

Experimental Study and Theoretical Analysis of Signage Legibility Distances as a Function of Observation Angle

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Signage systems are widely used in buildings to provide information for wayfinding, thereby assisting in navigation during normal circulation of pedestrians and, more importantly, exiting information during emergencies. An important consideration in determining the effectiveness of signs is establishing the region from which the sign is visible to occupants, the so-called Visibility Catchment Area (VCA). This paper attempts to factor into the determination of the VCA of signs, the observation angle of the observer using both experimental and theoretical analysis.

1. Introduction

Signage within complex building spaces is intended to provide occupants with information relating to wayfinding. A successful signage system can reduce the apparent complexity of an enclosure thereby improving wayfinding under both general circulation and emergency conditions. While inefficient signage may contribute to loss of commercial earnings in general circulation situations, it can have more serious consequences in emergency situations. It has been known for many years^{1,2} that in emergency situations occupant unfamiliarity with exit routes can significantly contribute to the resulting casualties³⁻⁷. To ensure reliable recognition and comprehension of signage information, safety signs are required to conform to certain design criteria specified in various national and international standards and guideline documents.

These documents usually contain basic requirements relating to the size of the sign, the size of the premises and the intended use of the premises^{8,9}. As an example consider the NFPA Life Safety Code Handbook⁸. This suggests that reflective signs that have a lettering height of 15.2 cm are legible for up to a distance of 30m. To extend the visibility of a sign the letter height can be increased, with a linear relationship existing between lettering height and visibility distance. These design criteria are generally based on data collected from standard eyesight tests, which involve participants viewing a sign of given size viewed with an observation angle of zero degrees (i.e. the sign is viewed straight on). This enables the determination of maximum viewing distances as a function of the letter height. However, in reality occupants may approach a sign from a multitude of angles (i.e. non zero observation angles), which in turn will influence the ability of the individual to resolve the sign. This influence on sign legibility has been virtually ignored to date.

Evacuation and pedestrian circulation models¹⁰ have also generally ignored the interaction of occupants with the wayfinding system; the implicit assumption in most of these techniques is that the occupants “know” the route. While this may be appropriate in many situations, it is clearly a simplification of reality. In order to produce realistic representation of evacuation and circulation in arbitrarily complex structures, it is necessary to represent the interaction between occupants and signage systems.

Recently the representation of the interaction between modelled agents and signage systems has been introduced into the buildingEXODUS evacuation model through the concept of the Visibility Catchment Area (VCA)^{11,12}. The VCA of a sign is defined as the region from where it is physically possible to visually receive and discern information from the sign. Within this model, the maximum viewing distance or the VCA termination distance, is currently arbitrarily set as the distance specified in regulations⁸.

In this paper we examine, through theoretical analysis and experimental study, the relationship between sign lettering size, observation angle and maximum viewing distance. Through this work we establish new maximum viewing distances as a function of observation angle. These results are then incorporated within the VCA concept and several demonstration applications of the new model are presented. It is important to note that this work does not include recognition of signage pictograms. Recognition of pictograms is expected to occur at greater distances than the legibility distance of text. Thus, the legibility distance represents a conservative or lower limit of the maximum recognition distance.

2. The Evacuation model

2.1. buildingEXODUS

The core software used in this paper is the buildingEXODUS V4.0 evacuation model. The basis of the model has frequently been described in other publications^{13,14} and so will only be briefly described here. EXODUS is a suite of software tools designed to simulate the evacuation of large numbers of people from complex enclosures. The version of the software used to simulate evacuation from the built environment is known as buildingEXODUS. The software takes into consideration people-people, people-fire and people-structure interactions. The model tracks the trajectory of each individual as they make their way out of the enclosure, or are overcome by fire hazards such as heat, smoke and toxic gases. The software has been written in C++ using Object Orientated techniques utilising rule base technology to control the simulation. Thus, the behaviour and movement of each individual is determined by a set of heuristics or rules. For additional flexibility these rules have been categorised into five interacting submodels, the OCCUPANT, MOVEMENT, BEHAVIOUR, TOXICITY and HAZARD submodels. These submodels operate on a region of space defined by the GEOMETRY of the enclosure.

