2

The Time Arrow of Radiation

After a stone has been dropped into a pond, one observes concentrically diverging ('defocusing') waves. Similarly, *after* an electric current has been switched on, one finds a retarded electromagnetic field that is coherently propagating *away* from its source. Since the fundamental laws of Nature, which describe these phenomena, are invariant under time reversal, they are equally compatible with the reverse phenomena, in which concentrically focusing waves (and whatever may be dynamically related to them – such as heat) would 'conspire' in order to eject a stone out of the water. Deviations of the deterministic laws from time reversal symmetry would modify this argument only in detail (see the Introduction). However, the reversed phenomena are never observed in Nature. In high-dimensional configuration space, the absence of dynamical correlations which would focus to create local effects characterizes the time arrow of thermodynamics (Chap. 3), or, when applied to wave functions, even that of quantum theory (see Sect. 4.3).

Electromagnetic radiation will here be considered to exemplify wave phenomena in general. It may be described in terms of the four-potential A^μ, which in the Lorenz gauge obeys the wave equation

$$-\partial^\nu \partial_\nu A^\mu(\boldsymbol{r}, t) = 4\pi j^\mu(\boldsymbol{r}, t) \ , \quad \text{with} \quad \partial^\nu \partial_\nu = -\partial_t^2 + \Delta \ , \tag{2.1}$$

using units with $c = 1$, the notations $\partial_\mu := \partial/\partial x^\mu$ and $\partial^\mu := g^{\mu\nu}\partial_\nu$, and Einstein's convention of summing over identical upper and lower indices. When an appropriate boundary condition is imposed, one may write A^μ as a functional of the sources j^μ. For two well known boundary conditions one obtains the *retarded* and the *advanced* potentials,

$$A^\mu_{\text{ret}}(\boldsymbol{r}, t) = \int \frac{j^\mu(\boldsymbol{r}, t - |\boldsymbol{r} - \boldsymbol{r}'|)}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}^3\boldsymbol{r}' \ , \tag{2.2a}$$

$$A^\mu_{\text{adv}}(\boldsymbol{r}, t) = \int \frac{j^\mu(\boldsymbol{r}, t + |\boldsymbol{r} - \boldsymbol{r}'|)}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}^3\boldsymbol{r}' \ . \tag{2.2b}$$

These two functionals of $j^\mu(\boldsymbol{r},t)$ are related to one another by a reversal of retardation time $|\boldsymbol{r}-\boldsymbol{r}'|$ – see also (2.5) and footnote 4 below. Their linear combinations are again solutions of the wave equation (2.1).

At this point, many textbooks argue somewhat mysteriously that 'for reasons of causality', or 'for physical reasons', only the retarded fields, derived from the potential (2.2a) according to $F^{\mu\nu}_{\text{ret}} := \partial^\mu A^\nu_{\text{ret}} - \partial^\nu A^\mu_{\text{ret}}$, occur in Nature. This condition has therefore to be *added* to deterministic laws such as (2.1), which historically did indeed emerge from the asymmetric concept of causality. This example allows us to formulate in a preliminary way what seems to be meant by this *intuitive notion of causality*: correlated effects (that is, *nonlocal* regularities, such as coherent waves) must always possess a *local* common cause in their past.[1] However, this asymmetric notion of causality is a major *explanandum* of the physics of time asymmetry. As pointed out in the Introduction, it cannot be derived from the deterministic laws by themselves.

The popular argument that advanced fields are not found in Nature because they would require improbable initial correlations is known from statistical mechanics, but totally insufficient (see Chap. 3). The observed retarded phenomena are precisely as improbable among *all possible* ones, since they describe equally improbable *final* correlations. So their 'causal' explanation from an initial condition would beg the essential question.

Some authors take the view that retarded waves describe emission, advanced ones absorption. However, this claim ignores the fact that, for example, moving absorbers give rise to *retarded* shadows, that is, retarded waves which interfere destructively with incoming ones. In spite of the retardation, energy may thus flow from the electromagnetic field into an antenna. When incoming fields are present (as is generically the case), retardation does not necessarily mean emission of energy (see Sect. 2.1).

At the beginning of the last century, Ritz – following simular ideas by Planck and others – formulated a radical solution of the problem by postulating the exclusive existence of retarded waves *as a law*. Such time-directed *action at a distance* is equivalent to fixing the boundary conditions for the

[1] In the case of a *finite* number of local effects resulting from *one* local cause in the past, this situation is often viewed as a 'fork' in spacetime (see Horwich 1987, Sect. 4.8). However, this *fork of causality* should not be confused with the *fork of indeterminism* (in configuration space and time), which points to different (in general global) *potential states* rather than to different events (see also footnote 7 of Chap. 3 and Fig. 3.8). The fork of causality ('intuitive causality') may also characterize deterministic measurements and the documentation of their results, that is, the formation and distribution of information. It is related to Reichenbach's (1956) concept of *branch systems*, and to Price's (1996) *principle of independence of incoming influences* (PI^3). Insofar as it describes the cloning and spreading of information, it represents an *overdetermination* of the past (Lewis 1986), or the *consistency of documents*. It is these correlations which let the macroscopic past appear 'fixed', while complete documents about *microscopic* history would be in conflict with thermodynamics and quantum theory.

electromagnetic field in a universal manner. The field would then *not* describe any degrees of freedom on its own, but just describe retarded forces.

This proposal, a natural generalization of Newton's gravitational force, led to a famous controversy with Einstein, who favored the point of view that retardation of radiation can be explained by thermodynamical arguments. Einstein, too, argued here in terms of an action-at-a-distance theory (see Sect. 2.4). At the end of their dispute, the two authors published a short letter in order to state their different opinions. After an introductory sentence, according to which retarded and advanced fields are equivalent "in some situations", the letter reads as what appears to be also a verbal compromise (Einstein and Ritz 1909 – my translation):[2]

> While Einstein believes that one may restrict oneself to this case without essentially restricting the generality of the consideration, Ritz regards this restriction as not allowed in principle. If one accepts the latter point of view, experience requires one to regard the representation by means of the retarded potentials as the only possible one, provided one is inclined to assume that the fact of the irreversibility of radiation processes has to be present in the laws of Nature. Ritz considers the restriction to the form of the retarded potentials as one of the roots of the Second Law, while Einstein believes that the irreversibility is exclusively based on reasons of probability.

Ritz thus conjectured that the thermodynamical arrow of time might be explained by the retardation of electromagnetic forces because of the latter's universal importance for all matter. However, the retardation of hydrodynamical waves (such as sound) would then have to be explained quite differently – for example, by again referring to the thermodynamical time arrow.

A similar but less well known controversy had already occurred in the nineteenth century between Max Planck and Ludwig Boltzmann. The former, at that time still an opponent of statistical mechanics, understood radiation as a genuine irreversible process, while the latter maintained that the problem is not different from that in kinetic gas theory: a matter of improbable initial conditions (Boltzmann 1897). These different interpretations became relevant, in particular, in connection with the quantum hypothesis: are quanta *caused*

[2] The original text reads: "*Während Einstein glaubt, daß man sich auf diesen Fall beschränken könne, ohne die Allgemeinheit der Betrachtung wesentlich zu beschränken, betrachtet Ritz diese Beschränkung als eine prinzipiell nicht erlaubte. Stellt man sich auf diesen Standpunkt, so nötigt die Erfahrung dazu, die Darstellung mit Hilfe der retardierten Potentiale als die einzig mögliche zu betrachten, falls man der Ansicht zuneigt, daß die Tatsache der Nichtumkehrbarkeit der Strahlungsvorgänge bereits in den Grundgesetzen ihren Ausdruck zu finden habe. Ritz betrachtet die Einschränkung auf die Form der retardierten Potentiale als eine der Wurzeln des Zweiten Hauptsatzes, während Einstein glaubt, daß die Nichtumkehrbarkeit ausschließlich auf Wahrscheinlichkeitsgründen beruhe.*"

by the emission process (as Planck had believed – later called quantum jumps – see Sects. 4.3.6 and 4.5), or inherent to light itself?

In Maxwell's classical *field theory*, the problem does not appear as obvious as in action-at-a-distance theories, since every bounded region of spacetime may contain 'free fields', which possess neither past nor future sources *in this region*. Therefore, one can consistently understand Ritz's hypothesis only cosmologically: *all* fields must possess advanced sources ('causes') somewhere in the Universe. While the examples discussed above demonstrate that the time arrow of radiation cannot merely reflect *the way* boundary conditions are posed, the problem becomes even more pronounced with the time-reversed question: "Do all fields also possess a *retarded source* (a *sink* in time-directed terms) somewhere in the future Universe?" This assumption corresponds to the *absorber theory of radiation*, a T-symmetric action-at-a-distance theory to be discussed in Sect. 2.4. The observed asymmetries would then require an unusual cosmic time asymmetry in the distribution of such sources.

