6

The Time Arrow in Quantum Cosmology

> Our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. (Stephen Weinberg 1977)
> – Well, but *which* theories?

The founders of quantum theory invented their theory as a theory of atoms, that was soon successfully applied also to other microscopic systems. Macroscopic objects were thought to require the established classical concepts even though they consist of atoms. This hardly consistent traditional point of view (that would also exclude quantum cosmology) seems to be slowly changing under the impact of more recent interpretations, which allow one to describe the world in terms of a universally valid quantum theory (Sect. 4.6).

Another obstacle to quantum cosmology is that a description of the whole Universe seems to require a 'theory of everything', which is elusive. While there are various mathematically deep and physically even plausible proposals for such a theory, physics is an empirical science. Physical cosmology should therefore only extrapolate empirically founded concepts and laws. Mathematical cosmological models may be important and interesting in their own right, and some of them may prove physically successful in the future, but reality has usually offered great conceptual surprises that could not have been foreseen by mathematical reasoning or pure logic.

Physical cosmology should not therefore rely on any details of unconfirmed unified quantum field theories, for example. Only the general framework of quantum theory may be regarded as empirically sufficiently founded to draw cosmological conclusions from it. This framework includes, first of all, the superposition principle and the unitarity of dynamics (in other words, a general wave function and a Schrödinger equation). In cosmology, this requires an answer to the fundamental problem of what quantum theory *means* in the absence of external observers or measurement devices. Physical cosmology must therefore depend on the interpretation of quantum theory (as discussed in Sect. 4.6) in an essential way. A pragmatic probability interpretation with respect to external observers is obviously ruled out, since the very concept of cosmology presumes an objective (though in principle hypothetical) reality. Quantum *field* theory has instead traditionally been used and confirmed as a method for calculating S-matrix elements, which describe probabilities for scattering events. This amounts to applying a collapse of the wave function after each elementary scattering process, and it would be insufficient for consistently describing objects which make up the Universe, such as condensed matter, complex systems (including measurement devices and observers), macroscopic fields, and global spacetime structure.

The general quantum framework is usually applied in the form of a 'quantization' of a classical theory (see Sect. 4.1.1) – in particular of the mechanics of particles, which are kinematically described as space points. By quantization I mean here[1] the application of the superposition principle to the elements of a classical configuration space (thus defining a wave function on it), and the construction of the corresponding quantum Hamiltonian by replacing variables and their canonical momenta by operators acting on wave functions. The second part is ambiguous because of the factor ordering problem.

We can now re-interpret this quantization procedure as the conceptual reversal of a physical decoherence process that led to the classical appearance of the system under consideration. This explains why this quantization cannot be expected to define a unique result, but requires further empirical input. The quantization of many-particle mechanics leads non-relativistically 'back' to a consistent and successful quantum theory: quantum mechanics. Some other 'particle' properties (such as spin or isotopic spin) have *no* similar classical correspondence. The quantization of classical *fields* in this canonical way leads to wave functionals on the configuration space for field amplitudes. It does not in general directly define a consistent quantum theory, although it can often be rendered consistent by a mere renormalization of its fundamental parameters. This is evidence that a fundamental quantum theory may be quite independent of any classical theory that could be quantized in this way. For example, relativistic quantum mechanics led to the discovery that field amplitudes of *not classically observed* fermion fields rather than particle positions define the correct arena for the wave function(al) – an approach that is somewhat misleadingly called a 'second quantization', since the fermion fields were first discovered as effective 'single-particle wave functions' (see Zeh 2003). The underlying fields (on space) define a local *basis* (the 'stage' for quantum

[1] This interpretation is quite different from the original and literal meaning of the term 'quantization' as a *discretization* of certain quantities. For example, 'light quanta' can be understood as a *consequence* of the eigenvalue problem in terms of wave functions for the amplitudes of free field modes, dynamically described as harmonic oscillators. These fundamental aspects of quantum theory are often hidden behind a collection of recipes to perform calculations (such as perturbation theory in terms of Feynman graphs). In particular, a 'quantization of time' (Sect. 6.2) does *not* require a quantum of time – just as the quantization of particle motion does not require a quantum of length (or a spatial lattice).

dynamics) that spans the required Hilbert space. This structure permits the formulation of *local dynamics* by means of a Hamiltonian density in spite of generically nonlocal states. It may therefore be useful – though also dangerous and certainly insufficient – to investigate mathematical models for a unified quantum field theory solely by investigating certain *classical* fields on three- or higher-dimensional spaces, rather than consistently taking into account their quantum nature from the beginning (for instance in terms of wave functionals of these fields as representing the true quantum reality).

Extrapolating unitary dynamics to the whole Universe requires an Everett type interpretation (see Sect. 4.6). Hugh Everett (1957) seems to have first seriously considered a wave function of the Universe,[2] that must then include *internal observers*. Although he may have had in mind the quantization of general relativity with its cosmological aspects, Everett applied his ideas, which were based on a time-dependent Schrödinger equation, to nonrelativistic quantum theory. His main interpretational obstacle was the entanglement arising from measurements described by means of von Neumann's unitary interaction (4.32). This led him to his 'extravagant' interpretation (in Bell's words) in terms of *many* quasi-classical 'branches' of the world, which are separately experienced, but are all assumed to exist in one superposition that defines the true and dynamically consistent quantum world. Beyond measurements proper and occasional interactions he does not seem to have regarded entanglement as particularly important (see Tegmark 1998).

The quantitative considerations reviewed in Sect. 4.3 demonstrate that *uncontrollable* 'measurement-like' interactions with the environment are essential and unavoidable for almost all systems under all realistic circumstances. Strong entanglement is, therefore, a *generic* aspect of quantum theory. The more macroscopic a system, the stronger its entanglement with its environment. The concept of a (pure) quantum state can be consistently applied only to the Universe as a whole (Zeh 1970, Gell-Mann and Hartle 1990). This seems to be a far more powerful argument for the need of quantum cosmology than an attempt to construct a unified quantum field theory.

The second pillar of physical cosmology is general relativity. It is empirically confirmed only as a classical theory, but this fact can be well understood by decoherence again (see Sects. 4.3.5 and 6.2.2). Exactly classical gravity would lead to inconsistencies with the uncertainty principle. Applying the quantization rules to the Hamiltonian formalism of general relativity (described in Sect. 5.4) leads to a non-renormalizable 'effective' quantum gravity that cannot be exact, but may be expected to be appropriate as a low energy limit. This readily allows us to discuss a number of important novel conceptual problems that must come up, in particular the need for a 'quantization of time' (Sect. 6.2).

[2] Thibault Damour (2006) has recently presented evidence that Everett was originally stimulated by remarks Albert Einstein made about quantum theory during his last seminar, given at Princeton in 1954.

The quantum state of the Universe must therefore include gravitational degrees of freedom (entangled with matter) in an essential way. However, many quantum cosmological aspects may be formulated on a quasi-classical background spacetime, using a given foliation parametrized by a time coordinate t. Global states can then be dynamically described by means of a time-dependent Schrödinger equation with respect to this coordinate time t. This formalism will be *derived* from quantum gravity (with its quantized concept of an *intrinsic* time) in Sect. 6.2.2 as an approximation. *Global* states (such as those of quantum fields) depend on a foliation (or a reference frame) even on flat spacetime, while the density matrix of any *local* system should be invariant under a change of foliation that preserves its local rest frame – a requirement that does not seem to have attracted much attention.

If the Quantum Universe is thus conceptually regarded as a whole, it does not decohere, since there is no further environment. Decoherence is meaningful only for *subsystems* of the Universe (or for subsets of variables), and with respect to observations by other subsystems (internal 'observer-participators'). If no real collapse of the wave function is assumed to apply, one is then *forced* to accept Everett's global wave function, which describes a superposition of at least all 'possible' outcomes of measurements and measurement-like processes that ever occurred in the Universe. This global quantum state may always be assumed to be pure, since a global density matrix could be consistently understood as representing incomplete information about such a pure state. A measurement that merely selects a subset from those states which diagonalize this density matrix would be equivalent to a classical measurement (as depicted in Fig. 3.5 – in contrast to Fig. 4.3).

The decoherence of subsystems by their environment according to a *global* Schrödinger equation leads dynamically to robust Everett branches. They represent dynamically autonomous *components* of the global wave function, which may factorize in the form $\phi_{\mathrm{obs1}}\phi_{\mathrm{obs2}}\ldots\psi_{\mathrm{rest}}$ with respect to 'observer states' that may describe objectivizable memory (see Sect. 4.3.2 and Tegmark 2000). This unitary evolution requires a fact-like arrow of time, corresponding to a cosmic initial condition of type (4.59). Branching into components which contain definite observer states has to be *taken into account* in addition to the unitary evolution as an *effective* dynamics in order to describe the history of the (quasi-classical) 'observed world' in quantum mechanical terms (see Sect. 4.6 and Fig. 4.3). However, this need *not* represent a modification of the fundamental dynamical laws, since this indeterminism affects the observer rather than the quantum world. The decrease of physical entropy characterizing the 'apparent collapse' experienced by the subjective observer may be negligible on a thermodynamical scale, and in comparison to the entropy increase by decoherence in the usual situation of a measurement. Yet it may have dramatic consequences for global phase transitions that describe a dynamical symmetry-breaking of the vacuum. This will now be discussed.