2.2. VCA Concept

building EXODUS V4.0 currently includes a method of representing the visibility of particular objects through the application of the VCA concept. The VCA of an object is defined as the region of space from where it is possible to visually receive information from that object; i.e. from where the information of the object is legible. The VCA of a sign attempts to address only the physical aspect of visibility, leaving the psychological and physiological aspects of sign recognition to the behaviour component of the model.

The algorithm uses a line of sight search method to determine the locations within the geometry that have visual access to the sign. Geometrically the VCA of a sign is assumed to correspond to the visibility polygon¹⁵ spanning outwards from a point. The calculation of the VCA considers the location of the sign, the size of the lettering on the sign, its height above the floor, the position and size of any obstructions and the observer height.

For simplicity the current algorithm used to determine the VCA makes use of the central point of the lower edge of the sign and a point in space at a height equal to that of the average occupant, which is a user defined parameter. In this work the default average occupant height is arbitrarily taken as 1.75m. Another feature which will influence the size and shape of the VCA is the observation angle. The observation angle is defined as the angle subtended by the observers line of sight to a normal line bisecting the surface of the sign. An observation angle of 0° means that the observer is viewing the sign straight on. There will be a maximum observation angle beyond which it will no longer be possible to resolve the sign and hence it will not be possible to detect the sign. Due to lack of data, in the current implementation¹², the extent to which the observation angle of the observer impacts the shape of the VCA is arbitrarily set to 85° .

3. A theoretical representation for the angular extent of the VCA

In the previous section, the method of representing the VCA of a sign implemented within the software was described. However, this method only approximates the VCA due to the assumption that the level of visibility afforded to the individual viewing a sign (i.e. the maximum distance from which a sign can be seen) was independent of the observation angle. This was primarily imposed on the approach due to the lack of reliable data linking observation angle with maximum viewing distance. Using this approach, the VCA for a sign would include a region of space defined by a near semi-circle, with a centre point located at the centre of the sign and a radius determined by the maximum viewing distance as defined by regulation. This region is represented in Figure 1 by the dashed semi-circular line.

It was suggested¹² that the size and shape of the VCA will be further influenced by the ability of an observer to resolve the angular separation of the sign. This is defined as the apparent angular separation of the ends of the sign as measured by a distant observer (i.e. angle φ in Figure 1). The angular separation of the sign will be dependent on the size of the sign (or more correctly the size of the letters on the sign), the distance of the observer from the centre of the sign and the observation angle. The observation angle is defined as the angle subtended by the observers line of sight to a normal line bisecting the surface of the sign (i.e. θ in Figure 1). An observation angle of 90° (i.e. viewing the sign side on) results in an angular separation (φ) of 0° , effectively making the sign invisible to the observer, while an observation angle of 0° (i.e. viewing the sign straight on) provides the maximum angular separation at fixed observation distance.

Clearly, there will be a minimum angular separation (φ_{\min}) beyond which it will no longer be possible to resolve the sign and hence there will be a maximum observation angle beyond which it will be impossible to detect the sign. In this work, the minimum angular separation (φ_{\min}) which can be resolved by the human eye is taken as a constant. For a sign of fixed size with an observer at a fixed distance from the centre of the sign, as the observation angle (θ) increases, the angular separation (φ) of the sign decreases until a maximum observation angle is reached beyond which it is no longer possible to resolve the angular separation of the sign (i.e. $\varphi < \varphi_{\min}$). Thus for a sign of given sign size, in order to resolve the angular separation of the sign, as the observation angle increases, the maximum viewing distance must decrease. Similarly, for a given viewing distance there will be a maximum observation angle beyond which the sign cannot be resolved. As the size of the sign increases, both the maximum viewing distance and the maximum observation angle increases.

