Analyzing Fuzzy System Reliability Based on the Vague Set Theory

Shyi-Ming Chen

Department of Computer Science and Information Engineering,
National Taiwan University of Science and Technology

12.1 Introduction

It is obvious that the reliability modeling is the most important discipline of reliable engineering (Kaufmann and Gupta, 1988). Traditionally, the reliability of a system's behavior is fully characterized in the context of probability measures. However, because of the inaccuracy and uncertainties of data, the estimation of precise values of probability becomes very difficult in many systems (Chen, 1996). In recent years, some researchers have used the fuzzy set theory (Zadeh, 1965) for fuzzy system reliability analysis (Cai *et al*, 1991a; Cai *et al*, 1991b; Cai *et al*, 1991c; Cai, 1996; Chen, 1994; Chen and Jong, 1996; Chen, 1996; Chen, 1997a; Cheng and Mon, 1993; Mon and Cheng, 1994; Singer, 1990; Wu, 2004).

Cai *et al.* (1991b) presented the following two assumptions for fuzzy system reliability analysis:

(1) Fuzzy-state assumption: At any time, the system may be either in the fuzzy success state or the fuzzy failure state.

(2) Possibility assumption: The system behavior can be fully characterized by possibility measures.

Cai (1996) presented an introduction to system failure engineering and its use of fuzzy methodology. Chen (1994) presented a method for fuzzy system reliability analysis using fuzzy number arithmetic operations. Chen and Jong (1996) presented a method for analyzing fuzzy system reliability using intervals of confidence. Chen (1996) presented a method for fuzzy system reliability analysis based on fuzzy time series and the α–cuts operations of fuzzy numbers. Cheng and Mon (1993) presented a method for fuzzy system reliability analysis by interval of confidence. Mon and Cheng (1994) presented a method for fuzzy system reliability analysis for components with different membership functions using non-linear programming techniques. Singer (1990) presented a fuzzy set approach for fault tree and

S.-M. Chen: *Analyzing Fuzzy System Reliability Based on the Vague Set Theory*, Computational Intelligence in Reliability Engineering (SCI) **40**, 347–362 (2007)
www.springerlink.com

reliability analysis. Suresh et al. (1996) presented a comparative study of probabilistic and fuzzy methodologies for uncertainty analysis using fault trees. Utkin and Gurov (1996) presented a general formal approach for fuzzy system reliability analysis in the possibility context. Wu (2004) presented a method for fuzzy reliability estimation using the Bayesian approach.

In this article, we present a method for analyzing fuzzy system reliability using the vague set theory (Chen, 1995; Gau and Buehrer, 1993), where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse [0, 1]. The grade of membership of an element x in a vague set is represented by a vague value $[t_x, 1 - f_x]$ in [0, 1], where t_x indicates the degree of truth, f_x indicates the degree of false, $1 - t_x - f_x$ indicates the unknown part, $0 \leq t_x \leq 1 - f_x \leq 1$, and $t_x + f_x \leq 1$. The notion of vague sets is similar to that of intuitionistic fuzzy sets (Atanassov, 1986). Both of them are generalizations of fuzzy sets (Zadeh, 1965). The proposed method can model and analyze fuzzy system reliability in a more flexible and convenient manner.

The rest of this article is organized as follows. In Section 2, we briefly review a method for fuzzy system reliability analysis from (Chen and Jong, 1996). In Section 3, we briefly review some definitions and arithmetic operations of vague sets from (Chen, 1995) and (Gau and Buehrer, 1993). In Section 4, we present a method for analyzing fuzzy system reliability based on the vague set theory. The conclusions are discussed in Section 5.

12.2 A Review of Chen and Jong's Fuzzy System Reliability Analysis Method

In this section, we briefly review a method for fuzzy system reliability analysis from (Chen and Jong, 1996).

In (Kaufmann and Gupta, 1988, pp. 184-208), the reliability $K(t)$ of a subsystem or system is represented by an interval of confidence $K(t) = [K_a(t), K_b(t)]$, where $K_a(t)$ and $K_b(t)$ are the lower and upper bounds of the survival function at time t (t = 0, 1, 2, …), respectively, and $0 \leq K_a(t) \leq K_b(t) \leq 1$. For example, Fig. 1 shows the lower and upper bounds of the survival function given subjectively by an expert.

Chen and Jong (1996) considered the situation in which there are uncertainties associated with the survival interval of confidence $[K_a(t), K_b(t)]$ at time t (t = 0, 1, 2, …). In such a situation, the reliability of a subsystem P_i can be represented by $[K_{ia}(t), K_{ib}(t)]/C_i(t)$, where $C_i\ (t)$ indicates the degree of certainty that the reliability of the subsystem P_i at time t lies in interval $[K_{ia}(t), K_{ib}(t)]$, $0 \le K_{ia}(t) \le K_{ib}(t) \le 1$, $0 \le C_i(t) \le 1$, and $i = 0, 1, 2, \ldots$. The values of $K_{ia}(t)$, $K_{ib}(t)$ and $C_i(t)$ at time t ($t = 0, 1, 2, \ldots$) are given by experts, respectively.

