Chattering Attenuation Using Linear-in-the-Parameter Neural Nets in Variable Structure Control of Robot Manipulators with Friction

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Abstract. Variable structure control is a recognized method to stabilize mechanical systems with friction. Friction produces non linear phenomena, such as tracking errors, limit cycles, and undesired stick-slip motion, degrading the performance of the closed-loop system. The main drawback of variable structure control is the presence of chattering, which is not suitable in mechanical systems. In this paper, we design a variable structure controller complemented with Linear-in-the-Parameter neural nets to attenuate chattering. Experimental validation applied to a three degree of freedom robot mechanical manipulator is shown to support the results.

1 Introduction

Friction is the resistance to motion, during sliding or rolling, that is experienced whenever one solid body moves tangentially over another with which it is in contact. Friction is undesirable in mechanical systems because can lead to tracking errors, limit cycles, and undesired stick-slip motion (cf. [1]). Control strategies for friction compensation have been proposed in [1]-[7], among others. In these papers the authors propose friction model based controllers to mitigate the friction effects. It is well-known that the phenomenon of friction is not yet completely understood and it is hard to model [8], therefore stabilization of mechanical systems through a feedback law with an imprecise friction compensation term may result in a considerable degree of uncertainty, thus not producing the expected motion. If the uncertainties are bounded, discontinuous robust control methods ([8]) provide simple and straightforward solutions to the friction compensation design, however, the system exhibits an infinitely fast switching of the input control called chattering ([9]) inducing fatigue in mechanical

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parts and the system could be damaged in a short time. For instance, in [8] and [10], friction compensator design involves chattering behavior where the chattering controller deals with the friction model uncertainties, which is a desired property in friction compensation.

This paper is intended to provide a solution to mechanical problems due to chattering without losing robustness properties given by variable structure controllers [11]. Neural nets have been used extensively in feedback control (see, for instance, [12]-[15]). Also, adaptive control theory has evolved as a powerful methodology for designing nonlinear feedback controllers for systems with uncertainties [16]. Using the advantage of chattering control to deal with uncertainty in the friction model ([8] and [10]), and utilizing a linear-in-the-parameter (LIP) neural net, a chattering friction compensation design is proposed, where a dynamic adaptation law for the parameters of the LIP neural nets is designed to attenuate the amplitude of the chattering once the control objective is achieved. In this way, chattering appears only when it is needed.

To the best knowledge of the authors, the chattering attenuation problem for the class of Variable Structure Control (VSC) introduced in this paper has not been reported. On the other hand, few results have appeared in research papers dealing with the chattering problem for sliding mode control: Parra-Vega *et al.* [17], for example, showed that adaptive and non-adaptive cases of variable structure robot control undergo chattering attenuation. Bartolini *et al.* [18] demonstrated that it is possible to eliminate chattering by generating a second-order sliding mode control using the first derivative of the control law as a control input instead of the actual control law. Another alternative used in control applications is to replace the signum function with a smooth approximation (*e.g.* tanh, sigmoid function, among others).

This paper is organized as follows: Section 2 presents the problem statement along with the dynamic model of mechanical manipulators and the previous result on chattering control developed by Orlov *et al.* [8]; Section 3 presents the neural nets chattering controller applied to a n-degrees-of-freedom robot manipulator where it is assumed that joint positions are the only information available for feedback, along with its stability analysis; Section 4 provides experimental results made for a three degrees of freedom mechanical manipulator using the neural nets chattering controller described in Section 3; and Section 5 presents some conclusions.

The following notations will be adopted throughout this paper. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of a symmetric positive definite matrix $A \in R^{n\times n}$, respectively, and $\|x\| = \sqrt{x^T x}$ represents the Euclidean norm of vector $x \in R^n$.

