Dynamical Synthesis of a Walking Cyclic Gait for a Biped with Point Feet

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Summary. This paper deals with a methodology to design optimal reference trajectories for walking gaits. This methodology consists of two steps: (i) design a parameterized family of motions, and (ii) determine the optimal parameters giving the motion that minimizes a criterion and satisfies some constraints within this family. This approach is applied to a five link biped, the prototype *Rabbit*. It has point feet and four actuators which are located in each knee and haunch. *Rabbit* is under-actuated in single support since it has no actuated feet and is overactuated in double support. To take into account this under-actuation, a characteristic of the family of motions considered is that the four actuated joints are prescribed as polynomials in function of the absolute orientation of the stance ankle. There is no impact. The chosen criterion is the integral of the square of torques. Different technological and physical constraints are taken into account to obtain a walking motion. Optimal process is solved considering an order of treatment of constraints, according to their importance on the feasibility of the walking gait. Numerical simulations of walking gaits are presented to illustrate this methodology.

1 Introduction

For more than thirty years walking robots and particularly the bipeds have been the objects of research. For example Vukobratovic and his co-author [1] have proposed in 1968 their famous Zero-Moment Point (*ZMP*), for the analysis of a biped gait with feet. In 1977, optimal trajectories [2] were designed for a bipedal locomotion using a parametric optimization. Formal'sky completely characterized the locomotion of anthropomorphic mechanisms in [3] in 1982. Sutherland and Raibert proposed their principle about virtual legs for walking robots in the paper [4] in 1983. Currently Humanoïds such as *Honda* biped in

[5] and *HRP2* biped in [6] (Humanoïd Robotics Project 2), which are probably on the technological point-of-view the most advanced biped robots, lead to many popular demonstrations of locomotion and interaction with their environment. In parallel, some research is done on legged robots with less degrees of freedom. Here it is worked with the control, the model and the reference trajectories to design walking bipedal gaits more fluid. See for example [7] where a biped with telescopic legs is studied, [8] where the famous dog *Aibo* from *Sony* is used to design biped gaits, [9] where an intuitive approach is developed for a biped locomotion or [10] where an accurate analysis of the gravity effects is made to give necessary and sufficient conditions to ensure a cyclic walking gait for a biped without feet.

In this paper, the efforts are focused on the design by a parametric optimization of a walking gait. This approach necessitates two steps: (i) design a parameterized family of motions, and (ii) determine the optimal parameters giving the motion that minimizes a criterion and satisfies some constraints within this family. The motion obtained is later used as a reference motion. This approach is applied to a planar five-link biped without feet and with four actuators only. The family of motions considered is composed of a single-support phase and a double-support phase, with no impact. The minimization criterion is the integral over the motion of the square of torques. Therefore it is a criterion of torque minimization. The originality of the present work is double:

- To overcome the underactuated characteristic of the biped, the four variables defined as polynomials in single support are function of another generalized coordinate, the absolute orientation at the stance leg ankle. This allows to define the configurations of the biped during the single support phase, while the dynamics of the not controlled degree of freedom are still unknown. In double support, two actuated joints are also prescribed as functions of the absolute orientation at the stance leg ankle, which is a polynomial function in time.
- There is a classification and a treatment of constraints according to their importance on the feasibility of the walking gait.

This paper does not address the stability of the motion obtained. The reader may refer to [11] which gives conditions of stability of the non controlled degree of freedom during the single support phase, and additionally a measure of this stability. It has been proved that the presence of the double support phase practically guarantees the stability.

This article is organized as follows: the dynamical model of the biped under interest is presented in Sect. 2 for the single and the double-support phase. Section 3 is devoted to the definition of the family of reference trajectories, their constraints and their parameters. The calculation of the criterion in torque during the single support and the double support, and the optimization process to determine the optimal parameters are presented in Sect. 4. Some simulation results are shown Sect. 5. Section 6 contains our conclusion and perspectives.

