Synchronization of Chaotic Neural Networks: A Generalized Hamiltonian Systems Approach

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Abstract. In this paper, we use a Generalized Hamiltonian forms approach to synchronize chaotic neural networks unidirectionally coupled. Synchronization is thus between the master and the slave networks with the slave network being given by an observer. In particular, we present two cases of study: the first is a second-order 3×4 **CNN** array, and the second is a **CNN** with delay. The chaotic **CNNs** are used as transmitter and receiver in encrypted information transmission.

1 Introduction

In recent years many complex network structures have been observed in diverse fields as physics, biology, economics, ecology, electronics and computer science. In particular, Cellular Neural Networks (**CNNs**) constitute an important example in such cases. **CNN** is a nonlinear system defined by coupling only identical simple dynamical systems called cells located within a prescribed sphere of influence, such as nearest neighbors [3]. **CNN** has broad applications in image and video signal processing, robotic and biological visions [30], and higher brain functions [18]. Many proceedings of workshop and special issues see e.g., [23]; [24]; [25]; [26] have been devoted to **CNNs.**

On the other hand, recently synchronization of complex dynamics (chaotic and hyperchaotic) has become a field of active research see e.g., [20];

C. Posadas-Castillo et al.: *Synchronization of Chaotic Neural Networks: A Generalized Hamiltonian Systems Approach*, StudFuzz **208**, 59–73 (2007)
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[17]; [27]; [28]; [29]; [10]; [4]; [5]; [22]; [21]; [1]; [7]; [11]; [12]; [13]; and references therein. Data encryption using chaotic dynamics was reported in the early 1990's as a new approach for signal encoding which differs from the conventional methods using numerical algorithms as the encryption key. One of the motivations for synchronization is the possibility of sending confidential information through chaotic signals for secure communications. The idea is use two highly dynamic nonlinear systems (as transmitter and receiver). So, the confidential information is imbedded into the transmitted chaotic signal by direct modulation, masking or another method. At the receiver end, if chaos synchronization can be achieved, then it is possible to recover the original information. The communication schemes based on chaos synchronization can be broadly categorized into three approaches. They include the chaotic masking scheme [8], the chaotic shift keying scheme [19]; [8]; [9], and the chaotic modulation scheme [31].

The main goal of this paper is to synchronize chaotic neural networks. This objective is achieved by using Generalized Hamiltonian forms and observer approach developed in [22]. Moreover, we proceed to illustrate this synchrony to transmit encrypted confidential information using a modified chaos-based communication scheme [16]; [14]. The synchronization method presents the following advantages: i) it is systematic, ii) it is useful to synchronize several well-known chaotic and hyperchaotic oscillators, iii) it does not require the computation of any Lyapunov exponent, and iv) it does not require initial conditions belonging to the same basin of attraction.

The paper is organized as follows: In Section 2, we give a brief review on chaos synchronization via Generalized Hamiltonian forms and observer approach. In Section 3, we apply this approach to synchronize chaotic neural networks using two numerical examples; a second-order 3×4 **CNN** array and a **CNN** with delay. In Section 4, we present the stability analysis related to the synchronization process. In Section 5, we apply the synchronization of chaotic neural networks to confidential communication for transmission and recovering of audio messages. Finally, in Section 6, we give some concluding remarks.

2 Review of Chaos Synchronization via Hamiltonian Forms and Observer Approach

Consider the following n-dimensional autonomous system

$$\dot{x} = f(x(t)), \qquad x \in \Re^n, \tag{1}$$

which provides an example of complex oscillator, whit f a nonlinear function of the state x. Following the approach provided in [22], many **CNN** models described by Eq. (1) can be written in the following "Generalized Hamiltonian" canonical form,

$$\dot{x} = J(x)\frac{\partial H}{\partial x} + S(x)\frac{\partial H}{\partial x} + F(x) \tag{2}$$