6.1 Phase Transitions of the Vacuum

Heisenberg (1957) and Nambu and Jona-Lasinio (1961) invented the concept of a vacuum that breaks symmetries of a fundamental Hamiltonian 'spontaneously' (in a fact-like way) – just as *most* actual states of physical systems do. This proposal was based on an analogy between the vacuum (the ground state of quantum field theory) and the phenomenological ground states of macroscopic systems, such as ferromagnets or solid bodies in general. Their asymmetric ground states lead to specific modes of excitation, which in quantum theory define quasi-particles (phonons, for example). The corresponding occupation number eigenstates span specific partial Hilbert spaces ('Fock spaces'). A symmetry-violating vacuum may similarly lead to *Goldstone bosons* or other collective modes, based on space-dependent oscillations of the order parameter about its macroscopic (collective) 'orientation' – see below.

A symmetry-breaking (quasi-classical) 'ground state' is in general not even an eigenstate of the fundamental (symmetric) Hamiltonian; it may only form an eigenstate of an effective (asymmetric) Fock space Hamiltonian. While non-diagonal elements of the exact Hamiltonian which connect states of different collective orientation of these many-body systems (lying in different Fock spaces), are usually extremely small, they would be essential to determine its exact eigenstates, since the diagonal elements for all states related by a symmetry transformation must be degenerate.

The symmetry-breaking vacuum was originally understood as part of the kinematics of a field theory, while the dynamics was then assumed to be completely defined by means of the Fock space Hamiltonian. Later, the analogy was generalized to allow for a *dynamical* phase transition of the vacuum during the early stages of the Universe. This may be induced by the variation of some global parameter (such as a rapid decrease of energy density, reflecting the expansion of the Universe). The arising 'unitarily inequivalent' different Fock spaces can then be interpreted as robust Everett branches or collapse components. Even the empirical P or CP-violating terms of the (effective) weak-interaction Hamiltonian may have *emerged* dynamically in this way by means of an apparent or genuine collapse of the wave function that led to a specific vacuum.

A popular model for describing symmetry-breaking in non-perturbative quantum field theory is the 'Mexican hat' or 'wine bottle potential' of the type $V(\Phi) = a|\Phi|^4 - b|\Phi|^2$ (with $a, b > 0$) for a fundamental complex matter field Φ (such as a *Higgs field*). It may possess a degenerate minimum on a circle in the complex plane, at $|\Phi| = \Phi_0 > 0$, say. The classical field configurations of lowest energy may then be written as $\Phi \equiv \Phi_0 e^{i\alpha}$, with an arbitrary phase α. They break the dynamical symmetry under rotations in the complex Φ-plane. These classical ground states correspond to different quantum mechanical vacuum states $|\alpha\rangle$ (for example described by narrow Gauß packets of α-eigenstates). One of them, $|\alpha_0\rangle$, say, is assumed to characterize our observed world (while the specific value of α_0 is in this case observationally meaningless).

A *dynamical* phase transition of the vacuum can now be described by assuming that the Universe was initially in the symmetric vacuum $|\Phi \equiv 0\rangle$. This may later become a 'false' vacuum (a *relative* minimum) through a change of the parameters a and b. The state of the observed universe is then assumed to undergo a transition into a specific Fock space vacuum $|\alpha_0\rangle$. If potential energy is thereby released in a 'slow roll' (similar to latent heat in a phase transition), it must be transformed into excitations (particle creation). Evidently, this symmetry-breaking process requires effective deviations from the Schrödinger equation – similar to a measurement process.

If the initial state is here assumed to be pure, a *unitary* evolution (similar to von Neumann's measurement) leads to a symmetric superposition of all asymmetric states. For example, the symmetric superposition of all Fock space vacua,

$$|0_{\mathrm{sym}}\rangle = C\int |\alpha\rangle \mathrm{d}\alpha \neq |\Phi \equiv 0\rangle \ , \tag{6.1}$$

may possess an even lower energy expectation value than $|\alpha\rangle$, and may thus represent an approximation to the ground state of the full theory. A globally symmetric superposition of type (6.1) would persist even when its components on the RHS contain or develop uncontrollable excitations *in their Fock spaces*, while these components then form dynamically independent Everett branches. The superposition itself describes *intrinsic complexity*, but not a global asymmetry. If $\pi_\alpha := \mathrm{i}\partial/\partial\alpha$ generates a gauge transformation, (6.1) describes a state obeying a gauge constraint, $\pi_\alpha|\psi\rangle = 0$ (see Sect. 6.2).

Each homogeneous *classical* state α_0 would permit excitations in the form of small space-dependent oscillations, $\alpha_0 + \Delta\alpha(\boldsymbol{r}, t)$. Quantum mechanically they describe massless *Goldstone bosons* (excitations with vanishing energy in the limit of infinite wavelength because of the degeneracy). Their degrees of freedom are thus *created* by the intrinsic symmetry breaking, and their observation demonstrates that the collective variables (including corresponding 'gauge' degrees of freedom) do not describe mere redundancies. These new variables may be thermodynamically extremely relevant. So it is remarkable that the most important cosmic entropy capacities are represented by zero-mass bosons: electromagnetic and gravitational fields (Zeh 1986a, Joos 1987). These capacities are not only relevant for physical entropy (such as in the form of heat), but also for the formation of entanglement between different spatial regions. This seems to be important for the 'arrow of quantum causality' (Sect. 4.6).

In contrast to the false vacuum, the symmetric superposition (6.1) would already describe a nonlocal state. If one neglects Casimir–Unruh type correlations (see Sect. 5.2), each vacuum $|\alpha\rangle$ may be written as a direct product of vacua on volume elements ΔV_k,

$$|\alpha\rangle \approx \prod_k |\alpha\rangle_{\Delta V_k} \ . \tag{6.2}$$

This non-relativistic approximation describes a pure vacuum state on each volume element (local subsystem) ΔV_k, while the superposition (6.1) would lead to 'mixed states' for them:

$$\rho_{\Delta V_k} \propto \int |\alpha_{\Delta V_k}\rangle\langle\alpha_{\Delta V_k}| \,\mathrm{d}\alpha \,, \tag{6.3}$$

formally representing Zwanzig projections $\hat{P}_{\mathrm{sub}}$. However, this density matrix would be meaningful only for an *external* observer of the global state (who could not live in one of the Fock spaces). It describes a canonical distribution of Goldstone bosons with infinite temperature (since then $\mathrm{e}^{-E/kT} \to 1$). Therefore, only a (genuine or apparent) collapse into *one* component α_0 gives rise to the pure (cold and not entangled) vacuum (6.2) experienced by an internal local observer who lives in this Fock space.

Order parameters such as α may differ in different spatial regions (similar to Weiss regions of a ferromagnet). If these regions are macroscopic, and thus decohere to become 'real' (see Sect. 4.3.1), they break translational symmetry (Calzetta and Hu 1995, Kiefer, Polarski and Starobinsky 1998, Kiefer et al. 2006). This scenario has now become 'standard' in quantum cosmology – although its interpretation varies. A homogeneous superposition of entangled *microscopic* inhomogeneities would represent 'virtual' symmetry breaking (in classical language circumscribed as 'vacuum fluctuations').

6.2 Quantum Gravity and the Quantization of Time

> Um sie kein Ort, noch weniger eine Zeit;
> Von ihnen sprechen ist Verlegenheit.
> (Mephisto advising Faust to time travel)

The compatibility of general relativity and quantum theory has often been questioned. This seems to be a prejudice, that derives from various roots:

Einstein's attitude regarding quantum theory is well known. He is even claimed to have remarked that a quantization of general relativity would be 'childish' – although he also emphasized the importance of reconciling his theory with quantum theory. Another position holds that gravitons may be unobservable in practice, and the quantization of gravity hence *not required* (von Borzeszkowski and Treder 1988). However, a classical gravitational field or spacetime metric is *inconsistent* with quantum mechanics, since it would always allow one in principle to determine the exact energy of a quantum object – in conflict with the uncertainty relations. This has been known since the early Bohr–Einstein debate (see Jammer 1974, for example), while other consistency problems regarding an exactly classical spacetime metric were raised by Page and Geilker (1982). Concepts of quantum gravity will turn out to be essential for cosmology and the definition of a master arrow of time. The classical *appearance* of spacetime cannot be regarded as an argument against its quantization, since these classical aspects may be understood within a universal quantum theory in a similar way to all other quasi-classical properties (Sect. 4.3.5).

One often finds also arguments that the canonical quantization of general relativity does not lead to a renormalizable theory, and must therefore be wrong. This argument would apply if quantum gravity was assumed to be an exact theory. However, it can only be expected to represent an 'effective theory' that describes specific low energy aspects of an elusive unified field theory. We know that QED, too, has to be modified and replaced by electroweak theory at high energies, while it remains an excellent and consistent description of all relevant phenomena at low energies. Its most general quantum aspects (described in terms of QED wave functionals) are in fact observed for laser fields in cavities. An analogous (though technically and conceptually more demanding) canonical method of quantizing general relativity leads to the Wheeler–DeWitt equation (6.4) below (DeWitt 1967). Why should the Einstein equations be saved from quantization, while the Maxwell equations are not? The conceptual consequences of quantum gravity, in particular those for cosmology, have turned out to be profound even at this level of a low energy approximation.

