10 Simulation models

After getting familiar with the principle of auralization, i. e., the separation of the acoustic problem in a model of signal transmission and binaural synthesis, and after having the sources characterized, we will now focus on the second key component of auralization. We consider the excitation signal as known and ready for convolution. Now, the propagation functions for sound and vibration must be measured or modelled. The task is to define and apply a theoretical approach to the propagation problem, either in free propagation or in a problem with boundary conditions. It is clear that not all methods listed in this chapter can be used for virtual reality applications. The computation time involved in the methods is to be discussed separately. In future more and more simulation methods, however, will be applicable. The focus, therefore, is set to the physical background of the simulation methods and not on computational constraints. In Chaps. 11–15, we will discuss in detail up-to-date simulations methods applicable to virtual reality systems.

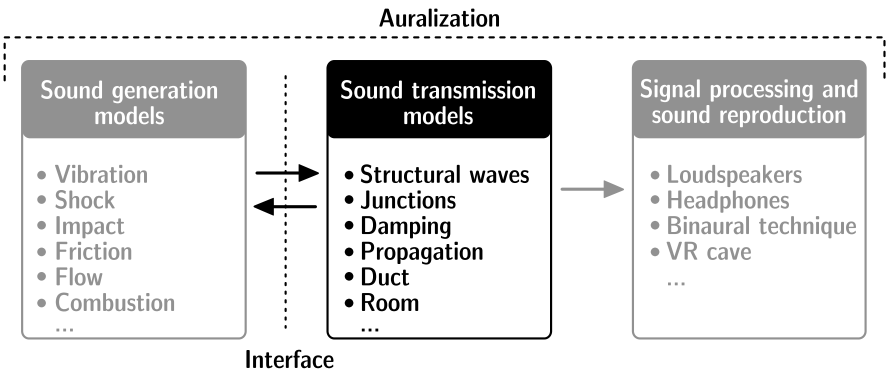

Fig. 10.1. Simulation models in acoustics

10.1 Simulation methods for sound and vibrational fields

Modelling of sound and vibrational propagation is one of the main problems in theoretical and numerical acoustics. All basic features of sound

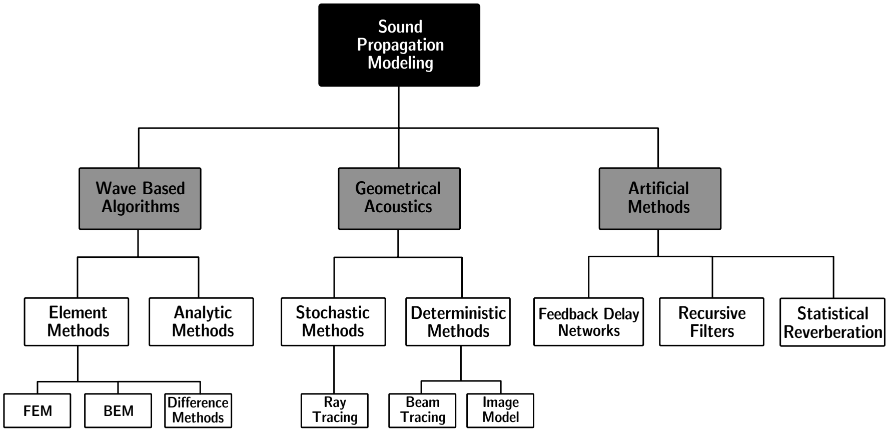

Fig. 10.2. Classification of models for simulating sound propagation

radiation (Chap. 2) and of sound fields (Chap. 3) come into play now. Nevertheless, specific methods particularly for nonanalytic approaches must be discussed in this chapter. Boundary conditions and field geometries mostly do not match the elementary conditions of standard coordinate systems such as Cartesian, spherical or cylindrical geometry. The basic solutions we found in Chaps. 2 and 3 are still interesting because they show the basic features of sound sources and propagation. The details and the fine structure in the results, however, can be obtained only when the real geometry and the conditions of the propagation space are taken into account with sufficient accuracy.

The accuracy of the models can be discussed on a physical basis and on psychoacoustic basis. The discussion on the physical basis is related to the size of objects in relation to the wavelength (diffraction), to the possibility of neglecting phase effects (high modal density), to the variety of wave types contributing to the transfer function and to elementary features of the signals simulated concerning the density of samples in the time and frequency domains.

A sound propagation or transmission problem can be described by Green's functions. They result from a formulation of the wave equation by using the potential function, $g(r|r_0)$. It corresponds to the sound field quantities by derivations in space and time (Skudrzyk 1971, Mechel et al. 2002):

$$\begin{aligned} p &= \rho_0 \dot{g} \,, \\ v &= -\nabla g \,. \end{aligned} \tag{10.1}$$

The benefit of Green's formulation is easily understood when it is applied to sources and spatial propagation paths from a point r_0 to a point r. For example, for a point source with volume flow Q (see Sect. 2.1),

$$Q = \lim_{a \to 0}\left(-4\pi a^2 \frac{\partial g}{\partial r} \right). \tag{10.2}$$

Its Green's function in free space is

$$g(r|r_0) = \frac{e^{-\mathrm{j}k|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|}. \tag{10.3}$$

Coming back to the acoustic radiation problem and to the Helmholtz equation with source term ($f = \mathrm{j}kZ_0q$; see Eq. (4.5)),

$$\Delta p(r) + k^2 p(r) = -f(r_0), \tag{10.4}$$

the Green's formulation leads to the Helmholtz–Huygens integral:[31]

$$\begin{aligned} p(r) = &\iiint f(r_0) g(r|r_0) \mathrm{d}V_0 \\ &+ \iint \left(g(r|r_0) \frac{\partial p(r_0)}{\partial n} - p(r_0) \frac{\partial g(r|r_0)}{\partial n} \right) \mathrm{d}S_0 \end{aligned} . \tag{10.5}$$

With this integral, the resulting sound pressure of various kinds of source distributions in a volume and any kind of reflections from boundaries on a surface surrounding the sources can be calculated. The integration surface can also represent a virtual surface where the sound field is expanded into elementary (secondary) sources (Huygens' principle). It is interesting that the surface source arrangement consists of monopoles and dipoles.

The discussion of Green's functions was, so far, related to harmonic signals. If we expand the radiation in the time domain by assuming an impulse excitation,[32] the tremendous importance of Green's functions becomes clear. They are *filters* transporting signals to the receiving point, expressed in the convolution of source functions with Green's functions (note the temporal relationship between source and receiver point):

$$p(r,t) = \int_{-\infty}^{t} f(r_0, t_0) g(r|r_0, t - t_0) \mathrm{d}t_0 . \tag{10.6}$$

---

[31] The derivations in the surface integral defined in the direction normal to the surface elements (Mechel et al. 2002).

[32] With impulse excitation, we consider a constant harmonic spectrum with zero phase, see Sect. 7.2.2.

By Fourier transformation, we obtain adequate solutions in the frequency domain. The kernel of the convolution can then be expressed in terms of source volume velocity, Q, which leads to the formulation of a transfer impedance,

$$Z = \frac{p(\omega, r)}{Q(\omega, r_0)}. \tag{10.7}$$

Also, the concept of a transfer function between the sound pressure at one point to the sound pressure of another point can be chosen, if the source characterization is based on a near-field pressure signal.

$$H = \frac{p(\omega, r)}{p(\omega, r_0)}. \tag{10.8}$$

Similarly, transfer functions and Green's functions for structural acoustics can be defined.

Which approach, Green's function, transfer impedance or transfer function, is preferable depends on the kind of source. Force or pressure sources are more straightforward to be coupled to Green's functions, velocity sources to transfer functions. Most easy to remember is the fact that multiplication of the source signal by the transmission function should yield a sound pressure signal in the end.