2 Dynamic Model

2.1 Presentation of the Biped and Notations

A planar five-link biped is considered and is composed by a torso and two identical legs with knee and point feet (see Fig. 1 for a diagram of the studied biped). There are four identical motors, which drive the haunches and the knees. We note $\Gamma = [\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4]^T$ the torque vector, $q = [\alpha, \delta^T]^T = [\alpha, \delta_1, \delta_2, \delta_3, \delta_4]^T$ the vector composed of the orientation of the stance leg and the actuated joint variables, and $X = [q^T, x_t, y_t]^T$ the vector of generalized coordinates. The components (x_t, y_t) define the position of the center of gravity of the trunk.

2.2 A Reduced Model

The optimization process to determined reference trajectories, which will be presented in the next sections leads to many CPU operations. Therefore the strategy was to use a reduced model that needs less computations. To obtain this reduced model, we consider that the contact between the leg tip 1 and

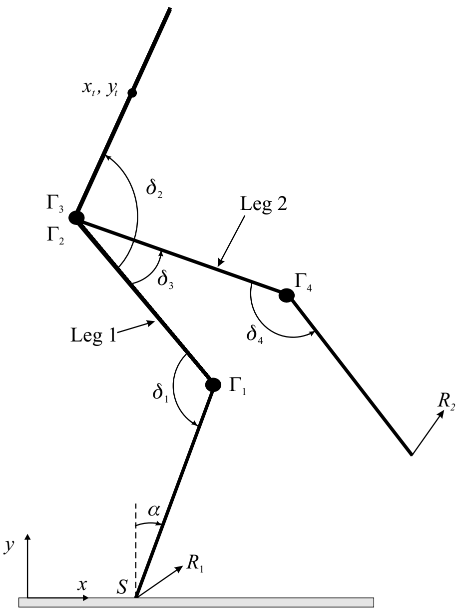

Fig. 1. Biped in the sagittal plane

the ground is acting as a pivot: there is no take off and no slipping. Then the biped configuration is described with vector q only. This model is reduced by comparison to a more general model that would be written with vector X. We obtain the reduced model by using Lagrange's equations:

$$A(\delta)\ddot{q} + H(q,\dot{q}) + Q(q) = D_\Gamma \Gamma + D_2(q)R_2 \tag{1}$$

where $A(\delta)(5\times5)$ is the symmetric positive inertia matrix of the biped. As the kinetic energy of the biped is invariant under a rotation of the world frame [12], and viewed that α defines the orientation of the biped, the 5×5-symmetric positive inertia matrix is independent of this variable, i.e. $A = A(\delta)$. Vector $H(q,\dot{q})(5\times1)$ represents the centrifugal, Coriolis effects, and $Q(q)(5\times1)$ is the gravity effects vector. $D_\Gamma(5\times4)$ is a constant matrix composed of ones and zeros. $D_2(q)$ is the 5×2-Jacobian matrix converting the ground reaction in the leg tip 2 into the corresponding joint torques.

Taking into account Coulomb dry and viscous frictions, Γ has the following form

$$\Gamma = \Gamma_u - \Gamma_s sign(D_{e\Gamma}^T\dot{q}) - F_v D_{e\Gamma}^T\dot{q} \tag{2}$$

where $\Gamma_s(4\times4)$ and $F_v(4\times4)$ are diagonal matrices representing respectively the dry friction and the viscous friction. Γ_u is the motors torque vector when considering the joint friction.

In the case of double support, the point foot 2 is in contact with the ground. Then the position variables q, the velocity variables $\dot{q}$, and the acceleration variables $\ddot{q}$ are constrained. In order to write these relations, we define the position, velocity and acceleration of the point foot 2 in an absolute frame. The position of the point foot 2 is noted $d_2(X)$. By differentiation of $d_2(X)$ we obtain the relation between the velocity $V_2 = (V_{2x}\ V_{2y})^T$ of the point foot 2 and $\dot{q}$,

$$V_2 = D_{e2}(q)^T\dot{q}\ . \tag{3}$$

By another differentiation we obtain the relation between the acceleration $\dot{V}_2 = (\dot{V}_{2x}\ \dot{V}_{2y})^T$ of the point foot 2 and $\ddot{q}$,

$$\dot{V}_2 = D_{e2}(q)^T\ddot{q} + \dot{D}_{e2}(q)^T\dot{q} = D_{e2}(q)^T\ddot{q} + C_{e2}(q,\dot{q})\ . \tag{4}$$