The construction of a unified theory certainly represents the major challenge to quantum field theory at a fundamental level. Such a theory must become important in the vicinity of spacetime singularities (inside black holes or close to the big bang), but may also have cosmological consequences. In the absence of any observational confirmation, the latter have to be regarded as 'mathematical cosmology', that remains physically entirely speculative. Candidate models are often studied just as classical theories – sometimes including certain 'quantum corrections'. The surprising claim that *M-theory* may eventually lead to an *explanation* of quantum theory (Witten 1997) seems to be based on an elementary misunderstanding of quantum mechanics and its empirical basis.

A second approach to overcome fundamental problems of quantum gravity is canonical loop quantum gravity (Ashtekhar 1987, Rovelli and Smolin 1990, Thiemann 2006b, Nicolai, Peeters, and Zamaklar 2005). As it does not necessarily require a unification with other field theories, it does not contain most of the speculative elements of string theories, for example. To some extent it may be regarded as a specific though non-trivial renormalization of general relativity. Since this includes a radical formal redefinition of many phenomenological concepts, mostly by means of active diffeomorphisms that would severely affect also *classical* general relativity, it may help to find the correct configuration space on which the ultimate wave function may be defined. However, the relation of its quantization procedures (such as 'Bohr compactification' – invented by the mathematician Harald August Bohr) to the empirically founded quantization concepts must be regarded as highly questionable.

If quantization can indeed be understood as the conceptual reversal of a physical process of decoherence, the thereby recovered superpositions of the (effective) classical quantities should at least define an *effective* quantum gravity – similar to quantum mechanics, which is an effective theory in spite of more general quantum field theory. Therefore, the Wheeler–DeWitt equation in its field representation (here defined in terms of three-geometries) appears as the method of choice for 'physical' quantum cosmology. Questions of interpretation related to those for the wave function in general then seem to be more urgent at this stage than the consequences of speculative attempts to solve consistency problems that arise at high energies or in connection with a complete renormalization procedure.

A major problem that nonetheless prevents many physicists from accepting the Wheeler–DeWitt equation as appropriately describing quantum general relativity is the absence of any time parameter in the case of a closed universe (see Isham 1992). According to the Hamiltonian formulation reviewed in Sect. 5.4, one would naively expect free gravity to be described by a time-dependent wave functional on the configuration space of three-geometries, $\Psi[{}^{(3)}G, t]$, dynamically governed by a Schrödinger equation, $\mathrm{i}\partial\Psi/\partial t = H\Psi$. However, there is no longer an external time parameter in a consistent quantum description, and nor are there *trajectories* of appropriate physical clock variables, which could give this time dependence an interpretation. Different three-geometries ${}^{(3)}G$ (classically the carriers of 'information' about *many-fingered* time – see Sect. 5.4) occur instead as *arguments* of these wave functionals. In the absence of parametrizable trajectories ${}^{(3)}G(t)$ – that is, of spacetime geometries ${}^{(4)}G$, neither proper times nor global time coordinates are available. Therefore, it appears conceptually quite consistent (see Zeh 1984, 1986b, Barbour 1986) that the quantized form of the Hamiltonian constraint, $H\Psi = 0$,[3] completely removes any time parameter t from the wave function of a kinematically closed (though not necessarily finite) universe. This consequence must be expected to remain valid in reparametrizable unified theories – even after renormalization.

If matter is again represented by a single scalar field Φ on space-like hypersurfaces defined by their three-geometries ${}^{(3)}G$, the Wheeler–DeWitt equation assumes the general form

$$H\Psi[\Phi,{}^{(3)}G] = 0\ . \tag{6.4}$$

[3] Only because of the (here quite inappropriate) Heisenberg picture in terms of particles is the equation $H\psi = E\psi$ usually called a *stationary* Schrödinger equation, while in wave mechanical terms it describes *static* solutions. Even 'vacuum fluctuations' represent static entanglement in the Schrödinger picture. Similarly, eigenvalues of the momentum operator are no more than *formally* analogous to classical momenta (which are defined as time derivatives). These conceptual subtleties will turn out to be essential for a consistent interpretation of the Wheeler–DeWitt equation.

Even though it represents dynamics, it does not describe a one-dimensional succession of states (or a history labelled by a parameter t). This is the natural quantum consequence of a classically missing *absolute time* (the absence of any preferred time parameter). In spite of the Hamiltonian constraint, the *classical* Hamiltonian equations would still define time-dependent (though reparametrizable) trajectories, which allow the unique (one-dimensional) ordering of states by means of physical clocks ('physical time'). While this does *not* apply to the dynamics (6.4) of quantum gravity any more, we shall see in Sect. 6.2.2 that one may approximately construct quasi-trajectories by means of a WKB approximation and using decoherence. Note that (6.4) describes the whole (Everett) quantum Universe, while branching components describing quasi-classical spacetimes would have to obey a stochastic quantum Langevin equation (see Sect. 4.6). However, the absence of *fundamental* trajectories in Hilbert space now leads to the problem of how to pose an 'initial' condition that would be able to explain the arrow of time that is already required for the irreversible process of decoherence, which is needed to justify the branching.

If time in a *closed mechanical* universe was according to Mach consistently defined by motion (as discussed in Chap. 1 regardless of general relativity), there could also be no meaningful time-dependent Schrödinger equation. Instead of an external or absolute time parameter t, one would have to refer to a physical clock variable u, say, that is part of this universe and has to be quantized, too. A time-dependent wave function $\psi(x,t)$ is thus replaced by an entangled wave function $\psi(x,u)$ (Peres 1980b, Page and Wootters 1983, Wootters 1984). In the conventional probability interpretation, $\psi(x,u)$ would describe a probability amplitude *for* 'physical time' u – not *at* a time u. Then why do we always observe states 'at' such a definite time rather than their superpositions? The answer is that one has to expect the relevant clock variable u to become quasi-classical for reasons explained in Sect. 4.3. For example, an assumed 'dust of test clocks' that measured proper times would according to (6.4) decohere any superposition of three-geometries which correspond to different intrinsic times. (This comes close to what really happens in our Universe – see Sect. 6.2.2.)

Equation (6.4) does not yet represent the Wheeler–DeWitt equation in a form that can be used. In practice, one has to represent the three-geometry ${}^{(3)}G$ by a metric $h_{kl}(x^1,x^2,x^3) = g_{kl}(x^0_0,x^1,x^2,x^3)$ with respect to a certain choice of coordinates (see Sect. 5.4). The wave functional $\Psi[h_{kl}]$ must then be invariant under spatial coordinate transformations. This is guaranteed by the three secondary momentum constraints, classically described as $H_i = 0$ (with $i = 1,2,3$). In their quantum mechanical form they must again be imposed as constraint operators acting on the wave function: $H_i\Psi[h_{kl}] = 0$, similar to the Hamiltonian constraint. If the momentum constraints are satisfied, the wave functional $\Psi[h_{kl}]$ represents a functional on three-geometries, $\Psi[{}^{(3)}G]$, only. This seems to require that the operators H_i commute in the weak sense, that is, their commutators must again define constraints. However, this may not be necessary if the effective theory lives in an Everett branch that breaks gauge symmetry – similar to an α-vacuum in (6.1). This would in turn require that the constraints define infinitesimal *physical* transformations – not just redundancies (see Giulini, Kiefer and Zeh 1995) as usually assumed for coordinate transformations. A similar conceptual problem occurs for many other gauge transformations.

General Literature: Kiefer 2007.

6.2.1 Quantization of the Friedmann Universe

A simple toy model of a quantum universe can be constructed by quantizing the classical Friedmann universe described in Sect. 5.3, presuming exact homogeneity and isotropy (Kaup and Vitello 1974, Blyth and Isham 1975). This leads to a reduced Wheeler–DeWitt equation for the two remaining variables, but in contrast to its classical counterpart it does not represent a reasonable approximation to reality. Symmetry requirements are much stronger in their quantum mechanical form than they are classically. For example, the rotation of a macroscopic spherical body produces a similar but microscopically different state, whereas a spherically symmetric *quantum* state is a symmetric superposition of all orientations, that would not be affected by this symmetry transformation any more. An exactly spherical quantum object therefore cannot possess any rotational degrees of freedom – compare the symmetric vacuum (6.1) or a superfluid in a spherical vessel. Rotational spectra are only found in the case of strong intrinsic symmetry breaking, as known for small molecules or deformed nuclei (Zeh 1967). Translations and rotations are thus *identity* operations when applied to a quantum Friedmann universe, which can therefore only be regarded as a very first step towards quantum cosmology (except perhaps very close to the big bang). The low-dimensional configuration space describing such a model is usually called a ‘mini-superspace’.