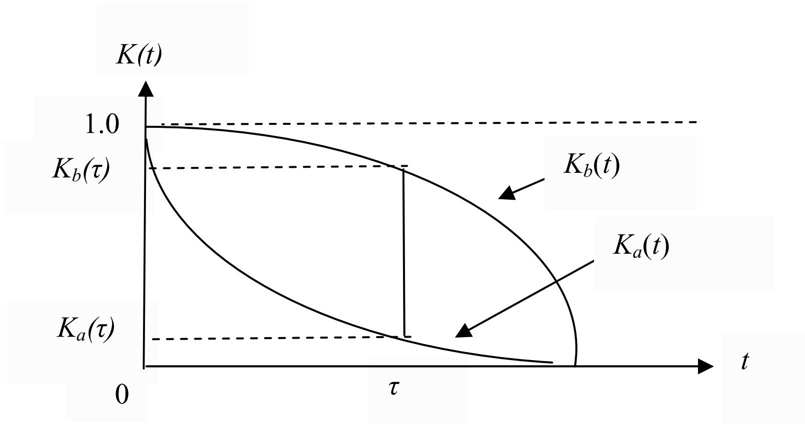

Fig. 1. Lower and upper bounds of the survival function

Chen and Jong (1996) presented a method for fuzzy system reliability analysis based on the interval of confidence, which is reviewed as follows. Let $[a_1, a_2]/c_1$ and $[b_1, b_2]/c_2$ be two survival intervals of confidence, where $0 \le a_1 \le a_2 \le 1$, $0 \le b_1 \le b_2 \le 1$, $0 \le c_1 \le 1$, and $0 \le c_2 \le 1$. The multiplication operation and the subtraction operation between the survival intervals of confidence $[a_1,a_2]/c_1$ and $[b_1,b_2]/c_2$ are defined as follows:

$$[a_1,a_2]/c_1 \otimes [b_1,b_2]/c_2 = [a_1 \times b_1, a_2 \times b_2]/\mathrm{Min}(c_1,c_2), \quad (1)$$

$$[a_1,a_2]/c_1 \ominus [b_1,b_2]/c_2 = [a_1 - b_2, a_2 - b_1]/\mathrm{Min}(c_1,c_2), \quad (2)$$

where $\otimes$ and $\ominus$ are the multiplication operator and subtraction operator between the survival intervals of confidence, respectively.

The complement of a survival interval of confidence $[b_1, b_2]/c_2$ is defined by

$$1 \ominus [b_1,b_2]/c_2 = [1, 1]/1 \ominus [b_1, b_2]/c_2$$
$$= [1 - b_2, 1 - b_1]/\mathrm{Min}(1, c_2) = [1 - b_2, 1 - b_1]/c_2. \quad (3)$$

It is obvious that if $a_1 = a_2 = a$ and $b_1 = b_2 = b$, then

$$[a_1,a_2]/c_1 \otimes [b_1,b_2]/c_2 = [a,a]/c_1 \otimes [b,b]/c_2$$
$$= [a \times b, a \times b]/\mathrm{Min}(c_1,c_2) = (a \times b)/\mathrm{Min}(c_1,c_2), \quad (4)$$
$$[a_1,a_2]/c_1 \ominus [b_1,b_2]/c_2 = [a,a]/c_1 \ominus [b,b]/c_2$$
$$= [a - b, a - b]/\mathrm{Min}(c_1,c_2) = (a - b)/\mathrm{Min}(c_1,c_2). \quad (5)$$

Because $[x,y]$ can be written as $[x,y]/1$, where $0 \le x \le y \le 1$, we can get

$$[a_1, a_2] \otimes [b_1, b_2] = [a_1, a_2]/1 \otimes [b_1, b_2]/1 \quad (6)$$
$$= [a_1 \times b_1, a_2 \times b_2]/\text{Min}(1, 1) = [a_1 \times b_1, a_2 \times b_2]/1 = [a_1 \times b_1, a_2 \times b_2],$$

$$[a_1, a_2] \ominus [b_1, b_2] = [a_1, a_2]/1 \ominus [b_1, b_2]/1 \quad (7)$$
$$= [a_1 - b_2, a_2 - b_1]/\text{Min}(1, 1) = [a_1 - b_2, a_2 - b_1]/1 = [a_1 - b_2, a_2 - b_1].$$

Consider the series system shown in Fig. 2, where the reliability of subsystem P_i at time t (t = 0, 1, 2, …) is represented by the survival interval confidence $[K_{ia}(t), K_{ib}(t)]/C_i(t)$, where $K_{ia}(t)$ and $K_{ib}(t)$ are the lower and upper bounds of the survival function of subsystem P_i at time t, respectively, $C_i(t)$ indicates the degree of certainty that the reliability of subsystem P_i at time t is $[K_{ia}(t), K_{ib}(t)]$, $0 \le K_{ia}(t) \le K_{ib}(t) \le 1$, $0 \le C_i(t) \le 1$, and $1 \le i \le n$. In this situation, the reliability of the series system shown in Fig. 2 at time t (t = 0, 1, 2, …) can be evaluated and is equal to

$$[K_{1a}(t), K_{1b}(t)]/C_1(t) \otimes [K_{2a}(t), K_{2b}(t)]/C_2(t) \otimes \ldots \otimes [K_{na}(t), K_{nb}(t)]/C_n(t)$$
$$= [K_{1a}(t) \times K_{2a}(t) \times \ldots \times K_{na}(t), K_{1b}(t) \times K_{2b}(t)$$
$$\times \ldots \times K_{nb}(t)]/\text{Min}(C_1(t), C_2(t), \ldots, C_n(t)). \quad (8)$$

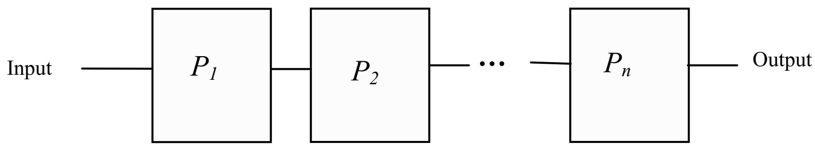

Fig. 2. A series system

Consider the parallel system shown in Fig. 3, where the reliability of subsystem P_i at time t (t = 0, 1, 2, …) is $[K_{ia}(t), K_{ib}(t)]/C_i(t)$, where $0 \le K_{ia}(t) \le K_{ib}(t) \le 1$, $0 \le C_i(t) \le 1$, and $1 \le i \le n$. Then, the reliability of the parallel system shown in Fig. 3 at time t can be evaluated and is equal to $[K_a(t), K_b(t)]/C_p(t)$, where