2.1 Retarded and Advanced Form of the Boundary Value Problem

In order to distinguish the indicated pseudo-problem that concerns only the definition of 'free' fields from the physically meaningful question, one has to investigate the general boundary value problem for hyperbolic differential equations (such as the wave equation). This can be done by means of Green's functions, defined as the solutions of the specific inhomogeneous wave equation with a point-like source:

$$-\partial^\nu \partial_\nu G(\boldsymbol{r}, t; \boldsymbol{r}', t') = 4\pi \delta^3(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t') \ , \tag{2.3}$$

and an appropriate boundary condition in space and time. Some of the concepts and methods to be developed below will be applicable in a similar form in Sect. 3.2 to the Liouville equations (Hamilton's equations applied to ensembles of states of mechanical systems). Using (2.3), a solution of the general inhomogeneous wave equation (2.1) may then be written as a functional of its sources:

$$A^\mu(\boldsymbol{r}, t) = \int G(\boldsymbol{r}, t; \boldsymbol{r}', t') j^\mu(\boldsymbol{r}', t') \, \mathrm{d}^3 r' \, \mathrm{d}t' \ , \tag{2.4}$$

where the boundary condition for $G(\boldsymbol{r}, t; \boldsymbol{r}', t')$ determines that for $A^\mu(\boldsymbol{r}, t)$, too. Retarded or advanced solutions are obtained from Green's functions G_{ret} and G_{adv}, which are given by

$$G_{\substack{\mathrm{ret}\\ \mathrm{adv}}}(\boldsymbol{r}, t; \boldsymbol{r}', t') := \frac{\delta(t - t' \pm |\boldsymbol{r} - \boldsymbol{r}'|)}{|\boldsymbol{r} - \boldsymbol{r}'|} \ . \tag{2.5}$$

The potentials A^μ_{ret} and A^μ_{adv} resulting from (2.4) are thus functionals of sources only on the past *or* future light cones of their argument, respectively.

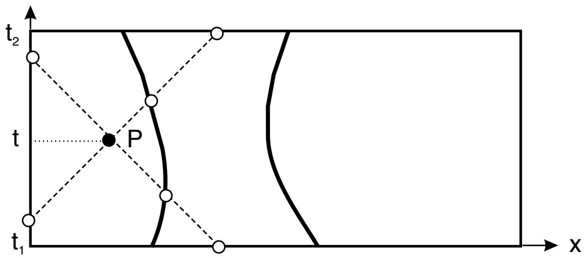

Fig. 2.1. Kirchhoff's boundary value problem, including initial, final and spatial boundaries. Sources (*thick world lines*) within the considered region and boundaries on both light cones (*dashed lines*) may in general contribute to the electromagnetic potential A^μ at the spacetime point P

By contrast, Kirchhoff's formulation of the boundary value problem allows one to express every specific solution $A^\mu(\boldsymbol{r},t)$ of the wave equation by means of any Green's function $G(\boldsymbol{r},t;\boldsymbol{r}',t')$. This can be achieved by using the three-dimensional Green theorem

$$\begin{aligned}\int_V \Big[G(\boldsymbol{r},t;\boldsymbol{r}',t')\Delta' A^\mu(\boldsymbol{r}',t') - A^\mu(\boldsymbol{r}',t')\Delta' G(\boldsymbol{r},t;\boldsymbol{r}',t')\Big]\,\mathrm{d}^3r' \\ = \int_{\partial V}\Big[G(\boldsymbol{r},t;\boldsymbol{r}',t')\nabla' A^\mu(\boldsymbol{r}',t') - A^\mu(\boldsymbol{r}',t')\nabla' G(\boldsymbol{r},t;\boldsymbol{r}',t')\Big]\cdot\mathrm{d}\boldsymbol{S}'\ ,\end{aligned} \tag{2.6}$$

where $\Delta = \nabla^2$ is the Laplace operator, and ∂V is the boundary of the spatial volume V. Multiplying (2.3) by $A^\mu(\boldsymbol{r}',t')$, and integrating over $\boldsymbol{r}'$ and t' from t_1 to t_2 – on the right-hand side (RHS) by means of the δ-functions, while using the Green theorem and twice integrating by parts with respect to t' on the left-hand side (LHS), one obtains by further using (2.1):

$$\begin{aligned}A^\mu(\boldsymbol{r},t) = &\int_{t_1}^{t_2}\!\!\int_V G(\boldsymbol{r},t;\boldsymbol{r}',t')j^\mu(\boldsymbol{r}',t')\,\mathrm{d}^3r'\,\mathrm{d}t' \\ &-\frac{1}{4\pi}\int_V \Big[G(\boldsymbol{r},t;\boldsymbol{r}',t')\partial_{t'}A^\mu(\boldsymbol{r}',t') - A^\mu(\boldsymbol{r}',t')\partial_{t'}G(\boldsymbol{r},t;\boldsymbol{r}',t')\Big]\,\mathrm{d}^3r'\Big|_{t_1}^{t_2} \\ &+\frac{1}{4\pi}\int_{t_1}^{t_2}\!\!\int_{\partial V}\Big[G(\boldsymbol{r},t;\boldsymbol{r}',t')\nabla' A^\mu(\boldsymbol{r}',t') - A^\mu(\boldsymbol{r}',t')\nabla' G(\boldsymbol{r},t;\boldsymbol{r}',t')\Big]\cdot\mathrm{d}\boldsymbol{S}'\,\mathrm{d}t' \\ &\equiv \text{‘source term’} + \text{‘boundary terms’}.\end{aligned} \tag{2.7}$$

if the event P described by $\boldsymbol{r}$ and t lies within the spacetime boundaries. Here, both (past and future) light cones may contribute to the three terms occurring in (2.7), as indicated in Fig. 2.1.

The formal T-symmetry of this representation of the potential as a sum of a source term and boundary terms in the past *and* future can be broken by the choice of Green's functions. When using one of the two forms (2.5), the

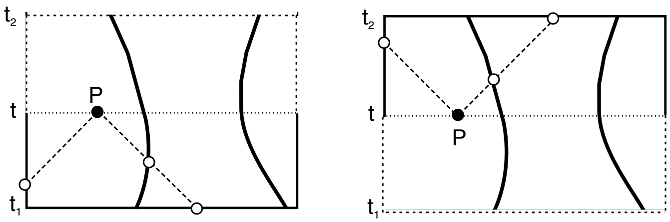

Fig. 2.2. Two representations of the same electromagnetic potential at time t by means of retarded or advanced Green's functions. They require data on partial boundaries (indicated by *solid lines*) corresponding to an initial or a final value problem, respectively

spacetime boundary required for determining the potential at time t assumes specific forms indicated in Fig. 2.2. Hence, the *same* potential can be written according to one or the other RHS of

$$\begin{aligned} A^\mu = \text{source term} + \text{boundary terms} &= A^\mu_{\text{ret}} + A^\mu_{\text{in}} \\ &= A^\mu_{\text{adv}} + A^\mu_{\text{out}} \ . \end{aligned} \tag{2.8}$$

For example, A^μ_{in} is here that solution of the *homogeneous* equations which coincides with A^μ for $t = t_1$. A^μ_{ret} and A^μ_{adv} vanish by definition for $t = t_1$ or $t = t_2$, respectively. *Any* field can therefore be described equivalently by an initial *or* a final value problem – with arbitrary boundary conditions. This result reflects the T-symmetry of the laws, while phenomenological causality is often used as an *ad hoc* argument for choosing G_{ret} rather than G_{adv}.

However, *two* free boundary conditions in the mixed form of Fig. 2.1 would in general not be consistent with one another, even if individually incomplete (see also Sects. 2.4 and 5.3). Retarded and advanced fields formally resulting from past and future sources, respectively, *do not add independently* (as sometimes assumed to describe a conjectured retro-causation) – they just contribute to different (or mixed) *representations* of the same field. In field theory, no (part of the) field 'belongs to' a certain source (in contrast to specific action-at-a-distance theories). Sources determine only the *difference* $A^\mu_{\text{out}} - A^\mu_{\text{in}}$ – similar to $T/i = S - 1$ in the interaction picture of the S-matrix. As can be seen from (2.8), this difference is identical to $A^\mu_{\text{ret}} - A^\mu_{\text{adv}}$. In causal *language*, where A^μ_{in} is regarded as given, the source 'creates' precisely its retarded field that has to be added to A^μ_{in} in the future of the source (where $A^\mu_{\text{adv}} = 0$).

