An Efficient Blind SIMO Channel Identification Algorithm Via Eigenvalue Decomposition*

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Abstract. An effective blind multichannel identification algorithm is proposed in this paper. Different from the Prediction Error Method, the new algorithm does not require the input signal to be independent and identical distribution, and even the input signal can be non-stationary. Compared with Least-Square Approach, the new algorithm is more robust to the overestimation of channel order. Finally, the experiments demonstrate the good performance of the proposed algorithm.

1 Introduction

Blind identification of Single-Input Multiple-Output (SIMO) systems has many applications, or potential applications in wireless communications, equalization, seismic data deconvolution, speech coding, image deblurring, echo cancellation[1-8], etc. For the FIR SIMO system, as long as the FIR channels do not share the common zeros and all channels are fully activated, the SIMO system can be identified by just second-order statistics of the output [1], which further makes the blind identification of SIMO systems so important. So many researchers paid much attention on this problem.

Because of the predominant advantage in computation cost and the weak requirement in data samples of the receiving signals, the second-order statistics (SOS)-based methods are very attractive and obtain much attention. Among them, the least-square approach (LSA) [1], the linear prediction methods (LP) [2] and the subspace methods (SS) [3]and are the three main classes. When the channel order is known, the channels can be very precisely estimated by SS-based approaches and LSA-methods, however, which are very sensitive to the estimation error of channel order. Contrastively, LP methods are not so accurate as the former two methods, but robust to the channel order overestimation. LP methods usually require the input signal is independent and identically distribution (i.i.d) while the other two methods is not limited by this requirement. Relatively, the LS approaches are a little simpler than SS ones.

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In this paper, we present a new blind identification algorithm for SIMO FIR system by improving the LS approaches. The proposed algorithm is simply based on generalized eigenvalue decomposition. The new algorithm can be easy implemented and is robust to the channel order overestimation than SS and LS approaches.

2 Problem Statement

The single-input m -output channel can be formulated as:

$$\boldsymbol{x}(t)=\sum_{\tau=0}^{L}\boldsymbol{h}(\tau)s(t-\tau)+\boldsymbol{n}(t),\ t=1,2,\cdots,T \tag{1}$$

where $\boldsymbol{x}(t)=\left(x_1(t),\cdots,x_m(t)\right)^T\in R^{m\times 1}$ is the observed signa1 vector, $s(t)$ is the input signal, $\boldsymbol{h}(\tau)=\left(h_1(\tau),\cdots,h_m(\tau)\right)^T,(\tau=0,\cdots,L)$ denotes the FIR channel impulse response. The order of the convolution is L. The additive noise is denoted as a vector $\boldsymbol{n}(t)=\left(n_1(t),\cdots,n_m(t)\right)^T\in R^{m\times 1}$. The blind identification problem can be stated as follows: Given the receiving signals $\left\{x_i(t)\middle|i=1,\cdots,m;t=1,\cdots,T\right\}$, we aim to determine the channels $\left\{h_i(\bullet)\right\}_{i=1}^{m}$ up to a nonzero scaling factor, i.e. $\left\{\hat{h}_i(\bullet)\right\}_{i=1}^{m}=c\left\{h_i(\bullet)\right\}_{i=1}^{m},(c\neq 0)$, then we can further recover the input signal $s(\bullet)$.

Xu, Tong, *et al* point out that if the channel order is known in advance, the necessary and sufficient identifiability condition of SIMO system (1) is that the FIR channels have no common zero [1]. So we assume that the FIR channels of system (1) do not share the common zeros.

3 Identification Equations

According to reference [1], we have the following equations:

$$\begin{aligned}x_i(t)&=h_i(t)\odot s(t),\\x_j(t)&=h_j(t)\odot s(t),\end{aligned} \tag{2}$$

where $\odot$ stands for *convolution operation*. Thus

$$h_j(t)\odot x_i(t)=h_j(t)\odot\left[h_i(t)\odot s(t)\right]=h_i(t)\odot\left[h_j(t)\odot s(t)\right]=h_i(t)\odot x_j(t),$$

i.e.,

$$h_j(t)\odot x_i(t)=h_i(t)\odot x_j(t),(i\neq j,i,j=1,\cdots,m) \tag{3}$$

From equation (3), we have

$$\left[X_i(L):-X_j(L)\right]\begin{bmatrix}\boldsymbol{h}_j\\\boldsymbol{h}_i\end{bmatrix}=0 \tag{4}$$

where $\boldsymbol{h}_k = (h_k(L), \cdots, h_k(0))^T$ and

$$X_k(L) = \begin{bmatrix} x_k(L) & x_k(L+1) & \cdots & x_k(2L) \\ x_k(L+1) & x_k(L+2) & \cdots & x_k(2L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x_k(T-L) & x_k(T-L+1) & \cdots & x_k(T) \end{bmatrix} \tag{5}$$

where $k = 1, \cdots, m$.