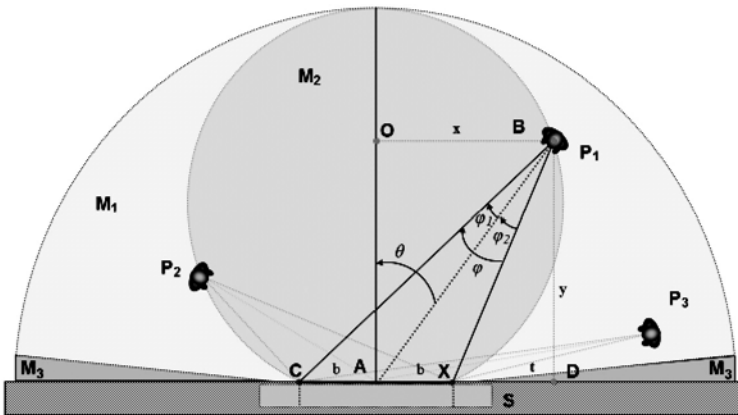


Figure 3: The geometric relationship between the observer and the sign.

Thus for an observer to be able to resolve a sign (i.e. make out the individual elements in the sign) at the maximum observation distance, the observation angle should be such that the angular separation of the individual elements making up the sign are greater than or equal to φ_{\min} .

To estimate φ_{\min} we use the maximum viewing distance for viewing signs with an observation angle of 0° (i.e. straight on) as specified in the NFPA Life Safety Code Handbook. For signs with lettering of 15.2 cm height, the maximum viewing distance is 30m ⁸. This produces a φ_{\min} of 0.29° .

Thus, in order to resolve the information on a sign at the maximum viewing distance, the observation angle should be such that the angular separation of the elements in the sign is greater than or equal to φ_{\min} or 0.29° . This problem can be described geometrically by considering the relative positions of the observer, the sign and the observation angle. Depicted in Figure 1 are the positions of a sign (S) and an observer (P_1). Angle φ represents the angular separation of an element of sign S from observer P_1 . In order for the observer to be able to read the lettering on the sign the distance from the sign must be such that the angle φ is greater than or equal to φ_{\min} or 0.29° . Therefore as the observation angle increases, the maximum distance AB must decrease in order to maintain the angular separation of the sign to φ_{\min} . By determining the length of the line AB within the constraints of the angular resolution of the eye the visibility catchment area of the sign can then be defined.

Known values	Description
φ	Angular separation of the sign; $\varphi = \varphi_1 + \varphi_2$
θ	Observation angle in relation to the normal line bisecting the surface of the sign
CX	Size of an element that an individual uses to resolve the sign (i.e. sign lettering)
b	b is set to half of the size of the recognisable element (CX)

Table 1: Variables used in formulation

The most efficient method of determining whether the sign is visible from a particular location within the geometry, given the considerations described above, would be to determine the geometrical shape that is formed by the maximum viewable distance from the sign. Considering the configuration of Figure 1, the known variables are listed in

Table 1. It can be shown that the geometrical shape of the visible region of the sign is given by the following formulation:

$$\left(\frac{b}{\sin(\varphi)}\right)^2 = x^2 + \left(y - \frac{b}{\tan(\varphi)}\right)^2 \quad (1)$$

This has the equivalent form of a circle with centre at point $(0, b/\tan(\varphi))$ and radius $b/\sin(\varphi)$. This circle defines the VCA of sign S that is formed of text elements of dimension CX (see Figure 1) assuming a constant angular separation of φ_{\min} degrees (i.e. 0.29° derived from the NFPA regulation). In this instance, we assume in this calculation that the human ability to resolve vertical components of the sign (i.e. the height of the text) is equivalent to their ability to resolve horizontal components.

Figure 1 depicts the catchment area of sign S generated using the original algorithm (area M_1) – which effectively ignores the dependence of VCA on observation angle. This image is overlapped by the catchment area of the formulation derived above, labelled as M_2 in the same figure. The restrictions imposed upon the VCA produced by the formulation are clearly evident, as is its circular appearance.