If the Hamiltonian (5.34) is quantized in the usual way in its field representation, canonical momenta have to be replaced by the corresponding differential operators. In general, this leads to a factor-ordering problem for the Hamiltonian, since arbitrary terms proportional to the commutators $[p, q]$ could always be added before quantization. Although (5.34) looks quite ‘normal’, its straightforward translation into a Wheeler–DeWitt equation,

$$\frac{\mathrm{e}^{-3\alpha}}{2}\left(\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\Phi^2} - k\mathrm{e}^{4\alpha} + m^2\mathrm{e}^{6\alpha}\Phi^2\right)\Psi(\alpha,\Phi) = 0\ , \tag{6.5}$$

is far from being trivial. For example, the result would have been different for quantization in terms of the expansion parameter a instead of its logarithm α. However, the choice used in (6.5) represents the invariant d’Alembertian with respect to DeWitt’s ‘superspace metric’ that is defined by the quadratic form of momenta describing the kinetic energy – see (6.14) below. This specific factor- ordering is analogous to that for a point mass in flat space when formulated in terms of non-Cartesian coordinates. The prefactor $e^{-3\alpha}/2$ can be omitted from (6.5).

Even though the Wheeler–DeWitt equation is a stationary Schrödinger equation, it is of hyperbolic type – similar to a Klein–Gordon equation with variable mass – for reasons explained in Sect. 5.4. This fact offers the surprising possibility of formulating an *intrinsic initial value problem* in spite of the absence of any time parameter (see Sect. 2.1). The logarithmic expansion parameter α may be regarded as a time-like variable with respect to this intrinsic dynamics. The wave function on a 'space-like' hypersurface in superspace (for example at a fixed value of α) then defines an intrinsic *dynamical state* according to this dynamics. This intrinsic quantum dynamics with respect to the 'variable' α can also be written in the form

$$-\frac{\partial^2}{\partial\alpha^2}\Psi(\alpha,\Phi) = \left[-\frac{\partial^2}{\partial\Phi^2} + V(\alpha,\Phi)\right]\Psi(\alpha,\Phi) =: H^2_{\mathrm{red}}\Psi(\alpha,\Phi) \ , \qquad (6.6)$$

in order to define a Klein–Gordon type *reduced Hamiltonian* H_{red}.[4] This dynamics is non-unitary, in particular as H^2_{red} is not in general a non-negative operator. The 'reduced norm', $\int |\Psi(\alpha,\Phi)|^2 d\Phi$, is thus not generally conserved as a function of α. Although there is a conserved formal 'relativistic' two-current density in this mini-superspace,

$$j := \mathrm{Im}\,(\Psi^*\nabla\Psi) \ , \qquad (6.7)$$

its direction depends on the sign of $i = \pm\sqrt{-1}$, which is physically meaningless in the absence of a time-dependent Schrödinger equation. There is in fact no reason even to expect a complex global solution of the real Wheeler–DeWitt equation.

The big bang and a conceivable big crunch would 'coincide' with respect to the intrinsic time variable α, while the expansion of the Universe becomes a tautology. The concept of a reversal of the cosmic expansion is an artifact of the classical description in terms of trajectories, such as in Fig. 5.7, while in more realistic models correlations of the expansion parameter α with quasi-classical physical clocks (including physiological ones) remain meaningful. So

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[4] In *loop quantum gravity*, this differential equation has been replaced by a difference equation with respect to $p := a^2$. This *discrete* new variable can then be extended to negative values in a regular way (Bojowald 2003). This doubling of space would represent a space reflection, inverting the sign of the volume measure, such that the deterministic propagation through $p = 0$ (even in the continuous case) could be visualized as 'turning space inside out'. However, even if this extension of the concept of space could be vindicated in some way, the dynamical meaning of negative values of p (such as representing 'pre-big-bang times') had to be carefully analyzed. Since the low entropy 'initial' condition may be expected to apply at $p = 0$ rather than at $p = -\infty$ for reasons of symmetry, the physical direction of time would always point into the direction of growing $|p|$ – thus replacing the big bang by a reversal of the physical arrow of time in the past (see also Laguna 2006).

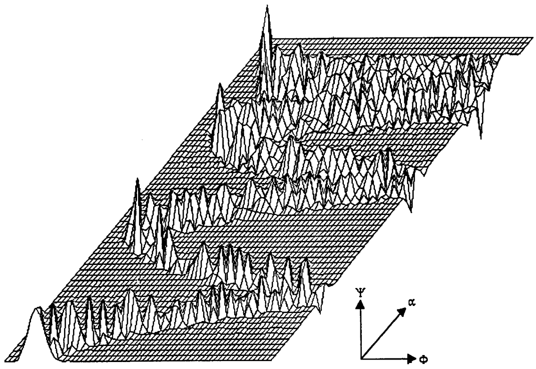

Fig. 6.1. Coherent 'wave tube' $\Psi(\alpha, \Phi)$ for the anisotropic indefinite harmonic oscillator (with $\omega_\Phi{:}\omega_\alpha = 7{:}1$) as a toy model of a periodically contracting and rebouncing quantum universe. It is here only plotted for the sector $\alpha > 0$ and $\Phi < 0$, since the solution is symmetric under reflections at both the α and the Φ axis (so wave tubes intersect at the right boundary). The intrinsic structure of the wave function is not completely resolved by the grid size used in this figure

what would 'happen' to a quasi-classical universe that were classically bound to recontract at some time?

One may discuss the quantum cosmological state in analogy to the 'stationary' wave function of a quantum 'particle' with fixed energy E, reflected from a spatial potential barrier (now a barrier in α). Since in timeless quantum gravity there is no reference phase $e^{-i\omega t}$, one cannot distinguish between incoming and outgoing partial waves by their proportionality to $e^{\pm ik\alpha}$.

Because of the hyperbolic nature of the Wheeler–DeWitt equation, narrow wave packets in Φ at fixed α lead to narrow 'wave tubes' that may approximately follow classical trajectories in mini-superspace (see Fig. 6.1). The case of positive spatial curvature, $k = +1$, is particularly illustrative. Its classical trajectories in mini-superspace would reverse direction with respect to α at some $\alpha_{\max}$ (Fig. 5.7). According to classical determinism, half of the trajectory (*defined* to represent the contracting universe) would be regarded as the dynamical successor of the other half (the expansion era). This deterministic relation is symmetric, since there is no absolute dynamical direction. The wave determinism described by the hyperbolic equation (6.6), on the other hand, propagates monotonically with α, and permits one to choose the whole initial condition (consisting of Ψ and $\partial\Psi/\partial\alpha$) on any 'space-like' hypersurface in superspace (e.g., at a small value of α). One could thus exclude precisely that part of the wave tube that would be required by classical determinism (Zeh 1988). How can these two forms of determinism (classical and quantum) be reconciled?

This dilemma can be resolved in analogy to conventional stationary states of quantum mechanics – though now including negative dynamical mass m_α

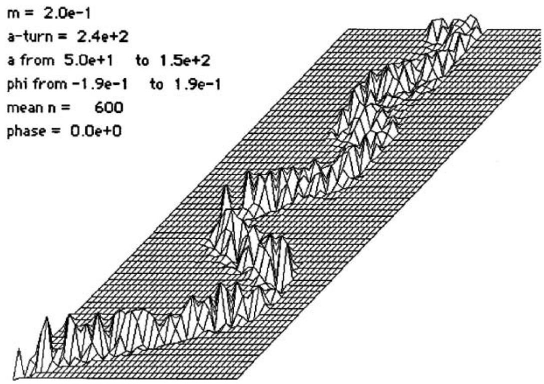

Fig. 6.2. Real-valued wave tube of the 'time'-dependent (damped) oscillator (6.5) in adiabatic approximation *without* taking into account the reflection at $a_{\max}$, i.e., only the first term of (6.12) is used. Expansion parameter $a = e^{\alpha}$ is plotted upward and the amplitude of a homogeneous massive scalar field Φ from left to right

for the time-like variable. The simplest toy model for what is classically a periodically contracting and rebouncing universe is a free motion between reflecting boundaries of a narrow rectangle in α and Φ, where quantum mechanical solutions, such as real-valued wave tubes, can be constructed as superpositions of products of trigonometric functions matching the boundary conditions. (The potential barriers would have to be positive infinite for Φ-boundaries and negative infinite for α-boundaries.) In order to allow nontrivial zero-energy solutions, $H\Psi = 0$, the box lengths L_i have to be commensurable when accounting for the mass ratio, that is, $L_\alpha/L_\Phi = \sqrt{-m_\Phi/m_\alpha}\,k/l$, with integers k and l. Similar stationary wave tubes may be constructed in analogy to Schrödinger's coherent states from anisotropic harmonic oscillators (Fig. 6.1). In this case, one needs an indefinite potential, $V(\Phi,\alpha) = \left[(\omega_\Phi^2\Phi^2 - \omega_\Phi) - (\omega_\alpha^2\alpha^2 - \omega_\alpha)\right]/2$, where 'zero point energies' have been subtracted. In the commensurable case, now defined by $\omega_\alpha/\omega_\Phi = l/k$, solutions to the constraint $H\Psi = 0$ may be obtained as superpositions of the factorizing eigensolutions $\Theta_{n_\alpha}(\sqrt{\omega_\alpha}\alpha)\Theta_{n_\Phi}(\sqrt{\omega_\Phi}\Phi)$ of H, with eigenvalues $E = E_\alpha + E_\Phi = -n_\alpha\omega_\alpha + n_\Phi\omega_\Phi = 0$, in the form

$$\Psi(\alpha,\Phi) = \sum_n c_n \Theta_{nk}(\sqrt{\omega_\alpha}\alpha)\Theta_{nl}(\sqrt{\omega_\Phi}\Phi)\ . \tag{6.8}$$

If the coefficients c_n are chosen to define an 'initial' Gaussian wave packet in Φ at 'time' $\alpha = 0$, centered at some $\Phi_0 \neq 0$ and with $\partial\Psi/\partial\alpha = 0$, say, the resulting tube-like solutions propagate in α, following classical Lissajous figures in mini-superspace – just as for the conventional oscillator (DeWitt 1967).