(1) $[K_a(t), K_b(t)] = 1 \ominus (1 \ominus [K_{1a}(t), K_{1b}(t)]) \otimes (1 \ominus [K_{2a}(t),$

$$K_{2b}(t)]) \otimes \ldots \otimes (1 \ominus [K_{2a}(t), K_{2b}(t)])] = 1 \ominus ([1 - K_{1b}(t), 1 - K_{1a}(t)]) \otimes ([1 - K_{2b}(t), 1 - K_{2a}(t)]) \otimes \ldots \otimes ([1 - K_{nb}(t), 1 - K_{na}(t)]).$$

(2) The value of $C_p(t)$ is evaluated as follows. Let X and Y be two real intervals in [0, 1], where $X = [x_1, x_2]$, $Y = [y_1, y_2]$, $0 \le x_1 \le x_2 \le 1$, and $0 \le y_1 \le y_2 \le 1$. Based on the similarity function S presented in (Chen and Wang, 1995), we can calculate the degree of similarity between the intervals X and Y, where $S(X, Y) = 1 - (|x_1 - y_1| + |x_2 - y_2|)/2$ and $0 \le S(X, Y) \le 1$. The larger the value of $S(X, Y)$, the more the similarity between the intervals X and Y. Because the reliability of the subsystem P_i at time t ($t = 0, 1, 2, \ldots$) is $[K_{ia}(t), K_{ib}(t)]/C_i(t)$, where $0 \le K_{ia}(t) \le K_{ib}(t) \le 1$, $0 \le C_i(t) \le 1$, and $1 \le i \le n$. Based on the similarity function S, we can get

$$S([K_{1a}(t), K_{1b}(t)], [K_a(t), K_b(t)]) = s_1,$$
$$S([K_{2a}(t), K_{2b}(t)], [K_a(t), K_b(t)]) = s_2,$$
$$\ldots$$
$$S([K_{na}(t), K_{nb}(t)], [K_a(t), K_b(t)]) = s_n,$$

where $0 \le s_i \le 1$ and $i = 1, 2, \ldots, n$. If s_j is the largest value among the values $s_1, s_2, \ldots$, and s_n, then let the value of $C_p(t)$ be equal to $C_j(t)$, where $1 \le j \le n$.

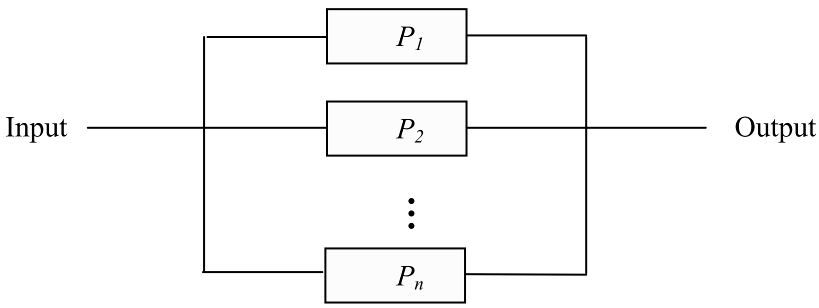

Fig. 3. A parallel system

Consider the series-parallel system shown in Fig. 4, where the reliability of the subsystem P_i at time t ($t = 0, 1, 2, \ldots$) is represented by $[K_{ia}(t), K_{ib}(t)]/C_i(t)$, where $0 \le K_{ia}(t) \le K_{ib}(t) \le 1$, $0 \le C_i(t) \le 1$, and $1 \le i \le 3$. Then, the reliability of the series-parallel system shown in Fig. 4 at time t (t = 0, 1, 2, …) can be evaluated and is equal to

$$[K_{1a}(t), K_{1b}(t)]/C_1(t) \otimes [1 - \prod_{i=2}^{3} (1 - K_{ia}(t)), 1 - \prod_{i=2}^{3} (1 - K_{ib}(t))]/C_p(t)$$

$$= [K_{1a}(t), K_{1b}(t)]/C_1(t) \otimes [1 - (1 - K_{2a}(t))(1 - K_{3a}(t)), 1 - (1 - K_{2b}(t))(1 - K_{3b}(t))]/C_p(t) = [K_{1a}(t)(1 - (1 - K_{2a}(t))(1 - K_{3a}(t)), K_{1b}(t)(1 - (1 - K_{2b}(t))(1 - K_{3b}(t))) / \mathrm{Min}(C_1(t), C_p(t)) = [K_{1a}(t) - K_{1a}(t)(1 - K_{2a}(t))(1 - K_{3a}(t)), K_{1b}(t) - K_{1b}(t)(1 - K_{2b}(t))(1 - K_{3b}(t))] / \mathrm{Min}(C_1(t), C_p(t)),$$

where the value of $C_p(t)$ is evaluated as follows:

Case 1: If $S([1 - \prod_{i=2}^{3} (1 - K_{ia}(t)), 1 - \prod_{i=2}^{3} (1 - K_{ib}(t))], [K_{2a}(t), K_{2b}(t)]) \geq S([1 - \prod_{i=2}^{3} (1 - K_{ia}(t)), 1 - \prod_{i=2}^{3} (1 - K_{ib}(t))], [K_{3a}(t), K_{3b}(t)])$, then let the value of $C_p(t)$ be equal to $C_2(t)$.

Case 2: If $S([1 - \prod_{i=2}^{3} (1 - K_{ia}(t)), 1 - \prod_{i=2}^{3} (1 - K_{ib}(t))], [K_{2a}(t), K_{2b}(t)]) < S([1 - \prod_{i=2}^{3} (1 - K_{ia}(t)), 1 - \prod_{i=2}^{3} (1 - K_{ib}(t))], [K_{3a}(t), K_{3b}(t)])$, then let the value of $C_p(t)$ be equal to $C_3(t)$.