2 Problem Statement

In the present paper we study controlled n-link mechanical manipulators described by interconnected second-order differential equations of the form [8]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau \tag{1}$$

where $q \in R^n$ is the position vector, $\tau \in R^n$ is the control input, $M(q)$, $C(q,\dot{q})$, $G(q)$ are smooth functions of appropriate dimensions, $M(q) = M^T(q) > 0$,

$$F(\dot{q}) = K_b\dot{q} + K_f \operatorname{sgn}(\dot{q}) \tag{2}$$

$$\operatorname{sgn}(\dot{q}) = [\operatorname{sgn}(\dot{q}_1), \operatorname{sgn}(\dot{q}_2), \ldots, \operatorname{sgn}(\dot{q}_n)]^T \tag{3}$$

$$\operatorname{sgn}(z) = \begin{cases} 1 & if \quad z > 0 \\ [-1,1] & if \quad z = 0 \quad \forall z \in R \\ -1 & if \quad z < 0 \end{cases} \tag{4}$$

and $K_b = diag\{k_{b_i}\}$ and $K_f = diag\{k_{f_i}\}$ are positive definite and diagonal matrices. Throughout, the precise meaning of solutions of the system (1) with discontinuous functions $F(\dot{q})$ and $\tau(q,\dot{q})$ are defined in Filippov's sense [8].

From the physical point of view, the position q represents the generalized coordinates, the control input τ is the vector of external torques, $M(q)$ is the inertia matrix, $C(q,\dot{q})\dot{q}$ is the vector of Coriolis and centripetal torques, $G(q)$ is the vector of gravitational forces, $F(\dot{q})$ represents the friction torques, where k_{b_i} and k_{f_i}, $i = 1,2,\ldots,n$ are the constant coefficients of viscous and Coulomb frictions, respectively. Because frictions are uncoupled among joints, we have assumed that the matrices K_b and K_f are diagonal.

Consider the following control law:

$$\tau = G(q) - K_d\dot{x} - K_p e - K_\alpha \operatorname{sgn}(e) \tag{5}$$

$$\dot{x} = -Lx + K_d e \qquad (6)$$

where $L \in R^{n\times n}$ is a symmetric positive definite matrix, $K_d \in R^{n\times n}$ is a symmetric positive semi-definite matrix, $K_p = diag\{k_{p_i}\} \in R^{n\times n}$ is a diagonal positive definite matrix, $K_\alpha = diag\{k_{\alpha_i}\} \in R^{n\times n}$ is a diagonal matrix such that $K_\alpha > K_f$, and $e = q - q_d$ represents the position error with respect to the constant desired position q_d. Equation (6) is a first-order linear compensator used to replace the velocity feedback (cf. [19]).

The control law (5)-(6), that belongs to the *variable structure controllers* family, is called a *chattering controller* because it generates no sliding mode, except at the origin, while exhibiting an infinite number of switches in a finite time interval ([8]).

Theorem 1 ([8]). *Let the friction manipulator (1)-(4) be driven by the switched position feedback controller (5)-(6) with the assumptions given above. Then, the closed loop system (1)-(6) is globally asymptotically stable at the equilibrium point* $(\dot{q}, e, x) = 0$.

The switched term in (5), represented by $K_\alpha \operatorname{sgn}(e)$, can be interpreted as LIP neural nets with dendrite weights equal to one, and with firing thresholds (the so called 'bias' terms) equal to zero. The cell inputs are the components of the vector e. The outputs are the components of the switched term. Because the dendrite weights are positive the neural nets correspond to *excitatory* synapses. Here, the activation functions are the so called *symmetric hard limit*. Representing the activation function by $\sigma(\cdot)$ we have:

$$y_i = k_{\alpha_i} \operatorname{sgn}(e_i) = k_{\alpha_i} \sigma(e_i); \quad i = 1,2,\ldots,n \qquad (7)$$

where y_i are the outputs of the LIP neural nets.

The problem to tackle is to find a training rule for each k_{α_i} such that the closed-loop system be globally asymptotically stable at the equilibrium point $(\dot{q}, e, x) = 0$ with the property that each k_{α_i} converges to zero as the system approaches the equilibrium point.