10.1.1 Reciprocity

Green's functions are reciprocal (Lyamshev 1959). Reciprocity is one of the most powerful tools in determining acoustic transfer functions. The problem of sound and vibrational transmission in a passive linear time-invariant system can be solved in both ways. Transfer functions, transfer impedances and Green's functions are identical when source and receiver points are interchanged. For an accurate description, we have to distinguish between sound propagation from an airborne source to a receiver point and vibroacoustic transmission from a force source exciting a structure that radiates to a receiver point (Fahy 1995).

Airborne sound reciprocity

The direct formulation consists of a real volume source emitting sound that is received in the field space. The ratio between the sound pressure at the receiver to the volume flow of the source[33] is $p_{\text{receiver}}/Q_{\text{source}}$. In the reciprocal

[33] = acoustic transfer impedance.

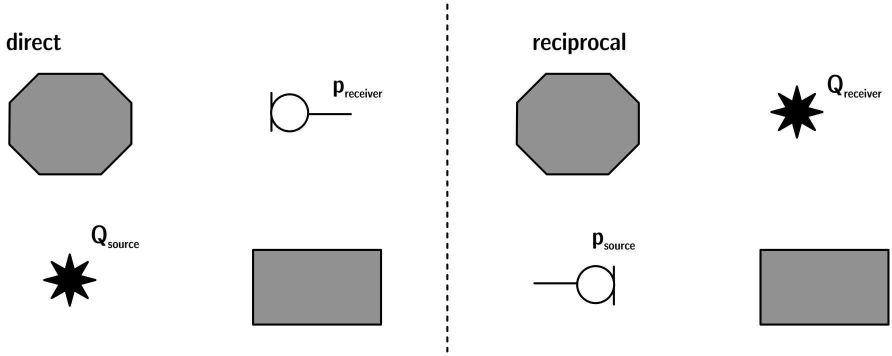

Fig. 10.3. Reciprocity of airborne sound propagation from a monopole source (after (Fahy 1995))

arrangement, this ratio is identical to the ratio of the pressure at the source point to the volume flow at the receiver, $p_{\text{source}}/Q_{\text{receiver}}$. However, specific reference conditions must be defined. The reference conditions concern the determination of the sound pressure which must be obtained in a mechanically blocked state ($Q=0$).[34] Thus,

$$\left.\frac{p_{\text{receiver}}}{Q_{\text{source}}}\right|_{Q_{\text{receiver}}=0} = \left.\frac{p_{\text{source}}}{Q_{\text{receiver}}}\right|_{Q_{\text{source}}=0}. \tag{10.9}$$

Vibroacoustic transfer function reciprocity

In a problem of a force source exciting a structure that radiates sound, the reciprocity relationship is given by

$$\left.\frac{p_{\text{receiver}}}{F_{\text{source}}}\right|_{Q_{\text{receiver}}=0} = \left.\frac{v_{\text{source}}}{Q_{\text{receiver}}}\right|_{F_{\text{source}}=0}. \tag{10.10}$$

We consider the sound pressure received from the direct formulation depending on the force injected on the sound-radiating structure. This ratio is identical to the ratio in the reciprocal approach between the volume flow at the receiver point in the fluid to the free ($F=0$) normal velocity on the structure.

[34] equivalent to an open-circuit situation for obtaining the voltage in terminated electric circuits.

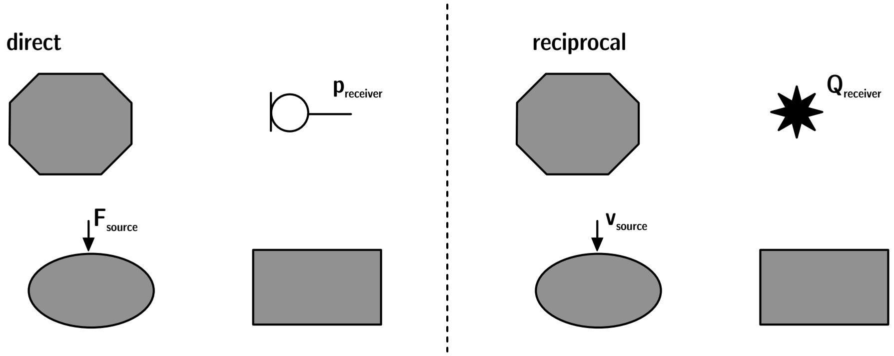

Fig. 10.4. Vibroacoustic transfer function reciprocity (after (Fahy 1995))

Vibroacoustic Green's function reciprocity

When the velocity of a vibrating surface element not the force is the input at the source side, the reciprocity relationship is given by

$$\left.\frac{p_{\text{receiver}}}{v_{\text{source}}\,\mathrm{d}S}\right|_{Q_{\text{receiver}}=0} = \left.\frac{p_{\text{source}}}{Q_{\text{receiver}}}\right|_{v_{\text{source}}=0} . \tag{10.11}$$

Here, we consider the sound pressure received from the direct formulation depending on the local normal velocity of a surface element dS of the sound-radiating structure. This ratio is identical to the ratio in the reciprocal approach between the sound pressure at the receiver point in the fluid to the blocked ($v=0$) sound pressure on the structure. This relationship is valid for all surface elements of the structure.

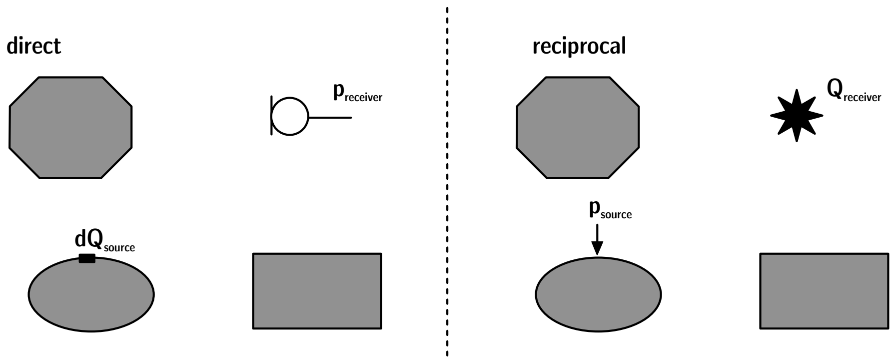

Fig. 10.5. Vibroacoustic Green's function reciprocity (after (Fahy 1995) with d$Q_{\text{source}} = v_{\text{source}}dS$)

After having discussed the principles of sound propagation between points in a fluid and between structures and fluid in analytic examples, we will focus in the next section on numerical methods in the frequency and time domains. These methods are applicable in a more general sense to any kind of geometric conditions of source, receiver and environment.

10.1.2 Frequency domain models

In frequency domain calculations, a constant frequency is considered. Hence, the discussion of transfer functions is based on harmonic signals.

$$s(t) = \underline{A} \cdot e^{j\omega t}, \tag{10.12}$$

$s(t)$ denoting any complex-amplitude harmonic signal of vibration, source volume velocity or sound pressure in the field point. In this case the wave equation reduces to the homogeneous Helmholtz equation (compare Eq. (4.5)), for instance, for the sound pressure in the field volume, $x \in \mathrm{V}$.

$$\Delta p(x) + k^2 p = 0\,. \tag{10.13}$$

To obtain the free (modal) response, we consider a source-free field domain, where the right-hand side of the equation is zero. Instead, this equation enables us to calculate the sound pressure relationships between field points, and this is the perfect approach for calculating transfer functions (Eq. (10.8)).

Depending on the geometry, as mentioned above, generally we cannot solve the Helmholtz equation straightforwardly, since the problem geometry and the boundary conditions do not match elementary coordinate systems. Instead, the problem can be solved by numerical methods. The most prominent approach is spatial discretization into small elements. In the discrete formulation, the Helmholtz equation can be transformed into a linear system of equations in the field space. The approach to solve the problem of wave physics is a) use of the Helmholtz equation in an integral formulation or b) the principle of energy conservation (energy minimum) in the Lagrange formulation as a variational problem. The first concept is used in the boundary element method, the latter in the formulation of the finite element method.