where *H(x)* denotes a smooth *energy function* which is globally positive definite in $\Re^n$. The column *gradient vector* of *H*, denoted by $\partial H/\partial x$, is assumed to exist everywhere. We use quadratic energy function $H(x) = 1/2\, x^T Mx$ with M being a, constant, symmetric positive definite matrix. In such a case, $\partial H/\partial x = Mx$. The square matrices, $J(x)$ and $S(x)$ satisfy, for all $x \in \Re^n$, the following properties, which clearly depict the *energy managing* structure of the system, $J(x) + J^T(x) = 0$ and $S(x) = S^T(x)$. The vector field $J(x)\,\partial H/\partial x$ exhibits the *conservative* part of the system and it is also referred to as the *workless* part, or *workless* forces of the system; and $S(x)$ depicting the *working* or *nonconservative* part of the system. For certain systems, $S(x)$ is *negative definite* or *negative semidefinite*. In such cases, the vector field is addressed to as the *dissipative* part of the system. If, on the other hand, $S(x)$ is positive definite, positive semidefinite, or indefinite, it clearly represents, respectively, the global, semi-global and local *destabilizing* part of the system. In the last case, we can always (although nonuniquely) descompose such an indefinite symmetric matrix into the sum of a symmetric negative semidefinite matrix $R(x)$ and a symmetric positive semidefinite matrix $N(x)$. And where $F(x)$represents a *locally destabilizing* vector field.

We consider a special class of Generalized Hamiltonian systems given by

$$\begin{aligned} \dot{x} &= J(y)\frac{\partial H}{\partial x} + (I + S)\frac{\partial H}{\partial x} + F(y), && x \in \Re^n \\ y &= C\frac{\partial H}{\partial x}, && y \in \Re^m \end{aligned} \tag{3}$$

where S is a constant symmetric matrix, not necessarily of definite sign. The matrix I is a constant skew symmetric matrix. The vector variable $y(t)$ is referred to as the system *output*. The matrix C is a constant matrix. The destabilizing vector field $F(y)$.

We denote the **estimate** of the state vector $x(t)$ by $\xi(t)$, and consider the Hamiltonian energy function $H(\xi)$ to be the particularization of H in terms of $\xi(t)$. Similarly, we denote by $\eta(t)$ the estimated output, computed in terms of the estimated state $\xi(t)$. The gradient vector $\partial H(\xi)/\partial \xi$ is, naturally, of the form $M\xi$ with M being a, constant, symmetric positive definite matrix.

A dynamic nonlinear **state observer** for the special class of Generalized Hamiltonian forms (3) is readily obtained as

$$\begin{aligned} \dot{\xi} &= J(y)\frac{\partial H}{\partial \xi} + (I+S)\frac{\partial H}{\partial \xi} + F(y) + K(y-\eta), \\ \eta &= C\frac{\partial H}{\partial \xi}, \end{aligned} \tag{4}$$

where K is a constant matrix, known as the *observer gain*. The **state estimation error**, defined as $e(t) = x(t) - \xi(t)$ and the output estimation error, defined as $e_y(t) = y(t) - \eta(t)$, are governed by

$$\begin{aligned} \dot{e} &= J(y)\frac{\partial H}{\partial e} + (I+S-KC)\frac{\partial H}{\partial e}, \quad && e \in \Re^n \\ e_y &= C\frac{\partial H}{\partial e}, && e_y \in \Re^m \end{aligned} \tag{5}$$

where the vector, $\partial H/\partial e$ actually stands, with some abuse of notation, for the gradient vector of the *modified* energy function, $\partial H(e)/\partial e = \partial H/\partial x - \partial H/\partial \xi = M(x-\xi) = Me$. We set, when needed, $I + S = W$.

Definition 1 (Complete synchronization problem) We say that the slave system (4) synchronizes with the master system (3), if

$$\lim_{t\to\infty} \|x(t) - \xi(t)\| = 0,$$

no matter which initial conditions $x(0)$ and $\xi(0)$ have. Where the state estimation error $e(t) = x(t) - \xi(t)$ represents the synchronization error.

3 Synchronization of Chaotic Neural Networks: Examples

In this section, we present two numerical examples of synchronization of chaotic neural networks, to this purpose, let us first briefly give a suitable material on CNN.

Definition 2 (CNN) A **CNN** is any spatial arrangement of **locally coupled cells**, where each cell is a dynamical system which has an **input**, and a **state** evolving according to some prescribed dynamical laws [3].

In three-dimensional lattice **CNN** architecture, mathematically each cell C_{ijk} at location (i, j, k) is a dynamical system whose states evolve according to some prescribed state equations, whose dynamics are **coupled** only among the neighboring cells lying within some prescribed **sphere of influence** S_{ijk}, centered at (i, j, k). In two-dimensional case, using a double subscript, the variables for an **isolated** cell are: input $u_{ij}(t) \in \Re^u$, threshold $z_{ij}(t) \in \Re^z$, state $x_{ij}(t) \in \Re^x$, and output $y_{ij}(t) \in \Re^y$. A **CNN** cell is said to be isolated if it is not coupled to any other cell (Fig. 1).