Physically, *spatial* boundary conditions represent an interaction with the (often uncontrollable) spatial environment. For infinite spatial volume ($V = \mathbb{R}^3$), when the light cone cannot reach ∂V within finite time $t - t_1$, or in a closed universe, one loses this boundary term in (2.7), and thus obtains the *pure* initial value problem (for $t > t_1$),

$$A^\mu = A^\mu_{\text{ret}} + A^\mu_{\text{in}} \equiv \int_{t_1}^{t} \int_{\mathbb{R}^3} G_{\text{ret}}(\boldsymbol{r}, t; \boldsymbol{r}', t') j^\mu(\boldsymbol{r}', t') \, \mathrm{d}^3 r' \, \mathrm{d}t' \tag{2.9}$$
$$+ \frac{1}{4\pi} \int_{\mathbb{R}^3} \Big[G_{\text{ret}}(\boldsymbol{r}, t; \boldsymbol{r}', t_1) \partial_{t_1} A^\mu(\boldsymbol{r}', t_1) - A^\mu(\boldsymbol{r}', t_1) \partial_{t_1} G_{\text{ret}}(\boldsymbol{r}, t; \boldsymbol{r}', t_1) \Big] \, \mathrm{d}^3 r' \, ,$$

and correspondingly the pure final value problem ($t < t_2$),

$$A^\mu = A^\mu_{\text{adv}} + A^\mu_{\text{out}} \equiv \int_{t}^{t_2} \int_{\mathbb{R}^3} G_{\text{adv}}(\boldsymbol{r}, t; \boldsymbol{r}', t') j^\mu(\boldsymbol{r}', t') \, \mathrm{d}^3 r' \, \mathrm{d}t' \tag{2.10}$$
$$- \frac{1}{4\pi} \int_{\mathbb{R}^3} \Big[G_{\text{adv}}(\boldsymbol{r}, t; \boldsymbol{r}', t_2) \partial_{t_2} A^\mu(\boldsymbol{r}', t_2) - A^\mu(\boldsymbol{r}', t_2) \partial_{t_2} G_{\text{adv}}(\boldsymbol{r}, t; \boldsymbol{r}', t_2) \Big] \, \mathrm{d}^3 r' \, .$$

The different signs at t_1 and t_2 are due to the fact that the gradient in the direction of the outward-pointing normal vector has now been written as a derivative with respect to t_1 (inward) or t_2 (outward).

So one finds precisely the retarded potential $A^\mu = A^\mu_{\text{ret}}$ if $A^\mu_{\text{in}} = 0$. (Only the 'Coulomb part', required by Gauß's law, must always be present by constraint. It can be regarded as the retarded *or* advanced consequence of the conserved charge.) In scattering theory, an initial condition fixing the incoming wave (usually described by a plane wave) is called a *Sommerfeld radiation condition*. Both conditions are to determine the actual situation. Therefore, the physical problem is not which of the two forms, (2.9) or (2.10), is *correct* (both are), but:

1. Why does the Sommerfeld radiation condition $A^\mu_{\text{in}} = 0$ (in contrast to $A^\mu_{\text{out}} = 0$) approximately apply in many situations?
2. Why are initial conditions more *useful* than final conditions?

The second question is related to the *historical nature* of the world. Answers to these questions will be discussed in Sect. 2.2.

The form (2.7) of the four-dimensional boundary value problem, characteristic of determinism in field theory, applies to partial differential equations of hyperbolic type (that is, with a Lorentzian signature $-{+}{+}{+}$). Elliptic type equations would instead lead to the Dirichlet or von Neumann problems, which require values of the field *or* its normal derivative, respectively, on a *closed* boundary (which in spacetime would have to include past *and* future). Only hyperbolic equations lead generally to 'propagating' solutions, which are compatible with free initial conditions. They are thus responsible for the concept of a *dynamical state* of the field, which facilitates the familiar concept of time.

The wave equation (with its hyperbolic signature) is known to be derivable from Newton's equations as the continuum limit of a spatial lattice of mass points, held at their positions by means of harmonic forces. For a linear chain, $m\mathrm{d}^2 q_i/\mathrm{d}t^2 = -k\big[(q_i - q_{i-1} - a) - (q_{i+1} - q_i - a)\big]$ with $k > 0$, this is the limit $a \to 0$ for fixed ak and m/a. The crucial restriction to 'attractive' forces ($k > 0$) may here appear surprising, since Newton's equations are *always* deterministic, and allow one to pose initial conditions regardless of the type or sign of the forces. However, only bound (here oscillating) systems possess a *stable* position (here characterized by the lattice constant a). In the same limit, an elliptic differential equation (with signature $++++$) would result for a lattice of variables q_i with repulsive forces ($k < 0$). This repulsion, though still representing deterministic dynamics, would cause the particle distances $q_i - q_{i-1}$ to explode immediately in the limit $k \to \infty$. The unstable solution $q_i - q_{i-1} = a$ is in this case the only eigensolution of the Dirichlet problem with eigenvalue 0 (derived from the *condition* of a bounded final state). Mathematically, the dynamically diverging solutions simply do not 'exist' any more in the continuum limit.

For second order wave equations, a hyperbolic signature forms the basis for all (exact or approximate) conservation laws, which give rise to the continuity of 'objects' in time (including the 'identity' of observers). For example, the free wave equation has solutions of a *conserved form* $f(z \pm ct)$, while the Klein–Gordon equation with a positive and variable 'squared mass' $m^2 = V(\boldsymbol{r}, t)$ has unitary solutions $\mathrm{i}\partial\phi(\boldsymbol{r}, t)/\partial t = \pm\sqrt{-\Delta + V}\phi(\boldsymbol{r}, t)$. This dynamical consequence of the spacetime metric, which leads to such 'wave tubes' (see also Sect. 6.2.1), is crucial for what appears as the inevitable 'progression of time' (in contrast to our freedom to move in space). However, the *direction* of this apparent flow of time requires *additional* conditions.

This section was restricted to the boundary value problem for fields in the presence of *given* sources. In reality, the charged sources depend in turn on the fields by means of the Lorentz force. The resulting coupled system of differential equations is still T-symmetric, while all consequences of the retardation regarding the *actual* electromagnetic fields, derived in this and the following section, remain valid. New problems will arise, though, from the *self-interaction* of point charges or elementary charged rigid objects (see Sect. 2.3).

2.2 Thermodynamical and Cosmological Properties of Absorbers

Wheeler and Feynman (1945, 1949) took up the Einstein–Ritz controversy about the relation between the two time arrows of radiation and thermodynamics. Their work essentially confirms Einstein's point of view, provided his 'reasons of probability' are replaced by 'thermodynamical reasons'. Statistical reasons by themselves are insufficient for deriving a thermodynamical arrow (see Chap. 3.) The major part of Wheeler and Feynman's arguments were again based on a T-symmetric action-at-a-distance theory, which is particularly well suited for presenting them in an historical context. From the point of view of local field theory (that is for good reasons preferred today), this picture may appear strange or even misleading. The description of their *absorber theory of radiation* will therefore be postponed until Sect. 2.4.

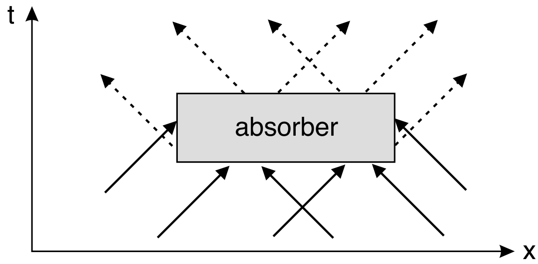

Fig. 2.3. Ideal absorbers do not contribute by means of G_{ret}. (*Arrows* represent the formal time direction of retardation)

In field theory, radiation is described by a continuum of variables, which may themselves require the application of thermodynamical concepts (as is well known for black body radiation). However, coupled harmonic oscillators are not ergodic, and so would not approach equilibrium. For this reason, radiation in a cavity consisting of reflecting walls was usually assumed to contain a small dust grain of coal in order to allow its spectral distribution to equilibrate by absorption and re-emission. I will here neglect the presence of reflecting bodies, and define absorbers (in the 'ideal' case assumed to possess infinite heat capacity) by the following phenomenological properties:

> A spacetime region is called an '(ideal) absorber' if any radiation propagating within its boundaries is (immediately) thermalized at the absorber temperature T $(= 0)$.