Meshing

Numerical models require spatial discretization by introducing a "mesh." A mesh is a discretized grid of surface and/or field points (nodes) and corresponding elements of groups of nodes. The elements can be rectangular,

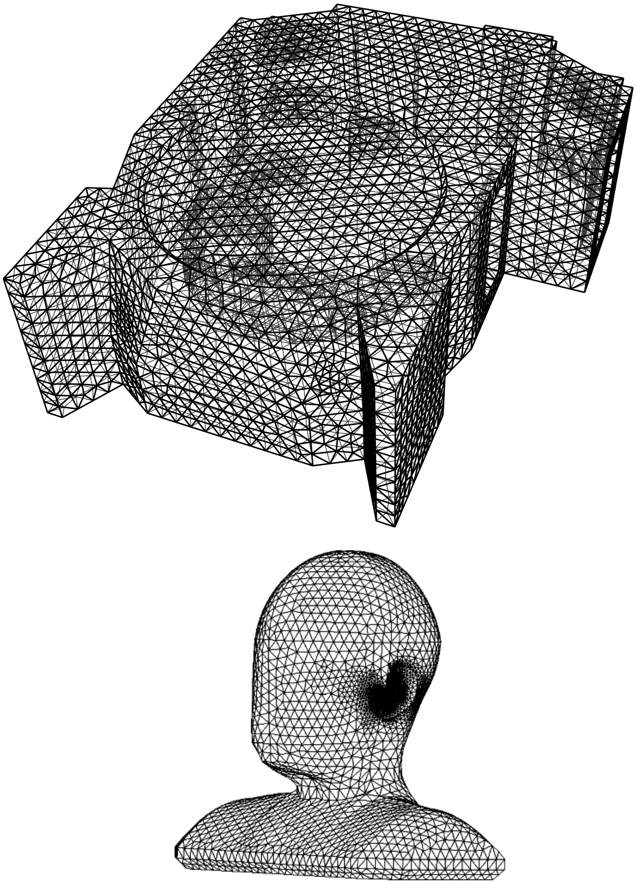

Fig. 10.6. Examples of meshes for numerical wave propagation analysis. A recording studio (top) and a customized dummy head (bottom)

triangular, tetrahedral, just to give a few examples. The volume or the boundary is meshed into elements depending on the method used. The degree of discretization depends on the local waviness of the sound or vibrational field. At high frequencies (small wavelengths), the discretization must be sufficiently large to allow interpolation between field points without too much loss of precision. The final limit, of course, is similar to the sampling theorem. The practical limit is roughly six nodes per wavelength.

Meshes must be strictly designed with respect to the numerical method used. For finite time differences, the mesh must be geometrically regular, whereas meshes for finite or boundary elements are more flexible in shape and size. However, in the latter case, mesh elements are crucially coupled to the specific formulation of the numerical wave model.

Boundary Element Method

The boundary element method (BEM) is explicitly related to Green's function, $G(r|r_0)$, where r_0 denotes a source position and r a set of field points. The radiation problem, thus, is rearranged into the Helmholtz–Kirchhoff integral equation and discretized. The Helmholtz–Kirchhoff integral is the source-free Helmholtz–Huygens integral (Eq. (10.5)):

$$p(r) = \iint \left(g(r|r_0) \frac{\partial p(r_0)}{\partial n} - p(r_0) \frac{\partial g(r|r_0)}{\partial n} \right) \mathrm{d}S_0 , \tag{10.14}$$

with Green's functions in 3-D free space:

$$g(r|r_0) = \frac{e^{-\mathrm{j}k|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|} , \tag{10.15}$$

which fulfil the far-field radiation (Sommerfeld) condition of vanishing sound pressure for $r \to \infty$.

The kernel of the integral thus contains monopole and dipole sources. The main application for BEM is radiation or equivalent radiation problems such as scattering. Radiation problems are characterized by boundary conditions (local impedances or admittances), including a vibrational velocity as a driving source. This integral is formulated in discretized form on a surface mesh and solved numerically in matrix algebra. The crucial point of the BEM formulation is the numerical nonuniqueness. It is worth mentioning that in contrast to FEM matrices, BEM matrices are full. In the famous Burton/Miller approach (Burton and Miller 1971), these problems are discussed in all detail. Another strategy for avoiding numerical problems is the so-called CHIEF point (combined integral equation formulation) method (chapter by Ochmann in (Mechel et al. 2002)). BEM matrix solvers are available; some codes are even in free software.[35]

The complexity of BEM can be roughly summarized as follows: A simulation which must be calculated up to a frequency of f required a mesh element size of at most $c/6f$. The resulting model size of a surface S is

$$N = \frac{36Sf^2}{c^2} . \tag{10.16}$$

[35] The reference software for acoustic BEM is "Sysnoise"™ http://www.lmsintl.com/SYSNOISE (renamed "Virtual.Lab Acoustics")

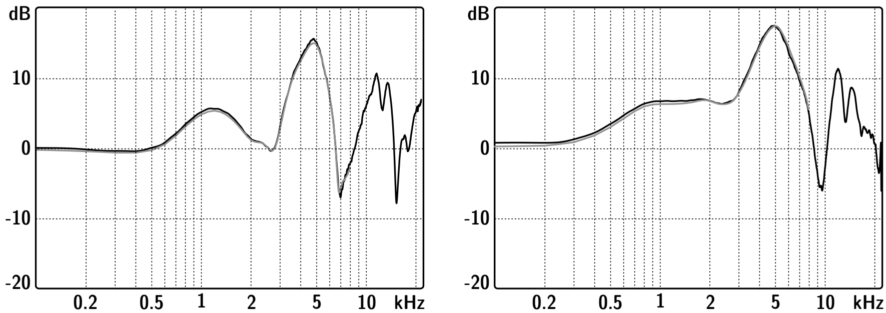

Fig. 10.7. BEM-calculated HRTF of a customized dummy head, left: Frontal incidence; right: 45° in the horizontal plane for the ipsilateral ear. Head model corresponding to Fig. 10.6 (bottom). Example follow the method described in (Fels et al. 2004), see also Sect. 6.3.1

The BEM matrix then contains N^2 entries. Solvers of PC software[36] today are capable of inverting a matrix of 8000 nodes in 60 seconds. This result holds for one frequency. In a problem of required frequency resolution of 1 Hz, the calculation time multiplies by f, which yields some $f/60$ hours for numerically generating a complete set of transfer functions for all field points.

Advanced techniques such as fast multipole BEM (Sakuma and Yasuda 2002; Yasuda and Sakuma 2003; Marburg and Schneider 2003) are developments that allow separating meshes into regions of high discretization and others with the effect of transfer propagation. The complex linking between mesh elements is thus rearranged in a hierarchical way.

Finite Element Method

Finite elements are created by discretization of a field volume into volume elements. In these elements, the energy formulation of the harmonic field equations is used. This is generally known as Hamilton's principle of minimum energy. Any disturbance of the system equilibrium[37] leads back to a stable and minimum energy state. Due to its general energetic formulation, this principle is used for mechanical problems of static load and deformation (also for crash test simulation), for fluid dynamics, heat conduction, electromagnetic or acoustic field problems, and it is also the basis for the finite element method (FEM) (Zienkiewicz 1977).

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[36] on a machine with a 2 GHz dual-core processor and sufficiently large RAM to keep the matrix problem in-core.

[37] by applying virtual displacement, for example.

The field space for the acoustic problem must be discretized into suitable volume elements. For each element, the relation between the forces and the displacements is introduced by using the variational approach. Thus the variational approach is used to identify the field quantities for minimum energy, element by element. The total energy is thus the sum of all element energies.

