3 Sound propagation

Sound radiation from sources often happens in free space. This applies to the sound that reaches the receiver directly over a free line of sight.[6] The dependence of sound intensity and sound pressure level on the propagation distance is an important law of sound propagation. The following table summarizes some basic distance laws.

Table 3.1. Free-field propagation from elementary sources in direction of maximum sound radiation

Type of source	Distance law of sound pressure level	Level reduction per distance doubling, dB
Monopole	$L = L_w - 20\log r - 11$	6
Multipole or other directional source, on axis	$L = L_w - 20\log r - 11 + L_D$	6
Circular rigid piston in baffle, on axis	$L = L_w - 20\log r - 11 + L_D$	6
Incoherent line above ground ($P' = P/\Delta x$) ($L'_w = L_w - 10\log\frac{\Delta x}{1\text{m}}$)	$L = L'_w - 10\log r - 3$	3

3.1 Reflection of plane waves at an impedance plane

Undisturbed free-field propagation is a good sound field model for anechoic environments. Any obstacle in the field, however, will interact with the incident sound pressure. Small or large objects or room boundaries influence the total sound pressure through reflection, scattering and diffraction.

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[6] Obviously this applies typically to outdoor sound propagation or to sound propagation in close distances between souce and receiver, so that no obstacle is in the propagation path.

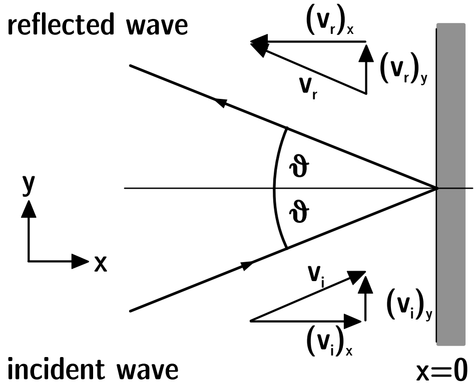


Fig. 3.1. Reflection of plane waves at impedance plane

Furthermore, properties of the medium such as inhomogeneity of sound velocity and viscous effects can introduce influences such as refraction and attenuation, respectively. A plane wave incident on a (infinitely) large smooth wall is reflected according to Snell's law. This can be described with a "specular" reflection.

The amplitude might be reduced and the phase changed. If the wave is incident at the angle ϑ,

$$\underline{p}_{\mathrm{i}}(x,y,t) = \hat{p}\mathrm{e}^{j(\omega t - kx\cos\vartheta - ky\sin\vartheta)}, \tag{3.1}$$

and the reflected wave is

$$\underline{p}_{\mathrm{r}}(x,y,t) = \hat{p}\underline{R}\mathrm{e}^{j(\omega t + kx\cos\vartheta - ky\sin\vartheta)}, \tag{3.2}$$

where $\underline{R}$ denotes the reflection factor, $\underline{R} = |\underline{R}|\mathrm{e}^{j\gamma}$. It is related to the wall impedance, $\underline{Z}$, by

$$\underline{R} = \frac{\underline{p}_r}{\underline{p}_i} = \frac{\underline{Z}\cos\vartheta - Z_0}{\underline{Z}\cos\vartheta + Z_0}. \tag{3.3}$$

$Z_0 = \rho_0 c$ is the characteristic impedance of air. The wall impedance, $\underline{Z}$, is defined as the ratio of sound pressure to the normal component of particle velocity, both determined at the wall. If the impedance is independent of the angle of incidence, we talk about "local reaction." The consequence of local reaction is that adjacent sections of the same wall surface are independent from each other, so that no tangential waves are transmitted along the wall surface. This is a good approximation for heavy walls, for walls with low bending stiffness (see Sect. 5.2) and for porous absorbers with high flow resistivity.

Particularly important is the absorption coefficient, α:

$$\alpha = \frac{|p_{\mathrm{i}}|^2 - |p_{\mathrm{r}}|^2}{|p_{\mathrm{i}}|^2} = 1 - |\underline{R}|^2 , \tag{3.4}$$

and the specific impedance

$$\underline{\zeta} = \frac{1}{Z_0}\left(\frac{\underline{p}}{\underline{v}_{\mathrm{n}}}\right)_{\mathrm{w}} = \frac{1}{\cos\vartheta}\frac{1+\underline{R}}{1-\underline{R}} . \tag{3.5}$$

For locally reacting surfaces, the absorption coefficient is

$$\alpha = \frac{4\,\mathrm{Re}(\underline{\varsigma})\cos\vartheta}{1 + 2\,\mathrm{Re}(\underline{\varsigma})\cos\vartheta + |\underline{\varsigma}|^2 \cos^2\vartheta} . \tag{3.6}$$