The *thermalization* referred to in this definition is based on the arrow of time given by the Second Law. For electromagnetic waves this can also be described by means of a complex refractive index when using the Maxwell equations. The sign of its imaginary part reflects the thermodynamical arrow. The definition means that no radiation can propagate within ideal absorbers, and in particular that no radiation may leave the absorbing region along forward light cones. This consequence can then be applied to the boundary value problem as follows (see also Fig. 2.3):

> By means of the retarded Green's function, (ideal) absorbers forming parts of a spacetime boundary contribute only thermal radiation at the absorber temperature T $(= 0)$.

Such a boundary condition simplifies the *initial* value problem considerably. If the space-like part ∂V of the boundary required for the retarded form of the boundary value problem depicted in Fig. 2.2 consists entirely of ideally absorbing walls (as is usually an excellent approximation for the relevant frequencies in a laboratory or other closed rooms), the condition $A^{\mu}_{\mathrm{in}} = 0$ applies shortly after the initial time t_1 that is used to define the 'incoming' fields in (2.7). So one finds precisely the retarded fields (including reflected waves) of sources which are present in the laboratory. On the other hand, absorbers on the boundary would *not* affect contributions to the Kirchhoff problem by means of G_{adv}; in the nontrivial case one has $A^\mu_{\mathrm{out}} \neq 0$. Therefore, in this laboratory situation the radiation arrow is a simple consequence of the thermodynamical arrow characterizing absorbers.

Do similar arguments also apply to situations outside absorbing boundaries, in particular in astronomy? The night sky does in fact appear black, representing a condition $A^\mu_{\mathrm{in}} \approx 0$, although the present Universe is transparent to visible light. Can the darkness of the night sky then be understood in a realistic cosmological model? For the traditional model of an infinitely old universe this was impossible, a situation called *Olbers' paradox* after one of the first astronomers who mentioned this problem. The total brightness B of the sky beyond the atmosphere would then be given by

$$B = 4\pi \int_0^\infty \rho L_{\mathrm{a}}(r) r^2 \,\mathrm{d}r \,, \tag{2.11}$$

where ρ is the number density of sources (mainly the fixed stars), while $L_{\mathrm{a}}(r) = \bar{L}/r^2$ is their mean apparent luminosity. In the static and homogeneous situation ($\bar{L}, \rho =$ constant) this integral diverges linearly, and the night sky should be infinitely bright. Light absorption by stars in the foreground would reduce this result to a finite but large value, corresponding to a sky as bright as the mean surface of a star. It would not help to take into account other absorbing matter, since this would soon have to be in thermal equilibrium with the radiation under these conditions.

Olbers' paradox was resolved by Hubble's discovery of the expansion of the Universe, which required a finite age of the order of 10^{10} years (following a *big bang*). An integral of type (2.11) with a finite upper limit would in general remain bounded. Since all wavelengths λ grow proportional to the expansion parameter $a(t)$, this leads according to Wien's displacement law, $T \propto \lambda^{-1}$, to the reduction of the apparent temperature T_{a} of all past sources. Stefan and Boltzmann's law for thermal radiation, $L \propto T^4$, then requires that the apparent brightness of the stars, L_{a}, decreases not only with the geometric factor r^{-2}, but also with the inverse fourth power of a. In a homogeneous expanding universe of finite age, the brightness of the sky is then given by

$$B \propto \int_0^{\tau_{\max}} \rho(t_0 - \tau)\bar{L}(t_0 - \tau) \left[\frac{a(t_0 - \tau)}{a(t_0)}\right]^4 \mathrm{d}\tau \,, \tag{2.12}$$

where t_0 means the present, while $\tau_{\max} \approx t_0$ is the age of the transparent universe. If neither the total number of stars nor their mean absolute luminosity, $\bar{L}$, have changed, the integrand is simply proportional to $a(t_0 - \tau)/a(t_0)$. This or similar models lead to a negligible contribution from star light. Indeed, if our present universe were static, times of the order of 10^{23} yr, that is, exceeding the Hubble age by a factor of 10^{13}, would be required in the integral (2.11) to produce a night sky as bright as the surface of a mean star (Harrison 1977).

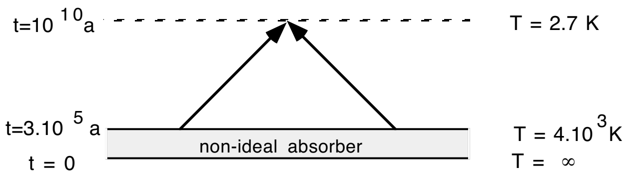

Fig. 2.4. The cosmological initial value problem for the electromagnetic radiation. The thermal contribution of the non-ideal absorber represented by the hot, ionized matter during the radiation era has now cooled down to the measured background radiation of 2.7 K (which can be neglected for most purposes)

While this conclusion resolves Olbers' original paradox, it is does *not* explain the cosmological condition $A^{\mu}_{\text{in}} \approx 0$, since (1) it *presumes* retardation, and (2) the nature of sources must have drastically changed during the early history of the Universe. In its 'radiation era', matter was ionized and almost homogeneous, representing a non-ideal absorber with a temperature of several thousand degrees (see Fig. 2.4) that can serve as an initial boundary. Because of the cosmic expansion, the thermal radiation of this absorber has cooled down to its now observed value of 2.73 K, compatible with the darkness of the night sky.

The cosmic expansion, which is vital for this low present temperature, is thus also essential for the non-equilibrium formed by the contrast between cold interstellar space and the hot stars. The latter are producing their energy by nuclear reactions under the control of gravitational contraction – see Chap. 5. The expansion of the Universe has therefore often been proposed as the *master arrow* of time. However, it would be inappropriate to use *causal* arguments to explain this connection. Even in a presumed contraction era of the Universe, absorbers would then retain their intrinsic arrow of time. In order to reverse it, the thermodynamical arrow would have to be reversed, too. The scenario of fields and phenomenological absorbers in an expanding universe is far too simple to describe a master arrow. This cosmological discussion will therefore be resumed in Sects. 5.3 and 6.2.

In a quite different approach, Hogarth (1962) had suggested that the opacity of intergalactic matter (cosmic absorbers) must have *changed* drastically during the evolution of the Universe in order to provide a time asymmetry that would explain the observed retardation of radiation. Inspired by Wheeler and Feynman's time-symmetric definition of absorbers (Sect. 2.4), he neglected the thermodynamical arrow of absorbers. However, even in thermal equilibrium, a time arrow may survive in the form of correlations between microscopic variables unless *enforced* otherwise (see the Appendix for an example).

The above conclusions regarding the retardation of electromagnetic radiation apply accordingly to all kinds of waves in interaction with matter obeying thermodynamics. Only gravitational waves might be sufficiently decoupled from absorbers, since even the radiation era must have been transparent to them. Ritz's conjecture of a *law-like* nature of retarded electrodynamics will therefore be reconsidered and applied to gravity[3] in Chap. 5.

2.3 Radiation Damping

This somewhat technical section describes an important application of retarded fields. Except for Dirac's radiation reaction of (2.22), which will be used in Sect. 2.4, its results are rarely needed for the rest of the book.

The emission of electromagnetic radiation by a charged particle that is accelerated by an external force requires the particle to react by *losing* energy. Similar to friction, this *radiation reaction*, described by an *effective equation of motion*, must change sign under time reversal. As will be explained, this can be understood as a consequence of the retardation of the field when acting on its own source, even though the retardation seems to disappear at the position of a point source. However, the self-interaction of point-like charges leads to singularities (infinite mass renormalization) which need care when being separated from that part of the interaction which is responsible for radiation damping. While these problems could be avoided if any self-interaction were eliminated by means of the action-at-a-distance theory (described in Sect. 2.4), others would arise in their place.

Consider the trajectory of a charged particle, represented by means of its Lorentzian coordinates $z^\mu(\tau)$ as functions of proper time τ. The corresponding four-velocity and four-acceleration are $v^\mu := \mathrm{d}z^\mu/\mathrm{d}\tau$ and $a^\mu := \mathrm{d}^2z^\mu/\mathrm{d}\tau^2$, respectively.[4] From $v^\mu v_\mu = -1$ one obtains by differentiation $v^\mu a_\mu = 0$ and $v^\mu \dot{a}_\mu = -a^\mu a_\mu$. In a rest frame, defined by $v^k = 0$ (with $k = 1, 2, 3$), one has $a^0 = 0$.