It has been shown theoretically that if the ability of an observer to resolve a sign is based on the assumption that the eye can resolve angular separations down to a constant minimum value (irrespective of observation angle), then the maximum viewing distance will decrease as the observation angle increases. This is an important result as the regulations implicitly assume that viewing distance is independent of observation angle. Furthermore, instead of the VCA being defined by a semi-circular region, as is implicitly assumed in regulation, from the above analysis we note that the VCA has a circular appearance with diameter approximately equivalent to the radius of the previously assumed semi-circular VCA.

In the next section, this theoretical finding is examined through a series of experimental trials designed specifically to examine this aspect of signage visibility.

4. Experimental method

The purpose of the trials was to test the theory presented in the previous section that the distance from which a sign can be perceived is dependent upon the angle at which it is approached. The experimental trials have been designed specifically to examine the distances from which individual participants are able to recognise the text (or some portion of it) within the sign for given observation angles.

The trials were completed by 48 volunteers, consisting of 29 males and 19 females, each of whom experienced the same number (15) of experimental conditions. The or-

der in which these conditions were experienced was varied in a systematic manner in order to minimise the influence of uncontrolled variables, e.g. learning. The vision of approximately 55% of the sample required constant correction in the form of spectacles or contact lenses, which were used during the trials. A detailed analysis of the results according to a number of variables (e.g. prior eye workload, text size, and use of visual correction) was produced; however the presentation of this material is beyond the scope of this paper.

The experiment was performed in a corridor 39 metres in length with consistent artificial illumination along its length. Three standard signs were used during these trials: two plastic signs and one photo luminescent sign. These signs varied in the letter size of the text, the case of the text and the background colour of the signs. The text within these signs differed in the height and the width of text, and the thickness of script that formed the text. Although the three signs used were of standard designs, it was felt that a variety of text types and signage designs were required in order to strengthen the credibility of the results produced.

Given the restricted nature of the corridor, the sign used in each trial was placed on a pivoting platform. Thus the observation angle was changed by varying the orientation of the sign to the observer rather than the observer to the sign. In this way each participant commenced the trial in the same location and approached the sign along the same path, irrespective of the observation angle. Five different observation angles were experienced by each participant: 0°, 30°, 60°, 70° and 80°. For each observation angle the observer would approach the sign until the lettering on the sign was legible. Therefore, each participant was equally exposed to 15 trial conditions (3 signs × 5 angles) in total.

5. The experimental results

The average viewing distance for each of the three signs at the five observation angles are shown in Table 2 for each of the categories. The results presented in Table 2 clearly demonstrate a relationship between the observation angle and the distance from which the text in the sign could be resolved: for all of the signs as the angle of observation is increased, the maximum viewing distance at which the text in the sign could be resolved decreased. From the experimental results it is apparent that the relationship between maximum viewing distance and observation angle is nonlinear and consistent for each of the three types of sign.

	Mean viewing distance at each observation angle (m)				
	0°	30°	60°	70°	80°
Sign 1	23.38	21.09	14.82	10.12	5.10
Sign 2	33.11	30.79	21.23	13.64	6.34
Sign 3	19.84	18.98	12.65	9.04	4.60

Table 2: Mean viewing distances of three signs at 5 different observation angles.

To further examine the relationship between observation angle and viewing distance the data is plotted using polar co-ordinates, with θ (the rotational ordinate) representing the observational angle and r (the radial measurement) representing the distance at which the text in the sign could be resolved. In Figure 2(a) the results are presented in this form for each of the signs examined and are reflected on the vertical axis. The validity of this action is based on the assumption that the observational angle is independent of the direction of the approach to the sign (i.e. whether they approach from the left or the right side). A solid curve passes through the average of the data-sets collected. It is apparent that although the size of the curve produced in each of these graphs is different, their general shape is similar: a circle is approximated by the curve connecting the averages of the five experimental conditions examined for each of the signs.