3 Neural Nets Chattering Controller

Considering the following control law:

$$\tau = G(q) - K_d \dot{x} - K_p e - \delta(t) K_\alpha sign(e) \quad (8)$$

$$\dot{x} = -Lx + K_d e \quad (9)$$

$$\dot{\delta} = -\alpha \log(1+\delta) + k_r \frac{(1+\delta)}{\log(1+\delta)+1} \|e\|^2 \quad (10)$$

where $\alpha, k_r \in R^+$ and $\delta(t) \in R$ is the adaptive term that will regulate the amplitude of the chattering term. In fact, the above controller is a LIP neural net with *dynamic* training implemented with a point of view similar to [14] and [15].

Again, $L \in R^{n\times n}$ is a symmetric positive definite matrix, $K_d \in R^{n\times n}$ is a symmetric positive semi-definite matrix, $K_p = diag\{k_{p_i}\} \in R^{n\times n}$ is a diagonal positive definite matrix, $K_\alpha = diag\{k_{\alpha_i}\} \in R^{n\times n}$ is a diagonal matrix such that $K_\alpha > K_f$, and $e = q - q_d$ represents the position error with respect to the constant desired position q_d.

Lemma 1 [20]: *Suppose the ordinary differential equation in (10) has initial condition* $\delta(t_0) \geq 0$, *then* $\delta(t) \geq 0$ *for all* $t \geq t_0$.

Our main result follows.

Theorem 2. *Let the friction manipulator (1)-(4) be driven by the switched position feedback controller (8)-(10) with the assumptions given above. Suppose that* $K_f = \beta K_\alpha$ *with* $\beta < 1$ *and* $0 \leq \delta(t) \leq \beta + 1$ *for all* $t \geq t_0$. *Then, the closed-loop system (1)-(4) and (8)-(10) is globally asymptotically stable at the equilibrium point* $(\dot{q}, e, x, \delta) = (\dot{e}, e, x, \delta) = 0$ *if* $\delta(t) \geq 0$ *and*

$$0 < k_r < \frac{1}{\beta+1} \lambda_{\min}\left(\begin{bmatrix} K_d L K_d & -\frac{1}{2} K_d L L \\ -\frac{1}{2}(K_d L L)^T & LLL \end{bmatrix} \right). \quad (11)$$

Remark 1: *Because* $\delta = 0$ *is an equilibrium point of the closed-loop system (1)-(4) and (8)-(10), the chattering amplitude vanishes as* $t \to \infty$ *.*

Proof. To this end, we follow the same line of reasoning given in [8]. Let us introduce the Lyapunov candidate function

$$V(\dot{q},e,x,\delta) = \frac{1}{2}\dot{q}^T M(q)q + \frac{1}{2}e^T K_p e + \frac{1}{2}(K_d e - Lx)^T (K_d e - Lx) + k_{\alpha_1}|e_1| + \ldots + k_{\alpha_n}|e_n| + (1+\delta)\log(1+\delta) . \quad (12)$$

This Lyapunov function is similar to the one proposed in [8] but the last term involves the dynamic adaptation law. This last term was also utilized in [20]. The time derivative of (12), along the trajectories of the closed loop system (1)-(4) and (8)-(10) yields:

$$\dot{V}(\dot{q},e,x,\delta) = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + e^T K_p \dot{e} + \dot{q}^T K_\alpha \operatorname{sgn}(e) + (K_d e - Lx)^T (K_d \dot{e} - L\dot{x}) + \dot{\delta}(\log(1+\delta)+1). \quad (13)$$

Employing the well-known property $\dot{q}^T[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})]\dot{q} = 0$, for all $q \in R^n$, and substituting the control law (8) into (13) we have

$$\dot{V}(\dot{q},e,x,\delta) = -\dot{q}^T K_b \dot{q} - \dot{q}^T K_f \operatorname{sgn}(\dot{q}) - e^T K_d \dot{x} - \dot{q}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) + (K_d e - Lx)^T (K_d \dot{e} - L\dot{x}) + \dot{\delta}(\log(1+\delta)+1). \quad (14)$$

From (9), the above equation is simplified to

$$\dot{V}(\dot{q},e,x,\delta) = -\dot{q}^T K_b \dot{q} - \dot{q}^T K_f \operatorname{sgn}(\dot{q}) - \dot{q}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) - (K_d e - Lx)^T L(K_d e - Lx) + \dot{\delta}(\log(1+\delta)+1). \quad (15)$$

