic steps (Mez 1998):

- 1. computing of the total surface of the spatio-temporal phenomenon studied at the moment t_{n+1} ,
- 2. obtaining maps of areas of progression and regression,
- 3. determining the preferred directions of progression or regression through the calculation of a coefficient of evolution by fuzzy logic or cellular automata,
- 4. obtaining a map of evolution,
- 5. obtaining the projected map.

In step 3, a specific mathematical formula was applied to compute the coefficient of evolution for each pixel of the analyzed image. This coefficient of evolution shows zones more or less favorable to the evolution. In order to compute this coefficient of evolution, all progression and regression maps obtained previously are required. The more recent maps will more heavily influence results. The basic principle of the reasoning mechanism adopted here is that an area next to a progression region has a higher probability of increasing than another one that is farther from this region. The same principle is applied for regression regions. The size of the regions must also be considered, since larger regions have a higher influence on its pixel neighbors than smaller regions.

Thus, for each pixel we determine two values: one of them determines a tendency of the pixel to progress; the other determines a tendency of the pixel to regress. The variables are defined as: *p*, the number of progression zones, *n* the number of geographic maps (for *n* times), *Dk* the distance between the *pixel i,j* to the zone *k*, *Sk* the surface of the zone *k* and *T* the temporal interval between the analysed map and the time to predict. We define the coefficient of progression of each pixel by:

$$
Coef_{\Pr \text{og}_{i,j}} = \sum_{t=1}^{n-1} \left(\frac{\sum_{k=1}^{p} \left(\frac{1}{D_k} S_k \right)}{T_t} \right)
$$
(4.2)

where *p* is the number of progression zones.

In a similar way the coefficient of regression of each pixel is given by:

$$
Coef_{\text{Re }g_{i,j}} = \sum_{t=1}^{n-1} \left(\frac{\sum_{k=1}^{r} \left(\frac{1}{D_k} S_k \right)}{T_t} \right)
$$
(4.3)

where *r* is the number of regression zones.

The coefficient of evolution results from a subtraction between the coefficients of progression and the previously calculated regression:

$$
Coef_{i,j} = Coef_{\text{Pr ogi},j} - Coef_{\text{Re gi},j}
$$
 (4.4)

The resulting coefficients were normalized resulting in a fuzzy set representing the membership function of the evolution function. This set was converted into gray levels resulting in a fuzzy image. The gray values in a range of values from 0 (black) to 255 (white) identify the trends of progress or regress of diverse areas in the total region. The more favourable regions to progress are associated with the greatest coefficients and the regions that are less favourable have smaller coefficients.

4.3.3 Prediction based on fuzzy logic

In order to compute this coefficient of evolution for each pixel, all progression and regression maps are required and the most recent maps will more heavily influence the results. It is also necessary to consider the surfaces of the progression and regression zones. Probably, the zone with the biggest surface will heavily influence a pixel being equidistant from a progression zone and from a regression zone. Furthermore, an area of closer proximity to a progression zone will have a more pronounced tendency to increase than an area near a regression zone and vice versa. Thus, these principles taken into consideration in order to compute a coefficient of evolution for each pixel of the image.

Thus, for each pixel we determine two values: one of them provides the tendency of the pixel to progress; the other provides the tendency of the pixel to regress. Let *p* be the number of progression zones, *n* be the number of geographic maps (for *n* times), *Dk* be the distance between the *pixel* i, j to the zone k , Sk be the surface of the zone k and T be the temporal interval between the analysed map and the time to predict. We define the coefficient of progression of each pixel by:

$$
Coef_{\text{Pr}\ o g_{i,j}} = \sum_{t=1}^{n-1} \left(\frac{\sum_{k=1}^{p} \left(\frac{1}{D_k} S_k \right)}{T_t} \right) \tag{4.5}
$$

In a similar way the coefficient of regression of each pixel is given by:

 $\mathcal{L}_{\mathcal{L}}$

$$
Coef \quad_{\text{Re } g_{i,j}} = \sum_{t=1}^{n-1} \left(\frac{\sum_{k=1}^{r} \left(\frac{1}{D_k} S_k \right)}{T_t} \right) \tag{4.6}
$$

where *r*, is the number of regression zones.