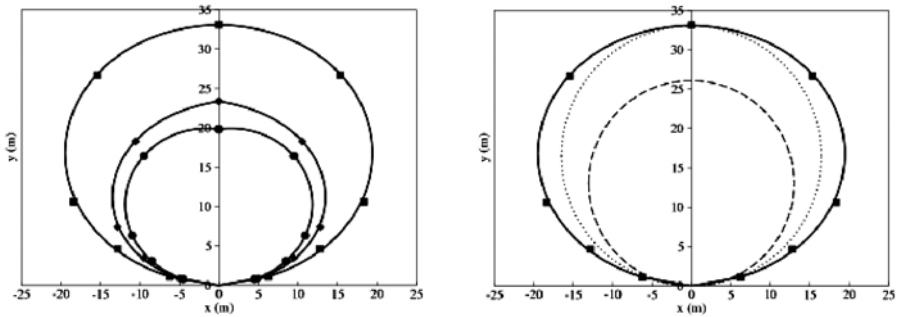


Figure 2: (a) The reflection of the original experimental data of Sign 1 (◆), Sign 2 (■) and Sign 3 (●), across the vertical axis, and (b) the comparison of the experimental VCA of Sign 1 (solid curve) and two VCAs of the same sign based on the theoretical model discussed in Section 4 using the maximum viewing distances suggested by BS 5499⁹ (dotted curve) and the NFPA Life Safety Code Handbook⁸ (dashed curve) respectively. In both cases, the safety factor of 2 is excluded.

The curves generated from the experimental data represent a slightly flattened circle; moreover, from Figure 2(b) this closely approximates the theoretical findings discussed in Section 3 and clearly contradicts the implicit assumption used within building regulations that the maximum distance from which a sign can be resolved is independent of the observation angle.

In the first edition of BS 5499⁹ the following formulation is provided relating the viewing distance (D) to the height of text (h)

$$D=250h \tag{2}$$

As mentioned previously, this formulation is based on the result of eye sight tests that people with normal (or corrected to normal) vision can reliably resolve a detail that

subtends an angle of 1 minute. This formulation also includes a small additional margin of extra difficulty in resolving some complex letters and a safety factor of 2.0 in order to guarantee a conservative estimate of the distance from which the sign can be resolved. Finally the coefficient is rounded off to two significant figures¹⁶. For instance, given the height of the text on Sign 2, the results produced by the formulation is

$$D=250 \times 0.066 = 16.5m,$$

which is approximately half of the measured average viewing distance (33.11m) for Sign 2 with observation angle of 0°. Given the incorporated safety factor of 2.0 and the other correctional factors mentioned, this approximates the findings of our experimental trials. The value describing the angular resolution of the eye demonstrated in this experiment is therefore consistent with the advice provided in the regulatory documentation. Alternatively, it should be noted that the NFPA Life Safety Code Handbook⁸ suggests a viewing distance of 30m for the exit lettering with a height of 15.2 cm. Again if the safety factor is taken into consideration, it approximates the relationship between sign size and average maximum viewing distance produced during the experimental trials, but it is a little more conservative compared to BS 5499⁹ standard (see Figure 2(b)).

It can be shown that the maximum viewing distances recorded during the trials approximate the values assumed in the NFPA and BS 5499 formulation, adding some credibility to the experimental conditions.

The results of the experiment indicate that the VCA of a sign approximates a circle. This confirms the initial hypothesis that a sign can be seen by an observer from a circular area located at a tangent to the surface of the sign. This is due to the constant nature of the angular resolution of the human eye and the non-linear relationship between the observational angle and maximum distance from which the sign can be resolved. Within building EXODUS the theoretical model describing the non-linear relationship between observation angle and maximum viewing distance has been implemented. This produces conservative results as it generates a circular VCA with the same maximum radius as the flattened circle generated from the experiment (VCA circle from theory lies within the flattened VCA circle produced by experiment).