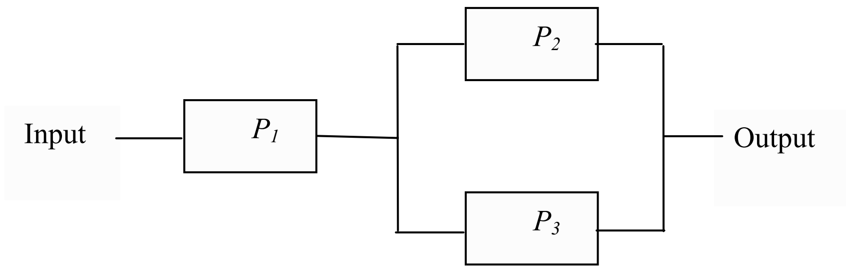

Fig. 4. A series-parallel system

The method presented in (Chen and Jong, 1996) is more flexible and more general than the one presented in (Kaufmann and Gupta, 1988, pp. 184-208) due to the fact that it allows the survival function of each subsystem at different times to be associated with different degrees of certainty between zero and one.

12.3 Basic Concepts of Vague Sets

In (Zadeh, 1965), Zadeh proposed the theory of fuzzy sets. Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$. The grade of membership of an element u_i in a fuzzy set is represented by a real value between zero and one, where $u_i \in U$. However, Gau and Buehrer (1993) pointed out that this single value combines the evidence for $u_i \in U$ and the evidence against $u_i \in U$. They also pointed out that it does not indicate the evidence for $u_i \in U$ and the evidence against $u_i \in U$, respectively, and it does not indicate how much there is of each. Furthermore, Gau and Buehrer also pointed out that the single value tells us nothing about its accuracy. Therefore, Gau and Buehrer (1993) presented the concepts of vague sets. Chen (1995) have presented the arithmetic operations between vague sets.

Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$, with a generic element of U denoted by u_i. A vague set $\tilde{A}$ in the universe of discourse U is characterized by a truth-membership function $t_{\tilde{A}}$, $t_{\tilde{A}}: U \rightarrow [0, 1]$, and a false-membership function $f_{\tilde{A}}$, $f_{\tilde{A}}: U \rightarrow [0, 1]$, where $t_{\tilde{A}}(u_i)$ is a lower bound of the grade of membership of u_i derived from the evidence for u_i, $f_{\tilde{A}}(u_i)$ is a lower bound of the negation of u_i derived from the evidence against u_i, and $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$. The grade of membership of u_i in the vague set $\tilde{A}$ is bounded by a subinterval $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ of $[0, 1]$. The vague value $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ indicates that the exact grade of membership $\mu_{\tilde{A}}(u_i)$ of u_i is bounded by $t_{\tilde{A}}(u_i) \leq \mu_{\tilde{A}}(u_i) \leq 1 - f_{\tilde{A}}(u_i)$, where $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$. For example, a vague set $\tilde{A}$ in the universe of discourse U is shown in Fig. 5.

If the universe of discourse U is a finite set, then a vague set $\tilde{A}$ of the universe of discourse U can be represented as follows:

$$\tilde{A} = \sum_{i=1}^{n} \left[t_{\tilde{A}}(u_i),\ 1 - f_{\tilde{A}}(u_i) \right] / u_i \,. \tag{9}$$

If the universe of discourse U is an infinite set, then a vague set $\tilde{A}$ of the universe of discourse can be represented as

$$\tilde{A} = \int_U \left[t_{\tilde{A}}(u_i),\ 1 - f_{\tilde{A}}(u_i)\right] / u_i \qquad u_i \in U. \tag{10}$$

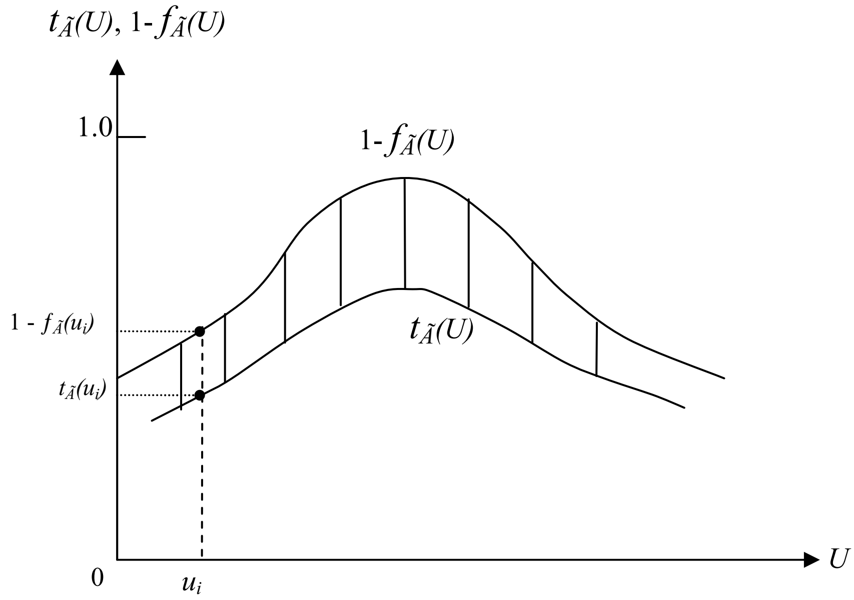

Fig. 5. A vague set

Definition 3.1: Let $\tilde{A}$ be a vague set of the universe of discourse U with the truth-membership function $t_{\tilde{A}}$ and the false-membership function $f_{\tilde{A}}$, respectively. The vague set $\tilde{A}$ is convex if and only if for all u_1, u_2 in U,

$$t_{\tilde{A}}(\lambda u_1 + (1-\lambda)u_2) \geq \text{Min}(t_{\tilde{A}}(u_1), t_{\tilde{A}}(u_2)), \tag{11}$$

$$1 - f_{\tilde{A}}(\lambda u_1 + (1-\lambda) u_2) \geq \text{Min}(1 - f_{\tilde{A}}(u_1), 1 - f_{\tilde{A}}(u_2)), \tag{12}$$

where $\lambda \in [0, 1]$.