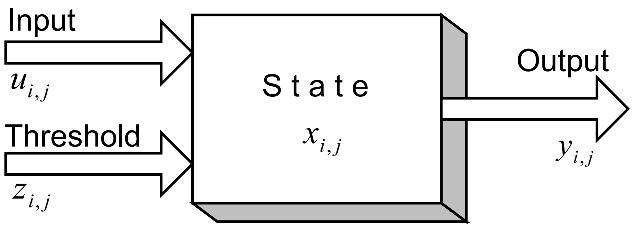

Fig. 1: Isolated cell: input u_{ij}, threshold z_{ij}, state $x_{ij}(t) \in \Re^x$, and output y_{ij} for a two-dimensional **CNN**.

In this work, we will assume that all isolated cells C_{ij} are identical, and that for simplicity we have that $z_{ij}(t)$ is a constant scalar. Besides, we assume that for any $x_{ij}(t_0)$ at $t = t_0$, any threshold $z_{ij}(t)$, and any input $u_{ij}(t)$, the state of each isolated cell C_{ij} is assumed to evolve for all $t > t_0$ as a nonautonomous set of ordinary differential equations

$$\begin{aligned} \dot{x}_{ij} &= f(x_{ij}, z_{ij}, u_{ij}), \qquad i = 1, 2, ..., M; \quad j = 1, 2, ..., N \\ y_{ij} &= g_{ij}(x_{ij}) \end{aligned}$$

where $g_{ij}(\cdot)$ is a nonlinear function of the state. However, in many cases the output of interest often coincides with the state, $y_{ij}(t) = x_{ij}(t)$.

The **standard CNN equations** used most widely in the literature, proposed in [2] for an M×N **CNN** array

$$\dot{x}_{ij} = -x_{ij} + z_{ij} + \sum_{kl \in S_{ij}(r)} a_{kl} y_{kl} + \sum_{kl \in S_{ij}(r)} b_{kl} u_{kl} \quad i = 1, 2, ..., M; \quad j = 1, 2, ..., N \tag{6}$$

$$y_{ij} = f(x_{ij}), \tag{7}$$

where $S_{ij}(r)$ is the sphere of influence of radius r; $\sum_{kl \in S_{ij}(r)} a_{kl} y_{kl}$ and $\sum_{kl \in S_{ij}(r)} b_{kl} u_{kl}$ are the local coupling, and

$$f(x_{ij}) = \frac{1}{2}\left(|x_{ij}+1| - |x_{ij}-1|\right) = \begin{cases} 1, & x_{ij} \geq 1 \\ x_{ij}, & |x_{ij}| < 1 \\ -1, & x_{ij} \leq -1 \end{cases}$$

For the particular case where $M = 3$ and $N = 4$, the Eqs. (6)-(7) assume the simpler form 3×4 **CNN** array

$$\begin{aligned} \dot{x}_1 &= -x_1 + a_{00} f(x_1) + a_{01} f(x_2) + b_{00} u_1(t), \\ \dot{x}_2 &= -x_2 + a_{0,-1} f(x_1) + a_{00} f(x_2) + b_{00} u_2(t), \\ y_1 &= f(x_1), \\ y_2 &= f(x_2). \end{aligned} \tag{8}$$

Example 1 [3] Consider the second-order nonautonomous **CNN**. If $a_{00} = 2, a_{0,-1} = -a_{0,1} = 1.2, b_{00} = 1, u_1(t) = 4.04 \sin\left(\frac{\pi}{2} t\right)$, and $u_2(t) = 0$; then Eq. (8) becomes

$$\begin{aligned} \dot{x}_1 &= -x_1 + 2 f(x_1) - 1.2 f(x_2) + 4.04 \sin\left(\frac{\pi}{2} t\right), \\ \dot{x}_2 &= -x_2 + 1.2 f(x_1) + 2 f(x_2), \end{aligned} \tag{9}$$

with nonlinear function

$$f(x) = \frac{1}{2}\left(|x+1| - |x-1|\right) = \begin{cases} 1, & x \geq 1 \\ x_{ij}, & |x| < 1 \\ -1, & x \leq -1 \end{cases} \tag{10}$$

Figure 2 shows a projection of the chaotic attractor of 3×4 **CNN** (9)-(10). The waveforms of $((x_1(t), x_2(t))$ corresponding to the $((x_1(t), x_2(t)) = (0.1, -0.1)$.