3.1.1 Examples of wall impedances

For perpendicular incidence, we can define some extreme cases. Real materials approximate these conditions quite well. For example, a heavy concrete wall represents a “hard wall,” and the ocean surface in underwater sound represents a “soft wall.” Some other examples are listed as follows (after (Kuttruff 2007)):

Matched wall $\underline{Z} = Z_0; \quad \underline{R} = 0; \quad \alpha = 1$

Hard wall $\underline{Z} = \infty; \quad \underline{R} = 1; \quad \alpha = 0$

Soft wall $\underline{Z} = 0; \quad \underline{R} = -1; \quad \alpha = 0$

Mass layer

At $x = 0$, we assume a layer of mass per surface area of m''. The reaction of this layer is exclusively inertia. Accordingly, the layer is characterized with neglected internal and mounting stiffness and losses. The force $p - p'$ on a surface area of 1 m^2 excites the layer to vibrations with the velocity $v' = v$. p, v denote the pressure and particle velocity on the incident side and p', v' on the transmission side of the mass layer.

One can easily show that

$$\underline{Z} = \left(\frac{\underline{p}}{\underline{v}}\right)_{x=0} = Z_0 + j\omega m'' . \tag{3.7}$$

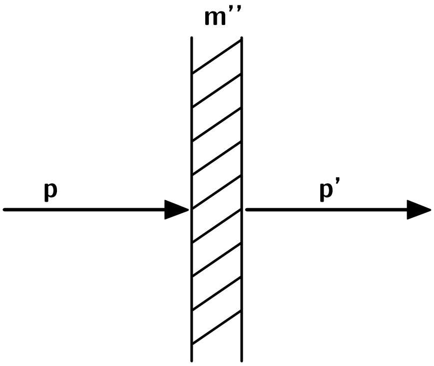


Fig. 3.2. Sound hitting a mass layer

The absorption coefficient is

$$\alpha = \frac{1}{1+\left(\frac{\omega m''}{2Z_0}\right)^2} . \tag{3.8}$$

If $\omega m'' \gg 2Z_0$, the equation can be simplified to

$$\alpha = \left|\frac{p'}{p}\right|^2 \approx \left(\frac{2Z_0}{\omega m''}\right)^2 . \tag{3.9}$$

For instance, assuming even a lightweight element such as a 6 mm glass pane with $m'' = 15\,\mathrm{kg/m^2}$, the term in brackets in Eq. (3.8) already exceeds 10 above 30 Hz.

Mass layer in front of a hard wall

We now consider the combined impedance from a mass layer mounted with an air gap to the rigid wall. At first, the air gap alone is considered. It results from choosing a new reference plane for $x = 0$. Shifting the reference plane this way may also be intended to place another object such as porous fabric at $x = 0$ (see below).

Before returning to $x = 0$, the reflected sound wave travels the twice the distance d. The reflection factor is thus

$$\underline{R} = |\underline{R}|\,\mathrm{e}^{-2jkd} , \quad |\underline{R}| = 1 \tag{3.10}$$

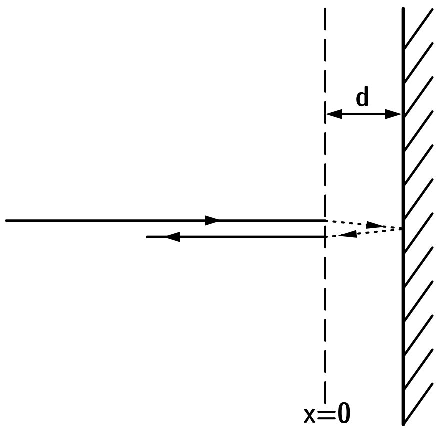


Fig. 3.3. Sound incident on a virtual air gap in front of a hard wall

and

$$\underline{Z} = -jZ_0 \cot(kd)\,. \tag{3.11}$$

When $kd \ll 1$, meaning the air gap is much smaller than the wavelength, we can approximate the cotangent function by $\cot(kd) \approx 1/kd$.

$$\underline{Z} \approx \frac{\rho_0 c}{jkd} = \frac{Z_0 c}{j\omega d}\,. \tag{3.12}$$

In this frequency range, the air gap apparently reacts as a spring with the stiffness per m^2

$$s'' = \frac{1}{n''} = \frac{Z_0 c}{d}\,. \tag{3.13}$$

We now add the mass layer at $x = 0$ by adding its impedance j$\omega m''$. Losses are added by a flow resistivity per m^2, w'', addressed to porous material placed in the air gap.