In every element, the so-called "shape functions," ψ, are defined to represent the sound pressures within the elements. At the nodes between the elements, the shape functions must fit continuously.

All elements' entries are combined into a so-called "stiffness" matrix, S,[38] a mass matrix, M, and a damping matrix, C. Furthermore, source contributions and boundary conditions are formulated and integrated into a matrix equation including S, M, and C, which is to be solved to obtain the sound pressures in the field space from the shape functions, read at their nodes.

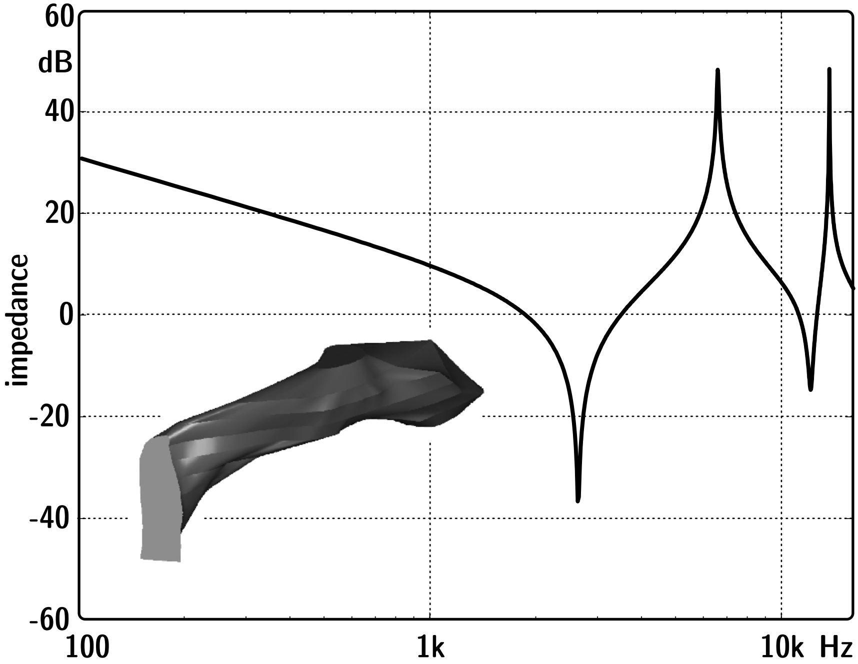


Fig. 10.8. Example of an ear canal impedance 20log($|\underline{Z}|/Z_0$) (for the CAD model shown, ear canal entrance and reference plane on the left) calculated using FEM

[38] Although other forces may be used here, the historic name is related to problems of static deformation by Hooke's forces.

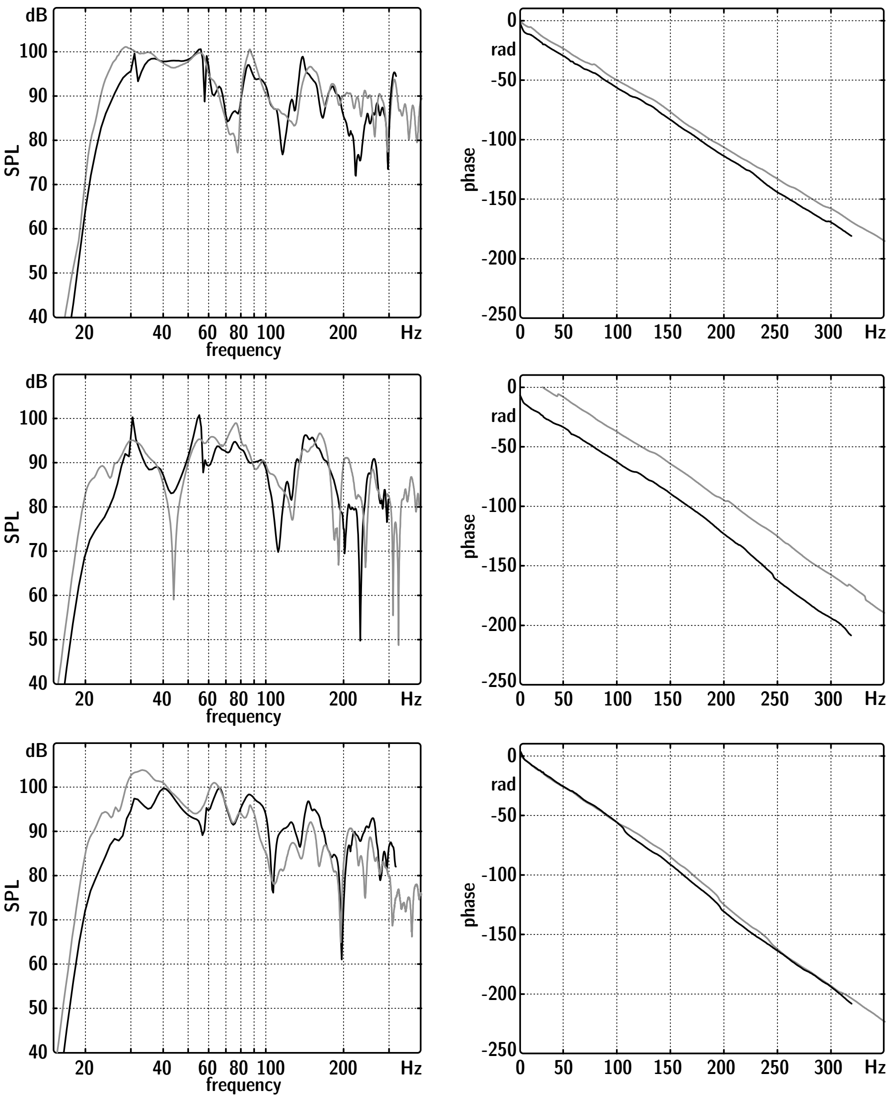


Fig. 10.9. Modulus and phase of results from FEM calculations and measurements in a recording studio (corresponding to the mesh shown in Fig. 10.6 (top), example after (Aretz 2007))

In FEM solvers, the direct solution to determine the eigenvalues of the matrix can be used, just using the matrix equation without further subspace conditions. In the indirect method, the problem is projected on a modal basis into an equivalent eigenvalue problem of orthogonal modes. The latter method has the great advantage that sources and boundary conditions can be studied in a second step. The numerical complexity is then given by the size of the modal basis and not by the FE mesh size.

State of the art is FE used for sound pressure calculation in small or midsize rooms for frequencies up to some kHz. Using PC software, typical mesh sizes are in a range of 100,000 nodes, and typical calculation times are of the order of magnitude of 5 minutes per frequency.

Modal approach and modal superposition

As in the finite element formulation, modes may serve as set of orthogonal function basis for expansion of broadband results into a series of modes. The modes as such can be described by using elementary second-order resonators (see also Sect. 4.1) with midband frequency, half-width or quality. This way of modal analysis is well known as powerful tool in measurement of complex systems. Also here, in simulation problems, a modal basis gives very important information about the system. Modal density and modal overlap can be studied and it can be decided to which extent the exact complex modal response is relevant. The transition point from separated modes to highly overlapping modes is of crucial importance. This was first discussed in Sect. 4.2.1, but it is generally interesting for all problems of acoustics and vibration.

Frequency spacing

The resolution of numerically determined spectra should be sufficiently high to identify all relevant modal details. Too high a resolution, on the other hand, is useless and just contains redundant information. The scale for defining reasonable frequency spacing is given by the width of modes. Each mode of width

$$\Delta_{f,mode} = \frac{\delta}{\pi} \tag{10.17}$$

shall be covered by at least two frequency lines. $\Delta_{f,mode}$ denotes the half-width of a typical resonance. This kind of line spectrum can approximate well a second-order band-pass function with a damping of δ. The decay time of this mode is approximately

$$T = \frac{6.9}{\delta}\,. \tag{10.18}$$

The required frequency spacing to model the system is thus

$$\Delta f = \frac{2\delta}{\pi} = \frac{4.4}{T}, \tag{10.19}$$

with T denoting the average decay time derived from the system impulse response.