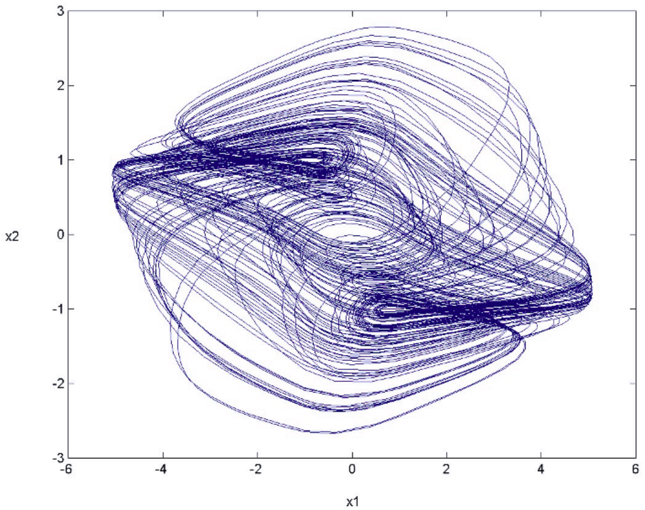

Fig. 2: Projection of the chaotic attractor of 3×4 **CNN** in the (x_1, x_2) plane.

The state equations describing the 3×4 **CNN** (9)-(10) in Hamiltonian canonical form with a destabilizing vector field (**master 3x4 CNN**) is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1.2 \\ 1.2 & 0 \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix} + \begin{bmatrix} 4.04 \sin\left(\frac{\pi}{2} t\right) - x_1 \\ -x_2 \end{bmatrix} \tag{11}$$

taking as the Hamiltonian energy function

$$H(x) = \int_0^{x_1} f(r_1) dr_1 + \int_0^{x_2} f(r_2) dr_2. \tag{12}$$

The destabilizing vector requires two signals for complete cancellation at the slave. Namely, the states $x_1(t)$ and $x_2(t)$. The output of the master (11) in this case, is then chosen as $y = (y_1, y_2)^T = (x_1, x_2)^T$. The matrices C, S, and I are found to be

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad I = \begin{bmatrix} 0 & -1.2 \\ 1.2 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

The pair (C, S) is observable, and hence detectable. An injection of the synchronization error $e_2(t) = x_2(t) - \xi_2(t)$ suffices to have an asymptotically stable trajectory convergence. The **slave** (**3x4 CNN**) would then be designed as follows

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1.2 \\ 1.2 & 0 \end{bmatrix} \begin{bmatrix} f(\xi_1) \\ f(\xi_2) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} f(\xi_1) \\ f(\xi_2) \end{bmatrix} + \begin{bmatrix} 4.04\sin\left(\frac{\pi}{2}t\right) - \xi_1 \\ -x_2 \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} e_2 \tag{13}$$

where $k = (k_1, k_2)^T$ is chosen in order to guarantee the asymptotic exponential stability to zero of the state reconstruction error trajectories (synchronization error). From (11) and (13) the synchronization error dynamics is governed by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2}k_1 - 1.2 \\ -\frac{1}{2}k_1 + 1.2 & 0 \end{bmatrix} \frac{\partial H(e)}{\partial e} + \begin{bmatrix} 2 & -\frac{1}{2}k_1 \\ -\frac{1}{2}k_1 & 2 - \frac{1}{2}k_1 \end{bmatrix} \frac{\partial H(e)}{\partial e}. \tag{14}$$

With initial states $(x_1(0), x_2(0)) = (0.1, -0.1)$ and $(\xi_1(0), \xi_2(0)) = (0.5, -0.5)$, and $k_1 = k_2 = 2$ we obtain the following numerical results. Figure 3 shows synchronization between: a) $x_1(t)$ and $\xi_1(t)$, b) $x_2(t)$ and $\xi_2(t)$; solid line $x_i(t)$ and dashed line $\xi_i(t)$, $i = 1, 2$. c) and d) illustrate the time behaviors of the synchronization error trajectories $e_i(t) = x_i(t) - \xi_i(t)$, $i = 1, 2$. e) and f) x_i versus ξ_i in phase space.