The four-current density of this point charge is given by

$$j^\mu(x^\nu) = e \int v^\mu(\tau)\delta^4\big[x^\nu - z^\nu(\tau)\big]\mathrm{d}\tau \; . \tag{2.13}$$

Its retarded field $F^{\mu\nu}_{\mathrm{ret}} = 2\partial^{[\mu}A^{\nu]}_{\mathrm{ret}} := \partial^\mu A^\nu_{\mathrm{ret}} - \partial^\nu A^\mu_{\mathrm{ret}}$ is known as the Liénard–Wiechert field. The retarded or advanced fields can be written in an invariant

[3] The retardation of gravitational waves has been indirectly confirmed by double pulsars (see Taylor 1994).

[4] While an orbit in space or configuration space would merely be passed backwards under time reversal ($t \to -t$), a worldline in spacetime *changes* according to $z^k(\tau) \to z^k(-\tau)$ and $z^0(\tau) \to -z^0(-\tau)$ (for $k = 1, 2, 3$). The reversal of the parameter τ is now only a consequence of the convention $\mathrm{d}t/\mathrm{d}\tau > 0$, but physically meaningless. As the derivative $v^\mu(\tau)$ – and accordingly also the current $j^\mu(\tau)$ – then get an additional minus sign under time reversal, the potentials A^μ and fields $F^{\mu\nu}$ inherit this transformation property for their respective indices (corresponding to $\boldsymbol{E} \to \boldsymbol{E}$ and $\boldsymbol{B} \to -\boldsymbol{B}$). In order to study questions of (ir)reversibility, one may often better use the simpler TP transformations, $z^\mu(\tau) \to -z^\mu(-\tau)$ for all μ – see the Introduction.

manner (see, for example, Rohrlich 1965) as

$$
\begin{aligned}
F^{\mu\nu}_{\text{ret/adv}}(x^\sigma) &= \pm\frac{2e}{\rho}\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{v^{[\mu}R^{\nu]}}{\rho} \\
&= \frac{2e}{\rho^2}v^{[\mu}u^{\nu]} + \frac{2e}{\rho}\left\{a^{[\mu}v^{\nu]} - u^{[\mu}v^{\nu]}a_u \pm u^{[\mu}a^{\nu]}\right\} ,
\end{aligned} \tag{2.14}
$$

with v^μ and a^μ taken at times τ_{ret} or τ_{adv}, respectively. In this expression,

$$
R^\mu := x^\mu - z^\mu\left(\tau_{\text{ret/adv}}\right) =: (u^\mu \pm v^\mu)\rho , \tag{2.15}
$$

with $u^\mu v_\mu = 0$ and $u^\mu u_\mu = +1$, is the light-like vector pointing from the retarded or advanced spacetime position z^μ of the source to the point x^μ where the field is considered. Obviously, ρ is the distance in space or in time between these points in the rest frame of the source, while $a_u := a^\mu u_\mu$ is the component of the acceleration in the direction of the unit spatial distance vector u^μ. Retardation or advancement are enforced by the condition of R^μ being light-like, that is, $R^\mu R_\mu = 0$.

On the RHS of (2.14), second line, the field consists of two parts, proportional to $1/\rho^2$ and $1/\rho$. They are called the generalized Coulomb field ('near-field') and the radiation field ('far-field'), respectively. Since the stress–energy tensor

$$
T^{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\alpha}F^\nu_\alpha + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right) \tag{2.16}
$$

is quadratic in the fields, it then consists of *three* parts characterized by different powers of ρ. For example, one has

$$
\begin{aligned}
T^{\mu\nu}_{\text{ret}} := T^{\mu\nu}(F^{\mu\nu}_{\text{ret}}) &= \frac{e^2}{4\pi\rho^4}\left(u^\mu u^\nu - v^\mu v^\nu - \frac{1}{2}g^{\mu\nu}\right) \\
&\quad + \frac{e^2}{2\pi\rho^3}\left\{a_u\frac{R^\mu R^\nu}{\rho^2} - \left[v^{(\mu}a_u + a^{(\mu}\right]\frac{R^{\nu)}}{\rho}\right\} \\
&\quad + \frac{e^2}{4\pi\rho^2}(a_u^2 - a^\lambda a_\lambda)\frac{R^\mu R^\nu}{\rho^2} ,
\end{aligned} \tag{2.17}
$$

where braces around pairs of indices define symmetrization, so for example, $v^{(\mu}R^{\nu)} := (v^\mu R^\nu + v^\nu R^\mu)/2$. Here, $T^{\mu\nu}$ is the ν-component of the current of the μ-component of four-momentum. In particular, T^{0k} is the Poynting vector in the chosen Lorentz system, and $T^{\mu\nu}\mathrm{d}^3\sigma_\nu$ is the flux of four-momentum through an element $\mathrm{d}^3\sigma_\nu$ of a hypersurface. If $\mathrm{d}^3\sigma_\nu$ is space-like (a volume element), this 'flux' describes its energy–momentum ('momenergy') content, otherwise it is the flux through a spatial surface element during an element of time.

The retarded field caused by an element of the world line of the point charge between τ and $\tau + \Delta\tau$ has its support between the forward light cones of these two points, that is, on a thin four-dimensional conic shell (see Fig. 2.5).

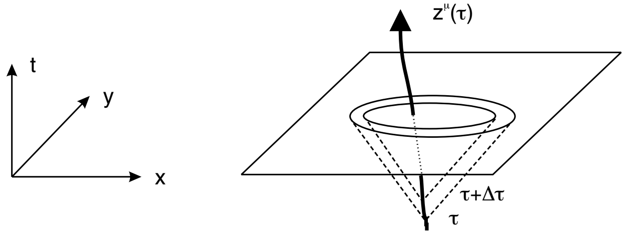

Fig. 2.5. The spacetime support of the retarded field of a world line element $\Delta\tau$ of a point charge is located between two light cones (co-axial only in the rest frame of the source). The flux of field momentum crosses light cones in the near-field region of the charge

The intersection of the cones with a space-like hyperplane forms a spherical shell (concentric only in the rest frame at time τ, and in the figure depicted two-dimensionally as a narrow ring). The integral of the stress–energy tensor over this spherical spatial shell,

$$\Delta P^\mu = \int T^{\mu\nu} \mathrm{d}^3\sigma_\nu \ , \tag{2.18}$$

is the four-momentum of the field on this hyperplane 'caused' by the world line element Δz^μ. In general, this momentum is not conserved along light cones, since (2.17) contains a momentum flux orthogonal to the cones, due to the dragging of the near-field by the charge. Therefore, Teitelboim 1970 suggested a time-asymmetric splitting of the energy–momentum tensor, which leads to an asymmetric electron dressing – valid *only* in connection with given F_{in}. However, the flux component orthogonal to the cones vanishes in the far-zone, where $T^{\mu\nu}$ is proportional to $R^\mu R^\nu$. In this region the integral (2.18) describes the four-momentum radiated *away* from the trajectory of the charge during the interval $\Delta\tau$,

$$\Delta P^\mu \underset{\rho\to\infty}{\longrightarrow} \Delta P^\mu_{\mathrm{rad}} = \frac{2}{3} e^2 a^\lambda a_\lambda v^\mu \Delta\tau =: \Re v^\mu \Delta\tau \ . \tag{2.19}$$

The quantity $\Re = 2e^2 a^\lambda a_\lambda/3$ is called the *invariant rate of radiation*. In the comoving rest frame ($v^k = 0$), one recovers the non-relativistic Larmor formula,

$$\Delta P^0_{\mathrm{rad}} = v_\mu \Delta P^\mu_{\mathrm{rad}} = \frac{2}{3} e^2 a^\lambda a_\lambda \Delta t = \frac{2}{3} e^2 \boldsymbol{a}^2 \Delta t \ . \tag{2.20}$$

This result confirms that the energy transfer into radiation in a positive interval of time cannot be negative – a consequence of the presumed retardation. An accelerated charged particle must *lose* energy to radiation, regardless of the direction of the driving *external* force.

Larmor's formula led to a certain confusion when it was applied to a charged particle in a gravitational field. Because of its dependence on acceleration, (2.19) is restricted to inertial frames. In general relativity, inertial frames are freely falling ones. According to the principle of equivalence, a freely falling charge should then *not* radiate, while a charge 'at rest' in a gravitational field (under the influence of non-gravitational forces) should do so. This problem was not understood until Mould (1964) demonstrated that the response of a detector to radiation depends on its acceleration, too (see also Fugmann and Kretzschmar 1991).

