A Mathematical Model for Energy-Efficient Coverage and Detection in Wireless Sensor Networks

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Abstract. The tradeoff between system lifetime and system reliability is a paramount design consideration for wireless sensor networks. In order to prolong the system lifetime, random sleep scheme can be adopted without coordinating with its neighboring nodes. Based on the random sleep scheme, an accurate mathematical model for expected coverage ratio and point event detection quality is put forward in this paper. Furthermore, the model also takes the border effects into account and thus improves the accuracy of performance and quality analysis. Our model is flexible enough to capture the interaction among the essential system parameters. Therefore, this model could provide beneficial guidelines for optimal sensor network deployment satisfying both the lifetime and reliability requirements. Additional simulation results confirm the correctness and effectiveness of our analysis.

1 Introduction

Recently, wireless sensor networks (WSNs) have attracted a great deal of research attention due to their wide-range potential applications, such as environment monitoring, surveillance, target detection and localization. The sensor nodes in such applications are often intended to be deployed in remote or hostile environment operating for a long time under limited battery power, so it is undesirable or impossible to recharge or replace the battery power of all sensors. How to extend system lifetime without sacrificing system reliability is a paramount design consideration for WSNs.

Fortunately, researchers have found that scheduling sensor nodes to alternate between sleep and active mode is an important method to conserve energy resources in sensor networks with high node density (high up to 20 nodes/m^3) [1],[2],[3]. However, dynamic management of node duty cycles may disrupt the performance of network, such as coverage, detection, etc. Hence, an important problem of dynamic management is to minimize the number of nodes that remain active, while still achieving satisfied quality of service (QoS) for applications.

In order to resolve the above problem, some interesting approaches have been recently proposed in literature [4], [5]. However, all these schemes require coordination among nodes by exchanging location or directional information obtained with the Global Positioning System (GPS) and the directional antenna technology. These

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hardware devices usually consume too much energy and the cost is too high for tiny sensors. Furthermore, coordination among nodes also takes additional energy. It is expected that scheduling algorithms could work without geography information.

Moreover, none of the aforementioned literature considered the border effects, that is, the points near the border of deployment area generally have less chance to be covered than the points in central area. When the proportion of the node's sensing range to the range of the deployment area is not small enough, the border effects should not be ignored ([6], [7]). In [6], a mathematical method was proposed to evaluate the number of nodes needed to reach the expected coverage ratio with the consideration of border effect. However, it can only apply to determine the number of active nodes, and when dynamic management of nodes duty cycle is adopted, the total number of nodes can not be derived from this model. In [7], a mathematical expression was formulated for expected k-coverage with the consideration of both the border effects and the uncoordinated node scheduling scheme. Though the border effects were considered in [6] and [7], the network performance of border area such as coverage and detection can not be predicted accurately from these models.

In this paper, we present a mathematical model for energy-efficient coverage and detection quality with the consideration of border effects. We base our analysis on random deployment since this deployment strategy is easy and inexpensive for sensor networks [8]. And for individual nodes, we adopt the model of random sleep scheme. In this model, nodes sleep and wake up randomly and independently of each other. The obvious advantage of this scheme is its simplicity for implementation, without incurring control overhead. Since the deployment and the sleep scheme we choose are random, it is more reasonable to study this problem from a probabilistic perspective.

The main contributions of this paper include:

(1) The model is flexible enough to capture the interaction among the system parameters such as sensor node numbers, random sleep ratio, etc. Hence, it can provide guidelines for optimal sensor network deployment.
(2) Our model can help to determine the sleep ratio for a desired coverage and acceptable detection quality.
(3) We pay more attention to the quality of service (QoS) that the sensor network provides for the border area. If the applications in border area demand high degree of accuracy, the QoS is desired to be upgraded to a higher level. In this case, how many nodes should we deploy? This problem is also answered analytically in our model.

The rest of the paper is organized as follows. In section 2 we present our network models and assumptions. Section 3 formulates the network coverage problem and section 4 considers point event detection. In section 5 we present the simulation results and section 6 concludes the paper.

2 Network Models and Assumptions

In this section, we present the notations and assumptions for our derivations. First of all, we assume that n sensor nodes are uniformly and independently distributed in a two-dimensional circular area Z with radius of R . For simplicity, we use the Boolean sensing model and assume that sensor's sensing range is a circular area centered at this sensor with a radius of r. In addition, all sensor nodes are supposed to have the same sensing radius and no two sensors are deployed exactly at the same location.