The coefficient of evolution results from a subtraction between the coefficients of progression and regression previously calculated:

$$
Coef_{i,j} = Coef_{Progi,j} - Coef_{Regi,j}
$$
 (4.7)

This table of coefficients will be normalized in such a way that the coefficient of evolution for each pixel of the image is determined by a membership function based on the time, the pixel location with regard to progression and regression zones and the surface of these zones. Thus, the more favorable regions to progress are associated with the greatest coefficients and the less favourable regions have smaller coefficients. Thus, the influence calculated for each pixel results in a fuzzy image. Such fuzzy images can help geographers to visualize the whole temporal phenomenon that is taking place. However in the final stage of the analysis, the geographer may be interested in less fuzzy, more clearly defined data. In this case, the imprecise data can be converted to "hard" data by applying the alpha-level sets that transform the fuzzy regions into distinct regions.

The fuzzy subset obtained can then be decomposed by means of its α-level sets in order to obtain the resulting map. A gradient rather than a line represents the boundary between these regions. This gradient may be interpreted as the degree to which each pixel of an image is part of a progression region of the forest.

We have done successive applications of this discrete approximation $(\alpha$ -cuts) to the coefficients of the evolution image in such a way that the final surface reaches the predicted surface. The result is a distinct set containing all the pixels, whose membership grades are greater than or equal the specified value of α .

4.3.4 Prediction based on cellular automata

The steps to the solution of the problem are described as follows in Fig. 4.7.

Fig. 4.7 Simulation steps using Automate cellular (AC)

The temporal GIS data was used to create a map called the situation map, which describes the forest area in accordance with their progression, regression or stability through time. The situation map was used to create the transition rules.

In this work, the totalistic rules were used, they are formed by the total quantity of neighbourhoods in some specific state.

After calculating the new value of the surface, the most recent image is used to start the prediction until the specified year.

The "situation map", in Fig. 4.9, was created to describe the forest areas in accordance with their progression, regression or stability through time. This map is formed by the combination of the temporal images (Fig. 4.8).

The Table 4.2 shows how the "situation map" was composed. It represents all the possible combinations that one pixel in the same position (i,j) with two states (0 or 1) can have when it is compared in three temporal images. The Moore's neighborhood with $r = 1$ was chosen.

That combination created three situations called stability, progression and regression. The digital level is indicated in the table to compose the map visualization (Fig. 4.9).

Fig. 4.8 Combination of temporal maps

Table 4.2 Composition of the situation map $(0 = \text{non forest}; 1 = \text{forest})$

1975	1987	1989	Situation	Digital level
			stability	250
			progression	50
			regression	200
			stability	
			regression	200
			regression	200
			progression	50
			progression	50

Fig. 4.9 Situation map (outside the industrial forest mask)

The forest was considered homogeneous with two states represented in the present diagram:

 $0 =$ non forest $1 =$ forest

The transition rules are the totalistic rules that consider the total quantity of cells in the state $= 1$, in the Moore's neighborhood. The situation map was adapted to different transition rules according to the zone. There are 3 zones for transition rules: stability zones, progression zones and regression zones.

Rule 1: Progression zones. If the neighbor's number equal 1 or 0 and the central pixel is 1, it will be in the next instant zero. If the neighbor's number is equal 2, the central pixel doesn't modify. If the number of neighbours is equal to 8, 7, 6, 5, 4 or 3 and the central pixel is equal to zero, it will be in the next instant 1.

Rule 2: Regression zones. If the number of neighbours is equal to 0, 1, 2, 3, 4, 5 or 6 and the central pixel is equal to 1 it will be in the next instant zero. If the neighbor's number is equal to 7 or 8, the central pixels don't modify.

Rule 3: Stability zones. In this case, there are no changes in the zones.

The practical application of both cellular automata and fuzzy logic based models consists in using specific algorithms in C++, which were developed by the authors themselves.

4.4 Results

The principal aim is to describe the evolution of the landscape of the "Reserve of Ticoporo" forest for the years 1994, 2000, 2005 and 2010. The geographic maps in Figure 4.6 are satellite images showing the recorded changes in the thresholds within the regional area at three times: 1975, 1987 and 1989. The prediction maps for the years 1994, 2000, 2005 and 2010 were obtained by applying α -level sets to the years of concern and are shown in the following figures. Since a satellite image from 1994 was provided, we have used it to validate the results.

The first three images have been processed taking into account the mask of the industrial forest. Using the methodology in Sect. 4.3.2, we have finally obtained the evolution image (fuzzy image), which shows the coefficients of evolution that combine the progression and regression data. The result is shown in Fig. 4.10, in which the darker a pixel is the more it will undergo deforestation (outside the geographic mask).