In general relativity, the principle of equivalence is only locally valid (see Rohrlich 1963). However, a *homogeneous* gravitational field (as would result from a homogeneous massive plane) is described by a flat spacetime, and thus globally equivalent to a *rigid field of uniform accelerations* a^μ on Minkowski spacetime. This field corresponds to a set of 'parallel' hyperbolic trajectories with constant (in time, but varying between trajectories) accelerations $a^\mu a_\mu$. These trajectories define accelerated rigid frames, since they preserve distances in comoving frames. Together with their proper times, the trajectories define the curved *Rindler coordinates* – see (5.16) and Fig. 5.5 in Sect. 5.2.

The equivalence principle can therefore be *globally* applied to a homogeneous gravitational field. This means that an inertial (freely falling) detector is *not* excited by an inertial charge, while a detector 'at rest' is. The latter would remain idle in the presence of a charge being 'equivalently at rest' (at a fixed distance in this case). A detector-independent definition of *total radiation* also turns out to depend on acceleration (as it should for consistency) because of the occurrence of spacetime horizons for truly uniform acceleration (see Boulware 1980 and Sect. 5.2).

The emission of energy according to (2.20) thus requires a deceleration of the point charge in order to conserve total energy. It should be possible to derive this consequence directly from the fundamental dynamical equations, which are governed by the Lorentz force,

$$\mathcal{F}^\mu_{\text{self}}(\tau) = eF^{\mu\nu}_{\text{ret}}\left[z^\sigma(\tau)\right]v_\nu(\tau) \ , \tag{2.21}$$

resulting from the particle's self-field. However, this expression leads to problems caused by the fact that the electromagnetic force acts only on the point charge, where the self-field is singular (its Coulomb part even with $1/\rho^2$), while part of the accelerated mass is contained in the energy of the comoving Coulomb field. Paul Dirac (1938) showed that the symmetric part $\bar{F}^{\mu\nu}$ of the retarded field,

$$F^{\mu\nu}_{\text{ret}} = \frac{1}{2}(F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{adv}}) + \frac{1}{2}(F^{\mu\nu}_{\text{ret}} - F^{\mu\nu}_{\text{adv}}) \equiv: \bar{F}^{\mu\nu} + F^{\mu\nu}_{\text{rad}} \ , \tag{2.22}$$

is responsible for the infinite mass renormalization, while the antisymmetric part, $F^{\mu\nu}_{\text{rad}}$, remains regular, and indeed describes the radiation reaction when treated properly.

In order to prove the second part of this statement, one has to expand all quantities in (2.14) up to the third order in terms of the retardation $\Delta\tau_{\text{ret}} = \tau_{\text{ret}} - \tau$, e.g.,

$$v^\nu(\tau_{\mathrm{ret}}) = v^\nu(\tau + \Delta\tau_{\mathrm{ret}}) = v^\nu(\tau) + \Delta\tau_{\mathrm{ret}} a^\mu(\tau) \\ + \frac{1}{2}\Delta\tau_{\mathrm{ret}}^2 \dot{a}^\mu(\tau) + \frac{1}{6}\Delta\tau_{\mathrm{ret}}^3 \ddot{a}^\mu(\tau) + \cdots \; . \tag{2.23}$$

All terms which are singular at the position of the point charge cancel from the antisymmetric field, and one obtains (see Rohrlich 1965, p. 142)

$$F^{\mu\nu}_{\mathrm{rad}} = -\frac{4e}{3}\dot{a}^{[\mu}v^{\nu]} \; . \tag{2.24}$$

The resulting PT-antisymmetric Lorentz self-force, the *Abraham four-vector*

$$\mathcal{F}^\mu_{\mathrm{rad}} := eF^{\mu\nu}_{\mathrm{rad}} v_\nu = \frac{2e^2}{3}(\dot{a}^\mu + v^\mu \dot{a}^\nu v_\nu) = \frac{2e^2}{3}(\dot{a}^\mu - v^\mu a^\nu a_\nu) \tag{2.25}$$

(using $a^\nu v_\nu = 0$ in the second step), should then describe the *radiation reaction* of a point charge. It leads to a nonlinear equation of motion (the *Lorentz–Abraham–Dirac or LAD equation*). However, while the second term on the RHS of (2.25) is in accord with (2.19), the $\dot{a}^\mu$ term is ill-defined (see below). Together with the singular mass renormalization term resulting from $\bar{F}^{\mu\nu}$, it describes the four-momentum transfer from the point charge itself to its comoving singular near-field.

In a rest frame (with $v^k = 0$ and $a^0 = 0$), one obtains

$$\mathcal{F}^0_{\mathrm{rad}} = -\frac{2e^2}{3}\boldsymbol{a}^2 \; , \qquad \mathcal{F}^k_{\mathrm{rad}} = \frac{2e^2}{3}\frac{\mathrm{d}a^k}{\mathrm{d}t} \; . \tag{2.26}$$

Therefore, the radiation reaction describes non-relativistically a force proportional to the change of acceleration, $\mathrm{d}a^k/\mathrm{d}t$, while its fourth component is the energy *loss* according to the non-negative invariant rate of radiation (2.19). The latter was originally defined by the energy flux through a distant sphere on the future light cone (Fig. 2.5). However, global conservation laws may be used only if all their contributions are taken into account. For example, one would not obtain an analogous conservation of *three-momentum* for the bare point charge and its far-field because of the aforementioned momentum flux orthogonal to the future light cone of the moving charge. For this reason, the uniformly accelerated charge may radiate with $\Re \neq 0$ even though the 'radiation reaction' $\mathcal{F}^\mu_{\mathrm{rad}}$ (including its ill-defined term) vanishes in this case, as can be seen separately for its two non-vanishing components, $\mathcal{F}^\mu_{\mathrm{rad}} a_\mu$ and $\mathcal{F}^\mu_{\mathrm{rad}} v_\mu$.

If the boundary condition $F^{\mu\nu}_{\mathrm{in}} = 0$ does not hold, the complete electromagnetic force acting on a point charge is given by

$$ma^\mu = \mathcal{F}^\mu = \mathcal{F}^\mu_{\mathrm{in}} + \mathcal{F}^\mu_{\mathrm{rad}} = \mathcal{F}^\mu_{\mathrm{out}} - \mathcal{F}^\mu_{\mathrm{rad}} \tag{2.27}$$

– cf. Sect. 2.1 and (2.22). Terms caused by the symmetric part of the self-field have now been brought to the LHS in the form of a mass renormalization $\Delta m a^\mu$. Equation (2.27) still exhibits T-symmetry, but the latter may be broken fact-like by the given *initial* condition $F^{\mu\nu}_{\mathrm{in}}$ (in contrast to the uncontrollable outgoing radiation contained in $F^{\mu\nu}_{\mathrm{out}}$). The LAD equation, based on (2.25) and the *first* RHS of (2.27), may then be written in the form

$$m(a^\mu - \tau_0 \dot{a}^\mu) = K^\mu(\tau) := \mathcal{F}^\mu_{\rm in} - \Re v^\mu \ , \tag{2.28}$$

where $\tau_0 = 2e^2/3mc^2$ is the time required for light to travel a distance of the order of the 'classical electron radius' e^2/mc.

Both terms of (2.28) that result from the radiation reaction (2.25) now change sign under time reversal (or the interchange of retarded and advanced fields). While the second one (now on the RHS) is the friction-type radiation damping $-\Re v^k$, required for the conservation of energy, the one now appearing on the LHS (called the *Schott term*) is proportional to the *third* time-derivative of the position in an inertial frame. A solution to the LAD equation (2.28) would thus require *three* initial vectors as integration constants (the initial acceleration in addition to the usual initial position and velocity). Evidently, information has been lost by differentiation in the expansion (2.23). Even for $\mathcal{F}^\mu_{\rm in} = 0$, the LAD equation (2.28) admits *runaway* solutions, non-relativistically in the form of an exponentially increasing *self-acceleration*, $a^k(t) = a^k(0)\exp(t/\tau_0)$.