Denote $\boldsymbol{h} \triangleq \left[\boldsymbol{h}_1^T, \cdots, \boldsymbol{h}_m^T\right]^T$, and we construct the following matrices:

$$X^i(L) = \left[\begin{bmatrix} 0 \cdots 0 & X_{i+1}(L) & -X_i(L) & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ \underbrace{0 \cdots 0}_{i-1 \text{ blocks}} & \underbrace{X_m(L) \quad 0 \quad \cdots \quad -X_i(L)}_{m-i+1 \text{ blocks}} \end{bmatrix}\right\} m-i \text{ blocks} \tag{6}$$

where $i = 1, \cdots, m$ .In equations (6), each block, e.g., 0 or $\{X_k(L), k = 1, \cdots, m\}$, has the size $(T-L+1) \times (L+1)$. In the noise free case, from SIMO system (1) we derive the following equations:

$$X(L) \cdot \boldsymbol{h} = 0 \tag{7}$$

where matrix $X(L)$ is $\{(T-L+1)[m(m-1)/2]\} \times [m(L+1)]$, and it is given by

$$X(L) = \left[\begin{bmatrix} X^1(L) \\ \vdots \\ \underbrace{X^{m-1}(L)}_{m \text{ blocks}} \end{bmatrix}\right\} \frac{m(m+1)}{2} \text{ blocks} \tag{8}$$

Now the blind identification problem (1) boils down to solving equations (7).

4 Blind Identification Algorithm

The solution of equation (7) is not unique. To find the practical solution, we usually add some appropriate constraints, e.g., $\|\boldsymbol{h}\|_2 = 1$ or $\boldsymbol{c}^H \boldsymbol{h} = 1$ for a constant vector $\boldsymbol{c}$. The LS approaches identify the channels of system (1) by solving the following optimization problem with constraints:

$$\begin{cases} \min J_1(\boldsymbol{h}) = \min\limits_{\boldsymbol{h}} \|X(L) \cdot \boldsymbol{h}\|_2^2, \\ st: \|\boldsymbol{h}\|_2 = 1. \end{cases} \tag{9}$$

Xu, Tong *et al* [1] use the *singular value decomposition* (SVD) or *fast subspace decomposition* (FSD) to solve optimization problem (9). Of course, we can replace the constraint $\|\boldsymbol{h}\|_2 = 1$ by constraint $\boldsymbol{c}^H \boldsymbol{h} = 1$ .Since the accurate channel order of system (1) is unknown and estimating it is a challenging work in practice. Usually what we can do is overestimating the order. Without the loss of generality, we overestimate the channels order of system (1) as $L_h\left(L_h \geq L\right)$. As mentioned in section 1, LSA algorithm is not robust to overestimation of channel order. To overcome this drawback, we attempt to improve the LSA algorithm, which intend to not only keep advantage of LSA algorithm, but also be robust to overestimation of channels order.

Denote the $\left[m\left(L_h+1\right)\right]\times 1$ vector $\hat{\boldsymbol{h}}$ to be the estimation of $\boldsymbol{h}$. Considering $L_h \geq L$, if $\hat{\boldsymbol{h}}$ satisfies $h_k\left(\tau\right)=0, \left(\tau = L+1,\cdots,L_h; k=1,\cdots m\right)$, the overestimation of channel order will have not any influence on the channel identification of system (1). Hence the desirable estimation $\hat{\boldsymbol{h}}$ of $\boldsymbol{h}$ should be

$$\begin{cases} \hat{\boldsymbol{h}} = \left[\hat{\boldsymbol{h}}_1^T,\cdots,\hat{\boldsymbol{h}}_m^T\right]^T, \\ \hat{\boldsymbol{h}}_k = c\left(\overbrace{0,\cdots,0}^{L_h-L},\boldsymbol{h}_k\right)^T = c\underbrace{\left(0,\cdots,0,h_k\left(L\right),\cdots,h_k\left(0\right)\right)^T}_{L_h+1}, k=1,\cdots,m, \end{cases} \tag{10}$$

where c is a nonzero constant. We construct the following $\left[m\left(L_h+1\right)\right]\times 1$ vector:

$$\begin{cases} \boldsymbol{\mu} = \left[\boldsymbol{\mu}_1^T,\cdots,\boldsymbol{\mu}_m^T\right]^T \in R^{m(L_h+1)}, \\ \boldsymbol{\mu}_k = \left(\mu^{L_h},\cdots,\mu,1\right)^T, k=1,\cdots,m; 0<\mu<1. \end{cases} \tag{11}$$