Definition 3.2: A vague set $\tilde{A}$ of the universe of discourse U is called a normal vague set if $\exists\ u_i \in U$, such that $1 - f_{\tilde{A}}(u_i) = 1$. That is, $f_{\tilde{A}}(u_i) = 0$.

Definition 3.3: A vague number is a vague subset in the universe of discourse U that is both convex and normal.

In the following, we introduce some arithmetic operations of triangular vague sets (Chen, 1995). Let us consider the triangular vague set Ã shown in Fig. 6, where the triangular vague set Ã can be parameterized by a tuple

<[(a, b, c); μ_1], [(a, b, c); μ_2]>. For convenience, the tuple <[(a, b, c); μ_1], [(a, b, c); μ_2]> can also be abbreviated into <[(a, b, c); μ_1; μ_2]>, where $0 \leq \mu_1 \leq \mu_2 \leq 1$.

Some arithmetic operations between triangular vague sets are as follows:

Case 1: Consider the triangular vague sets $\tilde{A}$ and $\tilde{B}$ shown in Fig. 7, where

$$\tilde{A} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]> = <[(a_1, b_1, c_1); \mu_1; \mu_2]>,$$

$$\tilde{B} = <[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]> = <[(a_2, b_2, c_2); \mu_1; \mu_2]>,$$

and $0 \leq \mu_1 \leq \mu_2 \leq 1$. The arithmetic operations between the triangular vague sets $\tilde{A}$ and $\tilde{B}$ are defined as follows:

$$\tilde{A} \oplus \tilde{B} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]> \oplus <[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]>$$

$$= <[(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mu_1], [(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mu_2]>$$

$$= <[(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mu_1; \mu_2]>, \quad (13)$$

$$\tilde{B} \ominus \tilde{A} = <[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]> \ominus <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]>$$

$$= <[(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mu_1], [(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mu_2]>$$

$$= <[(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mu_1; \mu_2]>, \quad (14)$$

$$\tilde{A} \otimes \tilde{B} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]> \otimes <[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]>$$

$$= <[(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mu_1], [(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mu_2]>$$

$$= <[(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mu_1; \mu_2]>, \quad (15)$$

$$\tilde{B} \oslash \tilde{A} = <[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]> \oslash <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]>$$

$$= <[(a_2/c_1, b_2/b_1, c_2/a_1); \mu_1], [(a_2/c_1, b_2/b_1, c_2/a_1); \mu_2]>$$

$= <[(a_2/c_1, b_2/b_1, c_2/a_1); \mu_1; \mu_2]>.$ (16)

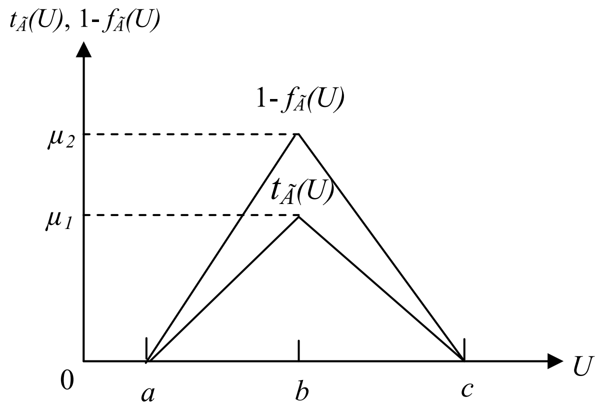

Fig. 6. A triangular vague set

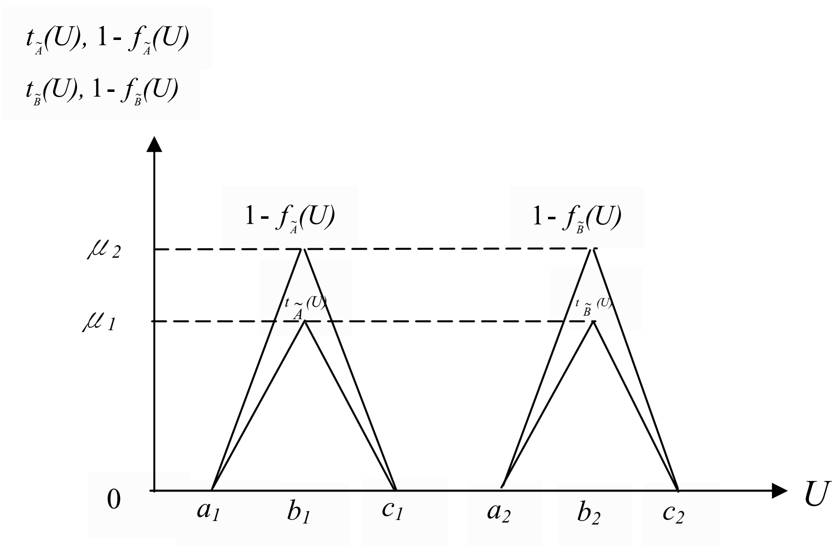

Fig. 7. Triangular vague sets $\tilde{A}$ and $\tilde{B}$ (Case 1)

Case 2: Consider the triangular vague sets $\tilde{A}$ and $\tilde{B}$ shown in Fig. 8, where

$$\tilde{A} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]>,$$
$$\tilde{B} = <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]>,$$

and $0 \le \mu_3 \le \mu_1 \le \mu_4 \le \mu_2 \le 1$.