Because of this formal information loss, the LAD equation is *not* a complete equation of motion. It can only represent a necessary condition for the motion of the point charge. In the free case, unphysical runaway solutions could simply be eliminated by fixing the artificial integration constant by the condition $a^k(0) = 0$. However, this would still lead to runaway as soon as an external force were turned on, since the formal solution of (2.28) with respect to a^μ is

$$ma^\mu(\tau) = \mathrm{e}^{\tau/\tau_0}\left[ma^\mu(0) - \frac{1}{\tau_0}\int_0^\tau \mathrm{e}^{-\tau'/\tau_0} K^\mu(\tau')\mathrm{d}\tau'\right] \ . \tag{2.29}$$

Therefore, Dirac suggested fixing the initial acceleration in terms of the *future force* according to $ma^\mu(0) = (1/\tau_0)\int_0^\infty \mathrm{e}^{-\tau'/\tau_0} K^\mu(\tau')\mathrm{d}\tau'$. The substitution $\tau' \to \tau' + \tau$ then leads to Dirac's equation of motion,

$$ma^\mu(\tau) = \int_0^\infty K^\mu(\tau + \tau')\frac{\mathrm{e}^{-\tau'/\tau_0}}{\tau_0}\mathrm{d}\tau' \ . \tag{2.30}$$

It represents a Newtonian (second order) equation of motion which depends on a force that acts ahead of time. How could this 'acausal' result be derived using retarded fields alone?

Moniz and Sharp (1977) demonstrated that the pathological behavior of this 'classical electron' is a consequence of a mass renormalization that exceeds the physical electron mass (so that the bare mass must be negative). If the point charge is replaced by a rigid charged sphere of radius r_0 in its rest frame, one obtains, by using the now everywhere regular retarded field, an equation of motion that was first proposed by Caldirola (1956), and later derived by Yaghjian (1992) as an approximation. It reads

$$m_0 a^\mu(\tau) = \mathcal{F}^\mu_{\rm in}(\tau) + \frac{2e^2}{3r_0}\frac{v^\mu(\tau - 2r_0) + v^\mu(\tau)v^\nu(\tau)v_\nu(\tau - 2r_0)}{2r_0} \ , \tag{2.31}$$

where m_0 is the bare mass. The retardation $2r_0$ in the arguments would change sign for advanced fields (consistent only in conjunction with given $\mathcal{F}^{\mu}_{\mathrm{out}}$). Taylor expansion of (2.31) with respect to $2r_0$, equivalent to (2.23), and using $v^{\mu}v_{\mu} = -1$ and its time derivatives leads in first order to a finite mass renormalization (4/3 of the electrostatic mass), and in second order back to the LAD equation (see Zeh 1999a). While (2.31) is analogous to a non-Markovian master equation (see Sect. 3.2), the LAD equation corresponds to its Markovian limit, valid for slowly varying fields. In this sense, the radiation reaction has to be calculated from the given history in order to *determine* the acceleration (rather than its derivative) towards the future (right derivative).

The self-force acting on the rigid 'electron' according to (2.31) is the difference (because of $v^{\mu}v_{\mu} = -1$) between a decelerating and an accelerating friction type force with different retardations. For positive bare and physical masses it does not lead to runaway, although it may possess complicated non-analytic solutions, in particular for forces varying on a time scale shorter than the light travel time within the charged sphere. Dirac's pre-acceleration of the center of mass can now be understood as a consequence of the *presumed* rigidity of the charged sphere, which requires forces of constraint acting ahead of time.

The most rigorous elimination of unphysical solutions from the LAD equation so far was proposed by Spohn (2000) – see also Rohrlich (2001), while the history of electron theory is discussed in Rohrlich (1997). It seems that the concept of a non-inertial point charge is inconsistent with classical electrodynamics, while external forces acting on a charge *distribution* would disturb its shape and structure. A quantum ground state of the electron may instead be protected against deformations by its discrete excitation spectrum. However, an explicit QED eigenstate would have to include nonlocal *quantum entanglement* between particle and field modes in an essential way (see Sect. 4.2).

General Literature: Rohrlich 1965, 1997, Levine, Moniz and Sharp 1977, Boulware 1980.

2.4 The Absorber Theory of Radiation

Ritz's retarded action-at-a-distance theory, mentioned at the beginning of this chapter, eliminates all electromagnetic degrees of freedom by postulating the *cosmological* initial condition $F^{\mu\nu}_{\mathrm{in}} = 0$ in order to fix all forces of electromagnetic origin. Since electromagnetic forces would then act only on the forward light cones of their sources, this theory cannot be compatible with Newton's third law, which requires their reactions. However, the reaction to a retarded action must be advanced.[5] In order to warrant energy–momentum conserva-

[5] In *field theory*, sources and fields interact *locally* in spacetime. For this reason the self-force (2.25) could not be derived from the flux of field momentum in the far-zone.

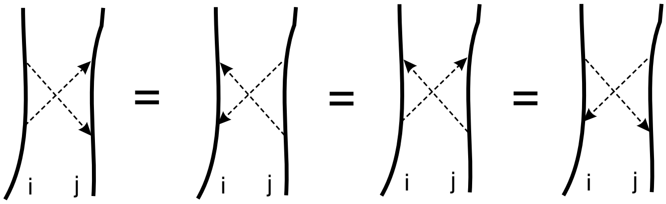

Fig. 2.6. Different interpretations of the same interaction term of the Hamiltonian for a pair of particles

tion, an action-at-a-distance theory has to be formulated in a T-symmetric way, as done by Fokker (1929) by means of his action

$$I = \int (T - V)\mathrm{d}t = \sum_i m_i \int \mathrm{d}\tau_i \qquad (2.32)$$
$$- \frac{1}{2} \sum_{i \neq j} e_i e_j \iint v_i^{\mu} v_{j\mu} \delta\big[(z_i^{\nu} - z_j^{\nu})(z_{i\nu} - z_{j\nu})\big] \mathrm{d}\tau_i \mathrm{d}\tau_j \ .$$

Here, indices i and j are particle numbers. A sum over $i \neq j$ defines a double sum excluding equal indices, while a sum over $i(\neq j)$ is meant as a sum over i only, excluding a given value j. In (2.32), the particle positions z_i^{μ} and velocities v_i^{μ} have to be taken at the proper time τ_i of the corresponding particle, for example $z_i^{\mu} = z_i^{\mu}(\tau_i)$.

Expanding the δ-function in the potential energy according to

$$\delta(\Delta z^{\nu} \Delta z_{\nu}) = \delta(\Delta z_0^2 - \Delta \boldsymbol{z}^2) = \frac{1}{2|\Delta \boldsymbol{z}|} \Big[\delta(\Delta z_0 - |\Delta \boldsymbol{z}|) + \delta(\Delta z_0 + |\Delta \boldsymbol{z}|)\Big] \quad (2.33)$$

(with $\Delta z^{\nu} = z_i^{\nu} - z_j^{\nu}$) preserves its symmetric form. By integrating either over τ_i or over τ_j, one obtains, respectively, the first or second of the following expressions (first two graphs of Fig. 2.6):

$$\frac{e_i}{2} \int \Big[A^{\mu}_{\mathrm{ret},j}(z_i^{\sigma}) + A^{\mu}_{\mathrm{adv},j}(z_i^{\sigma})\Big] v_{i\mu} \mathrm{d}\tau_i \equiv \frac{e_j}{2} \int \Big[A^{\mu}_{\mathrm{adv},i}(z_j^{\sigma}) + A^{\mu}_{\mathrm{ret},i}(z_j^{\sigma})\Big] v_{j\mu} \mathrm{d}\tau_j \ . \qquad (2.34)$$

$A^{\mu}_{\mathrm{ret},j}$ and $A^{\mu}_{\mathrm{adv},j}$ are the retarded and advanced potentials of the j th particle according to (2.2a) and (2.2b). However, if the integral is always carried out with respect to the particle on the backward light cone of the other one, one obtains, in spite of the preserved T-symmetry of the theory, only contributions in terms of retarded potentials (third graph):

$$\frac{e_i}{2} \int A^{\mu}_{\mathrm{ret},j}(z_i^{\sigma}) v_{i\mu} \mathrm{d}\tau_i + \frac{e_j}{2} \int A^{\mu}_{\mathrm{ret},i}(z_j^{\sigma}) v_{j\mu} \mathrm{d}\tau_j \ , \qquad (2.35)$$

and analogously, but time reversed, for the advanced potentials (fourth graph). Einstein seems to have been referring to this equivalence of different *forms* of the interaction in his letter with Ritz (quoted in the introduction to this chapter).