Statistical Energy Analysis SEA

Statistical energy analysis, SEA, was introduced by Lyon and Maidanik in the early 1960s (Lyon 1975). The model of coupled resonators serves well for understanding the basic principles. On this basis, any complex system of coupled resonances, in problems of airborne sound or structural vibration, or both coupled can be described by using energy balance and energy flow. At this stage of abstraction, SEA offers a powerful technique for calculating sound pressure levels and so power flow between the subsystems of a complex structure.

The general approach can be compared with the situation of water reservoirs and a connecting tube system. Water pumps are “sources,” losses from damping and radiation can be modelled as water loss in porous ground or vapourization, respectively. Coupling is given by connecting tubes with certain cross section and capacity.

The amount of water represents energy in a subsystem, water flow produced by a pump represents sound power injected into a system; connecting tubes are similar to sound power transfer between systems. By using SEA the amount of water in each basin can be calculated for steady-state conditions.

The difficult task, however, must still be discussed. Because calculation of sound energy in a subsystem is a more complex problem than dealing with water, we must define proper conditions of energy analysis. The crucial point of SEA is energy stored in resonators (modes) and statistical modal overlap in certain frequency bands. The subsystems contain modes of all kind, compressional waves, flexural waves on plates etc. Typically subsystems are defined for each medium, material or shape separately. They should be clearly separated so that the energy exchange by coupling is small.

All data used in SEA calculations are frequency-dependent since energy, energy flow and boundary conditions are defined for frequency bands. The more modes are present in the frequency bands considered, the more precise the calculation.

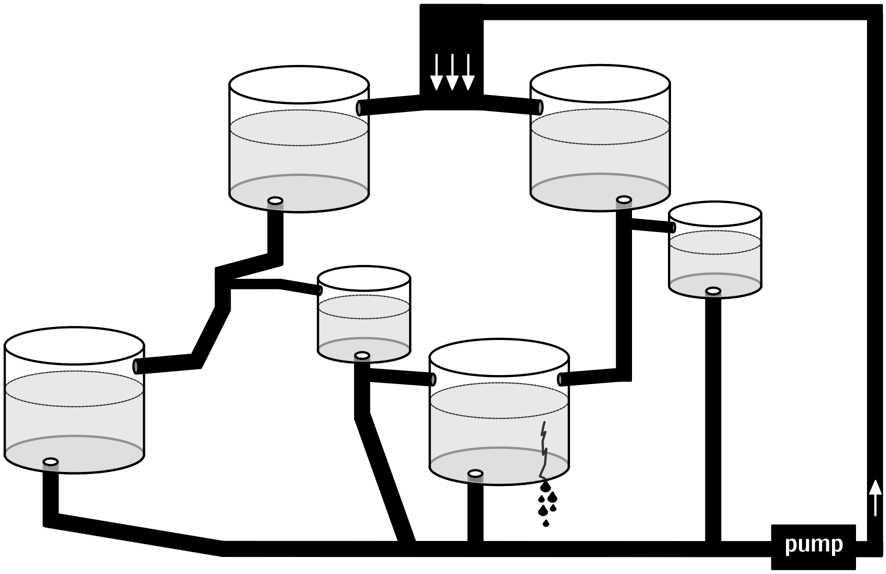


Fig. 10.10. Water flow in a system of basins and tubes

The power extracted from a subsystem i is given by

$$\Pi_i = \omega\eta_i E_i , \tag{10.20}$$

where η_i is the damping loss factor and E_i the steady-state energy stored in modes of spectral density v_i. When two subsystems are coupled, conservation of energy and balance leads to equilibrium when

$$\Pi_{ik} = \omega v_i \eta_{ik} \left(\frac{E_i}{v_i} - \frac{E_k}{v_k} \right). \tag{10.21}$$

The effective energy transport is, thus, expressed by the ratio of absolute energy in the frequency band and the modal density. This is appropriate in statistical sense since the probability of energy exchange in modal coupling depends on several factors including interaction of normal and tangential modes, fluid-structure coupling and geometric factors. It is essential that subsystems are independent, weakly coupled and contain statistically many independent modes.

Furthermore, reciprocity can be found in SEA, and this is a powerful tool to describe coupling loss factors.

$$v_i \eta_{ik} = v_k \eta_{ki} \tag{10.22}$$

With this concept we can formulate the energy injected into a subsystem by accounting for a source and all power input from other systems.

$$\Pi_{i,\text{input}} = \omega\eta_i E_i + \sum_k \omega v_k \eta_{ik}\left(\frac{E_i}{v_i} - \frac{E_k}{v_k}\right) \tag{10.23}$$

The equation of modal energies of all subsystems, coupling and power flow can thus be expressed in matrix form, such as

$$\mathbf{\Pi} = \mathbf{CE}\,, \tag{10.24}$$

which for three subsystems is expanded in the form

$$\mathbf{C} = \begin{pmatrix} \omega v_1(\eta_1 + \sum\limits_{k\neq 1}\eta_{1k}) & -\omega v_1\eta_{12} & -\omega v_1\eta_{13} \\ -\omega v_2\eta_{21} & \omega v_2(\eta_2 + \sum\limits_{k\neq 2}\eta_{2k}) & -\omega v_2\eta_{23} \\ -\omega v_3\eta_{31} & -\omega v_3\eta_{32} & \omega v_3(\eta_3 + \sum\limits_{k\neq 3}\eta_{3k}) \end{pmatrix} \tag{10.25}$$

Typical applications in acoustic engineering involve hundreds or thousands of subsystems. Free software is available on the Internet.[39]

When the energy density or the sound intensity is known, the sound pressure can be estimated on the basis of a specific sound field type. In diffuse field condition in a room, for example, the squared sound pressure is related to the energy density by using Eq. (1.31).

10.1.3 Time domain models

Wave propagation can be simulated in the time domain as well. We do not start the discussion with harmonic signals (Dirac pulse in frequency domain), but with its "opposite." the temporal Dirac pulse. Pulse propagation can now be studied in a mesh from node to node (finite difference model) or on a large scale assuming special wave types (ray tracing). The results are wave fronts propagating in time. Reflections, diffraction and other propagation effects can be calculated. At chosen field points, impulse responses can be obtained.

[39] The reference (but not free) software is "AutoSEA2"™:
http://www.esi-group.com/SimulationSoftware/Vibro_acoustics.
For free software, see, for example: http://opensea.mub.tu-harburg.de/

Waveguides

The imagination of wave propagation in one dimension is perfectly represented by waveguides. They introduce propagation delay and attenuation due to divergence and damping. A spherical wave (omnidirectional) also can be described by waveguides. Points of reflections due to interfaces between different impedances are also easy to implement by connecting waveguides (delay lines) with transfer functions of reflection and transmission factors. At this point, it is obvious that waveguides contain forth- and backtravelling waves and thus are bidirectional.

In 2-D or 3-D wave propagation, the waveguide model is mapped to a corresponding CAD model, such as a room. The delay lines then are geometrically fixed at some point or patches on the walls. Otherwise, the combinations of geometric paths would increase exponentially. With geometric concentration of the waveguide nodes, a finite number of node connections results (Krämer 1994).

Waveguides are well described and studied in application to physical modelling of musical instruments and vocal tracts (Välimäki et al. 1993; Välimäki 1995; Fant 1970). In these cases, the transmission system is separated into adjacent tubes of varying cross section. The famous "Kelly–Lochbaum" model of the vocal tract explains the formation of formants in vowels.