A point event E that occurs within Z can be detected if it lies within at least one active sensor node's sensing range. So we define a point's neighboring area $\mathbb{N}$ as a region that any sensor node, if it is located within the region, can cover this point. When border effects are not taken into account, for all points in Z, $\mathbb{N}_Z = \pi r^2$. If border effects are considered, we divide the area Z into two parts as shown in Figure 1 for the convenience of analyzing this problem.

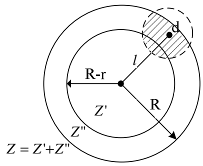

Fig. 1. Illustration of central area, border area and neighboring area

The central area Z' that is concentric with Z, has a radius of $R-r$. Obviously, for any point in Z', its neighboring area is $\mathbb{N}_{Z'} = \pi r^2$. However, for an arbitrary point d in border area Z'', only the shadowed part has the probability of being deployed with sensor nodes. Hence, $\mathbb{N}_d < \pi r^2$.

In our analysis, we assume that all sensors have the same sensing period T and the same sleep ratio α ($0 \le \alpha \le 1$) that defines the percentage of time the sensor is in sleep state. Each sensor node determines independently for each common time unit called slot to be inactive with probability α.

3 Network Coverage Analysis

First, according to our assumptions, since nodes are deployed with a uniform distribution, the probability that a sensor node falls on a point's neighboring area is $\phi = \mathbb{N}/\pi R^2$. Hence, it is well-known that the number of nodes within $\mathbb{N}$ conforms to a binomial distribution $B(n, \mathbb{N}/\pi R^2)$.

Hence, the probability that an arbitrary point is covered by at least one node is

$$\mathrm{P} = \sum_{k=1}^{n} C_n^k \phi^k (1-\phi)^{n-k} = 1-(1-\phi)^n \tag{1}$$

Considering the random sleep scheme, the probability of a point event E is covered by at least one active sensor is

$$\mathrm{P}' = 1-\sum_{k=0}^{n} \alpha^k C_n^k \phi^k (1-\phi)^{n-k} = 1-\left[1-(1-\alpha)\phi\right]^n \tag{2}$$

For each point in Z', the probability that a sensor node falls on its neighboring area is the same: $\phi_{Z'} = \pi r^2 / \pi R^2 = r^2 / R^2$. According to Formula 2, the probability of being covered is also same: $1 - \left[1 - (1-\alpha)(r/R)^2\right]^n$. Then the expected coverage ratio of area Z' is

$$\mathrm{P}'_{Z'} = 1 - [1 - (1-\alpha)\phi_{Z'}]^n = 1 - \left[1 - (1-\alpha)(r/R)^2\right]^n \tag{3}$$

In particular, the neighboring areas of points in Z'' have various values, determined by the distance l between the point and the center of Z as shown in Figure 1. In order to evaluate the average coverage ratio of area Z'', we have to compute the average probability of being covered for all points in Z''. For any point d in Z'', its neighboring area $\mathbb{N}_{d\in Z''}$ can be calculated using the formula proposed in [6]:

$$\begin{aligned}\mathbb{N}_{d\in Z''} = {} & \frac{1}{2}\pi(R^2 + r^2) + r^2 \arcsin\frac{R^2 - r^2 - l^2}{2lr} + \frac{R^2 - r^2 - l^2}{2l}\sqrt{r^2 - \left(\frac{R^2 - r^2 - l^2}{2l}\right)^2} \\ & - R^2 \arcsin\frac{R^2 - r^2 + l^2}{2lR} - \frac{R^2 - r^2 + l^2}{2l}\sqrt{R^2 - \left(\frac{R^2 - r^2 + l^2}{2l}\right)^2}\end{aligned} \tag{4}$$

Then, the average neighboring area of all points in Z'' is

$$\overline{\mathbb{N}_{Z''}} = \iint_{Z''} \mathbb{N}_{d\in Z''} d_\sigma \Big/ \pi[R^2 - (R-r)^2] = \int_{R-r}^{R} 2\pi l \mathbb{N}_{d\in Z''} d_l \Big/ \pi[R^2 - (R-r)^2] \tag{5}$$

So the average coverage ratio of Z'' is

$$\mathrm{P}'_{Z''} = 1 - \left[1 - (1-\alpha)\overline{\phi_{Z''}}\right]^n = 1 - \left[1 - (1-\alpha)(\overline{\mathbb{N}_{Z''}}/\pi R^2)\right]^n \tag{6}$$

Furthermore, we are also interested in the average coverage ratio of whole area Z when all the deployed sensors are active. Based on this value, we can explore how much the QoS will be disrupted under the random sleep scheme and the quantitative quality differences between the central area Z' and border area Z''.