$$\underline{Z} = w'' + j\left(\omega m'' - \frac{Z_0 c}{\omega d}\right) \tag{3.14}$$

The reader may recognize that this impedance belongs to a resonator with resonance frequency

$$\omega_0 = c\sqrt{\frac{\rho_0}{m'' d}}\,. \tag{3.15}$$

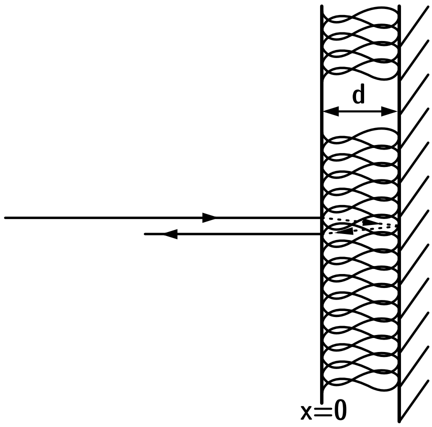


Fig. 3.4. Sound hitting a mass layer with air gap and porous filling

Porous layer in front of a hard wall

The impedance of the air gap is now combined with a purely resistive component, w'':

$$\underline{Z} = w'' - jZ_0 \cot(kd) \,. \qquad (3.16)$$

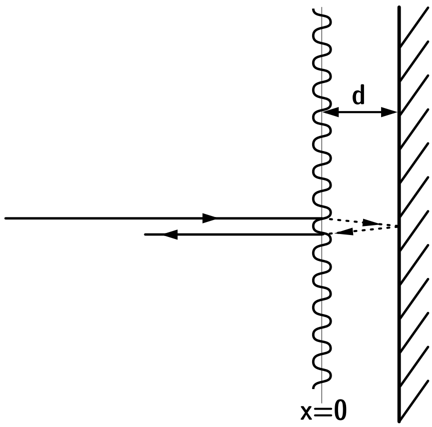


Fig. 3.5. Sound hitting a curtain in front of a wall

3.2 Spherical wave above impedance plane

The total field at R contains a contribution from the direct sound travelling along the vector $\vec{r}_0$ and another component reflected from the plane. In contrast to a plane wave reflection, for spherical wave incidence, the impedance plane is hit at various angles and the total reflection must be integrated:

$$p = \frac{j\omega\omega_0 \hat{Q} e^{-jkr_0}}{4\pi r_0} - \frac{\omega^2 \rho_0 \hat{Q}}{4\pi c} \int_{\vartheta} J_0(kr \sin\vartheta) e^{-jk(z+z_0)\cos\vartheta} \underline{R}(\vartheta) \sin\vartheta d\vartheta. \quad (3.17)$$

The first term represents the direct sound and the latter the contribution of the reflection. An approximation of the entire expression assuming a constant angle of incidence, ϑ_0, is

$$p = \frac{j\omega\omega_0 \hat{Q} e^{-jkr_0}}{4\pi r_0} + \frac{j\omega\omega_0 \hat{Q} e^{-jkr_1}}{4\pi r_1} \underline{R}(\vartheta_0)\,. \quad (3.18)$$

This equation assumes a constant angle and, thus, a plane wave. The contribution of the reflection can be related to another point source, called an "image source" (see also Sect. 11.3), which is apparently located below the surface and radiates a spherical wave whose amplitude is reduced by $\underline{R}(\vartheta_0)$. Although a spherical wave is present, the reflection is calculated for constant angle, ϑ_0. Apparently, for the moment of reflection, the sound

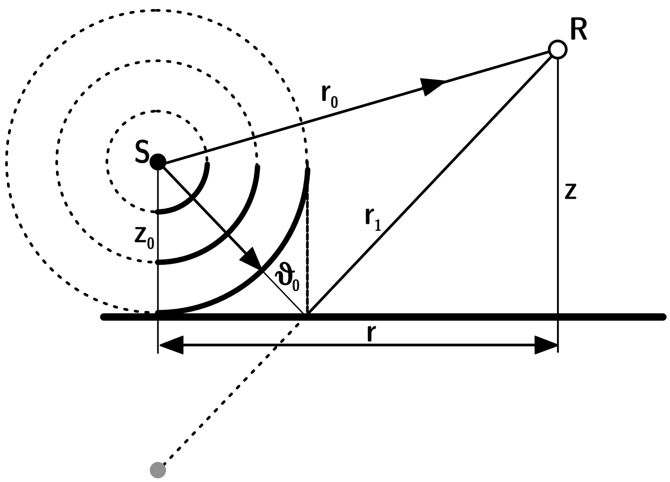


Fig. 3.6. Spherical wave reflection above impedance plane

field model is switched to a plane wave.[7] This "image source model" is even exact when $R = 1$ or $R = -1$. For other reflection factors, the approximation was identified sufficiently accurately for z, z_0 and $r \gg \lambda$ (Suh and Nelson 1999); see also Sect. 11.3.3.