Frequency-dependent losses are included adding digital low-pass filters into the delay loop. FIR and IIR filter networks are used, partly involving sophisticated phase models and subsample shifts for better adjusting the actual geometric relations between nodes (Karjalainen 2005). The geometric conditions and corresponding wave divergence must be specifically included, except for 1-D application such as for wind instruments and vocal tracts.

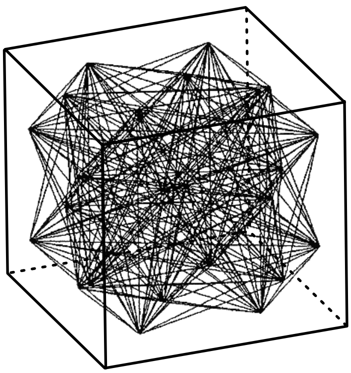


Fig. 10.11. Mapping room geometry on a set of coupled delay lines (after (Krämer 1994))

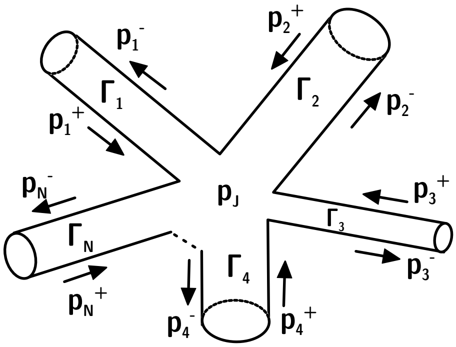


Fig. 10.12. Nodes coupling delay lines (after (Karjalainen 2005))

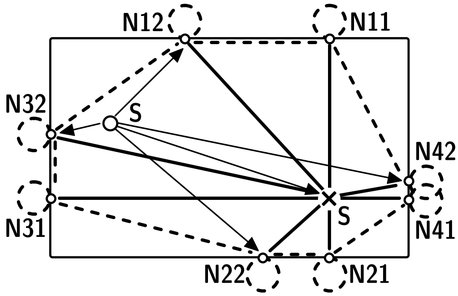


Fig. 10.13. Room model with waveguide network (after (Karjalainen 2005))

Geometrical acoustics

This model, already introduced for calculating basic features of room acoustics in Sect. 4.3, can be applied to various other purposes. Geometrical acoustics is easily understood by using the analogy of geometrical optics. A laser beam representing a straight line carrying light energy is well known. We now interpret sound propagating as rays, too. Rays are reflected, refracted or diffracted (at least in first order). Rays carry sound energy, and the quantity hitting a receiver area or volume determines the sound energy received. Because rays are not describing near-field wave effects, they are related to long-distance approximation of quasi-plane waves. In this way, any field geometry, spherical or cylindrical wave fields can be modelled by sending rays in an appropriate arrangement. Fields of application of geometrical acoustics are

– room acoustics
– outdoor sound propagation
– underwater acoustics
– ultrasound

For auralization, room acoustics and noise immission prognosis are the most interesting fields. Accordingly, geometrical acoustics is well developed in these areas.

Ray construction is the key to geometrical acoustics. In elementary definition, rays will travel along the path with shortest travel time (Fermat's principle). Depending on the medium, this might lead to straight lines and specular reflections. In layered media such as in the atmosphere with height-dependent temperature or a wind profile, the rays are bent by refraction. But as in geometrical optics and discussion of lenses, refraction can well be included in the model. The strategy to construct rays is twofold:

– forward geometric construction from the source to the receiver
– reconstruction from the receiver to the source.

Basically these two approaches are equivalent, which is inherently given in the law of reciprocity (source and receiver may be interchanged). But the approaches also show partly extremely diverging advantages and disadvantages, which are discussed in Chap. 11.

When the ray propagation is known, impulse responses containing Dirac pulses of delays and energies weighting are constructed. Impulse responses also may contain specific impulse responses of edge diffraction or boundary reflection factors.

Radiosity

A geometric model, too, consisting of elements of energy radiation over distances and via observation angles is "radiosity." The concept is irradiation and reradiation of energy from surface elements.[40] It is essential that the energy is diffusely scattered. With these prerequisites, the integral equation between the total irradiation strength received from all other surface elements can be formulated. This model is also known in illumination simulation on diffusely reflecting surfaces.

According to (Kuttruff 1971), the energy portion irradiating the surface element dS' at r from the other surface elements located at r' is given by

$$B(r,t)=\frac{1}{\pi}\iint_S (1-\alpha(r'))B(r',t-\frac{R}{c})\frac{\cos\vartheta\cos\vartheta'}{R^2}\mathrm{d}S'+B_\mathrm{d}(r,t)\,. \tag{10.26}$$

B_d is the direct sound intensity. Lambert's law of diffuse scattering is included in this integral equation. Note that exact timing is also included due to retardation by $t-R/c$.

[40] often called "patches."

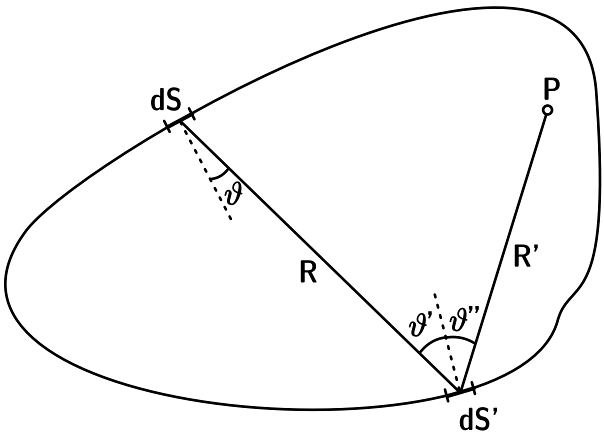


Fig. 10.14. Radiosity based on Kuttruff's integral equation (after (Kuttruff 2000))

The pure calculation of energy irradiation on the walls is interesting, but the energy at field point r in the interior field space is more relevant. This is achieved by

$$w(r,t) = \frac{1}{\pi c} \iint_S (1-\alpha(r'))B(r',t-\frac{R'}{c})\frac{\cos\vartheta''}{R'^2}\mathrm{d}S' + w_\mathrm{d}(r,t)\,. \tag{10.27}$$

w_d denotes the direct sound (see Eq. (4.40)). In discretized form, the Integral (10.27) is exactly the basis for acoustic radiosity. It is used for calculating the total energy density at the receiving point, r, via a direct path and reverberant paths under conditions of diffuse reflections. If not only the boundary (in patches) is discretized, but also the time, the temporal process of energy transition from wall to wall and to the receiver can be calculated (room impulse response).

10.2 Two-port models

Many acoustical/mechanical and electrical equations are formally similar, for example,

$$U = L\frac{\mathrm{d}I}{\mathrm{d}t} \qquad F = m\frac{\mathrm{d}v}{\mathrm{d}t} \qquad I = C\frac{\mathrm{d}U}{\mathrm{d}t}\,. \tag{10.28}$$

Hence it is worthwhile to use these analogies for solving mechanical or acoustical problems with techniques known in the analysis of electrical circuits. The power of methods in analyzing electrical circuits must be seen in coupling current and voltage in complex circuits and in identifying major and minor current flow paths. The model, therefore, is excellently qualified to analyze force and velocity ratios over in mechanical systems, force feedback and resonance systems. When comparing the equations, including the concentrated elements of mechanical masses, springs and resistors and the corresponding equations of voltage, current and the typical electronic elements, two possibilities are found:

Table 10.1. Electromechanical analogies

	Analogy I		Analogy II	
Voltage U	⟷	Force F	⟷	Current I
Current I	⟷	Velocity v	⟷	Voltage U
Electr. impedance	⟷	Mech. impedance	⟷	Electr. conductivity
Z_{el}		Z_{m}		Y_{el}
Resistance R	⟷	Friction losses w	⟷	Conductivity $1/R$
Inductivity L	⟷	Mass m	⟷	Capacity C
Capacity C	⟷	Spring n	⟷	Inductivity L

Analogy I is impedance conserving and analogy II is conserving the circuit plan. This means that an equivalent mechanical circuit developed from an electrical circuit will have the same general structure. Some examples are shown in Fig. 10.15. It is worth mentioning that electroacoustic transducers and, thus, coupled electrical-mechanical devices can also be modelled.