The average neighboring area of all points in Z can be obtained by:

$$\begin{aligned}\overline{\mathbb{N}_Z} & = \left\{\pi r^2 \times \pi(R-r)^2 + \overline{\mathbb{N}_{Z''}}\left[\pi R^2 - \pi(R-r)^2\right]\right\} \Big/ \pi R^2 \\ & = \left\{\pi r^2 (R-r)^2 + \overline{\mathbb{N}_{Z''}}\left[R^2 - (R-r)^2\right]\right\} \Big/ R^2\end{aligned} \tag{7}$$

Hence, according to formula 1 and 7, the average coverage ratio of Z without adopting the sleep scheme is

$$\overline{\mathrm{P}}_Z = 1 - (1-\overline{\phi})^n = 1 - (1 - \overline{\mathbb{N}_Z}/\pi R^2)^n \tag{8}$$

In this paper, we are only concerned with 1-coverage. Our model can be easily extended to k-coverage.

4 Point Event Detection Analysis

In this section, we propose a mathematical model to analyze the probability of detection delay for the point event. Even the points in area Z are all covered, it may not be guaranteed that all the point events are detected instantaneously when they occur due to the random sleep scheme we adopt. Now consider an arbitrary point covered by at least one node in Z . As Figure 2 illustrates, we call the period from time t_1 to t_3 the Worst Case Sleep Time (WCST), during which all nodes within this point's neighboring area happen to be in sleep state, but unfortunately an event occurs during this time period (As shown in Figure 2, the event occurs at time t_2). Hence, a detection delay t_d $(t_d = t_3 - t_2)$ unavoidably occurs for the reason that the event can only be detected when at lest one node wakes up at time t_3. [9] also considered this scenario, but they only targeted large scale wireless sensor networks.

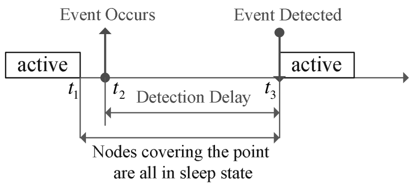

Fig. 2. Illustration of point event detection delay

Intuitively, increasing the number of nodes deployed in the area, or decreasing the sleep ratio of each node can decrease delay time. But, how many nodes we should deploy? How to choose the sleep ratio? Due to the border effects, we need not only to guarantee the detection quality of point events in central area Z', but also to pay much attention to analyze the detection delay probability of the point events in area Z'', if the applications require a high degree of accuracy of detection in the whole region Z .

Based on Figure 2 and above analysis, it is necessary to calculate the conditional probability (denoted by $P_{S/C}$) that a point is not covered by any active node even it could be covered

$$P_{S/C} = \frac{\sum_{k=1}^{n} \alpha^k C_n^k \phi^k (1-\phi)^{n-k}}{1-(1-\phi)^n} = \frac{[1-(1-\alpha)\phi]^n - (1-\phi)^n}{1-(1-\phi)^n} \tag{9}$$

Then, the probability of a given point event is uncovered for at least τ slots is

$$\begin{aligned} P_{t_d}(t_d \geq \tau) &= P_{S/C} - \sum_{i=1}^{\tau} \left\{ \left[1-(1-\phi)^n\right]^{-1} \sum_{k=1}^{n} \alpha^{ik} (1-\alpha^k) C_n^k \phi^k (1-\phi)^{n-k} \right\} \\ &= P_{S/C} - \left[1-(1-\phi)^n\right]^{-1} \sum_{i=1}^{\tau} \left\{ \left[1-(1-\alpha^i)\phi\right]^n - \left[1-(1-\alpha^{i+1})\phi\right]^n \right\} \end{aligned} \tag{10}$$

From previous section we can calculate $\phi_{Z'}$ and $\overline{\phi_{Z''}}$. Hence, if node number n and sleep ratio α are known in advance, the detection delay of point event can be evaluated analytically. Equivalently, given other parameters, the number of sensors to be deployed can also be estimated.

5 Simulation Results

In this section, we demonstrate that the analytical results are consistent with simulation results. In our simulation, locations of nodes are generated conforming to uniform random distribution over a circular area Z with radius R . The area Z is divided into many small grids with size 0.1×0.1 .The node's sensing range r is set to 10. Then, R can be determined by the value of r/R (1, 0.5, and 0.1 respectively). The period T is chosen to $10s$, and let time slot be $1s$.