Example 2: Time-delay oscillators represent examples of high-dimensional chaos generators. Now, the system considered is a cell equation in Cellular Neural Networks with delay [15]. Its model is given by

$$\dot{x}(t) = 0.001x(t) - 3.8\left(|x_\tau + 1| - |x_\tau - 1| \right) + 2.85\left(\left|x_\tau + \frac{4}{3}\right| - \left|x_\tau - \frac{4}{3}\right| \right) \tag{15}$$

where $x_\tau = x(t - \tau)$. Its solution space is infinite-dimensional, with initial condition as any continuous function defined on the closed interval $[\tau, 0]$. By considering $\tau = 1$ and initial condition as a constant function equal to 0.5 on $[-1, 0]$, and initial state $x(0) = -1$. Figure 4 shows a projection of the chaotic attractor of the cellular neural network with delay in the (x, x_τ) plane.

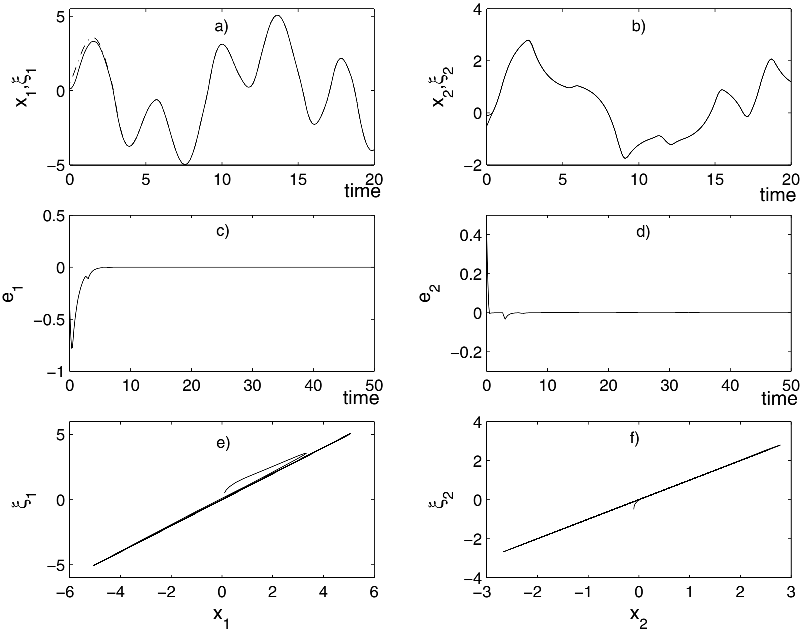

Fig. 3: a) and b) Synchronization between x_i and ξ_i ; solid line x_i and dashed line $\xi_i(t),\ i=1,2.$ c) and d) the behaviors of the synchronization error trajectories $e_i(t)=x_i(t)-\xi_i(t),\ i=1,2$. e) and f) x_i versus ξ_i in phase space.

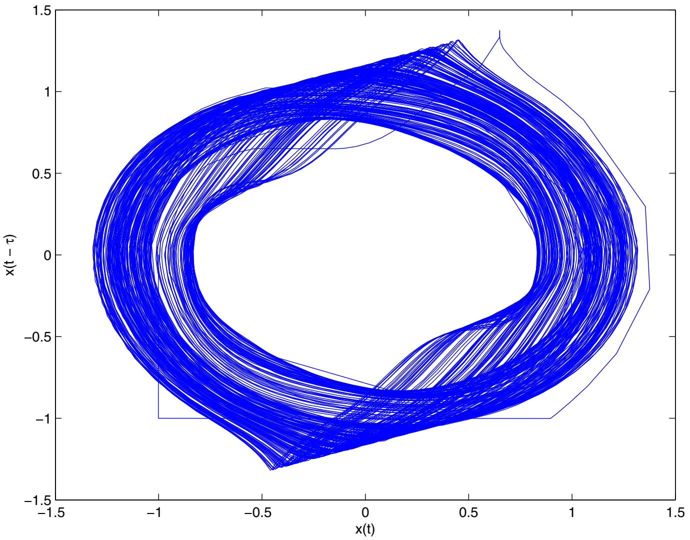

Fig. 4: Phase space dynamics for the Cellular Networks with delay projected onto the (x, x_τ) plane.