6. Implementation of proposed algorithm into the evacuation software

The algorithm described in Section 3 has been implemented in prototype form within building EXODUS. Here we demonstrate the performance of the algorithm using an example, which assumes a complex compartment with many internal obstacles. The geometry used in this example is the supermarket layout used in previous analysis of the VCA¹². The geometry will only be briefly described in this section as a fuller account can be found in previous publications¹². The supermarket contains an array of internal shelving components, tills and a café in the southern part of the geometry. Four main

exit points (exits 3, 4, 5, 6) are located at the south side of the building. Four emergency exits (exits 1, 2, 7, 8) are available: two on the east side and two on the west side. The total free area of the supermarket has been calculated within the model to be approximately 2927 m² after the shelving and other furnishings have been taken into account. Signage is provided by exit signs located above each of the exits (main and emergency) and by two sets of four connected signs at the cross aisles.

The majority of the shelving extends to a height of 2.5m. However, there are some shelves with a height of 1.8m and the tills and tables in the café area have a height of 1.2m. The emergency exit signage is positioned at a height of 2.2m above the floor. All the remaining features are at ceiling height, thus preventing any visibility access past them. The height of the shelving and furnishings is taken into account when calculating the VCA of each exit. The width of each door is assumed to be 2.5m. The signs are assumed to have lettering of 15.2cm corresponding to a visibility cut off distance of 30m as suggested by the NFPA Code⁸ and the observer is assumed to have the default height of 1.75m.


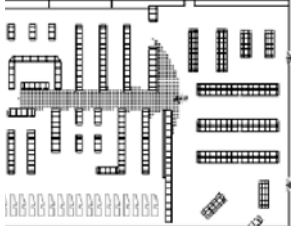
	Existing VCA method	Prototype VCA method
VCA of sign S6		
Area covered	196.75m ²	142.75m ²
Percent coverage	6.72%	4.88%

Table 3: VCA comparison of sign S6 between the existing and prototype methods

The VCA of the signage system is determined using both methods. Using the existing method, the combined VCA of all the signs is 2006.25m², while using the prototype algorithm produces a combined VCA of 1896.0m²; thus, the existing method over estimates the VCA by some 6% (or 110m²). Presented in Table 3 is an example of the difference between the VCA produced by both methods for an interior sign S6.

In a simple example, the differences between the VCA produced by the two techniques was shown to be significant (see a comparison between area M₁ and area M₂ in Figure 1). However, these differences are somewhat diminished as the complexity of the compartment is increased through the introduction of internal obstacles. This is due to the presence of the obstacles intercepting and preventing the propagation of the VCA. In

this way, the presence of the obstacles masks some of the over estimation produced by earlier method.

We note that the average Total Evacuation Time was 1 min 23 secs using the existing technique and 1 min 21 secs using the new algorithm. The average individual evacuation time was 34.2 seconds and 33.8 second for the old and new approach respectively. The average congestion experienced by an individual was 10.9 and 9.9 seconds while the average distance travelled was 28.7 m and 29.5 m for the old and new approach respectively.

As is to be expected, the reduction in VCA generated by the new algorithm has resulted in a greater number of occupants utilising the normally used (or main) exits – i.e. exits 3-6. On average there are some 17 additional people utilising the main exits when the new algorithm is used to determine the VCA. As a result there is a slight increase in the average distance travelled, generated by a larger section of the population not utilising the nearer emergency exits. In this case, the slight decrease in the number of occupants using the emergency exits has resulted in a slight decrease in the levels of congestion experienced (at the emergency exits) which in turn has resulted in a slight decrease in both the average overall evacuation time and the average personal evacuation time. Thus the differences in the key results produced by the incorporation of the new technique of calculating the VCA in this example are small and self consistent.

7. Conclusion

In this paper we have demonstrated both theoretically and through experimental trials that the maximum viewing distance is dependent on the viewing angle and that as the viewing angle increases, the maximum viewing distance decreases in a non-linear manner. This is the result of the angular separation of the sign (or more precisely the angular separation of the lettering on the sign) decreasing as the angle of observation increases at fixed observation distance and the human eye possessing a lower limit to its angular resolving abilities. Furthermore, when the viewing angle is taken into consideration, the VCA associated with the sign describes an area defined by a flattened circle which is tangent to the surface of the sign with minor radius equal to the previously defined semi-circle or half of that if the safety factor is considered.