4.4.1 Space-time environment dynamics from satellite images since 1989 to 1994

Before assessing the results of the temporal projections, we give a statistical evaluation of the spatial dynamics in Ticoporo, according to the last two acquired images from the years of 1989 and 1994 (binary images indicated as "bin"). Table 4.3 shows the statistical results of evolution (in percentage) and their spatializations from Fig. 4.10. The legend of this figure translates the grayscale into four possible combinations of evolution: regression of the forest from 89 to 94 (F89b-NF94b), progression of the forest (NF89-F94b), forest in 89 and forest in 94 (F89b-F94b), non-forest in 89 and non-forest in 94 (NF89b-NF94b).

Fig. 4.10 Evolution zones of the forest

To evaluate the experimentation's results for the modelling of the landscape changes at various temporal periods, we again used the method introduced by Pontius (2004). Indeed, this method applied to research of the LUCC (*Land Uses & Land Cover Change*) program is exportable. In our case, it makes it possible to establish a rigorous statistical comparison of known and/or simulated environmental data through time. This makes it possible to estimate the relevance of the used methods for space projections. This also provides space and statistical dimensions changes to landscape changes over one defined time period. This evaluation begins from a former, known state.

Fig. 4.11 Spatial comparison between satellite image from 1989 to 1994 (left) and the main axes of penetration (right)

On the other hand, the cartography (Fig. 4.11) authorizes a perception of spatio-temporal dynamics in the central part of the forest where more and more isolated forest scraps (F89-F94, black) appear compartmentalized. This phenomenon is all the more perceptible as the not-forests spaces in white (NF89-NF94) seem to increase toward the south through time; the demarcate white areas overall from the northwest to the southeast. The right-hand side of the figure schematizes the rectilinear axes of this penetration well.

$\frac{0}{0}$	1989-NF 1989-F		total 1994	Gain	$\frac{0}{0}$	Gain	Loss	Total change	Swap	Absolute value of net change
1994-NF	34.8	6.9	41.7	6.9	NF	6.9	5.6	12.5	11.2	1.3
1994-F	5.6	52.7	58.3	5.6	F	5.6	6.9	12.5	11.2	1.3
total 1989	40.4	59.6	100.0							
Loss	5.6	6.9								

Table 4.3 Statistical states of forest (F) and non-forest (NF) in % between 1989- 1994 from satellite images

With Table 4.3, one obtains an evaluation and a comparison between the forest states starting from known data: dynamics between two images Spot –binarized– of 1989 and 1994. The left side of the translated table is expressed in percentages, then the proportions of Forest and Non-Forest in the time interval and the losses and the profits are added up for each date. The right side gives an account of the extent of spaces in the reserve affected by these changes (total changes) and the rate of the permutations occurring between these two environmental objects –F and NF– (Swap). The nomenclature comprises only two stations, and the "profits and losses" are relatively close.

Comparing the evolution before 1975 to 1989 (Fig. 4.6), when deforestation was extreme, and the five year interval 1989-1994 (Table 4.1; see Sect. 4.2.3), one observes an attenuation of the rate of deforestation. The cross matrix of Table 4.3 establishes the proportion of forest to pastures (NF) (52.7% per 34.8%) in 1994. 87.5% of spaces of the reserve thus remained unchanged, whereas only 12.5% permuted between these two stations of nomenclature. What corresponds in detail to a double phenomenon: a deforestation, which reaches 6.9%, while reforestation is 5.6%. The total rate of permutation remains relatively unimportant 11.2% (difference between the total change –12.5– and the absolute value of the net change - 1.3-), which corresponds to 14,386 hectares. These results indicate a relative statistical stability for 1989 to 1994, which is in conformity with the cartography of Fig. 4.6 with few modifications between spaces of pastures (NF) and the forest (F).

4.4.2 Validation of the predicted model for 1994

The real image present some small regions. The method applies a filter to the image, for this reason the noise does not appear in the two predicted images.

Fig. 4.12 Comparison between the real satellite image and the predict maps for 1994 by fuzzy logic (FL) and cellular automata (CA) approaches (Forest, black; Non-Forest, white)

Fig. 4.12 juxtaposes the cartography of two space projections by fuzzy logic (FL) and the cellular automats (CA) for the year 1994 beside the real satellite image of 1994 (Spot image binarized; left). Generally, the two models implemented (FL and CA) seem to provide cartographic results that reveal a rather strong similarity. The two modeled images seem to have almost entirely eliminated the small forest islands scattered throughout the central part of the area towards the northwest (whereas the real image shows that they still exist). Both also show the increase of the new axes of pasture penetration from the peasants (linear-shaped axes corresponding to a type of deforestation), which have a very perceptible energy from the center of the image towards the south. These projections highlight the particular behavior of these peasants - principal agents of deforestation.