In contrast to Schrödinger's time-dependent coherent states, which follow classical trajectories without changing their shape, these 'upside-down oscil-

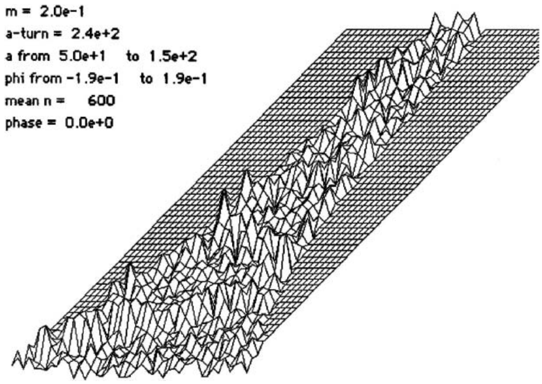

Fig. 6.3. Same as Fig. 6.2, but including the contribution of the reflected part: second term of (6.12). The coherent wave tube assumed to represent the expanding universe is here hardly recognizable against the background of the diffuse contribution representing the recontracting universe(s)

lators' display a rich intrinsic structure. Wave packets on different parts of a trajectory must also interfere with one another whenever they overlap – in particular close to the classical turning points. Interference between intersecting tubes would be suppressed by taking into account additional variables (higher-dimensional configuration spaces), since a projection onto mini-superspace – that is, tracing out all other variables – represents decoherence (see Sect. 6.2.2).

All components of (6.8) satisfy the usual boundary condition of normalizability in Φ and α. This choice is responsible for the reflection of wave tubes at the potential barriers. Although unusual for a conventional time parameter, it is consistent with the role of α as a dynamical variable. Such boundary conditions (if applied to the complete Quantum Universe) might even determine the solution of the Wheeler–DeWitt equation uniquely – provided there is a solution for its fixed zero eigenvalue at all. The degeneracy of the oscillator model (6.7), which allows the choice of 'initial' narrow wave packets, is evidently pathological. Narrow wave tubes can in general only be expected to arise as robust *branches* of the complete solution. They may not have to obey the boundary conditions individually.

An approximate solution can also be constructed for the Wheeler–DeWitt equation with a Friedmann Hamiltonian (6.5) – see Fig. 6.2 (Kiefer 1988). The oscillator potential with respect to Φ may here be assumed to be weakly α-dependent over many classical oscillations in Φ (shown in Fig. 5.7) except for small values of α. If α-dependent oscillator wave functions $\Theta_n(\Phi)$ – similar to those used in (6.8) – are now defined by the eigenvalue equation

$$\left(-\frac{\partial^2}{\partial\Phi^2} + m^2 \mathrm{e}^{6\alpha}\Phi^2\right)\Theta_n\left(\sqrt{m\mathrm{e}^{3\alpha}}\Phi\right) = (2n+1)m\mathrm{e}^{3\alpha}\Theta_n\left(\sqrt{m\mathrm{e}^{3\alpha}}\Phi\right) , \quad (6.9)$$

one may expand a solution of (6.5) in terms of them as

$$\Psi(\alpha, \Phi) = \sum_n c_n(\alpha)\Theta_n(\sqrt{m e^{3\alpha}}\Phi) \ . \tag{6.10}$$

In the adiabatic approximation with respect to α (that may here be based on the Born–Oppenheimer expansion in terms of the inverse Planck mass – see Banks 1985 and Sect. 6.2.2), this leads to decoupled equations for the coefficients $c_n(\alpha)$:

$$\left[+\frac{\partial^2}{\partial \alpha^2} + 2E_n(\alpha)\right] c_n(\alpha) = 0 \ . \tag{6.11}$$

For positive spatial curvature, $k = 1$, the effective potentials

$$2E_n(\alpha) := (2n+1)m e^{3\alpha} - k e^{4\alpha}$$

become negative for $\alpha \to +\infty$. Even though $V(\alpha, \Phi)$ is positive almost everywhere in this limit - see (5.35), the Φ-oscillations are drawn into the narrow region $V < 0$ (in the vicinity of the α-axis – see Fig. 5.7) by damping.

So if one requires square integrability for $\alpha \to \infty$, only the exponentially decreasing partial wave solutions of (6.11) are admitted. Wave packets in Φ consisting of many oscillator eigenstates with quantum numbers $n \approx n_0$, say, may then be used to form wave tubes in α and Φ following the classical paths of Fig. 5.7 – see Fig. 6.2. However, in the Friedmann model, wave tubes cannot remain narrow wave packets in Φ when reflected at $a_{\max} = e^{\alpha_{\max}}$, since the turning point of the nth partial wave, $a_{\max,n} = (2n+1)m$, depends strongly on n. For values of a sufficiently below $a_{\max,n}$, the coefficients $c_n(\alpha)$ can according to Kiefer be written by means of a 'scattering' phase shift (caused by the reflection) in the form of a sum of incoming and outgoing (though real) waves. In the lowest WKB approximation one obtains ('asymptotically' in this sense)

$$c_n(\alpha) \propto \cos\left[\phi_n(\alpha) + n\Delta\phi\right] + \cos\left[\phi_n(\alpha) - n\Delta\phi + \frac{\pi}{4}{a_n}^2\right] \ . \tag{6.12}$$

Here,

$$\phi_n(\alpha) := \left(\frac{a_n}{4} - \frac{a}{2}\right)\sqrt{a(a_n - a)} + \left[\arcsin\left(1 - \frac{2a}{a_n}\right) - \frac{\pi}{2}\right]\frac{a_n^2}{8} - \frac{\pi}{4} \tag{6.13}$$

is a function of α and α_n, while the integration constant $\Delta\phi$ is the phase of the corresponding classical Φ-oscillation at its turning point in α. If coefficients are chosen, when substituting (6.12) into (6.10), such that the first cosines describe a (narrow) coherent oscillator wave packet, the 'scattering phase shifts' $\pi {a_n}^2/4$ of the second cosine terms cause the reflected wave to spread widely (see Fig. 6.3). Only for pathological potentials, such as the indefinite harmonic oscillator (6.8), or for integer values of $m^2/2$ in the specific model (6.4), can the phase shift differences be omitted as multiples of π.

Therefore, even the WKB approximation, which would suppress any dispersion of the wave packet, cannot in general describe an expanding and recollapsing quasi-classical universe by means of ('initially' prepared) wave packets that propagate as narrow tubes beyond the turning point. The concept of a universe deterministically expanding and recollapsing along a certain trajectory in superspace is as incompatible with quantum cosmology as the concept of an electron orbit in the hydrogen atom is with quantum mechanics. Many quasi-trajectories (wave tubes) describing expanding universes have to be superposed in order to obtain one quasi-classical contraction era (and vice versa). Decoherence has to select very different superpositions of partial waves $c_n\Theta_n$ in (6.10) to define robust branches in opposite eras. The reflection at a_{max} describes a quasi-stochastic quantum process – just as in a quantum scattering event. Compatibility problems for boundary conditions can thus affect only the total Wheeler–DeWitt wave function (the superposition of *all* branches). They would *not at all* occur for a non-normalizable Wheeler–DeWitt wave function that represents forever expanding universes ($k = 0$ or -1 in the case of $\Lambda = 0$).

All these simple models are far from being realistic. They are not only unable to describe statistical aspects or decoherence – they also neglect the important coupling between cosmic degrees of freedom and microscopic ones. In particular, the latter's ground states ('zero point motion') must in general depend on α and Φ. The cosmic variables are then subject to extreme decoherence (Barvinsky et al. 1999). Simplified models, as discussed above, may nonetheless appropriately describe certain important *conceptual* aspects of quantum cosmology.

General Literature: Ryan 1972, Kiefer 1988.

6.2.2 The Emergence of Classical Time

> If classical time emerges, it cannot emerge in classical time

A we have seen in Sect. 5.4, there is no dynamically preferred time parameter in general relativity or other reparametrization invariant classical theories. Moreover, the dynamical succession of global states, which may be conveniently described by means of a time coordinate, depends on the choice of a foliation. The resulting invariant spacetime geometry nonetheless defines many-fingered *physical* time, that is, absolute proper times as local controllers of motion, while any foliation represents a trajectory in superspace (a global history) that may then also be parametrized.

In *quantum* gravity, no global time parameter is generally available any longer, since there are no trajectories (that is, no one-dimensional successions of classical states with their physical clocks). Two given three-geometries are then not dynamically connected by a definite four-geometry (a spacetime).

Neither world lines nor their proper times, which could control Schrödinger or master equations for *local* systems, are defined, and three-geometry is no longer a reliable 'carrier of information about time'.