Invoking (10), the above equation is reduced to

$$
\begin{aligned}
\dot{V}(\dot{q},e,x,\delta) &= -\dot{q}^T K_b \dot{q} - \dot{q}^T K_f \operatorname{sgn}(\dot{q}) - \dot{q}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) \\
&\quad - (K_d e - Lx)^T L (K_d e - Lx) + k_r (1+\delta)\|e\|^2 \\
&\quad - \alpha \log(1+\delta)(\log(1+\delta)+1) \\
&= -\dot{q}^T K_b \dot{q} - \dot{q}^T K_f \operatorname{sgn}(\dot{q}) - \dot{q}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) \\
&\quad - \begin{bmatrix} e \\ x \end{bmatrix}^T \begin{bmatrix} K_d L K_d & -\frac{1}{2} K_d L L \\ -\frac{1}{2}(K_d L L)^T & LLL \end{bmatrix} \begin{bmatrix} e \\ x \end{bmatrix} \\
&\quad + k_r (1+\delta)\|e\| - \alpha \log(1+\delta)(\log(1+\delta)+1) \\
&\leq -\dot{q}^T K_b \dot{q} - \dot{q}^T K_f \operatorname{sgn}(\dot{q}) - \dot{q}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) \\
&\quad - (\lambda_{\min}(Q) - k_r(1+\delta))\|e\|^2 - \lambda_{\min}(Q)\|x\|^2 \\
&\quad - \alpha \log(1+\delta)(\log(1+\delta)+1)
\end{aligned}
$$

where

$$
Q = \begin{bmatrix} K_d L K_d & -\frac{1}{2} K_d L L \\ -\frac{1}{2}(K_d L L)^T & LLL \end{bmatrix} \tag{16}
$$

From (11) we forward to

$$
\begin{aligned}
\dot{V}(\dot{q},e,x,\delta) &\leq -\dot{q}^T K_b \dot{q} - \dot{q}^T K_f \operatorname{sgn}(\dot{q}) - \dot{q}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) \\
&\quad - \lambda_{\min}(Q)\|x\|^2 - \alpha \log(1+\delta)(\log(1+\delta)+1) \\
\dot{V}(\dot{q},e,x,\delta) &\leq -\dot{e}^T K_b \dot{e} - \dot{e}^T K_f \operatorname{sgn}(\dot{e}) - \dot{e}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) \\
&\quad - \lambda_{\min}(Q)\|x\|^2 - \alpha \log(1+\delta)(\log(1+\delta)+1).
\end{aligned}
$$

By virtue of

$$
\begin{aligned}
&- \dot{e} K_f \operatorname{sgn}(\dot{e}) - \dot{e}^T K_\alpha (\delta - 1)\operatorname{sgn}(e) \\
&\leq \sum_{i=1}^{n} |e_i| \left(-k_{f_i} + |\delta(t) - 1| k_{\alpha_i}\right)
\end{aligned}
$$

and taking into account that $0 \leq \delta(t) \leq \beta + 1$ for all $t \geq t_0$, we have

$$\begin{aligned} \dot{V}(\dot{q},e,x,\delta) \;\leq\; & -\dot{e}^T K_b \dot{e} - \lambda_{\min}(Q)\|x\|^2 \\ & -\alpha \log(1+\delta)(\log(1+\delta)+1) \;\; \leq 0 \end{aligned} \tag{17}$$

Since $V(\dot{q},e,x,\delta)$ is positive definite and $\dot{V}(\dot{q},e,x,\delta)$ is a negative semi-definite decreasing function, it follows that the equilibrium point $(\dot{q},e,x,\delta)=(\dot{e},e,x,\delta)=0$ of the closed-loop system (1)-(4) and (8)-(10) is uniformly stable, *i.e.*, $x(t),e(t),\dot{e}(t),\delta(t)\in L_\infty$. From (17), we can easily show that the squares of $x,\dot{e},\delta$ are integrable with respect to time t; *i.e.*, $x(t),\dot{e}(t),\delta(t)\in L_2$. Next, Barbalat's lemma implies that $\dot{e}(t)\to 0, x(t)\to 0$ and $\delta(t)\to 0$. If $x(t)\to 0$ then $\dot{x}(t)\to 0$, and from (9), it follows that $e(t)\to 0$. This concludes our proof. ■