Table 4.4 shows a clear statistical over-estimation of deforestation for the two projections compared to the real image, as is indicated by the rate of timbering. It is nearly 52% for fuzzy logic and nearly 50% for the cellular automats, whereas in reality it is 56%. The FL simulation model overestimates by 4.31% (5,533 ha) and the AC model overestimates by 6.21% (7,977 ha) in comparison with the real deforestation. In other words, the model of the cellular automats would be a little less relevant than fuzzy logic.

However, the two projections are fairly similar to each other, for example the difference in the deforestation rate is only 1.9%, which corresponds to 2,443 hectares. These models appear to want to give a scenario of relative stability for the phenomenon of spatial deforestation considering the small gap between the two projection models for 1994.

Fig. 4.13 Space-time dynamic states of Ticoporo between FL and AC in comparison with the reality in 1994

Fig. 4.13 corresponds to the cartographic projections of the two models for the year 1994 with a detailed nomenclature of the operated changes. The method used is identical to that described in the preceding paragraph (i.e Fig. 4.10). Thus, each of the two space projections includes the four possible combinations of dynamic environmental between reality (noted 94b) and projections in 1994 (noted "94 FL" for fuzzy logic and "94 AC" for the cellular automats).

The square matrix of Table 4.5 allows for the comparison of the predictive results of the two models for the year 1994 with the real image of 1994 (binary image) on the same bases of the nomenclature "Forest-1; Non-forest-0". Statistically, the two predictive models used demonstrate relatively little difference between them with respect to the field reality. Thus for forest spaces, the difference between the predicted totals and the real totals are established at 56.5 compared with 58.3% for FL and at 55.2% compared with 58.3% for AC, that is to say a variation of prediction from only 1.8% for the FL and of 3.1% for CA. The variations for nonforest spaces are identical.

$\frac{0}{0}$	Real image 94		
1994	Forest-1	$non-forest-0$	total predicted
$FL - forest-1-$	51.4	5.1	56.5
FL- non forest-0-	7.0	36.5	43.5
total reality	58.3	41.7	100.0
$\frac{0}{0}$	Real image 94		
1994	Forest-1	$non-forest-0$	total predicted
$AC - forest-1$	49.6	5.6	55.2
AC - non forest-0-	8.7	36.1	44.8

Table 4.5 Comparison of predictions and the ground reality for 1994 (FL and AC)

Thus for forest spaces, the difference between the predicted totals and the real totals are established to 56.5 compared with 58.3% with the FL and to 55.2% compared with 58.3% with AC, that is to say a variation of prediction from only 1.8% for the FL and of 3.1% for CA the variations are identical for nonforest spaces.

Moreover, the total rates of prediction posted by FL and AC are also correspond, because they present only 1.3% of difference between them for the forest and only 0.3% for pastures (NF).

Fig. 4.14 Differences between the image spot of 1994 and the projections in 1994: fuzzy logic (left) and cellular automata (right-hand side)

Figure 4.14 represents the arithmetic difference, pixel with pixel, between the known reality through the binarised Spot image of 1994 and the two types of space projection for the same year. The legend's format is standard in white, a correct prediction at the same time for forested spaces and for grazing ground spaces; in black, an erroneous prediction in terms of probability for these same spaces. We can also refine these results with the method expressed spatially by Figs. 4.8 and 4.11, which define the "areas of progression and areas of regression".

In terms of the set's themes, the origin of the shift in results between the projected images and the real image (binarised) also come from the quasisystematic removal (in both cases) of the many small scattered forest scraps, which are still perceptible in the central part of the forest reserve as in the northeast corner of the image. In other words, these discontinuous, small forest islands of variable sizes, although minority on a become pastoral space dominating, would have a probability less strong than envisaged to be destroyed contrary to the result provided by the methods of predictive modelling. This stage of the analysis, one can put forth the assumption of following explanation for the fuzzy logic model : if the principle stated in phase 1 (to predict a value of total surface by applying to the analytical data a method of adapted regression linear) seems overall true, the absence of introduction of rules of behaviours to the predictive models reduced some their capacities to be extrapolated to become it of these small forest small islands for, on the contrary, marking more that of the largest solid masses.