In a reflecting plane, plane waves or spherical waves can be separated into elementary sources (Huygens' principle). These sources together build up an interference field that produces a plane or spherical wave. This model is the background for the simplicity in Eq. (3.18) (which is exact when $|\underline{R}| = 1$).

If, however, the plane has discontinuities in geometry or impedance, the Huygens superposition will be disturbed and the reflected field shows scattering and diffraction. Scattering is related to the reflection by an object, while diffraction is related to the boundary of an object.

3.3 Scattering

3.3.1 Object scattering

Sound waves may hit obstacles. Depending on the size of the objects compared with the wavelength, the scattered field has large amplitudes in the forward direction ("forward scattering"), in the reverse direction ("reflection") or in any other direction following a specific distribution. The exact formulation and solution of the scattered field amplitude is a difficult problem, except in academic cases of objects such as spheres, cylinders, etc.

An efficient strategy for addressing the problem in practical cases is to map the scattered field to an equivalent field created by a spherical scatterer. By this approach, the scattering cross section is defined. For an incidence plane wave with intensity I_0, the scattering cross section is ($\lambda \ll a$)

$$Q = \frac{P_s}{2I_0}. \tag{3.19}$$

In a more general approach, the theoretical model is based on superposition of the undisturbed incident field, p_0, and the scattered wave, p_s,

$$p = p_0 + p_s \tag{3.20}$$

[7] Plane wave approximation.

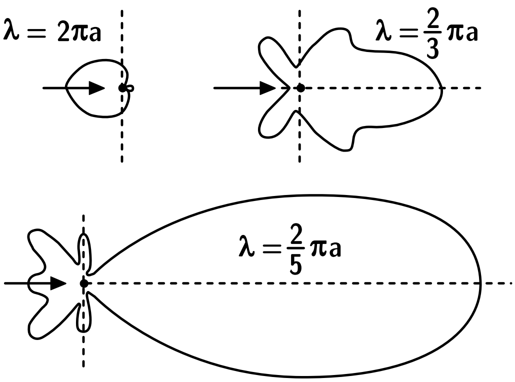


Fig. 3.7. Sound scattering at a sphere with radius *a* (after (Morse and Ingard 1968))

p_s must fulfil the boundary condition at the object's surface. With $Z = \infty$ the normal component of the particle velocity must be zero, and, thus, the normal component of the pressure gradient, too. From this follows

$$\frac{\partial p_s}{\partial n} = \frac{\partial p_0}{\partial n}, \tag{3.21}$$

and for the particle velocity of the scattered wave,

$$(v_s)_n = \frac{1}{j\omega\rho_0}\frac{\partial p_s}{\partial n}. \tag{3.22}$$

The radiated field generated from this velocity distribution is to be calculated by using the models of Sect. 2.4 in an equivalent radiation problem.

3.3.2 Surface scattering

Sound reflection at rough, corrugated walls is also described by scattering. We still assume a large wall, but its surface corrugations in length and depth are not small compared with the wavelength. A plane wave incident on the wall will interact in a way that the local phases of the elementary (Huygens) sources form a complicated total field of scattered sound.

As illustrated in Fig. 3.8, the wall can be assumed smooth at low frequencies if the depth, *h*, and the length, *a*, of the corrugation profile are significantly smaller than $\lambda/2$. When corrugations are of the order of magnitude of the wavelength, a complicated scattered field will develop. At high frequencies, the fine structure of the corrugations will lead to a specular type of reflection again.

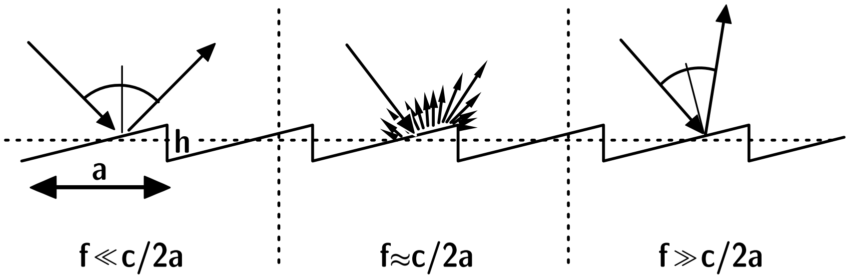


Fig. 3.8. Scattering caused by surface corrugations

Note that scattering from a rough surface may lead to sound paths with oblique angles of incidence and reflection and with delayed arrival, compared with the specular sound path.