Then the contact constraints for the point foot 2 with the ground are given by the three vector-matrix equations:

$$\begin{cases} d_2(X) = const\ , \\ \quad V_2 = 0\ , \\ \quad \dot{V}_2 = 0\ . \end{cases} \tag{5}$$

These vector-matrix equations (5) mean that the position of the point foot 2 remains constant, and then the velocity and acceleration of the point foot 2 are zero.

During the double-support phase, both legs are in contact with the ground. Then the dynamic model is formed of both vector-matrix equations (1) and (5). During the single-support phase on leg 1, the dynamic model is simply written as (1) with the ground reaction for foot 2 in the air is $R_2 = (0\,0)^T$.

Model (1) allows us to compute the torques and the dynamic model of α easier (10). However, it is not possible to take into account a single-support on the leg 2 with (1). Furthermore we cannot calculate the ground reaction with model (1) only. We add the two following equations, obtained from the Newton's second law at the center of mass G of the biped

$$\begin{cases} M\ddot{x}_G = R_{1x} + R_{2x} \\ M\ddot{y}_G = R_{1y} + R_{2y} - Mg \end{cases} \tag{6}$$

where M is the mass of the biped and (x_G, y_G) are the coordinates of G.

3 Definition of the Walk and Its Constraints

Our objective is to design a cyclic bipedal gait. There are two aspects for this problem. The definition of a parameterized family of reference trajectories and the method to determine a particular solution in this restricted space. This section is devoted to the definition of the parameterized family of reference trajectories. The optimal process to choose the best solution of parameters from the point of view of a given criterion will be described in the next section. The parameterized family of reference motions is such that one degree of freedom, which changes monotonically during a step composed of a single-support phases and a double-support phases, will be used as a variable to define the other degrees of freedom. These special solutions lead to a particular simple dynamical model of the biped in single support which can be calculated from (1). An impactless bipedal gait is considered because in [13] numerical results proved that the insertion of an impact with this walking gait for the studied biped is a very difficult challenge. The condition found to obtain no impact was simply that the velocity of free foot must reach the ground with zero velocity. After the choice of parameters, the constraints will be determined. In the following, indices “ss” and “ds” respectively indicate the single-support phase and the double-support phase.

3.1 Restrictions of Motion Considered in Single Support

During the single support, the biped has five degrees of freedom. With the four actuators for the biped, only four output variables can be prescribed. Then the biped is underactuated in single support. In previous experiments, see for example, [7, 14, 15], researchers observed that for most of walking gaits of biped robots the ankle angle α of the stance leg changes absolutely monotonically during the single-support phase. Therefore, it is possible to use the angle variable α instead of time t as an independent variable during the single-support phase of the bipedal gait. As a consequence α like time will have to be monotonic. But this choice will not eliminate potentially optimal motions in the space in which we seek for solutions, since so far all motions observed were satisfying this property. Thus the four joint variables δ_j are prescribed as polynomial functions of the ankle angle α, $\delta_{j,ss}(\alpha)$ $(j = 1, \ldots, 4)$. The behavior of α is governed by the dynamic model (1). To deal with the underactuation the advantage of this approach is that the complete set of configurations is defined during the motion of the biped and it is not necessary to anticipate a duration for the single-support phase, which is the result of the integration of (1). The order of these polynomial functions (7) is chosen at four to specify initial, final and intermediate configurations, plus initial and final joint velocity variables,

$$\delta_{j,ss}(\alpha) = a_{j0} + a_{j1}\alpha + a_{j2}\alpha^2 + a_{j3}\alpha^3 + a_{j4}\alpha^4 \; . \tag{7}$$