4.4.3 Prospected scenarios of fuzzy logic and cellular automata for years 2000, 2005 and 2010

The modelling on steps of selected times 2000, 2005 and 2010 uses the same databases –the binary satellite images–. The two types of projections carried out do not seem to visually confirm (Fig. 4.15) what we had previously detected with the analysis of projection over the year 1994 (see Sect. 4.4.1; Fig. 4.11). The cellular automata method over-estimates the phenomenon of deforestation compared to the fuzzy logic method. It appears that the reverse dominates. To be convinced of this, it is enough to compare the central parts of the six small images: they became completely white with fuzzy logic, therefore a projection which appears radical apparently without nuance and undoubtedly far away from reality, whereas the method of the cellular automata appears less brutal, in other words, more adjusted to the rate/rhythm of transformation of the landscape since one still distinguishes pieces isolated from forest in center-south space of the image.

4.5 Statistical validation of spatio-temporal projections by fuzzy logic and cellular automata

4.5.1 The state of the "Reserve of Ticoporo" forest estimated to the year 2010

The validation of the model is a comparison of the results of the two projections date for date since 1994, but it is not compared with known

results, because there have been no Spot or Landsat images without cloud cover since 1994.

by CA: Predicted image for 2000; Predicted image for 2005; Predicted image for 2010.

Fig. 4.15 Resulting maps to fuzzy logic (FL) and cellular automata (CA) approaches (years 2000, 2005, 2010)

Table 4.6 and Fig. 4.16 shows the statistical results acquired for the projections for the selected years 2000 - 2005 - 2010 by the two methods: fuzzy logic ("Fuz") and cellular automats (CA). If the scores reached vary somewhat for each projected date, the two types of projection have a common point: the process of deforestation and/or creation of new spaces of pasture are continuous from 1975 to 2010. The forest permutation in pastures appears inexorably linked at the neighbourhood level to the timbering rate included/understood spreading from 42 to 46%, respectively for the cellular automats and fuzzy logic.

Table 4.6 Statistics of the space-time dynamics for the test site of Ticoporo for the years of 2000, 2005 and 2010

				2000-Fuz 2000-CA 2005-Fuz 2005-CA 2010-Fuz 2010-CA		
Forest (ha)				54,821.69 59,706.99 51,779.77 56,491.12 58,986.69 54,507.11		
Non Forest (ha)				73,628.87 68,743.57 76,670.79 71,959.44 69,463.87 73,943.45		
Timbering Rate %	42.68	46.48	40.31	43.98	45.92	42.43
All (ha)				128,450.56 128,450.56 128,450.56 128,450.56 128,450.56 128,450.56		

Fig. 4.16 Changes occurred with Ticoporo since 1975 to 2010 as well as the timbering rate

Table 4.7 and the diagrams (Fig. 4.15) are the result of the two types of projections in terms of deforestation scenarios. The phenomenon seems attenuated compared to the first period (1975-1994), seeing as the increase of devastation would reach 13,150 ha according to fuzzy logic or 17,629 ha according to cellular automata by the year 2010. The method of fuzzy logic seems to over-estimate deforestation for the years 2000 and 2005 compared to that of the cellular automats (24 and 31% against 18 and 28%, calculated in hectares, that is a difference in 814 ha in 2000, 429 ha in 2005), and in contrast to 2010 with respectively 22 against 34% of additional cuts compared to 1994 (an additional 280 ha for the cellular automats).

since 1994	1994-FL			-1994CA 2000-FL 2000-CA 2005-FL 2005-CA 2010-FL 2010-CA			
Deforestation (Def.)				5,533.57 7,977.20 17,314.64 12,429.34 20,356.56 15,645.21 13,149.64 17,629.22			
Average/year		2.886	2.072	1.851	1.422	822	1.102
Def. since 94 (%)		24.00	18.66	31.73	28.54	22.02	34.05
Def./vear (%)		4.00	3.11	2.88	2.59	1.38	2.13

Table 4.7 State of the forest of Ticoporo until 2010. Projection Fuzzy logic and cellular automata models

Unfortunately, interpretation cannot refine these statistical results more by comparing them with known situation on the ground of each projected date (variable space and known sets of themes), because there are no satellite images for the period of interest (not since 1994) due to cloud cover inhibiting the evaluation of the relevance of these space-time projections.