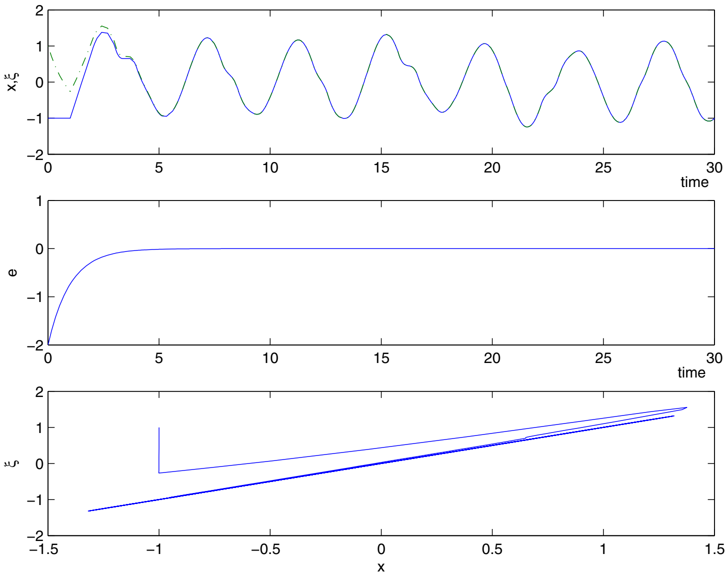

Fig. 5: Synchronization between the states $x(t)$ (solid line) and $\xi(t)$ (dashed line) (top of figure). Synchronization error (middle of figure). x versus ξ in phase space (bottom of figure).

The **CNN** with delay system (15) in Generalized canonical form (as master) is given by

$$\dot{x}(t) = 0.001\frac{\partial H(x)}{\partial x} - 3.8\left(|x_\tau + 1| - |x_\tau - 1|\right) + 2.85\left(\left|x_\tau + \frac{4}{3}\right| - \left|x_\tau - \frac{4}{3}\right|\right) \tag{16}$$

taking as Hamiltonian energy function

$$H(x) = \frac{1}{2}x^2 \tag{17}$$

with $\partial H(x)/\partial x = x$. It is clear that the system (16) is observable. The observer (as slave) for dynamics (16) is designed as

$$\dot{\xi}(t) = 0.001\xi(t) - 3.8\left(|\xi_\tau + 1| - |\xi_\tau - 1|\right) + 2.85\left(\left|\xi_\tau + \frac{4}{3}\right| - \left|\xi_\tau - \frac{4}{3}\right|\right) + k\,e(t), \tag{18}$$

where $e(t) = x(t) - \xi(t)$. From (16) and (18) the synchronization error dynamics is governed by

$$\dot{e}(t) = (0.001 - k)e(t). \tag{19}$$

Figure 5 depicts the synchronization between the state trajectories $x(t)$ (solid line) and $\xi(t)$ (dashed line) (top of figure), the time behavior of the synchronization error trajectory $e(t) = x(t) - \xi(t)$ (middle of figure), and x versus ξ (bottom of figure). When $x(0) = -1$ and $\xi(0) = 1$, and $k = 1$ are chosen.

4 Synchronization Stability Analysis

Now, we give conditions for asymptotic stability of the synchronization errors (14) and (19) between chaotic dynamics (11)-(13) and (16)-(18), respectively.

Theorem 1 [22] The state $x(t)$ of the nonlinear system (3) can be globally, exponentially, asymptotically estimated by the state $\xi(t)$ of an observer of the form (4), if the pair of matrices (C,W), or the pair (C,S), is either observable or, at least, detectable.

An observability condition on either of the pairs (C,W), or (C,S), is clearly a sufficient but not necessary condition for asymptotic state reconstruction. A necessary and sufficient condition for global asymptotic stability to zero of the estimation error is given by the following theorem.

Theorem 2 [22] The state $x(t)$ of the nonlinear system (3) can be globally, exponentially, asymptotically estimated, by the state $\xi(t)$ of the observer (4) if and only if there exists a constant matrix K such that the symmetric matrix

$$[W - KC] + [W - KC]^T = [S - KC] + [S - KC]^T = 2\left[S - \frac{1}{2}(KC + C^T K^T)\right]$$

is negative definite.

In particular, the matrix $2\left[S - \frac{1}{2}(KC + C^T K^T)\right]$ is negative definite (stability synchronization condition holds) for Example 1, if we choose k_1 and k_2 such that

$$k_1 \geq \sqrt{2k_2 - 4}, \; k_2 \geq 2,$$

i.e., if $k_1 = k_2 = k$, then $k \geq 1.6568$. And for Example 2, the synchronization error is stabilized at the origin for $k > 0.001$.