These results are valuable in their own right as they more accurately define the visibility limits of signs. In addition, the method of determining the VCA of signs has been implemented with the buildingEXODUS evacuation model providing a more accurate way of determining the visibility of signs in complex geometries. The impact that the new developments may exert when combined with the other factors evident during a simulated evacuation have been shown to be sensitive to the complexity of the geometry and the scenario modelled. While the overall differences in the key evacuation indicators (e.g. average total evacuation time and average personal evacuation time) resulting from

the introduction of the new developments may on occasion be small, it is essential to correctly represent these subtleties if the model is to correctly represent reality.

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References

1. J. Sime: *Escape Behaviour In Fire: ,Panic‘ Or Affiliation?*, PhD Thesis, Department of Psychology, University Of Surrey (1984).
2. S. Gwynne, E.R. Galea, M. Owen, and P.J. Lawrence: *Escape as a Social Response*, Published by the Society of Fire Protection Engineers (1999).
3. P.M. Weinspach, J. Gundlach, H.G. Klingelhofer, R. Ries, and U. Schneider: *Analysis of the Fire on April 11th, 1996, Recommendations and Consequences for Dusseldorf Rhein-Ruhr-Airport*, Staatskanzlei Nordrhein-Westfalen, Mannesmannufer 1 A, 40190 Düsseldorf, Germany (1997).
4. Report of the Tribunal of Inquiry on the Fire at the Stardust, Artane, Dublin, 14th Feb 1981 (1981).
5. R.L. Best: *Reconstruction of a Tragedy: The Beverly Hills Supper Club Fire*, Southgate, Kentucky, May 28, NFPA (1977).
6. Summerland Fire Commission Report, Douglas: Isle of Man Fire Report (1974).
7. W. Grosshandler: *The Station Nightclub Fire, Federal Advisory Committee*, Building and Fire Research Laboratory, National Institute of Standards and Technology, December 3, 2003, http://wtc.nist.gov/media/Final_RI_Station_Nightclub_Status_12-3.pdf.
8. NFPA, *Life Safety Code Handbook*, National Fire Protection Association, Quincy, MA (1997).
9. BS5499-1:1990, *Fire Safety Signs, Notices and Graphic Symbols. Specification for fire safety signs*, ISBN 0 580 18830 2, UK (1990).
10. S. Gwynne, E.R. Galea, M. Owen, P.J. Lawrence, and L. Filippidis: *Review of Modelling Methodologies Used in the Simulation of Evacuation*, Journal of Building and the Environment, 34, pp. 441-749 (1999).
11. L. Filippidis, S. Gwynne, E.R. Galea, and P.J. Lawrence: *Simulating the Interaction of Pedestrians with Wayfinding Systems*, Proceedings of the 2nd International Conference on Pedestrian and Evacuation Dynamics, E.R.Galea (Ed.), CMS Press, Greenwich, London, ISBN 1904521088, pp. 39-50 (2003).
12. L. Filippidis, E.R. Galea, S. Gwynne, and P.J. Lawrence: *Representing the Influence of Signage on Evacuation Behaviour with Evacuation Models*, to appear in the Journal of Fire Protection Engineering, Sage Publications (2005).

13. E.R. Galea, S. Gwynne, P.J. Lawrence, L. Filippidis, and D. Blackshields: *buildingEXODUS V4.0 User Guide and Technical Manual*, University of Greenwich, London (2004).
14. S. Gwynne, E.R. Galea, P.J. Lawrence, and L. Filippidis: *Modelling Occupant Interaction with Fire Conditions Using the buildingEXODUS Evacuation Model*, *Fire Safety Journal*, 36, pp. 327-357 (2001).
15. D. Avis, G.T. Toussaint: *An Optimal Algorithm for Determining the Visibility of a Polygon from an Edge*, *IEEE Trans. On Computers*, C-30, No.12, pp. 910-914 (1981).
16. J. Creak: *Viewing Distances, Means of Escape* (1997).