For some corrugation types, analytic or numerical solutions are available. The bandwidth of the sound waves determines whether the total scattered field at the observation point can be approximated by energetic formulations or if distinct spectral and directional scattering lobes will occur (Cox and D'Antonio 2004). Scattering theory is best understood when it is expanded into spatial wave decomposition; the zero-order component (zero-order lobe) represents the specular reflection component. Higher order lobes direct the sound in nonspecular directions.

The energies of reflections are normalized with respect to the incident plane wave, as shown in Fig. 3.10:

$$E_{\text{spec}} = (1-\alpha)(1-s) \equiv (1-a)\,, \quad E_{\text{total}} = (1-\alpha) \tag{3.23}$$

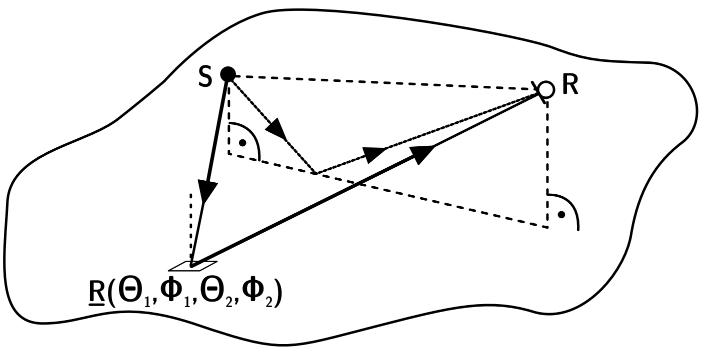


Fig. 3.9. Delayed sound paths from surface scattering

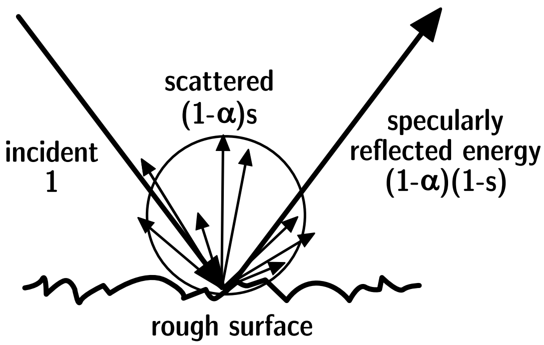


Fig. 3.10. Energy reflected from a corrugated surface into a scattered and specularly reflected portion. Definition of the total reflected energy $(1-\alpha)$, the scattered energy $(1-\alpha)s$ and specularly reflected enery $(1-\alpha)(1-s)$ (Vorländer and Mommertz 2000)

where a is the "specular absorption coefficient." It is an apparent absorption coefficient because the energy is scattered rather than absorbed. From these equations, the energy portion scattered, s, can be determined by

$$s = \frac{a-\alpha}{1-\alpha} = 1 - \frac{E_{\text{spec}}}{E_{\text{total}}}. \tag{3.24}$$

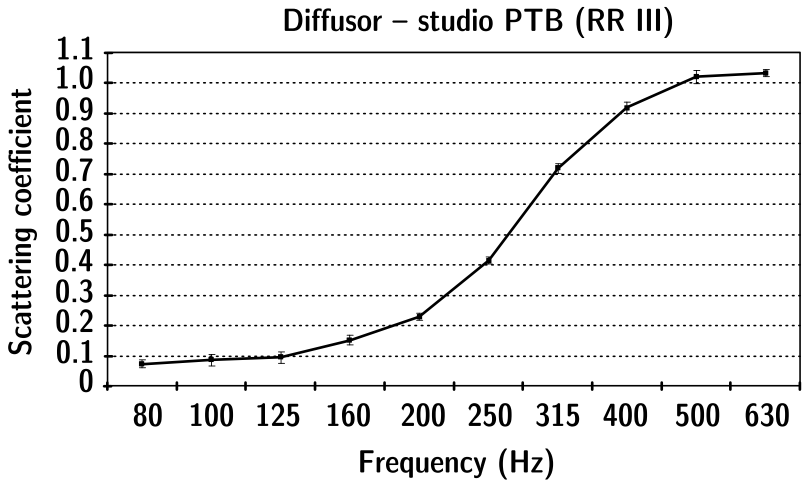


Fig. 3.11. Random-incidence scattering coefficients measured on a scale-model sample of a diffusing ceiling structure (PTB, Round Robin III, used in (Bork 2005a))

Furthermore, measurement data are available to serve as input data for simulation software (see Sect. 11.2 and Annex). These data describe the energetic amount of scattering (scattering coefficient) compared with the zero-order scattering lobe (specular component).