As explained in Chap. 4, the Schrödinger equation is exact only as a *global* equation. Its restriction to matter would require the global foliation of a classical spacetime. While this could still be chosen to proceed just locally (thus defining a 'finger of time'), the Wheeler–DeWitt equation describes entangled dynamics for global quantum states of matter *and* three-geometry.

How can the traditional concept of time (either in the form of many-fingered time, or as a parameter for the dynamics of global states) be recovered from the Wheeler–DeWitt equation? This requires concepts and methods discussed in Sect. 4.3, where quasi-classical quantities were shown to emerge dynamically and irreversibly by means of decoherence, but the dynamics has to be appropriately modified to suit the timeless Wheeler–DeWitt equation: classical time cannot emerge *in* classical time. Similarly, classical spacetime cannot have entered existence in a global quantum 'event' (which would have to presume time).

In the local field representation, the general Wheeler–DeWitt equation can be explicitly written in its gauge-dependent form (DeWitt 1967) as

$$-\frac{16\pi}{m_{\mathrm{P}}^2}\sum_{klk'l'} G_{klk'l'}\frac{\delta^2\Psi}{\delta h_{kl}\delta h_{k'l'}} - \frac{m_{\mathrm{P}}^2}{16\pi}\sqrt{h}(R-2\Lambda)\Psi + H_{\mathrm{matter}}\Psi = 0\ , \quad (6.14)$$

when disregarding factor ordering. Here, Ψ is a functional of the *six* independent functions $h_{kl} = g_{kl}$ $(k,l = 1,2,3)$, which represent the spatial metric on a hypersurface. The letter h (without indices) means their determinant, R their spatial Riemann curvature scalar. Λ is the cosmological constant, while $m_{\mathrm{P}} := 1/\sqrt{G}$ is the Planck mass. The hamiltonian density, H_{matter}, also depends on the metric by means of its kinetic energy terms. The matrix $G_{klk'l'} := (h_{kk'}h_{ll'} + h_{kl'}h_{lk'} - 2h_{kl}h_{k'l'})/2\sqrt{h}$ with respect to the six symmetric *pairs* of indices kl is DeWitt's 'superspace metric'. It has the *locally* hyperbolic signature $-+++++$.

The Wheeler–DeWitt equation (6.14) can assume this local form only because of its gauge degrees of freedom. Their elimination requires the wave functional to obey the three momentum constraints (see Sect. 5.4), in their quantum mechanical form written as $H_k\Psi = (\delta\Psi/\delta h_{kl})|_l = 0$, where $|_l$ is the covariant derivative with respect to the spatial metric h_{kl}. They represent *three* functional differential equations for the functional $\Psi[h_{kl}]$, which depends on six variables h_{kl} at each point. Integration of the Wheeler–DeWitt equation (6.14) under the constraints would then leave *two* functions as 'integration constants'. These two degrees of freedom at each space point may be regarded as representing the two physical components (polarizations) of the gravitational field. The momentum constraints are analogous to Gauß's law in electrodynamics, which also forms a constraint on the initial data when written in terms of the potential $\boldsymbol{A}$. However, the momentum constraints are 'secondary': they can only in special situations be solved analytically. In the superspace region that describes Friedmann-type universes, all but one of the infinity of negative kinetic energy terms of the Wheeler–DeWitt equation represent gauge degrees of freedom (see footnote 4 of Chap. 5). The remaining physical dynamics is *globally* hyperbolic.

I shall now assume that a solution of these coupled equations exists. Since the quantity $m_{\mathrm{P}}^2/32\pi$ that appears in (6.14) as a formal dynamical mass is very large compared to dynamical masses contained in H_{matter}, one may conveniently analyze the solution by means of a Born–Oppenheimer approximation in analogy to molecular physics (Banks 1985). The matter wave function will then adiabatically depend on the massive gravity variables – even though there is no time-dependence. This situation resembles *small* molecules, which are usually found in their energy eigenstates (giving rise to rotational and vibrational bands) rather than in states representing quasi-classical motion. However, as a *novel* aspect of quantum cosmology, the matter degrees of freedom must now also describe observers, while molecules or other microscopic systems are observed from outside. Because of the adiabatic correlation of the observer with the quantum state of geometrodynamics (and that of other macroscopic variables), this quasi-classical state appears 'given' to him (see Sect. 4.6).

In order to obtain a semiclassical dynamical description of spacetime geometry, one may use the *ansatz*

$$\Psi[h_{kl}, x] = \exp\left[\mathrm{i}S_0(h_{kl})\right]\chi(h_{kl}, x) \;, \tag{6.15}$$

where x represents all matter degrees of freedom. S_0 is defined as a solution of the Hamilton–Jacobi equation of geometrodynamics (Peres 1962), with a self-consistent source term that is given as the expectation value $\langle\chi|T_{\mu\nu}|\chi\rangle$ of the matter states χ that are to be calculated along classical trajectories described by S_0. This is analogous to the description of *large* molecules, where the heavy atomic nuclei or ions are dynamically described by time-dependent classical orbits resulting from an effective potential that arises from an expectation value for electron wave functions, which in turn depend adiabatically on the nuclear positions. If the global boundary conditions are appropriate to justify the WKB approximation, the matter wave function χ may indeed depend adiabatically on the massive variables h_{kl} in the relevant regions of configuration space. As this spatial metric describes the three-geometry as the carrier of information about time along every WKB trajectory (similar to geometric optics), *the dependence of* χ *on* h_{kl} *may be regarded as a generalized physical time dependence* of the matter states.

Since the classical Hamilton–Jacobi equations describe an ensemble of dynamically independent trajectories in the configuration space of the three-geometries, the remaining equations for the matter states χ can be integrated along them. This procedure becomes particularly convenient after the exponential $\exp(\mathrm{i}S_0)$ has been raised to the usual second order WKB approximation that includes a 'prefactor' which warrants the conservation of probability. In its local form (6.14), the Wheeler–DeWitt equation is reduced by the *ansatz* (6.15) to a *Tomonaga–Schwinger equation* (Lapchinsky and Rubakov 1979, Banks 1985):

$$\mathrm{i} \sum_{klk'l'} G_{klk'l'} \frac{\delta S_0}{\delta h_{kl}} \frac{\delta \chi}{\delta h_{k'l'}} = H_{\mathrm{matter}} \chi \,. \tag{6.16}$$

Its LHS is (the local component of) a derivative of χ in the direction of the gradient ∇S_0 in the configuration space of three-geometries. Written as $\mathrm{i}\nabla S_0 \cdot \nabla \chi =: \mathrm{i d}\chi/\mathrm{d}\tau$, it defines a many-fingered time parameter τ *for all trajectories* (that is, for all different spacetimes in the ensemble described by the specific solution S_0). Because of its dependence on the WKB approximation, τ may be called a 'WKB time'. Since the WKB wave function in general describes an extended superposition, its individual quasi-trajectories (corresponding to narrow wave tubes), correlated to their specific matter states χ, can only represent Everett *branches* of a general quantum universe. They are indeed decohered from one another by the adiabatic dependence on gravity of the uncontrollable microscopic degrees of freedom contained in χ (Sect. 4.3.5).

Equation (6.16) thus represents an effective *time-dependent* Schrödinger equation for matter. Higher orders of the WKB approximation lead to corrections to this Schrödinger dynamics (Kiefer and Singh 1991). The local fingers of time – represented by this local form – may be combined by integrating (6.16) over three-space (Giulini 1995). This integration elevates the local inner product $\nabla S_0 \cdot \nabla \chi$, that is, a sum over $k'l'$ by means of the Wheeler–DeWitt metric, to a global one. In this way it defines the progression of a reparametrizable *common global* dynamical time, valid for *all* spacetimes which are described by the Hamilton–Jacobi function S_0. Since τ is defined for all WKB trajectories (thus defining 'simultaneous' three-geometries on them), it also defines a *foliation of superspace* (Giulini and Kiefer 1994). Only the definition of spacetime coordinates requires lapse and shift functions N and N_k (Sect. 5.4), which then also define 'velocities' $\dot{h}_{kl}$ with respect to coordinate time according to

$$\dot{h}_{kl} = -N G_{klk'l'} \frac{\delta S_0}{\delta h_{k'l'}} + N_{k|l} + N_{l|k} \,. \tag{6.17}$$

The *complex ansatz* (6.15) is obviously essential for the result (6.16). The correct Wheeler–DeWitt wave function will in general have to be approximated by a superposition of *several* such WKB components. There is no reason to expect a complex solution for the complete Wheeler–DeWitt wave function that describes the Quantum Universe. So one may, in particular, have to replace (6.15) by its real part, $\Psi \to \Psi + \Psi^*$. The two terms may then *separately* obey (6.16) and its complex conjugate to an excellent approximation, similar to the various WKB trajectories (or wave tubes) which have to be superposed to form the extended wave front of geometric optics described by an appropriate Hamilton–Jacobi solution S_0 (or some higher order approximation S_1). This dynamical separation can again be understood as a consequence of decoherence by the microscopic matter degrees of freedom (Halliwell 1989, Kiefer 1992). It is interesting, as it represents a simple but fundamental example of *gauge symmetry breaking by Everett branching*. Note, however, that in contrast to the complex classical field in Sect. 6.1, the complex form of (6.16) characterizes superpositions in Hilbert space – not in the configuration space of fields.