Fig. 4.17 Projections based on FL model (up) and CA model (down) from 1994 to 2010

Figure 4.17, in which the two types of projections are shown separately, shows a very different behaviour between the two timbering rates from deforestation for two consecutive periods (2000-1994; 2005-2000; 2010- 2005) according to the projections: a decidedly more stability for fuzzy logic and a regular but decreasing pattern for cellular automata.

Table 4.8 Differences between the two models for projected estimations in time

CA-FL 1994 CA-FL 2000	CA-FL 2005	$CA-FL2010$
$-2,443.63$ Ha $-4,885.30$ Ha	4,711.35 Ha	-4,479.58 Ha

Related to the year 1994, the projection by fuzzy logic appears to underestimate the deforestation by 5,533 hectares (that is to say -7.67 % of existing forest space in 1994), that of cellular automata still more with 7,977 hectares (-11.06 %). But compared with the 1975 forest state, the same percentages decrease to the respective values of 4.31% and 6.21%. In addition, the difference between the two projections (CA - FL for same year -1994- adds up to a little more than 2,443 hectares. It is a relatively minor difference (3.40% compared to the forest state estimated in 1994) in comparison to the same

differences operated for the other years; indeed these last years would reach figures higher than 4,400 hectares (6.1% of the forest total of 1994). We also note the inversion of direction of this calculation between on the one hand, the years 2000-2005, and on the other hand, the year 2010.

Forest stable 1994 – 2010 Non Forest 1994 Projection : deforestation 2000 Projection : deforestation 2005 Projection : deforestation 2010

Fig. 4.18 Result of the space projections cumulated for 1994 to 2010 by fuzzy logic (left) and cellular automata (right)

This cartographic result spatially combines three temporal cumulated projections of fuzzy logic and cellular automata by superposition. This cartography will confirm that even the heart of the *forest Reserve,* which is mainly affected by this continuous phenomenon of deforestation, is well. The fuzzy logic method proceeds in a spatial way (like an areola), whereas cellular automata function much more in a staircase.

Our observations on the ground, show that the displacement of the migrating peasants, who are major participants in the deforestation, is always toward the southeast. These individuals, who are involved in the radical permutation "forest-pastures," claiming pasture land along the axes directed north-west-south-, this penetration into the surrounding forest is already well marked on the 1989 image. Logically, these projections thus appear as a continuity of the process that started around 1975.

In addition, always in the heart of the solid mass (more precisely in the south-western centre), a black, untouched area remains, of a rather imposing size. In other words, in the space of 16 years, this means that the peasants have not entirely destroyed this forest unit. The advance thus appears to be blocked by the presence of an insuperable river.

4.6 Conclusion and outlook

The predictive methods of fuzzy logic and cellular automata make it possible to answer partly the question of short-term and medium-term conditions for a tropical forest environment whose existence is radically threatened by a migrating peasant population, who use a cut and burn technique to win pasture land from the rain forest. The present method combines the remote observation of a face tropical pioneer front with the means of the processing of multi-date satellite images to fuzzy logic and cellular automata. This achieves the objectives of description, cartography, and temporal projections of the deforestation dynamics in progress for various steps of the selected times (1994 to 2010). The method is founded on the diachronic analysis of the space-time evolutions contained in a sequence of chart sets of themes resulting from satellite images (Landsat and Spot) from three years: 1975-1987-1989. With a simplified nomenclature "*forest - non forest*", the model of evolution takes into account the contemporary history of the face by analyzing the changes the of the existing forest areas for each image. These last, two methods - fuzzy logic and cellular automata make it possible to manage the inherent uncertainties and inaccuracies, which surround the modelling of the evolution of this unstable and dynamic forest environment.

A detailed statistical estimate is carried out specifying the compared contributions of the two implemented methods. We can also relate the statistical estimates of the Department of the Environment of Venezuela to this study, specifying that "in 1980, 39% of the forest surface of Ticoporo was destroyed". Since 1987, a comparable area to the east is provided by the space image processing for comparison. The evaluation of the 1994 projection (this being the most recent date possible due to the increased rate of nebulosity in this intertropical area since then, which has inhibited the capture of a new image), provides an encouraging estimate for the use of these tools and these space-time methods, which have been described in this article. The temporal validation of these two methods of modelling has yet to be confirmed. The acquisition of satellite images without cloud cover is, however, essential to carry out in assessment and validate the temporal modelling methods for this validation.

However, one can expect that this space-time modelling, in the future, will be enriched by new rules for explicit behaviours, thus integrating more of the ground data available, including all the factors impacting environmental, economic and social processes of evolution. The results of extrapolation would be without doubt refined by it.

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