4 Application to an Industrial Robot Manipulator

4.1 Experimental Setup

The experimental setup designed in the research laboratory of CITEDI-IPN involves a three degrees-of-freedom (3-DOF) industrial robot manipulator manufactured be Amatrol, it is shown in Figure 1. This mechanical system presents Coulomb friction [8]. The base of the mechanical robot has a horizontal revolute joint, q_1, whereas two links have vertical revolute joints q_2 and q_3. The nominal parameter values of the mechanical manipulator are summarized in Table 1. A worm gear set, a helicon gear set and a roller chain are used for torque transmission to joints q_1, q_2 and q_3, respectively; there is a DC gear motor for each joint with a reduction ratio of 19.7:1 for q_1 and q_2 and 127.8:1 for q_3. The ISA Bus servo I/O card from the company *Servo To Go* is employed for the real time control system and it mainly consists of eight channels of 16-bit D/A outputs, 32 bits of I/O, and an interval timer capable of interrupting the PC. The controller is implemented using C++ programming language running on a 486 PC. Position measurements of each articulation of the robot are obtained using the quadrature encoder channel available on each DC gear-motor, connected to the I/O card, and programmed to provide the encoder signal processing every millisecond; the resolution of the encoders is 52 x 10^{-3} rad, 62 x 10^{-3} rad and 34 x 10^{-3} rad for q_1, q_2 and q_3, respectively. Along with this, a digital oscilloscope is used to store the control signal. Linear power amplifiers are installed en each servomotor which apply a variable torque to each joint. These amplifiers accept control inputs from the D/A converter in the range of ±10 volts. See Figure 2 for the hardware setup configuration. The dynamic model of the robot in the form of (1) is given in [8]. However, for control implementation, we only require $G(q)$.

Table 1. Nominal parameter values fo the mechanical manipulator

Description	Notation	Value	Units
Lenght of link 1	l_1	0.297	m
Lenght of link 2	l_2	0.297	m
Mass of link 1	m_1	0.38	Kg
Mass of link 2	m_2	0.34	Kg
Gravity acceleration	g	9.8	m/s^2

4.2. Experimental Results

The regulator performance was studied experimentally. The experiment was performed with the 3-DOF robot manipulator required to move in space from the origin $q_1(0) = q_2(0) = q_3(0) = 0$ to the desired position $q_{d1} = q_{d2} = q_{d3} = \pi/2$ [rad]. The initial velocities $\dot{q}(0) \in R^3$ and $\delta(0)$ were set to zero, respectively.

The control goal was achieved by implementing the control (8)-(10) where ([8])

$$G(q) = g\begin{bmatrix} 0 \\ m_1 l_1 \cos q_2 + m_2 l_1 \cos q_2 + m_2 l_2 \cos(q_1 + q_2) \\ m_2 l_2 \cos(q_2 + q_3) \end{bmatrix},$$

and the controller gains selected as follows:

$$K_p = diag\{15,40,40\}, \quad K_d = diag\{5,5,5\},$$
$$K_\alpha = diag\{3,3,3\}, \quad L = diag\{10,10,10\},$$

and $K_r = 2$ and $\alpha = 8$. The physical constant parameters, $m_i, l_i, i = 1,2$ are given in table 1.

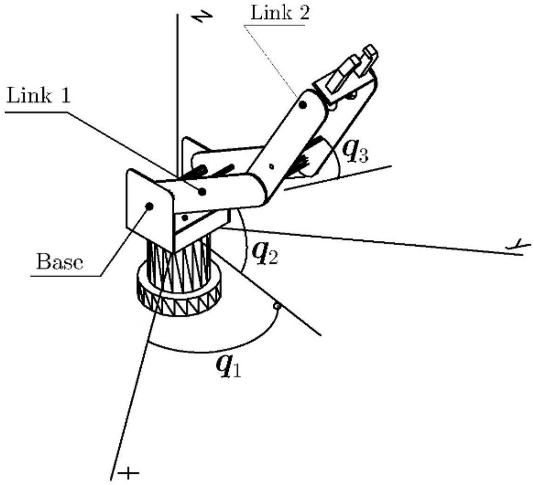

Fig. 1. Schematic diagram of the robot.