The Born–Oppenheimer approximation with respect to the Planck mass is not always the most appropriate one. Macroscopic matter variables may be described by a WKB approximation, too, while certain geometric modes (graviton states) may not. These variables could then be shifted between S_0 and χ (see Vilenkin 1989). For example, Halliwell and Hawking (1985) applied the WKB approximation only to the monopoles α and Φ which characterize the quantum Friedmann universe (6.5). Their WKB solution would describe an ensemble of trajectories $a(t)$, $\Phi(t)$ in mini-superspace (Sect. 5.4). Wave functions for the (nonlocal) higher multipole amplitudes of matter and geometry can then again be obtained by means of a Tomonaga–Schwinger equation, similar to (6.16). This choice of *nonlocal* variables offers the advantage of separating physical degrees of freedom (in this case the pure tensor modes) from the remaining pure gauge modes. As mentioned in Sect. 5.4, the physical tensor modes all appear in the kinetic energy with a dynamical mass of the same sign as the matter modes, while only the monopole variable a (or $\alpha = \ln a$) has negative dynamical mass. In this model, the monopoles a and Φ are very efficiently decohered by the tensor modes (Zeh 1986b, Kiefer 1987, Barvinsky et al. 1999). Decoherence of local 'fluctuations' may lead to the formation of large scale structures of the Universe (Kiefer, Polarski and Starobinsky 1998).

The Tomonaga–Schwinger equation (6.16) justifies a dynamical time parameter on the basis of the timeless Wheeler–DeWitt equation, but not yet an arrow of time. Its solutions require 'initial' conditions with respect to the global time parameter τ. In order to describe our observed time-asymmetric Universe, these initial conditions must be responsible for the arrow(s) of time (in particular 'quantum causality', that has already been used when referring to decoherence). However, they cannot be freely postulated any more, but must be obtained from the complete Wheeler–DeWitt wave function, that depends on its own boundary conditions. For them, the hyperbolic nature of the Wheeler–DeWitt equation is essential. As discussed in Sect. 6.2.1, initial conditions may be posed for it at any fixed value of a time-like variable in superspace, such as α. While the initial condition for χ has to characterize an early Everett branch, the total wave function, which gives rise to the time arrow of this branching, must depend on a general boundary condition (perhaps just normalizability). 'Before' anything may evolve in classical time – as assumed when applying (6.16) in the forward direction, classical time must itself emerge from an appropriate initial condition that is a consequence of boundary conditions for the solution of (6.14). For example, if the dynamics in the form (6.6) is generalized by means of higher multipoles as

$$H = \frac{\partial^2}{\partial \alpha^2} + H^2_{\text{red}} = \frac{\partial^2}{\partial \alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + V_i(\alpha, x_i) \right] + V_{\text{int}}(\alpha, \{x_i\}) \ , \quad (6.18)$$

where the x_i are now *all other* variables, the potential V_{int} may be highly asymmetric under a reversal of the sign of α. In particular, it could have a simple structure for $\alpha \to -\infty$, as indicated in (6.5), which might give rise to a 'simple' initial condition (SIC) in α (see Conradi and Zeh 1991). This may even explain the symmetric initial vacuum of Sect. 6.1. In the absence of any theory that describes the big bang singularity, we can only *assume* an appropriate simple structure of the total wave function in this limit (just as discussed for $t \to 0$ at the end of Chap. 4).

Inasmuch as the Tomonaga–Schwinger equation along a WKB trajectory in superspace describes *measurements*, it is practically useless for calculating 'backwards' in global time τ. One would have to know all (observed and unobserved) final *branches* of the total matter wave function χ (such as those that have unitarily arisen during measurements in the past). In the 'forward' direction, this global Schrödinger equation for matter has to be replaced by a master equation if decoherence of the quantum state of matter by gravity is relevant (see Sect. 4.3.4). This may explain the oft-proposed gravity-induced collapse as an apparent one in the 'usual' manner.

This limited applicability of a time-dependent Schrödinger or master equation that is based on WKB time would become particularly important for a recontracting universe (Sect. 5.3). If a master equation can be derived for all or most WKB trajectories with respect to that direction of τ that represents increasing a, it cannot remain valid along a classical spacetime history that leads to recontraction – cf. Fig. 6.3. Page (1985) and Hawking (1985) understandably arrived at the opposite conclusion when they described a recontracting universe by using WKB trajectories beyond the turning point (see the discussion following Zeh 1994). They thereby interpreted their semi-classical Feynman paths as representing an *ensemble of possible* cosmic histories that they justified by 'initial quantum uncertainties'. Their further treatment then neglects the final condition in τ that would be part of the initial condition in α, and give rise to formal recoherence along the trajectory even if quasi-trajectories of geometry *were* defined beyond the turning point.

Quantum cosmology requires a *consistent realistic interpretation* of quantum theory (Everett's, for example). It is often uncritically applied by using some pragmatic (for this purpose insufficient) interpretation, including a traditional concept of time. Let me therefore briefly mention consequences of a timeless wave function on Bohm's quantum mechanics, which were first discussed for different reasons by Julian Barbour (1994a,c):

In addition to a time-dependent Everett wave function, Bohm's theory postulates the existence of an ensemble of trajectories in a classical configuration space that describes particles and fields (see Sect. 4.6). If quantum gravity is taken into account, this configuration space must include three-geometries. In a time-less theory, the trajectories degenerate into an ensemble of fixed states (points), assumed to possess 'statistical weights' according to $|\Psi|^2$. In contrast to Bohm's time-dependent theory, this is *no longer an initial condition* that would have to be preserved by the presumed unobservable dynamics for the Bohm trajectories.

The structure of the Wheeler–DeWitt wave function in the range of applicability of a WKB approximation then statistically favors those classical states which lie on *apparent trajectories*. This result is very similar to Mott's (1929) description of α-particle tracks in a cloud chamber, where the 'stationary' (static) wave function suppresses configurations that describe droplets not approximately lying along particle tracks. Barbour calls these preferred states 'time capsules', since they represent consistent memories (without corresponding histories). In Barbour's words: "time is in the instant" (in the state) "– the instant is not in time" (in a history). If all classical states in the ensemble are regarded as 'real' (precisely as all past and future states are assumed to form a real one-dimensional history in the conventional block universe description), they now form a multi-dimensional rather than a one-dimensional continuum. One may even say that time is *replaced* by the wave function in this picture.

In contrast to the Everett interpretation, Bohm's theory presumes these classical configurations as part of fundamental reality, which must include observers. Each electron in a molecule, for example, is then assumed to possess a definite position in every actual state (though not any velocity or momentum). Since this particle position is *not* part of a memorized or documented (real or apparent) history according to this interpretation, we are only led to *believe* that it 'actually exists' as a wave function. The intrinsic dynamics of the static Wheeler-DeWitt wave function has the consequence that the electron's effects on measurement devices are dynamically 'caused' by *all* its positions in the support of the wave function (in dependence of the latter's amplitude) – not by a one-dimensional history. This picture would explain why the arena for the wave function is a classical configuration space, although most problems and disadvantages of Bohm's theory (see Zeh 1999b) persist, and even new ones may arise. Why should there be arbitrary global simultaneities representing actual elements of reality, while 'actuality' seems to be meaningful only with respect to local observers?

General Literature: Anderson 2006, Kiefer 2007.

6.2.3 Black Holes in Quantum Cosmology

During the early days of general relativity, the spacetime region behind a black hole horizon was regarded as meaningless, since it is inaccessible to observers in the external region. From their positivistic point of view, it would 'not exist'. Later one realized that world lines, including those of observers, can be smoothly continued beyond the horizon, where they would hit the singularity within finite proper time. The new conclusion, that the internal

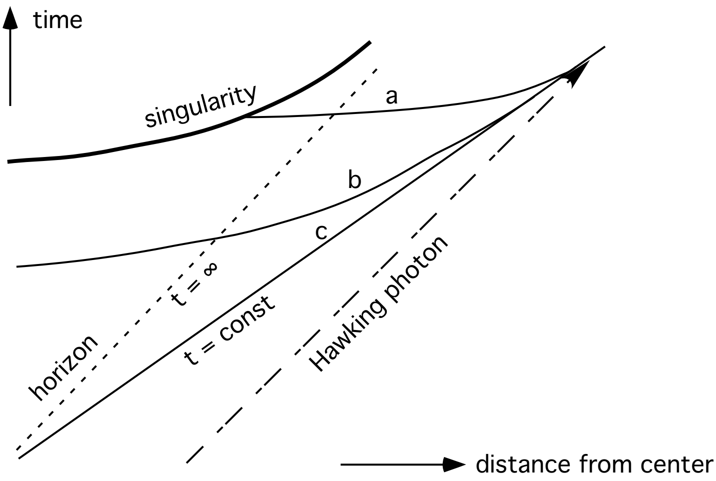

Fig. 6.4. Various kinds of simultaneities for a spherical black hole in a Kruskal type diagram: (a) hitting the singularity, (b) entering only the regular internal region, (c) completely remaining outside (Schwarzschild coordinate t). *Any* Schwarzschild time, for example $t = t_{turn}$, may be identified with $t = 0$ (a horizontal line in the diagram) regardless of the time of the observed collapse. No horizon forms on the Schwarzschild simultaneities, which are complete for the external universe. (From Zeh 2005c)

regions of black holes are physically 'regular' except at the singularity (hence for limited time only), seems to apply as well to Bekenstein–Hawking black holes until they disappear (see Sect. 5.1). However, arguments indicating a genuine (possibly dramatic) quantum nature of the event horizon have also been raised ('t Hooft 1990, Keski-Vakkuri et al. 1995, Li and Gott 1998).