To make $\hat{\boldsymbol{h}}$ be robust to overestimation of channel order and satisfy expression (10) as possible as it can, we solve the following optimization problem with constraints:

$$\begin{cases} \min J\left(\hat{\boldsymbol{h}}\right) = \min\limits_{\hat{\boldsymbol{h}}}\left[\left\|X\left(L_h\right)\cdot\hat{\boldsymbol{h}}\right\|_2^2 + \hat{\boldsymbol{h}}^T\left(diag\left(\boldsymbol{\mu}\right)\right)^l\hat{\boldsymbol{h}}\right], \\ st: \left\|\hat{\boldsymbol{h}}\right\|_2 = 1, \end{cases} \tag{12}$$

where l is a positive integer. Because $0<\mu<1$, it is easy to know that $1>\mu>\cdots>\mu^{L_h}$ and $1>\mu^l>\cdots>\mu^{lL_h}$. So under the constraints $\left\|\hat{\boldsymbol{h}}\right\|_2 = 1$ and $X\left(L_h\right)\hat{\boldsymbol{h}} = 0$, minimizing $\hat{\boldsymbol{h}}^T\left[diag\left(\boldsymbol{\mu}\right)\right]^l\hat{\boldsymbol{h}}$ will force $\hat{\boldsymbol{h}}$ to approximately satisfy expression (10) in some degree.

The constraint $\left\|\hat{\boldsymbol{h}}\right\|_2 = 1$ means $\hat{\boldsymbol{h}}^T\hat{\boldsymbol{h}} = 1$. Thus the optimization problem (12) can be formulated into the following one without constraint:

$$\begin{aligned}\min_{\hat{\boldsymbol{h}}} \mathrm{e}\left(\hat{\boldsymbol{h}}\right) &= \min_{\hat{\boldsymbol{h}}} \frac{\left[\left\| X\left(L_h\right)\cdot\hat{\boldsymbol{h}}\right\|_2^2 + \hat{\boldsymbol{h}}^T \left(diag\left(\boldsymbol{\mu}\right)\right)^l \hat{\boldsymbol{h}}\right]}{\hat{\boldsymbol{h}}^T\hat{\boldsymbol{h}}} \\ &= \min_{\hat{\boldsymbol{h}}} \frac{\hat{\boldsymbol{h}}^T\left[X^T\left(L_h\right)X\left(L_h\right)+\left(diag\left(\boldsymbol{\mu}\right)\right)^l\right]\hat{\boldsymbol{h}}}{\hat{\boldsymbol{h}}^T\hat{\boldsymbol{h}}}\end{aligned} \tag{13}$$

From expression (13), we have

$$\mathrm{e}\left(\hat{\boldsymbol{h}}\right)\cdot\hat{\boldsymbol{h}}^T\hat{\boldsymbol{h}} = \hat{\boldsymbol{h}}^T\left[X^T\left(L_h\right)X\left(L_h\right)+\left(diag\left(\boldsymbol{\mu}\right)\right)^l\right]\hat{\boldsymbol{h}} \tag{14}$$

For equation (14), calculating the derivative of the two sides with respect to $\hat{\boldsymbol{h}}$, we get

$$\frac{\partial\,\mathrm{e}\left(\hat{\boldsymbol{h}}\right)}{\partial\hat{\boldsymbol{h}}}\cdot\hat{\boldsymbol{h}}^T\hat{\boldsymbol{h}} + 2\,\mathrm{e}\left(\hat{\boldsymbol{h}}\right)\hat{\boldsymbol{h}} = 2\left[X^T\left(L_h\right)X\left(L_h\right)+\left(diag\left(\boldsymbol{\mu}\right)\right)^l\right]\hat{\boldsymbol{h}} \tag{15}$$

Let $\frac{\partial\,\mathrm{e}\left(\hat{\boldsymbol{h}}\right)}{\partial\hat{\boldsymbol{h}}} = 0$, from equation (15), we have

$$\frac{\partial\,\mathrm{e}\left(\hat{\boldsymbol{h}}\right)}{\partial\hat{\boldsymbol{h}}}\cdot\hat{\boldsymbol{h}}^T\hat{\boldsymbol{h}} = 2\left\{\left[X^T\left(L_h\right)X\left(L_h\right)+\left(diag\left(\boldsymbol{\mu}\right)\right)^l\right] - \mathrm{e}\left(\hat{\boldsymbol{h}}\right)\cdot I\right\}\hat{\boldsymbol{h}} = 0 \tag{16}$$