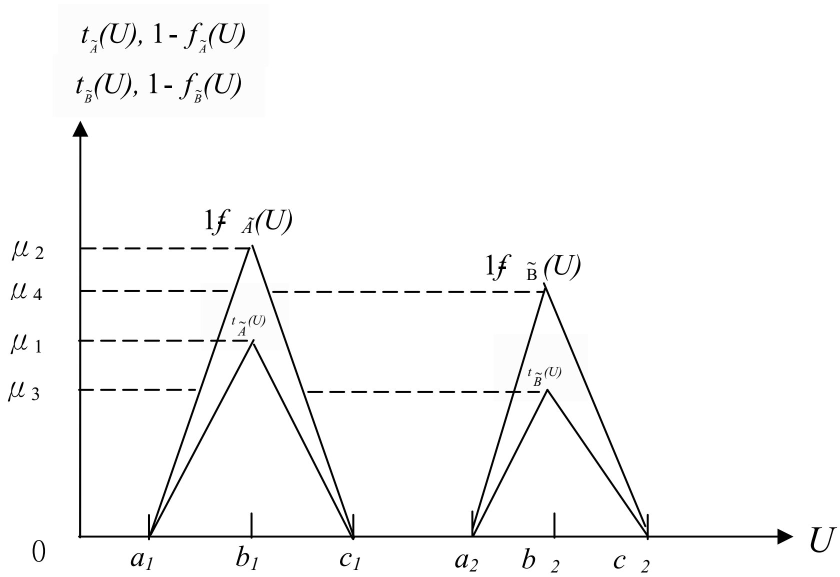

Fig. 8. Triangular vague sets $\tilde{A}$ and $\tilde{B}$ (Case 2)

The arithmetic operations between the triangular vague sets $\tilde{A}$ and $\tilde{B}$ are defined as follows:

$$\tilde{A} \oplus \tilde{B} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]> \oplus <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]>$$
$$= <[(a_1 + a_2, b_1 + b_2, c_1 + c_2); \text{Min}(\mu_1, \mu_3)], [(a_1 + a_2, b_1 + b_2, c_1 + c_2); \text{Min}(\mu_2, \mu_4)]>, \quad (17)$$

$$\tilde{B} \ominus \tilde{A} = <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]> \ominus <[(a_1, b_1, c_1); \mu_1],$$

$[(a_1, b_1, c_1); \mu_2]>$

$= <[(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mathrm{Min}(\mu_1, \mu_3)], [(a_2 - c_1, b_2 - b_1, c_2 - a_1);$

$\mathrm{Min}(\mu_2, \mu_4)]>, \quad (18)$

$\tilde{A} \otimes \tilde{B} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]> \otimes <[(a_2, b_2, c_2); \mu_3],$

$[(a_2, b_2, c_2); \mu_4]>$

$= <[(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mathrm{Min}(\mu_1, \mu_3)], [(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2);$

$\mathrm{Min}(\mu_2, \mu_4)]> \quad (19)$

$\tilde{B} \oslash \tilde{A} = <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]> \oslash <[(a_1, b_1, c_1); \mu_1],$

$[(a_1, b_1, c_1); \mu_2]>$

$= <[(a_2/c_1, b_2/b_1, c_2/a_1); \mathrm{Min}(\mu_1, \mu_3)], [(a_2/c_1, b_2/b_1, c_2/a_1);$

$\mathrm{Min}(\mu_2, \mu_4)]>. \quad (20)$

12.4 Analyzing Fuzzy System Reliability Based on Vague Sets

In this section, we introduce a method for analyzing fuzzy system reliability based on vague sets (Chen, 2003), where the reliabilities of the components of a system are represented by triangular vague sets defined in the universe of discourse [0, 1].

Consider a series system shown in Fig. 2, where the reliability $\tilde{R}_i$ of the subsystem P_i is represented by a triangular vague set $<[(a_i, b_i, c_i); \mu_{i1}; \mu_{i2}]>$, where $0 \le \mu_{i1} \le \mu_{i2} \le 1$, and $1 \le i \le n$. Then, the reliability $\tilde{R}$ of the series system shown in Fig. 2 can be evaluated as follows:

$\tilde{R} = \tilde{R}_1 \otimes \tilde{R}_2 \otimes \ldots \otimes \tilde{R}_n = <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]> \otimes$

$<[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]> \otimes \ldots \otimes <[(a_n, b_n, c_n); \mu_{n1}; \mu_{n2}]>$

$= <[(\prod_{i=1}^{n} a_i, \prod_{i=1}^{n} b_i, \prod_{i=1}^{n} c_i); \mathrm{Min}(\mu_{11}, \mu_{21}, \ldots, \mu_{n1});$

$\mathrm{Min}(\mu_{12}, \mu_{22}, \ldots, \mu_{n2})]>. \quad (21)$

Furthermore, consider the parallel system shown in Fig. 3, where the reliability $\widetilde{R}_i$ of the subsystem P_i is represented by a triangular vague set $<[(a_i, b_i, c_i); \mu_{i1}; \mu_{i2}]>$, where $0 \le \mu_{i1} \le \mu_{i2} \le 1$, and $1 \le i \le n$. Then, the reliability $\widetilde{R}$ of the parallel system shown in Fig. 3 can be evaluated as follows:

$$\widetilde{R} = 1 \ominus \prod_{i=1}^{n} (1 \ominus \widetilde{R}_i) = 1 \ominus (1 \ominus <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]>)$$

$$\otimes (1 \ominus <[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]>) \otimes \ldots \otimes (1 \ominus <[(a_n, b_n, c_n); \mu_{n1}; \mu_{n2}]>)$$

$$= 1 \ominus <[(1 - c_1, 1 - b_1, 1 - a_1); \mu_{11}; \mu_{12}]> \otimes <[(1 - c_2, 1 - b_2, 1 - a_2);$$

$$\mu_{21}; \mu_{22}]> \otimes \ldots \otimes <[(1 - c_n, 1 - b_n, 1 - a_n); \mu_{n1}; \mu_{n2}]>$$

$$= 1 \ominus <[\prod_{i=1}^{n} (1 - c_i), \prod_{i=1}^{n} (1 - b_i), \prod_{i=1}^{n} (1 - a_i)); \mathrm{Min}(\mu_{11}; \mu_{21}, \ldots, \mu_{n1});$$
$$\mathrm{Min}(\mu_{12}; \mu_{22}, \ldots, \mu_{n2}))>$$

$$= <[(1 - \prod_{i=1}^{n} (1 - a_i), 1 - \prod_{i=1}^{n} (1 - b_i), 1 - \prod_{i=1}^{n} (1 - c_i));$$
$$\mathrm{Min}(\mu_{11}; \mu_{21}, \ldots, \mu_{n1}); \mathrm{Min}(\mu_{12}; \mu_{22}, \ldots, \mu_{n2}))> \qquad (22)$$