However, the Euler–Lagrange equations resulting from (2.32) automatically lead to T-symmetric forces which comply with Newton's third law:

$$ma_i^\mu = \frac{e_i}{2} \sum_{j(\neq i)} \left[F^{\mu\nu}_{\mathrm{ret},j}(z_i^\sigma) + F^{\mu\nu}_{\mathrm{adv},j}(z_i^\sigma) \right] v_{i,\nu} \ . \tag{2.36}$$

According to (2.8), this would correspond to the cosmic boundary condition $F^{\mu\nu}_{\mathrm{in}} + F^{\mu\nu}_{\mathrm{out}} = 0$ in Maxwell's theory. Equations (2.36) differ from the empirically required ones,

$$ma_i^\mu = e_i \sum_{j(\neq i)} F^{\mu\nu}_{\mathrm{ret},j}(z_i^\sigma) v_{i,\nu} + \frac{e_i}{2} \left[F^{\mu\nu}_{\mathrm{ret},i}(z_i^\sigma) - F^{\mu\nu}_{\mathrm{adv},i}(z_i^\sigma) \right] v_{i,\nu} \ , \tag{2.37}$$

not only by the replacement of half the retarded by half the advanced forces, but also by the missing radiation reaction $\mathcal{F}^\mu_{\mathrm{rad},i}$ (Dirac's asymmetric self-force). While the problem of a mass renormalization has disappeared, (2.36) seems to be in drastic conflict with reality. Moreover, it contains a complicated dynamical meshing of the future with the past that does not in any obvious way permit the formulation of an initial-value problem.

The two equations of motion, (2.36) and (2.37), differ precisely by a force that would result from the sum of the asymmetric fields of *all* particles, $F^{\mu\nu}_{\mathrm{rad,total}} = \sum_j (F^{\mu\nu}_{\mathrm{ret},j} - F^{\mu\nu}_{\mathrm{adv},j})/2$. Since the retarded and advanced fields appearing in this expression possess identical sources, their difference solves the *homogeneous* Maxwell equations, and thus represents a *free* field in spite of the dependence of the retarded and advanced fields on the sources. Therefore, this sum of differences may be assumed to vanish for *all* times as a 'boundary' condition. As there are no retarded fields at the beginning of the Universe, this would require $\sum_j F^{\mu\nu}_{\mathrm{adv},j}(t_{\mathrm{big\ bang}}) = 0$ as a very restrictive *global constraint on all sources* that will ever arise; it can hardly be exactly valid.

If the condition $F^{\mu\nu}_{\mathrm{rad,total}} = 0$ did apply, the advanced effects of all charged matter in the Universe would precisely double the retarded forces in (2.36), cancel the advanced ones, and imitate a self-interaction that is responsible for radiation damping. This is an example of the equivalence of apparently quite different dynamical representations of *deterministic* theories, such as causal or teleological, local or global ones.

Instead of referring to a cosmic initial condition, Wheeler and Feynman (1945) tried to explain the vanishing of the sum of asymmetric fields by the assumption that the total charged matter in the Universe behaves as an 'absorber' in a sense that is very different from that used in Sect. 2.2. They required that the symmetric field $\bar{F}$ resulting from *all* particles, which would according to (2.36) determine the force on an additional 'test particle', should vanish *for statistical reasons* (by destructive interference) in a presumed empty space surrounding all matter of this 'island universe'. This assumption,

$$\sum_j \bar{F}_j^{\mu\nu} := \sum_j \frac{1}{2}\left[F_{\mathrm{ret},j}^{\mu\nu} + F_{\mathrm{adv},j}^{\mu\nu}\right] \longrightarrow 0\ , \tag{2.38}$$

constitutes their cosmic *absorber condition*. Since the retarded or advanced fields vanish by definition in the asymptotic past or future, respectively, so must their time-reversed partner because of (2.38), and hence also their asymmetric combination. Wheeler and Feynman then concluded by means of the homogeneous Maxwell equations that the total asymmetric field would vanish *everywhere*. This is just the required 'boundary' condition.

However, the consistency of this procedure is very questionable. A similar problem would arise for an expanding and recollapsing Universe that were sandwiched between two *thermodynamically opposite* radiation eras (absorbers with opposite thermodynamical arrows of time) – see Sect. 5.3. As explained in Sect. 2.1, the compatibility of double-ended (two-time) boundary conditions is highly nontrivial – similar to an eigenvalue problem. This consistency problem is particularly severe for a universe that remains optically transparent and thus preserves information contained in the radiation such as light (Davies and Twamley 1993).

In contrast to the physical absorbers of Sect. 2.2, the new *absorber condition* is symmetric under time reversal. This fact led to many misunderstandings. For example, rather than adding the vanishing antisymmetric term to (2.36), one might as well subtract it in order to obtain the time-reversed representation

$$m a_i^\mu = e_i \sum_{j(\neq i)} F_{\mathrm{adv},j}^{\mu\nu}(z_i^\sigma) v_{i,\nu} - \frac{e_i}{2}\left[F_{\mathrm{ret},i}^{\mu\nu}(z_i^\sigma) - F_{\mathrm{adv},i}^{\mu\nu}(z_i^\sigma)\right] v_{i,\nu}\ . \tag{2.39}$$

Although it is as correct as (2.37) under the absorber condition, (2.39) describes *advanced* actions and a radiation reaction that leads to reverse damping (exponential acceleration).

Therefore, Wheeler and Feynman's absorber condition cannot explain the observed radiation arrow. Neither (2.37) nor (2.39) would describe the local empirical situation, which requires in general that only a limited number of 'obvious sources' contribute noticeably to the *retarded* sum (2.37). Otherwise, retardation would never have been recognized. This means that the retarded contribution of all 'other' sources (those which form the true universal absorber) must interfere destructively (see Fig. 2.7):

$$\sum_{i\,\in\,\mathrm{absorbers}} F_{\mathrm{ret},i}^{\mu\nu} \approx 0 \qquad \text{'inside' universal absorber}\,. \tag{2.40}$$

This is possible (except for the remaining thermal radiation) if the absorber particles approach thermal equilibrium by means of collisions *after* having been accelerated by retarded fields. Therefore, one *cannot* expect

$$\sum_{i\,\in\,\mathrm{absorbers}} F_{\mathrm{adv},i}^{\mu\nu} \approx 0 \qquad \text{'inside' universal absorber} \tag{2.41}$$

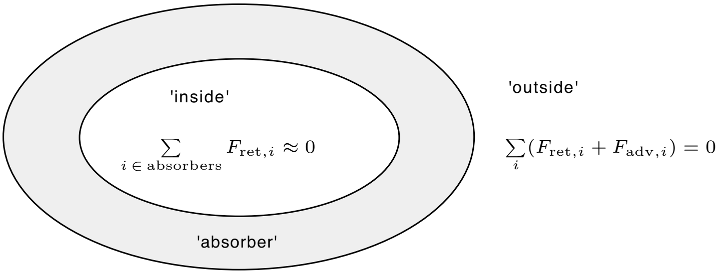

Fig. 2.7. T-symmetric ('outside') and T-asymmetric ('inside') absorber conditions of a model Universe with action-at-a-distance electrodynamics

to hold in a symmetric way. Since $F^{\mu\nu}_{\text{ret}}$ contributes only on the forward light cones, Fig. 2.7 reduces to Fig. 2.4.

In order to justify the applicability of (2.37) in contrast to that of (2.39), one still needs the *asymmetric* condition that has been derived in Sect. 2.2 from the thermodynamical arrow of time under certain cosmological assumptions. This means that any motion of absorber particles is dissipated as heat *after* it has been induced. While in field theory the field may be regarded as 'matter' with its own thermodynamical state, action-at-a-distance theory ascribes thermodynamical properties only to the sources. In the former description, the relation between electromagnetic and thermodynamical arrows is just an example of the *universality* of the thermodynamical arrow (see Sect. 3.1.2).

With these remarks I also hope to put to rest objections raised by Popper (1956) against the thermodynamical foundation of the radiation arrow – see also Price (1996), page 51. The only 'unusual' aspect of electromagnetic fields (when regarded as matter) is their weak coupling, which may greatly delay their thermalization in the absence of absorbers (see also Sect. 5.3.3). It is this very property that allows light and radio waves to serve as information media.

Therefore, the time-reversal-symmetric 'absorber condition' (2.38) leads to the equivalence of various forms of electrodynamics, but *cannot* explain the time arrow of radiation. In action-at-a-distance theories, there is no free radiation, while the radiation reaction is the effect of advanced forces 'caused' by future absorbers. If the Universe remained transparent for all times in some direction, an appropriately beamed emitter should not draw any power according to the absorber theory. As it always seems to do so (Partridge 1973), the absorber theory may even be ruled out empirically. If similarly applied to *gravitational* fields, it might also be in conflict with the observed energy loss of double pulsars.

General Literature: Wheeler and Feynman 1945, 1949, Hoyle and Narlikar 1995.