Let us note that it would be possible to prescribe other variables as Cartesian variables. But to avoid the problems of singularity of the inverse geometric model in the single-support phase, we prefer to work with angular variables only. However some authors, for example [2, 16], use Cartesian coordinates of the hip for the definition of the bipedal gait. The joint variables are then prescribed. However since the biped is underactuated the evolution of the angle α must be such that the biped motion satisfies the dynamic model. Considering the relations (7) we introduce for the variables of the reference motion $q = q(\alpha)$ the following temporal derivatives

$$\begin{aligned} \dot{q}(\alpha, \dot{\alpha}) &= q^* \dot{\alpha} \\ \ddot{q}(\alpha, \dot{\alpha}, \ddot{\alpha}) &= q^* \ddot{\alpha} + q^{**} \dot{\alpha}^2 \end{aligned} \tag{8}$$

where the notation $()^*$ means partial derivative with respect to α, and the $\dot{()}$ represent derivation with respect to time. Then we have $q^* = [1\, \delta_1^*\, \delta_2^*\, \delta_3^*\, \delta_4^*]^T$ and $q^{**} = [0\, \delta_1^{**}\, \delta_2^{**}\, \delta_3^{**}\, \delta_4^{**}]^T$. By calculating the angular momentum of the biped at the fixed point S (see Fig. 1), we obtain the general form

$$\sigma = \sum_{i=1}^{4} f_i(\delta_1, \delta_2, \delta_3, \delta_4)\dot{\delta}_i + f_5(\delta_1, \delta_2, \delta_3, \delta_4)\dot{\alpha} \; . \tag{9}$$

We can obtain two first order differential equations on σ and α (see [15])

$$\begin{cases} \dot{\sigma} = -Mg\,(x_G(\alpha) - x_S) \\ \dot{\alpha} = \dfrac{\sigma}{f(\alpha)} . \end{cases} \tag{10}$$

M is the biped mass, g the acceleration of gravity, $x_G(\alpha)$ and x_S are respectively the horizontal component of the positions of the biped's mass center and of the foot of the stance leg. The first equation of (10) comes from the dynamic momentum equation at S when eliminating q from (7). The second equation of (10) follows from (9) when eliminating q and $\dot{q}$ using (7) and (8). This differential system (10) is equivalent to the first line of (1). By identification, it is possible to determine $f(\alpha)$ and $x_G(\alpha)$ from (1). The simple model (10) completely defines the dynamic behavior of the biped in single support for the reference motion. From (10) we can deduce that (see [17])

$$\dot{\sigma} = \frac{d\sigma}{d\alpha}\dot{\alpha} = \frac{d\sigma}{d\alpha}\frac{\sigma}{f(\alpha)} = \frac{1}{2}\frac{d\sigma^2}{d\alpha}\frac{1}{f(\alpha)} = -Mg\left(x_G(\alpha) - x_S\right).$$

Finally this calculation leads to the relation due to [17]

$$\frac{d\sigma^2}{d\alpha} = -2Mg\left(x_G(\alpha) - x_S\right) f(\alpha)\,. \tag{11}$$

If α is strictly monotone, the integration of (11) gives

$$\sigma^2 - \sigma_{iss}^2 = -2Mg \int_{\alpha_{iss}}^{\alpha} \left(x_G(s) - x_S\right) f(s) ds \tag{12}$$

where σ_{iss} is the angular momentum at the beginning of single support characterized by the initial value α_{iss}. Then the dynamic of the biped is completely defined from (10) as function of $\Phi(\alpha) = \sigma^2 - \sigma_{iss}^2 = \dot{\alpha}^2 f^2(\alpha) - \dot{\alpha}_{iSS}^2 f^2(\alpha_{iSS})$ such as

$$\dot{\alpha} = -\frac{\sqrt{\Phi(\alpha) + f(\alpha_{iSS})2\dot{\alpha}_{iSS}^2}}{f(\alpha)}\,. \tag{13}$$

$\ddot{\alpha}$ is obtained from the second equation of (10)

$$\ddot{\alpha} = \frac{\dot{\sigma} f(\alpha) - \sigma \dot{f}(\alpha)}{f^2(\alpha)} = -\frac{Mg\left(x_G(\alpha) - x_S\right) + \frac{df(\alpha)}{d\alpha}\dot{\alpha}^2}{f(\alpha)}\,. \tag{14}$$

From the solution of the differential equation in α (11) and using relations (13) and (14) the numerical simulation to find the optimal motion and the calculation of constraints will be easier.