In the following, we use an example to illustrate the fuzzy system reliability analysis process of the proposed method.

12.4.1 Example

Consider the system shown in Fig. 9, where the reliabilities of the subsystems P_1, P_2, P_3 and P_4 are represented by the triangular vague sets $\widetilde{R}_1$, $\widetilde{R}_2$, $\widetilde{R}_3$ and $\widetilde{R}_4$, respectively, where

$$\widetilde{R}_1 = <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]>, \ \widetilde{R}_2 = <[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]>,$$

$$\widetilde{R}_3 = <[(a_3, b_3, c_3); \mu_{31}; \mu_{32}]>, \ \widetilde{R}_4 = <[(a_4, b_4, c_4); \mu_{41}; \mu_{42}]>,$$

$0 \le \mu_{i1} \le \mu_{i2} \le 1$ and $1 \le i \le 4$. Based on the previous discussion, we can see that the reliability $\widetilde{R}$ of the system shown in Fig. 9 can be evaluated as follows:

$$\widetilde{R} = [1 \ominus (1 \ominus \widetilde{R}_1) \otimes (1 \ominus \widetilde{R}_2)] \otimes [1 \ominus (1 \ominus \widetilde{R}_3) \otimes (1 \ominus \widetilde{R}_4)]$$

$$= [1 \ominus (1 \ominus <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]>) \otimes (1 \ominus <[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]>) \otimes [1 \ominus (1 \ominus <[(a_3, b_3, c_3); \mu_{31}; \mu_{32}]>) \otimes (1 \ominus <[(a_4, b_4, c_4); \mu_{41}; \mu_{42}]>)]$$

$$= [1 \ominus <[(1 - c_1, 1 - b_1, 1 - a_1); \mu_{11}; \mu_{12}]> \otimes <[(1 - c_2, 1 - b_2, 1 - a_2); \mu_{21}; \mu_{22}]>] \otimes [1 \ominus <[(1 - c_3, 1 - b_3, 1 - a_3); \mu_{31}; \mu_{32}]> \otimes <[(1 - c_4, 1 - b_4, 1 - a_4); \mu_{41}; \mu_{42}]>]$$

$$= [1 \ominus <[((1 - c_1)(1 - c_2), (1 - b_1)(1 - b_2), (1 - a_1)(1 - a_2)); \mathrm{Min}(\mu_{11}; \mu_{21}); \mathrm{Min}(\mu_{12}, \mu_{22})]>] \otimes [1 \ominus <[((1 - c_3)(1 - c_4), (1 - b_3)(1 - b_4), (1 - a_3)(1 - a_4)); \mathrm{Min}(\mu_{31}; \mu_{41}); \mathrm{Min}(\mu_{32}, \mu_{42})]>]$$

$$= <[(1 - (1 - a_1)(1 - a_2), 1 - (1 - b_1)(1 - b_2), 1 - (1 - c_1)(1 - c_2)); \mathrm{Min}(\mu_{11}; \mu_{21}); \mathrm{Min}(\mu_{12}, \mu_{22})]>] \otimes <[(1 - (1 - a_3)(1 - a_4), 1 - (1 - b_3)(1 - b_4), 1 - (1 - c_3)\ (1 - c_4)); \mathrm{Min}(\mu_{31}; \mu_{41}); \mathrm{Min}(\mu_{32}, \mu_{42})]>$$

$$= <[(a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2, c_1 + c_2 - c_1c_2); \mathrm{Min}(\mu_{11}; \mu_{21}); \mathrm{Min}(\mu_{12}, \mu_{22})]> \otimes <[(a_3 + a_4 - a_3a_4, b_3 + b_4 - b_3b_4, c_3 + c_4 - c_3c_4); \mathrm{Min}(\mu_{31}; \mu_{41}); \mathrm{Min}(\mu_{32}, \mu_{42})]>$$

$$= <[((a_1 + a_2 - a_1a_2)(a_3 + a_4 - a_3a_4), (b_1 + b_2 - b_1b_2)(b_3 + b_4 - b_3b_4), (c_1 + c_2 - c_1c_2)(c_3 + c_4 - c_3c_4)), \mathrm{Min}(\mu_{11}, \mu_{21}, \mu_{31}, \mu_{41}); \mathrm{Min}(\mu_{12}, \mu_{22}, \mu_{32}, \mu_{42})]>$$

$$= <[(a_1a_3 + a_1a_4 - a_1a_3a_4 + a_2a_3 + a_2a_4 - a_2a_3a_4 - a_1a_2a_3 - a_1a_2a_4 + a_1a_2a_3a_4, b_1b_3 + b_1b_4 - b_1b_3b_4 + b_2b_3 + b_2b_4 - b_2b_3b_4 - b_1b_2b_3 - b_1b_2b_4 + b_1b_2b_3b_4, c_1c_3 + c_1c_4 - c_1c_3c_4 + c_2c_3 + c_2c_4 - c_2c_3c_4 - c_1c_2c_3 - c_1c_2c_4 + c_1c_2c_3c_4); \mathrm{Min}(\mu_{11}, \mu_{21}, \mu_{31}, \mu_{41}); \mathrm{Min}(\mu_{12}, \mu_{22}, \mu_{32}, \mu_{42})]>. \quad (23)$$