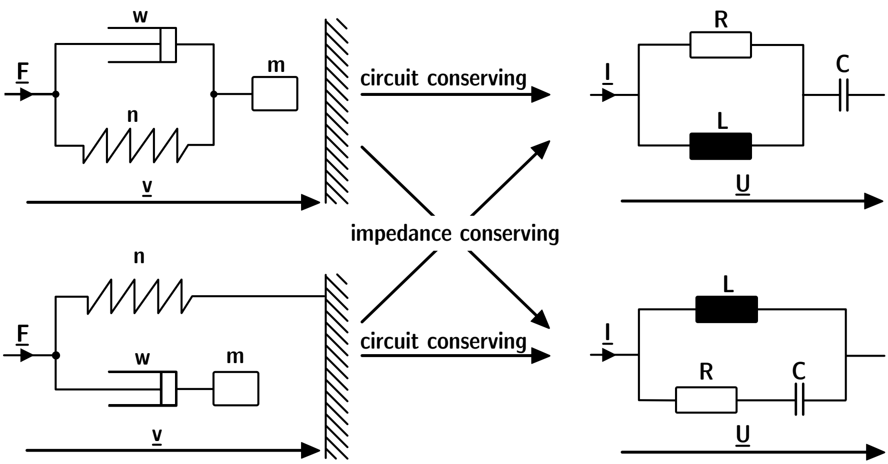


Fig. 10.15. Equivalent electromechanical circuits

Concentrated elements are valid in one-dimensional or multi-dimensional orthogonal signal transmission. Particularly masses, springs and losses should act as clearly separated elements. A mass, thus, should not show any internal spring or waveguide behaviour, for instance.

Waveguide elements are delay lines corresponding to a specific kind of wave propagation, mostly the plane wave.

From the definition of circuit elements, the step toward system modelling is quite easy. A so-called “two-port” is defined as a system with two input terminals and two output terminals. Between these terminals, a voltage is defined as a difference in the potential electric field. In the language of mechanics and acoustics, this difference is to be interpreted as force or velocity, depending on the analogy used. The inside circuit hidden in the two-port is a priori unknown. Its circuit plan and the concentrated elements are not even necessary for modelling purposes. Instead, the transfer function and the transfer impedance serve as descriptors. Matrix formulations, too, help in connecting two-ports in complex networks. Transfer functions, transfer impedances, matrix elements, etc., are spectral data that can be used in coupling two-ports and in simulating networks. If the network represents a system acting between a source and a receiver, it can be used perfectly for calculating auralization filters.

Passive circuits are reciprocal. If they are fed by excitation signals from the left or from the right side, the ratio of the open-circuit forces (infinite mechanical load) to the velocities on the opposite side,

$$\frac{F_{2\mathrm{o}}}{v_1} = \frac{F_{1\mathrm{o}}}{v_2}, \tag{10.29}$$

remains invariant. Infinite mechanical loads are achieved by blocking the motion.

Similarly, electromechanical transducers can be modelled. They require circuits on the electrical and on the mechanical/acoustical side. The inner transducer effect (electrodynamic, electrostatic, piezoelectric, etc.) is modelled by a transformer or by a gyrator.

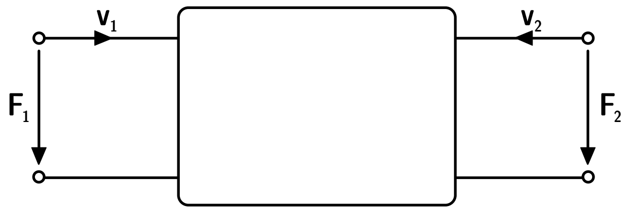


Fig. 10.16. Mechanical two-port, analogy I (force/voltage)

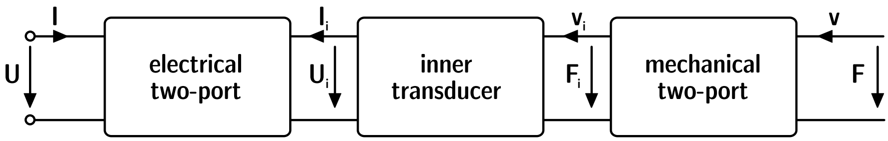


Fig. 10.17. Network representing an electromechanical transducer (analogy I)

Reciprocity also holds here, for instance, in the form of

$$\begin{aligned} \frac{F_{v=0}}{I} &= \pm \frac{U_{I=0}}{v} \\ \frac{v_{F=0}}{I} &= \mp \frac{U_{I=0}}{F} \end{aligned} \tag{10.30}$$

with the upper sign denoting the rule for electric field transducer (force/voltage transduction), and the lower for magnetic field transducers with (force/current transduction). The left side represents an actuator (or loudspeaker), the right side a sensor (microphone).

Electroacoustic transducers such as loudspeakers and microphones are modelled as mechanical transducers with division of the force by their membrane area, S. The force, thus, becomes a pressure and the velocity a volume flow. This is represented exactly by a transformer with the ratio $1{:}S$. The port with volume flow and pressure can finally be coupled with radiation impedances, Eq. (2.18), for instance.

It is worth mentioning that the two-port model can be extended into multiports, if the paths are clearly separated. One very efficient way of separation in terms of linear combination is a modal basis. This way, even distributed fields can be used as input and output data, if the fields are clearly defined in their modal contribution factors.

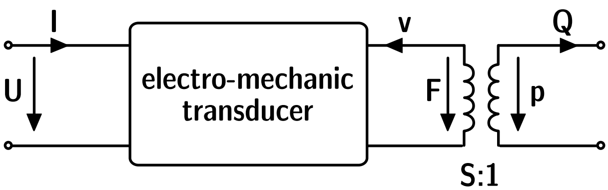


Fig. 10.18. Electroacoustic two-port (analogy I)

10.2.1 Transfer path models

Complex networks can be established on the basis of general two-port theory. They can be used for airborne sound propagation, vibration propagation, structure-borne sound radiation and auralization. The energy transmitted in the network is separated into paths, as illustrated in Fig. 10.19. Ideal or real force or velocity sources are connected directly to the two-port network. The difference between ideal and real sources is the impedance coupling between the source and the transmission system.

For airborne sound paths, the source signals are coupled with transfer functions to the receiver (monaural or binaural). Modelling feedback is generally not required. Structure-borne should be modelled with feedback, and this requires network analysis in a first step and signal flow calculation in a second step.