In the first set of experiments r/R is set to 1, and the number of nodes n is varied from 1 to 10 with an increment of 1. We first obtain the simulation results of coverage ratio (denoted by $SC-Z$) of whole area Z when deployed nodes are all active. Then, the random sleep scheme is adopted. We measure the coverage ratio of both central area Z' and border area Z'' under different combinations of n $(1,2,3\ldots,10)$ and α (0.3, 0.6), denoted by $SC-Z'-0.3$, $SC-Z'-0.6$, $SC-Z''-0.3$, and $SC-Z''-0.6$ respectively. The coverage ratio is obtained as follows. The simulation coverage ratio for a single time slot is obtained by calculating the proportion of the number of covered grids to the total number of grids in the area. For each deployment, 1000 time slots are examined. Besides, we generated 100 deployments for every combination of parameters, and get the average simulation results as shown in Figure 3(a). Comparing with the analytical results ($AC-Z$, $AC-Z'-0.3$, $AC-Z'-0.6$, $AC-Z''-0.3$, $AC-Z''-0.6$), we observe that the simulation results match the analytical curves well.

In the second set of experiments, for a given node number, we study the detection delay probability of central area Z' and border area Z'' under different combination of sleep ratio $\alpha(0.3,0.6)$ and time slots $\tau(1,2,\ldots,20)$, denoted by $SD-Z'-0.3$, $SD-Z'-0.6$, $SD-Z''-0.3$ and $SD-Z''-0.6$ respectively. The node number is selected intentionally. As Figure 3(a) shows, when 8 deployed sensor nodes are all waking up, the whole area Z is almost fully covered. Then we can explore how much the detection quality will be disrupted under the different α values, and the quantitative quality differences between the central area and border area. For each time slot τ , when random sleep scheme is applied, the area coverage ratio P'_{τ} can be obtained by calculating the proportion of the number of the covered grids to the total number of grids in this area. Then, $\mathrm{P}_{S/C}$ can be estimated by the long run average of $1-\mathrm{P}'_{\tau}$. Every grid is assumed as a point event. For each grid, we record the number of experiments where the detection delay is larger than or equal to $1s,2s,3s,\ldots,20s$ respectively. The simulation results shown in Figure 3(b) are averages over 100 runs.

We also conducted additional experiments with $r/R = 0.5$ and $r/R = 0.1$ to examine the accuracy of our theoretical results, and the simulation results are showed in Figure 4 and Figure 5.

Our observations from simulation are summarized as follows:

1) The simulation results are very close to the analytical results, which validates the correctness of our derivations.
2) The QoS of central area Z' outperforms that of border area Z'' on both coverage and detection quality.
3) The coverage ratio increases with the increasing number of deployed nodes. For a given nodes number, coverage ratio increases with the decrease of α.
4) For a given node number, the probability of detection delay increases with the increase of α.

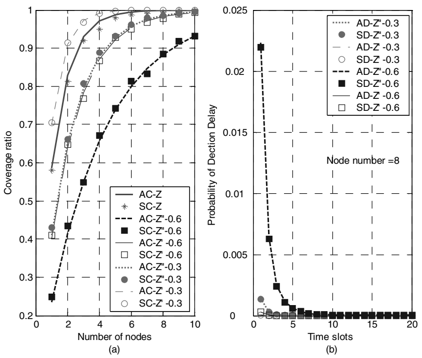

Fig. 3. Comparing analytical results with simulation results **(r/R=1)**

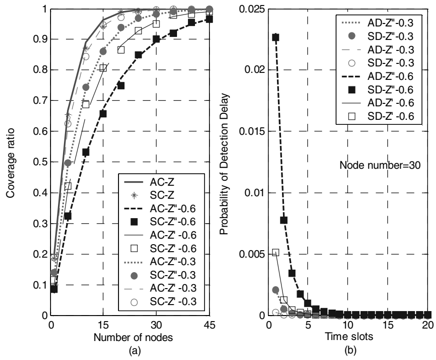

Fig. 4. Comparing analytical results with simulation results **(r/R=0.5)**

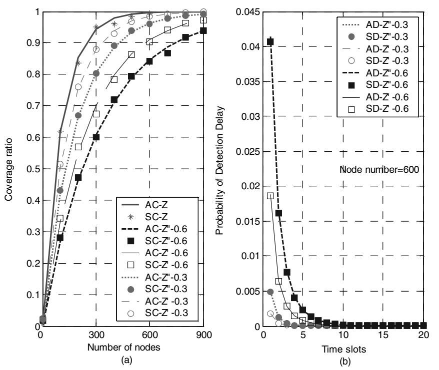

Fig. 5. Comparing analytical results with simulation results **(r/R=0.1)**

6 Conclusions

In this paper, we presented an accurate mathematical model for energy-efficient coverage and detection with the consideration of border effects. The correctness and effectiveness of our analytical model are justified through extensive simulation experiments. This model enables us to analyze the tradeoff between network lifetime and system reliability of wireless sensor networks more effectively, and provides guides for optimal sensor network deployment.

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