The authors of [17] showed that system (10) behaves like an inverted pendulum. Therefore the only possible non-monotone behavior would be that the biped fall back if the initial velocity of single support is not sufficient. The condition to ensure the monotony of α has been added as a constraint in the optimization process, see (18).

3.2 Restrictions of Motion Considered in Double Support

In double support, the biped has three degrees of freedom. With its four actuators, the biped is over actuated. Hence the motion of the biped is completely defined with three prescribed degrees of freedom. For a question of convenience for the use of the inverse geometric model, the ankle angle α and both joint variables, δ_j ($j = 1, 2$) are prescribed. A polynomial function in time of third-order (15) is chosen to define α. In a concern to be homogeneous with the single support phase we define both joint angular variables δ_j, as polynomial functions of third-order in α. Then initial and final configurations, and initial and final velocities can be defined for these three prescribed variables. The duration of the double-support phase is a parameter. Hence we get

$$\begin{cases} \alpha(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \ , \\ \\ \delta_j(\alpha) = a_{j0} + a_{j1}\alpha + a_{j2}\alpha^2 + a_{j3}\alpha^3. \end{cases} \tag{15}$$

It should be noted that there is no differential equations needed for the definition of the motion, since the biped is over-actuated in double support.

3.3 Optimization Parameters

A boundary value problem has to be solved to design this cyclic bipedal gait with successive single and double-support phases. This boundary value problem depends on parameters to prescribe the initial and final conditions for each phase. Taking into account the conditions of continuity between the phases and the conditions of cyclic motion we will enumerate now in detail the minimal number of parameters which are necessary to solve this boundary value problem on a half step k (a half step is considered as a single support and a double support).

1. Seven parameters are needed to define the initial and final configurations in double support. The parameters α_{ids}, $\delta_{1,ids}$, θ_{ids}, α_{fds}, $\delta_{1,fds}$, θ_{fds} and d, the distance between both tips of stance legs in double support are chosen. The use of the absolute orientation of the trunk, θ (see Fig. 2) instead of $\delta_{2,fds}$ is easier and does not change the problem.
2. Time T_{ds} of the double support is given as a parameter.
3. The initial velocity of the biped in single support is prescribed by only three parameters $\dot{\alpha}_{iss}$, $\delta^*_{1,iss}$, $\delta^*_{2,iss}$. The velocities $\delta^*_{3,iss}$ and $\delta^*_{4,iss}$ are deduced taking into account the null velocity of the leg tip which takes off.
4. The final velocity of the biped in single support is prescribed by only three parameters $\dot{\alpha}_{fss}$, $\delta^*_{1,fss}$, $\delta^*_{2,fss}$. The velocities $\delta^*_{3,fss}$ and $\delta^*_{4,fss}$ are deduced taking into account the absence of impact of the swing leg tip on the ground, which is equivalent to a null velocity of this tip.
5. With the chosen order for the polynomial functions (7) (fourth order) it is necessary to specify five conditions for each function $\delta_{j,ss}$, $j = 1, \ldots, 4$. Then the fifth coefficient is calculated by defining an intermediate configuration. Let intermediate configuration in single support be determined with the five following parameters α_{int}, $\delta_{1,int}$, θ_{int} and the coordinates