The uniformity of the directional scattering distribution (diffusion coefficient) is also of interest, but this should not be confused with the scattering coefficient (Cox and D'Antonio 2004; Cox et al. 2006). The directional distribution of the sound scattered from the surface of the object is obtained analogously to the way one might test the uniformity of loudspeaker radiation. Thus, a free-field polar response must be calculated or measured. The diffusion coefficient is then a single number describing the uniformity of the polar response. If the energy is scattered uniformly in all directions, then the diffusion coefficient is one. If all the energy is scattered in one direction, then the diffusion coefficient is zero. The diffusion coefficient is usually determined in one-third octave bands and is frequency dependent.

The limiting case is the ideal diffuse reflection according to Lambert's cosine law. The intensity of a Lambert scatterer depends on the cosine of the scattering angle, ϑ, and the distance from the wall element dS. It is independent of the angle of incidence.

$$\left|\vec{I}(\vartheta)\right| = (1-\alpha)\frac{B\mathrm{d}S}{\pi r^2}\cos\vartheta \,. \tag{3.25}$$

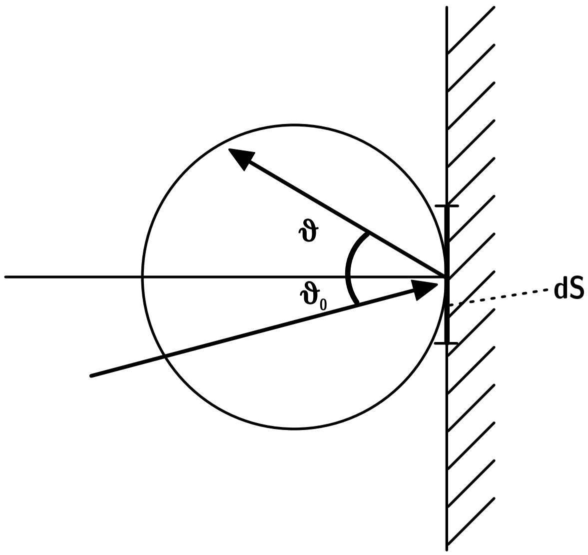


Fig. 3.12. Probability distribution of scattered sound (Lambert's law)

B denotes the irradiation strength on the wall; α the part of the incident energy $B\mathrm{d}S$ which is not reflected from the wall. This kind of scattering distribution creates a constant illumination effect on a detector with fixed sensor area, such as a membrane or, in the optical analogy, such as a camera or an eye with a fixed aperture. A white sheet of paper thus seems to have a brightness independent of the observation angle. Another example of a Lambert scatterer is the light-reflecting moon which looks more such as a disc than a sphere, since the light scattered from the surface near the moon's apparent midpoint (at a direction normal to the observer) has the same effective brightness as the light scattered from the apparent circumference, with an observation angle of almost $\pi/2$.

3.4 Diffraction

Diffraction occurs at objects with free edges, at corners and edges in a room, or at boundaries between materials with two different impedances. The diffraction wave is apparently radiated from the edges or perimeter of an object. Its intensity is negligibly small if the object is small compared with the wavelength. In this case, the incident wave remains unaffected. As the object gets larger with respect to wavelength, a shadow region first appears and then grows clearer and sharper. The shadow results from a total cancellation of the incident wave by the diffraction wave.

As for scattering, the calculation of the diffracted field is analytically possible for geometrically simple obstacles (sphere, circular disc, cylinders, free edge on a screen, slits or holes in a screen, etc.).

Diffraction models must be taken into account when sound propagation is predicted for large distances in urban areas, for instance, or in open-plan offices, just to give a few examples. Noise barriers are a typical example for the application of engineering models of diffraction. It can be shown (Maekawa 1968) that the diffracted wave from the edge and corresponding insertion loss of a vertical screen can be approximated by using the detour, d, of the diffraction (see Fig. 3.13):

$$\Delta L \approx 10 \log(2\pi^2 \frac{d}{\lambda}) \tag{3.26}$$

Because the sizes of many objects encountered in daily life are of the order of magnitude of the wavelength, diffraction is easily noticeable, although hard to predict by calculation. Diffraction influences binaural hearing, the sound transmission through doors or windows when they are not completely sealed and the orchestra sound from an orchestra pit in an opera house.