While a consistent quantum description of black holes has not as yet been presented, attempts were mostly based on semiclassical methods. (For an overview see Kiefer 2007.) When combined with quantum cosmology, they may lead to important novel consequences, which seem to revive the early doubts in the meaning and existence of black hole interiors.

Consider the Schwarzschild metric (Fig. 5.1) as far as it is relevant for a black hole formed by collapsing matter, such that the Kruskal regions III and IV do not occur (Fig. 5.3a). Its dynamical (3+1) description in terms of three-geometries depends in an essential way on the choice of a foliation (see Fig. 6.4, or the Oppenheimer–Snyder model described in Box 32.1 of Misner, Thorne and Wheeler 1973). Three-geometries which intersect the event horizon may spatially extend to the singularity at $r = 0$, and thus render the global quantum states that they carry prone to dynamical indeterminism or consequences of a future theory that may avoid singularities. In contrast, a foliation according to Schwarzschild time t would describe regular three-geometries for $t < \infty$, which could be continued *in time* beyond $t = \infty$ by means of the new

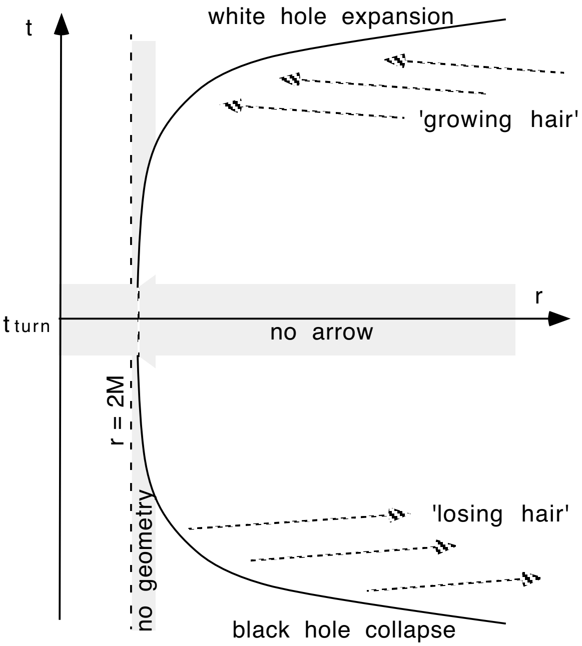

Fig. 6.5. Classical trajectory of a collapsing dust shell or the surface of a collapsing star (*solid curve*) in a thermodynamically symmetric recontracting universe. It is represented here in compressed Schwarzschild coordinates as used in Fig. 5.1, with the Schwarzschild metric now being valid only to the right of the star's surface. Because of the scale compression, light rays appear almost horizontal in the diagram. For $t > t_{\mathrm{turn}}$, advanced radiation from the formal past would focus onto the black hole, which must now re-expand and grow hair in this scenario, while observers would experience time in the opposite direction. No horizon ever forms: the region $r < 2M$ (which is later than $t = \infty$) would arise only if gravitational collapse continued forever in a classical manner. Because of the drastic quantum effects close to the turning point of a Friedmann universe (see Fig. 6.3), there are in general only 'probabilistic' connections between quasi-classical branches in the expansion and contraction eras of the Universe. (From Kiefer and Zeh 1995)

time coordinate r (with physical time growing with decreasing r for $r < 2M$). According to this foliation, the black hole interior with its singularity would always remain in our formal future, and the singularity must be irrelevant for Hawking radiation. In the pair creation picture, the negative-energy partner is absorbed to the spacetime region close to what appears as a horizon until this is completely transformed into radiation. Therefore, this foliation seems to be appropriate for the formulation of a cosmological boundary condition (in superspace), that may explain the master arrow of time.

For further discussion now assume that the expansion of the classical universe on which this diagram is based is reversed at a finite Schwarzschild time

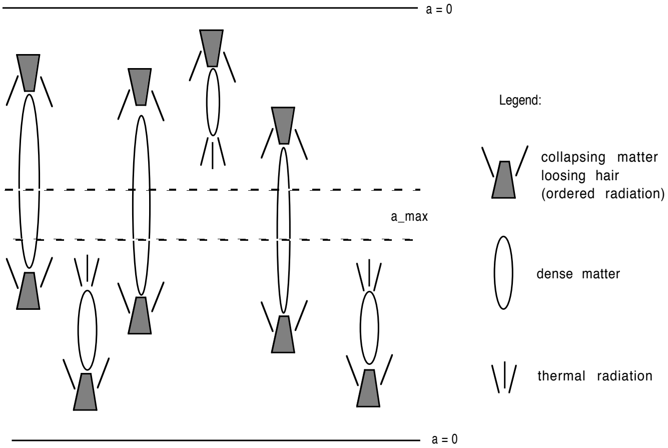

Fig. 6.6. Quasi-classical picture (using Schwarzschild coordinates) of a thermodynamically T-symmetric quantum universe which contains black holes, white holes, and black-and-white holes that re-expand by anti-causal effects. Instead of horizons and singularities, there are merely spacetime regions of large curvature ('dense matter') in this scenario. Because of their strong time dilation, they may serve as a short cut in proper time between big bang and big crunch (or between the presumed eras of opposite arrows of time). 'Information-gaining systems' could not thereby survive as such. In quantum cosmology there is no unique connection between quasi-classical histories (Everett branches) represented by the upper and lower halves of the figure, but there is no need for a violation of conservation laws

$t = t_{\text{turn}}$ that is much larger than the time of the effective gravitational collapse (losing hair – see Fig. 6.5). No horizon yet exists on the Schwarzschild simultaneity $t = t_{\text{turn}} < \infty$. If the cosmic time arrow does change direction (while the quasi-classical universe passes through an era of thermodynamical indefiniteness), the gravitationally collapsing matter close to the expected horizon will very soon (in terms of its own proper time) enter the era where radiation is advanced in the sense of Chap. 2. The black hole can then no longer 'lose hair' by emitting retarded radiation; it must instead 'grow hair' in an anti-causal manner (Fig. 6.6). According to a 'time-reversed no-hair theorem' it has to re-expand when the Universe starts recontracting (Zeh 1994, Kiefer and Zeh 1995).

This scenario does not contradict the geometrodynamical theorems about a monotonic growth of black hole areas, since no horizons ever form. A classical spacetime will not even exist close to the 'turning of the tide'. Here, decoherence is competing with recoherence before being replaced by it. Only region I of Fig. 5.1 is then realized. Events which appear 'later' than t_{turn} in the classical picture are 'earlier' in the sense of the intrinsic dynamics of the Wheeler–DeWitt equation (6.6) – and therefore also in the thermodynamical sense if this is based on an intrinsic initial condition. This quantum cosmological model describes an *apparent* (quasi-classical) two-time Weyl tensor or similar condition (see Fig. 6.6). In quantized general relativity, the two apparently different boundaries are identical, and thus represent *one and the same* boundary condition. The problem of their consistency (Sect. 5.4) is reduced to the intrinsic 'final' condition of normalizability for $a \to \infty$.

The description used so far in this section does not apply directly to a forever-expanding universe, where the arrow would preserve its direction along a complete quasi-trajectory from $a = 0$ to $a = +\infty$. The Wheeler–DeWitt wave function is then *not* normalizable for $a \to \infty$. However, one may require this wave function to vanish on *all* somewhere-singular three-geometries by a symmetric generalization of the Weyl tensor hypothesis. Such a condition has been confirmed to apply to a simple quantum model of a collapsing thin spherical matter shell (Hájíček and Kiefer 2001). In more realistic cases it would again lead to important thermodynamical and quantum effects close to event horizons (Zeh 1983), and drastically affect (or even exclude) the possibility of continuing a quasi-classical spacetime beyond them. These consequences would be unobservable in practice by external observers, since the immediate vicinity of a future horizon remains outside their backward light cones for all finite future. In order to receive information from the vicinity of a future horizon, one has to come dangerously close to it, and thus participate in the extreme time dilatation (see Fig. 5.2, where the light cone structure is made evident, while distances are strongly distorted).

These conclusions seem again to throw serious doubts on the validity of a classical continuation of spacetime into black hole interiors (see also Kiefer 2004 or Zeh 2005a). Event horizons in classical general relativity may signal the presence of drastic thermodynamical and quantum effects rather than representing 'physically normal' regions of spacetime. While their observable consequences depend on the world lines of detectors or observers (their acceleration, in particular), global quantum states, such as a specific 'vacuum', are invariantly defined – though not invariantly observed (Sect. 5.2). These global states may define an objective arrow of time, including 'quantum causality' (responsible for decoherence), by means of a fundamental boundary condition for the Wheeler–DeWitt wave function.