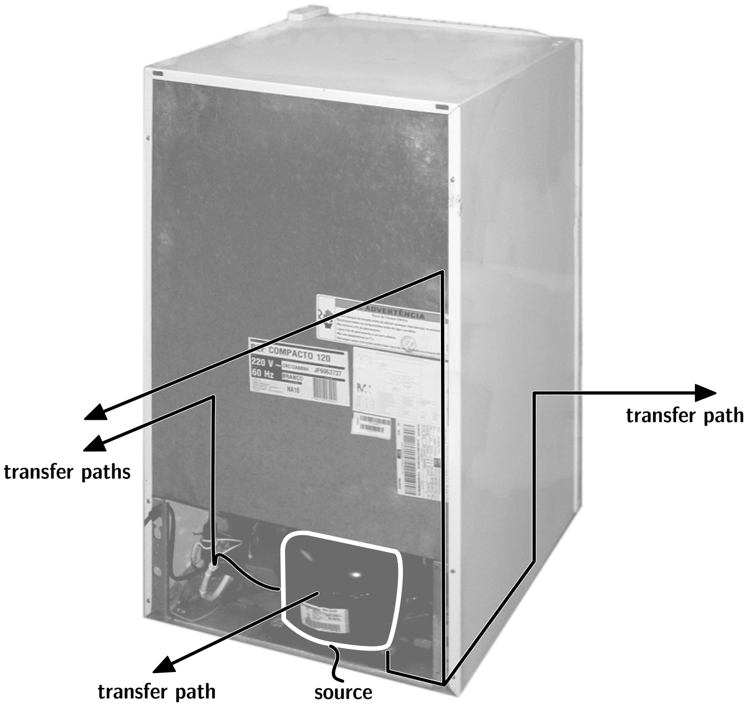


Fig. 10.19. Transfer path separation. Illustrated with the example of a refrigerator

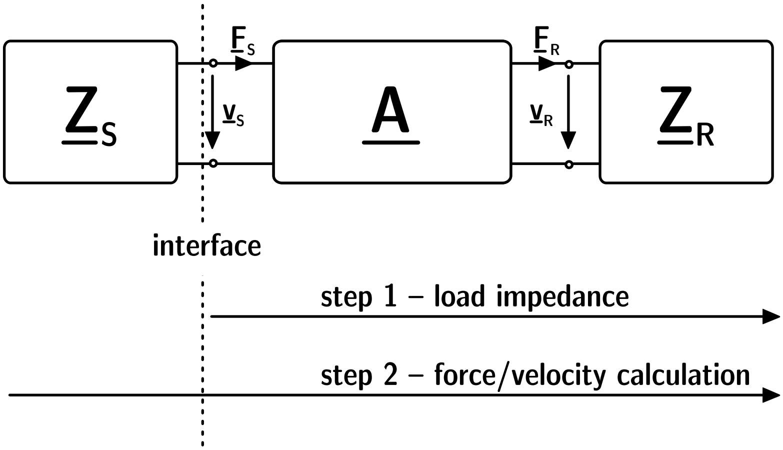


Fig. 10.20. Two-port network of source impedance $\underline{Z}_S$, transfer matrix $\underline{\mathbf{A}}$ and receiver impedance, $\underline{Z}_R$ in analogy II (force/current)

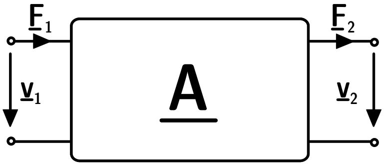
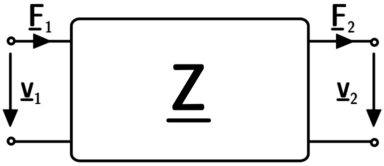
As illustrated in Fig. 10.20, in the first step, the two-port network must be analyzed for the total impedance at the interface to the source. Only when this load impedance is known, can the power output of the source be calculated, in dependence on the inner impedance of the source. This general principle must be applied to simple one-dimensional networks and to complex matrix formulations.

The matrix $\underline{\mathbf{A}}$ (chain matrix) contains the mechanical impedances, admittances and other (dimensionless) parameters. The exact meaning of the matrix elements depends on the analogy used (see Sect. 10.2). The matrix can also be rearranged into a pure impedance matrix, $\underline{\mathbf{Z}}$:

$$\begin{pmatrix} \underline{F}_1 \\ \underline{F}_2 \end{pmatrix} = \begin{pmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{pmatrix} \cdot \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix} = \underline{\mathbf{Z}} \cdot \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix}. \tag{10.31}$$

The two-port parameters $\underline{Z}_{11}$ to $\underline{Z}_{22}$ are called input and output impedances, respectively, while $\underline{Z}_{12}$ and $\underline{Z}_{21}$ are called transfer impedances. $\underline{\mathbf{A}}$ can be easily transformed into $\underline{\mathbf{Z}}$. For reciprocal networks, det $\underline{\mathbf{A}} = 1$ holds. The task in transfer path analysis is determining the matrix elements; see also Chap. 14.

Table 10.2. Two-port matrix conversion, after (Dohm 2004)

Chain form	Impedance form
	
$\begin{pmatrix} \underline{F}_1 \\ \underline{v}_1 \end{pmatrix} = \begin{pmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{pmatrix} \cdot \begin{pmatrix} \underline{F}_2 \\ \underline{v}_2 \end{pmatrix} = \underline{\mathbf{A}} \cdot \begin{pmatrix} \underline{F}_2 \\ \underline{v}_2 \end{pmatrix}$	$\begin{pmatrix} \underline{F}_1 \\ \underline{F}_2 \end{pmatrix} = \begin{pmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{pmatrix} \cdot \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix} = \underline{\mathbf{Z}} \cdot \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix}$
$\begin{pmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{pmatrix}$	$\underline{\mathbf{A}} = \begin{pmatrix} \dfrac{\underline{Z}_{11}}{\underline{Z}_{21}} & \dfrac{\det \underline{\mathbf{Z}}}{\underline{Z}_{21}} \\ \dfrac{1}{\underline{Z}_{21}} & \dfrac{\underline{Z}_{22}}{\underline{Z}_{21}} \end{pmatrix}$
$\underline{\mathbf{Z}} = \begin{pmatrix} \dfrac{\underline{A}_{11}}{\underline{A}_{21}} & \dfrac{\det \underline{\mathbf{A}}}{\underline{A}_{21}} \\ \dfrac{1}{\underline{A}_{21}} & \dfrac{\underline{A}_{22}}{\underline{A}_{21}} \end{pmatrix}$	$\begin{pmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{pmatrix}$

The prerequisite of concentrated elements should be discussed in more detail. The elastic and viscous effects of large construction elements such as dampers or springs, cannot always be separated into concentrated elements. At small wavelengths, too, they act as waveguides of propagating or standing waves. As in modelling electromagnetic waveguides, they can be modelled as quasi-continuous networks of parallel circuits, each representing a small interval Δx which is appropriate for one-dimensional wave propagation; see also Sect. 10.1.3, Waveguides.

10.3 Other models

Not all models for simulating sound and vibration can be described in detail here. If the sources and the sound propagation need not be separated or if stimuli are to be created on the basis of an experimental approach mixed with a technical parameter model, sound synthesis is a possible tool.

Sound synthesis can be an adequate model for creating mixtures of tones and noise with specific harmonic, stochastic and temporal content for subjective testing. The approach is similar to transfer path models with the difference that not transfer paths but signal content is separated (in the analysis) and recombined in the synthesis. This model can be applied in noise control as well as in modelling musical sounds.

For studies of the propagation paths without the need for predicting propagation spectra exactly, wave-front synthesis by using finite time differences offer an insight into the field of travelling waves. In mesh-based time domain models, there is no a priori assumption of wave types. The waves are developing on the basis of the mesh itself (waveguide mesh), provided the degrees of freedom of motion and forces are integrated in the model equations. The mesh must be uniform.

The equations are discretized acoustic field equations (Eqs. (1.11) and (1.12)). They yield (here, in one-dimensional form)

$$-\frac{\Delta p}{\Delta x} = \rho_0 \frac{\Delta v}{\Delta t}, \tag{10.32}$$

$$-\rho_0 \frac{\Delta v}{\Delta x} = \frac{1}{c^2} \frac{\Delta p}{\Delta t} + \rho_0 q. \tag{10.33}$$

The problem, however, is that the wave propagation, its speed and its interaction with boundaries can be solved exactly only for 1-D cases. Examples of 2-D problems were presented as approximations, but FTD in 3-D domains suffers from severe artefacts such as dispersion, unless corrections are introduced (Savioja and Välimäki 2000). The reason is the geometrically impossible condition of creating a perfectly uniform (isotropic) mesh. Furthermore, specific impedance boundary conditions other than ideally hard (or soft) surfaces cannot be implemented. The finite time difference method, however, is useful for illustration and animation of wave-front propagation.