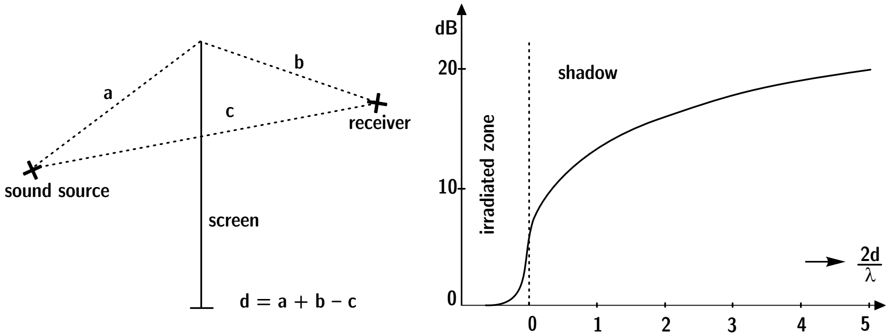


Fig. 3.13. Estimating the insertion loss of a screen

3.5 Refraction

Based on Fermat's principle, sound waves take the path with the shortest travel time. For transmission of a plane sound wave from air with characteristic impedance Z_0 into another medium with characteristic impedance Z', we use the refraction index,

$$n = \frac{c'}{c} = \frac{\sin \vartheta'}{\sin \vartheta}. \tag{3.27}$$

for calculating the geometric conditions.

The amplitude of the refracted wave into the medium with Z' follows from

$$T = \frac{2Z'}{Z'+Z_0}, \tag{3.28}$$

while for the reflection factor, Eq. (3.3) still holds.

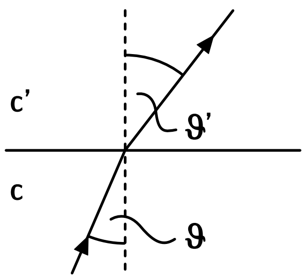


Fig. 3.14. Refraction at the boundary between two media

This approach can also be used to derive curved transmission paths in layered media. If the sound speed changes gradually, for instance, with temperature at various atmospheric elevations or with temperature and salt concentration in the ocean, the effect can be described by using small layers of constant sound speed. For the boundary between two adjacent layers with sound speed c and $c + \mathrm{d}c$,

$$\frac{c+\mathrm{d}c}{c} = \frac{\sin(\vartheta + \mathrm{d}\vartheta)}{\sin\vartheta} \approx 1 + \frac{\cos\vartheta}{\sin\vartheta}\mathrm{d}\vartheta . \tag{3.29}$$

This effect corresponds to a curved sound path with reciprocal radius:

$$\frac{1}{r_k} = \frac{1}{c}\frac{\partial c}{\partial n}, \tag{3.30}$$

where n denotes the normal direction of the sound wave. The curvature is the bigger, the larger the sound speed gradient in the direction normal to the propagation. This effect can lead to curved sound rays in the atmosphere. In some weather conditions (temperature increasing with height), upward radiated sound may be bent down to reach the ground again. Long-distance sound propagation is affected greatly. The same will be observed with wind speed profiles. In outdoor sound propagation, therefore, refraction is an essential aspect of sound field modelling.

3.6 Attenuation

Attenuation is another effect of long-distance sound propagation. It should be noted that long-distance sound propagation may happen outdoors, of course, but also indoors. If a sound wave is observed during its propagation for some seconds, it is clear that it has travelled several hundred metres; independent of whether the problem is outdoor or indoor, the latter involves numerous wall reflections.

Several attenuative effects lead to a complex wave number, $\underline{k}'$, and to an exponential decrease of the sound pressure and intensity, described by

$$\underline{p}(x,t) = \hat{p}\,\mathrm{e}^{-\frac{m}{2}x}\,\mathrm{e}^{j(\omega t - kx)} = \hat{p}\,e^{j(\omega t - \underline{k}'x)}, \tag{3.31}$$

where m denotes the energetic attenuation coefficient and

$$\underline{k}' = \frac{\omega}{c} - j\frac{m}{2}. \tag{3.32}$$

The intensity along the x coordinate in a plane wave is

$$I(x) = I_0 \mathrm{e}^{-mx} \tag{3.33}$$

or expressed in effective level loss,

$$D = 4{,}34\, m \quad \mathrm{dB/m}\,, \tag{3.34}$$

with the level loss after propagation over the distance x

$$L = L_0 - D \cdot x\,. \tag{3.35}$$

The reason for attenuation is viscosity, heat conduction and thermal relaxation (Bass et al. 1995). All effects irreversibly extract energy from the sound wave and feed other energy reservoirs, for instance, translational, rotational or vibratory modes of water molecules. The amount of water in the medium – humidity – has a crucial influence on the attenuation.