5 Confidential Communication

Finally, we apply the Hamiltonian synchronization of chaotic neural networks to transmit encrypted information. In particular, we use the modified chaos communication scheme (MCCS) for signal information masking with single transmission channel [16]; [14]. Figure 6 shows the MCCS (using previous Example 1) where: $m(t)$ is the confidential information to be hidden and transmitted, $x_2(t)$ is the chaotic signal of the network for masking purpose, $s(t) = x_2(t) + m(t)$ is the transmitted signal, and $m^{'}(t) = s(t) - \xi_2(t)$ the recovered information. It was reported in [14] that due to $m(t)$ is also injected into the transmitter, the MCCS is able to recover faithfully the hidden information even if a noise level is present through the transmission channel.

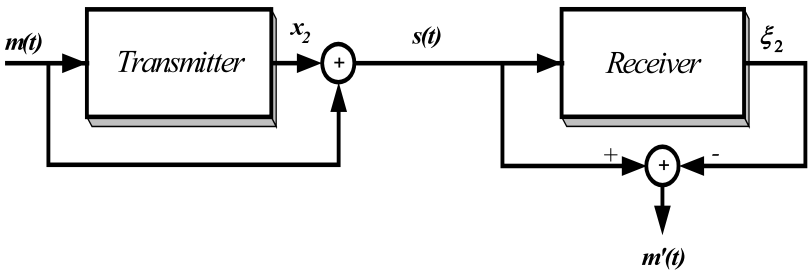

Fig. 6: Modified chaos-based communication scheme for signal masking using a single transmission channel.

Figure 7 illustrates the secret message communication of an audio message using the Example 1: the confidential message to be hidden and transmitted $m(t)$ (top of figure), the transmitted chaotic signal $s(t) = x_2(t) + m(t)$ (middle of figure), and the recovered audio message $m^{'}(t)$ at the receiver (bottom of figure).

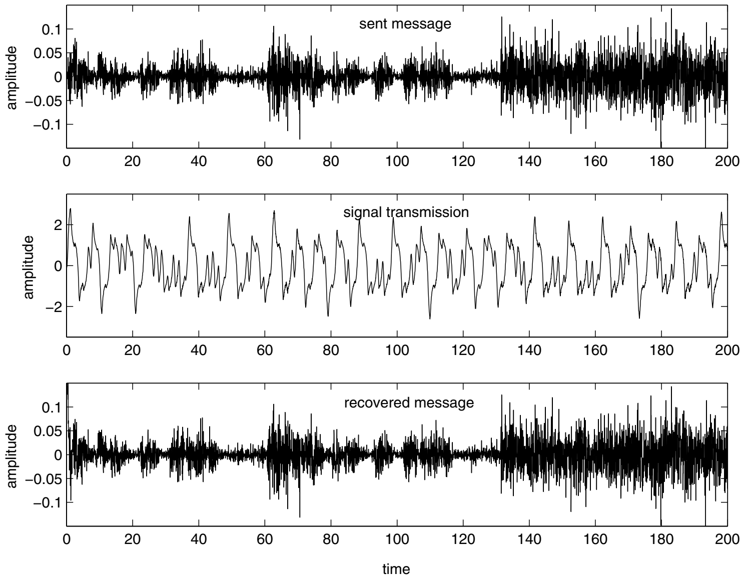

Fig. 7: Transmission and recovering of an audio message: Confidential message to be hidden and transmitted (top of figure). Transmitted chaotic signal $s(t) = x_2(t) + m(t)$ (middle of figure). Recovered audio message $m'(t)$ at the network receiver (bottom of figure).

6 Concluding Remarks

In this paper, we have presented the synchronization problem of chaotic neural networks from the perspective of Generalized Hamiltonian forms and observer design. The approach allows one to give a simple design procedure for the slave **CNN**. We have shown that synchronization of chaotic **CNNs** is possible from this viewpoint. The approach can be easily implemented on experimental setups. Moreover, we have shown based on chaotic **CNNs** synchronization the transmission of encrypted confidential information.

In a forthcoming work we will be concerned with a physical implementation of **CNN** with electronic circuits, and the synchronization of large chaotic neural networks and possible applications.

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