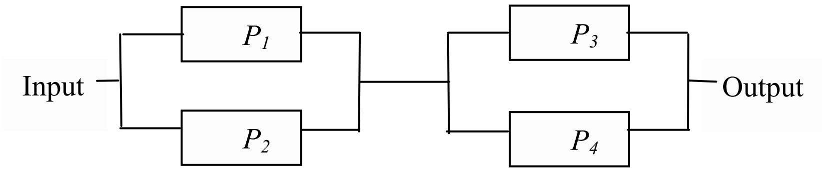

Fig. 9. A system with four subsystems P_1, P_2, P_3 and P_4

12.5 Conclusions

In this chapter, we have presented a method for analyzing fuzzy system reliability based on the vague set theory, where the components of a system are represented by triangular vague sets defined in the universe of discourse [0, 1]. The grade of membership of an element x in a vague set is represented by a vague value $[t_x, 1 - f_x]$ in [0, 1], where t_x indicates the degree of truth, f_x indicates the degree of false, $1 - t_x - f_x$ indicates the unknown part, $0 \le t_x \le 1 - f_x \le 1$, and $t_x + f_x \le 1$. The proposed method can model and analyze fuzzy system reliability in a more flexible and convenient manner.

References

Atanassov, K (1986) Intuitionistic fuzzy sets. Fuzzy Sets and Systems 20: 87-96.

Cai, KY, Wen, CY, Zhang, ML (1991a) Fuzzy variables as a basis for a theory of fuzzy reliability in possibility context. Fuzzy Sets and Systems 42: 145-172.

Cai, KY, Wen, CY, Zhang, ML (1991b) Posbist reliability behavior of typical systems with two types of failure. Fuzzy Sets and Systems 43: 17-32.

Cai, KY, Wen, CY, Zhang, ML (1991c) Fuzzy reliability modeling of gracefully degradable computing systems. Reliability Engineering and System Safety 33: 141-157.

Cai, KY (1996) System failure engineering and fuzzy methodology: An introductory overview. Fuzzy Sets and Systems 83: 113-133.

Chen, SM (2003) Analyzing fuzzy system reliability using vague set theory. Inter national Journal of Applied Science and Engineering 1: 82-88, 2003.

Chen, SM (1994) Fuzzy system reliability analysis using fuzzy number arithmetic operations. Fuzzy Sets and Systems 64: 31-38.

Chen, SM, Jong, WT (1996) Analyzing fuzzy system reliability using interval of confidence. International Journal of Information Management and Engineering 2: 16-23.

Chen, SM, Wang, JY (1995) Document retrieval using knowledge-based fuzzy information retrieval techniques. IEEE Transactions on Systems, Man, and Cybernetics 25: 793-803.
Chen, SM (1995) Arithmetic operations between vague sets. Proceedings of the International Joint Conference of CFSA/IFIS/SOFT'95 on Fuzzy Theory and Applications, Taipei, Taiwan, Republic of China, pp. 206-211.
Chen, SM (1996) New method for fuzzy system reliability analysis. Cybernetics and Systems: An International Journal 27: 385-401.
Chen, SM (1997a) Fuzzy system reliability analysis based on vague set theory. Proceedings of the 1997 IEEE International Conference on Systems, Man, and Cybernetics, Orlando, Florida, U. S. A., pp. 1650-1655.
Chen, SM (1997b) Similarity measures between vague sets and between elements. IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics 27: 153-158.
Chen, SM, Shiau, YS (1998) Vague reasoning and knowledge representation using extended fuzzy Petri nets. Journal of Information Science and Engineering 14: 391-408.
Cheng, CH, Mon, DL (1993) Fuzzy system reliability analysis by interval of confidence. Fuzzy Sets and Systems 56: 29-35.
Gau, WL, Buehrer, DJ (1993) Vague sets. IEEE Transactions on Systems, Man, and Cybernetics 23: 610-614.
Kandel, A (1986) Fuzzy Mathematical Techniques with Applications. Addison-Wesley, Massachusetts, U. S. A.
Kaufmann, A, Gupla, MM (1988) Fuzzy Mathematical Models in Engineering and Management Science. North-Holland, Amsterdam, The Netherlands.
Mon, DL, Cheng, CH (1994) Fuzzy system reliability analysis for components with different membership functions. Fuzzy Sets and Systems 64: 147-157.
Singer, D (1990) A fuzzy set approach to fault tree and reliability analysis. Fuzzy Sets and Systems 34: 145-155.
Song, Q, Chissom, BS (1993a) Fuzzy time series and its models. Fuzzy Sets and Systems 54: 269-277.
Song, Q, Chissom, BS (1993b) Forecasting enrollments with fuzzy time series — Part I. Fuzzy Sets and Systems 54: 1-9.
Song, Q, Chissom, BS (1994) Forecasting enrollments with fuzzy time series — Part II. Fuzzy Sets and Systems 62: 1-8.
Suresh, PV, Babar, AK, Raj, VV (1996) Uncertainty in fault tree analysis: A fuzzy approach. Fuzzy Sets and Systems 83: 135-141.
Utkin, LV, Gurov, SV (1996) A general formal approach for fuzzy reliability analysis in the possibility context. Fuzzy Sets and Systems 83: 203-213.
Wu, HC (2004) Fuzzy reliability estimation using Bayesian approach. Computers & Industrial Engineering 46: 467-493.
Zadeh, LA (1965) Fuzzy sets. Information and Control 8: 338-353.
Zimmermann, HJ (1991) Fuzzy Set Theory and Its Applications. Kluwer Academic Publishers, Boston, U. S. A.