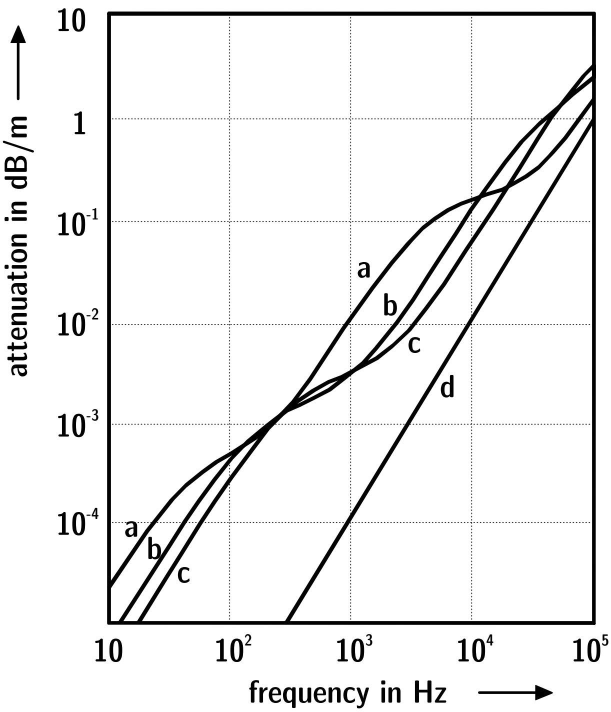


Fig. 3.15. Typical attenuation curves (after (Kuttruff 2007)) of humid air a) 10%, b) 40%, c) 80%, d) classical theory ($\sim \omega^2$)

3.7 Doppler effect

One of the acoustic effects best known to the public is the Doppler effect, the frequency shift perceived when cars, police or fire-brigade horns are passing by at high speeds. Moving sound sources or receivers cause a change in the received rate of sound pressure maxima and minima and, thus of frequency at the receiver. Differing from the analogy in electromagnetic waves, the acoustic Doppler effect depends on the actual movement and not just on the relative movement. If the receiver is moving with velocity V toward the source, the received frequency is higher than the radiated. Inserting $x = x_0 - Vt$ into a harmonic sound pressure equation yields

$$\underline{p} = \hat{p}\mathrm{e}^{\mathrm{j}[(\omega + kV)t - kx_0]}. \tag{3.36}$$

The perceived frequency is thus

$$f' = \frac{\omega + kV}{2\pi} = f\left(1 + \frac{V}{c}\right). \tag{3.37}$$

If, however, the sound source is moving relative to the medium in direction of the receiver, at first another sound speed is to be accounted for, $c' = c - V$ and $k' = \omega/(c - V)$. This, together with the changed distance $x = x_0 - Vt$, yields

$$\underline{p} = \hat{p}\mathrm{e}^{\mathrm{j}[(\omega + k'V)t - k'x_0]}, \tag{3.38}$$

with the effective frequency

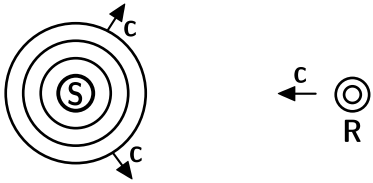
$$f' = \frac{\omega + k'V}{2\pi} = f\left(1 + \frac{V}{c - V}\right) = \frac{f}{1 - \frac{V}{c}}. \tag{3.39}$$

In the latter case only movement with $V < c$ leads to a registration of regular harmonic sound at the receiver. If $V > c$, the sound signal at the receiver is compressed to a shock wave. The distinction from relative movement can be better understood by discussing the extreme cases ($V \to c$) with respect to a harmonic signal. The rate of received pressure maxima determines the received frequency.

Table 3.2. Four cases of the Doppler effect at relative speed *c0*

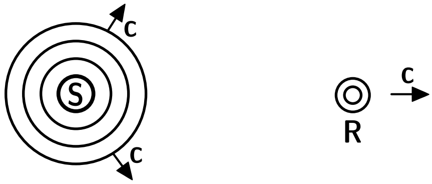
1) Moving receiver toward the source

$$f' = f\left(1+\frac{V}{c}\right) \xrightarrow{V\to c} 2f$$



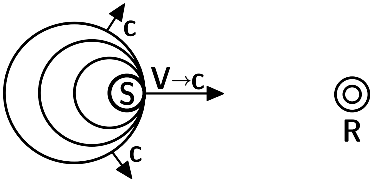
2) Moving receiver away from the source

$$f' = f\left(1-\frac{V}{c}\right) \xrightarrow{V\to c} 0$$



3) Moving source toward the receiver

$$f' = \frac{f}{1-\frac{V}{c}} \xrightarrow{V\to c} \infty$$



4) Moving source away from the receiver

$$f' = \frac{f}{1+\frac{V}{c}} \xrightarrow{V\to c} f/2$$