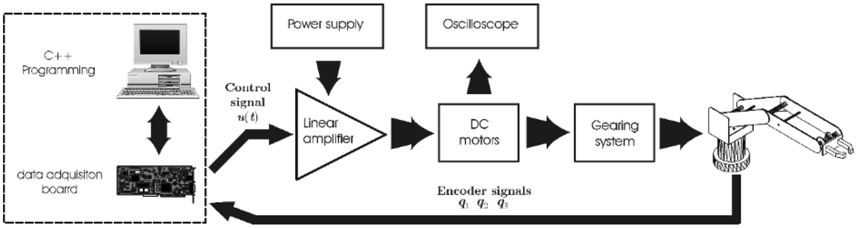

Fig. 2. Hardware setup.

The resulting joint positions and input torques are depicted in Figures 3 and 4, respectively. Figure 3 shows that joint positions converge to the desired position for the closed loop system [(1), (8)-(10)], whereas the fast switching due to LIP terms vanishes as t tends to ∞ (see Figure 4). Also, from Figure 3, a finite time convergence of the articulated positions to their desired positions is appreciated in about 3.2 seconds. The applied control inputs present chattering that is attenuated in about 8 seconds (see Figure 4). This chattering attenuation is good in mechanical systems, and was the main objective of the present paper. Finally, Figure 5 presents the time evolution of $\delta(t)$. It should be noted that the dynamic of $\delta(t)$ (10) is slower (smooth and slow variation) than [(1)-(4), (7), (8)].

5 Conclusions

We have developed a variable structure controller with chattering attenuation for robot manipulators in the presence of friction. The manipulator is governed by a second order differential equation with a right-hand discontinuous side admitting discontinuous terms to account for friction phenomena. The proposed controller uses Linear-in-the-Parameter Neural nets to attenuate the chattering signal inherent to variable structure systems without losing the robustness of the function framework. Effectiveness of the design is supported by the experiments made for a three degrees-of-freedom robot manipulator with frictional joints.

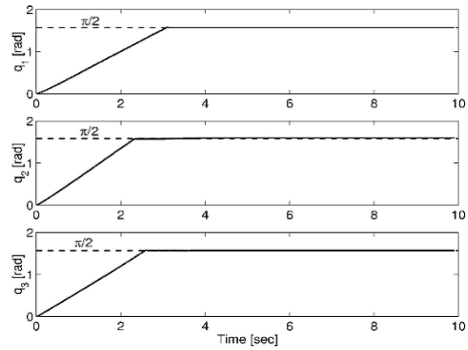

Fig. 3. Joint Positions.

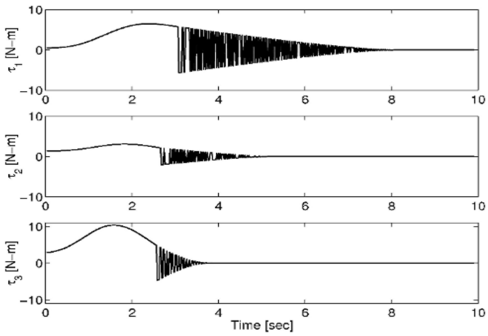

Fig. 4. Input Torques.

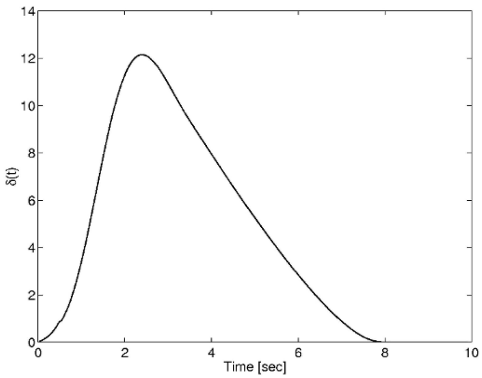

Fig. 5. Time evolution of $\delta(t)$.

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