Equation (16) means that one can estimate $\hat{\boldsymbol{h}}$ by doing the eigenvalue decomposition with respect to matrix $\left[X^T\left(L_h\right)X\left(L_h\right)+\left(diag\left(\boldsymbol{\mu}\right)\right)^l\right]$, and the eigenvector corresponding smallest value is just the estimation of $\hat{\boldsymbol{h}}$. So we obtain the proposed algorithm as follows:

1) Input the received signals $\boldsymbol{x}(t) = \left(x_1(t),\cdots,x_m(t)\right)^T, t = 1,\cdots,T$. Set μ, integer l and the channel order L_h.
2) Construct the matrix $X\left(L_h\right)$ and $\boldsymbol{\mu}$.
3) Compute the eigenvalues and corresponding eigenvectors of matrix.
4) The eigenvector corresponding smallest value is just the estimation $\hat{\boldsymbol{h}}$ of $\boldsymbol{h}$.

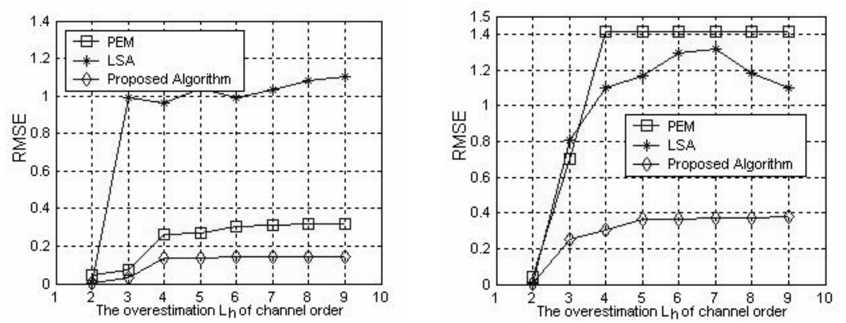
5 Numerical Experiments and Result Analysis

Root-mean-square-error (RMSE) is employed as a performance measure of channel estimation. Usually, when RMSE<0.8, the channels are well identified; when RMSE>1.0, the estimation of channels is not reliable. The input signal is supposed to be independent and identical distribution in the experiment. Computer simulations were conducted to evaluate the performance of the proposed algorithm in comparison with Least-Squares Approach (LSA) and Prediction Error Method (PEM). In the following two experiments, the related parameters of the proposed algorithm are set as: $T=1000$, $\mu=0.99$ and $l=2$ experientially. All input signals are i.i.d Gaussian signals generated by Matlab command $randn(\bullet)$. The channel coefficients are listed below.

$$h_1(z) = \text{-}0.4326\text{+}0.1253z^{-1}\text{-}1.1465z^{-2},$$
$$h_2(z) = \text{-}1.6656\text{+}0.2877z^{-1}\text{+}1.1909z^{-2}.$$

Table 1. The overestimation of channel order and corresponding RMSE for i.i.d input signal

L_h	2	3	4	5	6	7	8	9
LSA	9.0984e-016	0.99	0.97	1.04	0.10	1.03	1.08	1.10
PEM	0.0424	0.08	0.26	0.27	0.31	0.31	0.32	0.32
Our	9.3014e-006	0.03	0.14	0.14	0.14	0.14	0.14	0.14



(a) Noise free (b) The received signals are added white Gaussian noise and the SNR is 40dB

Fig. 1. Performance comparison between LSA, PEM and the proposed algorithm

From Table 1 and Fig.1(a), when the order of channel is accurately given, LSA can obtain the precise estimation of channels. But for overestimation case without noise, we can see that both PEM algorithm and the proposed algorithm well identify the channels, but LSA does not do this. Additionally, Fig.1(b) shows the comparison result in the same simulation environment except adding white Gaussian noise to the receiving signals. All SNRs are 40dB. In this situation, we can see that only the proposed algorithm get the relatively satisfactory estimation (Fig.1(b)).

6 Conclusion

Based on matrix eigenvalue decompostion, an effective blind multichannel identification algorithm is proposed in this paper. Different from the Prediction Error Method, the new algorithm does not require the input signal to be independent and identical distribution, and even the input signal can be non-stationary. Compared with Least-Square Approach, the new algorithm is more robust to the overestimation of channel order and much faster.

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