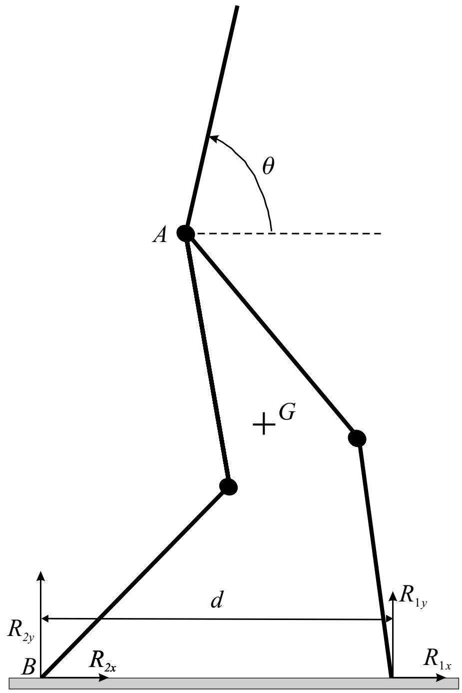

Fig. 2. Biped in the sagittal plane (the point G is the center of mass of the biped)

($x_{p,int}$ and $y_{p,int}$) of the swing leg tip. The angle α_{int} is fixed equal to $\dfrac{\alpha_{iss} + \alpha_{fss}}{2}$.

Then finally the vector of parameters has eighteen coordinates

$$p = [T_{ds}, \alpha_{ids}, \delta_{1,ids}, \theta_{ids}, \alpha_{fds}, \delta_{1,fds}, \theta_{fds}, d, \dot{\alpha}_{iss}, \delta^*_{1,iss}, \ldots$$

$$\delta^*_{2,iss}, \alpha_{fss}, \delta^*_{1,fss}, \delta^*_{2,fss}, \delta_{1,int}, \theta_{int}, x_{p,int}, y_{p,int}] \; .$$

3.4 Constraints

Constraints have to be considered to design nominal gait. We will present them according to their importance on the feasibility of the walking gait.

- First, no motion is possible if the distance $d(A, B)$ between the tip of leg 2 and the hip joint, for initial and final configurations of the double support and the intermediate configuration of the single support, is such that

$$d(A, B) > 2 \times l \tag{16}$$

where l is the common length of the femur and the tibia. In other words, there is no solution with the geometrical model to compute δ_3 and δ_4.

- Constraint (16) is also taken into account during the motion of the biped in double support. The maximum value of $d(A, B)$ in function of α is considered.
- The mechanical stops of joints for initial, intermediate and final configurations of each phase and during the motion are

$$\begin{cases} -260° < (\delta_2)_{min}\,, & (\delta_2)_{max} < -110° \\ -260° < (\delta_2 - \delta_3)_{min}\,, & (\delta_2 - \delta_3)_{max} < -110° \\ -230° < (\delta_1)_{min}\,, & (\delta_1)_{max} < -127° \\ -230° < (\delta_4)_{min}\,, & (\delta_4)_{max} < -127° \end{cases}$$

The notation $()_{max}$ and $()_{min}$ stands respectively for the maximum and minimum value over one step.

- In double support the monotony condition for variable α is imposed

$$\max_{t \in [0, T_{ds}]} \dot{\alpha}(t) < 0\,. \tag{17}$$

- In single support, the monotony condition for variable α is imposed by the inequality

$$\Phi_{min} + f(\alpha_{iss}) 2\dot{\alpha}_{iss}^2 > 0 \tag{18}$$

where $\Phi_{min} = \min_{\alpha \in [\alpha_{iss}, \alpha_{fss}]} \Phi(\alpha)$.

- In single support it is fundamental to avoid the singularity $f(\alpha) = 0$ to simulate one step. Then we define the following constraint

$$\min_{\alpha \in [\alpha_{iss}, \alpha_{fss}]} f(\alpha) > 0\,. \tag{19}$$

Now the following constraints can be violated during the optimization process to simulate a half step. However they are important for experimental objectives. The optimization process will ensure their verification.

- Each actuator has physical limits such that

$$\begin{cases} \left(|\Gamma_1^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_1|)\right)_{max} < 0 \\ \left(|\Gamma_2^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_2|)\right)_{max} < 0 \\ \left(|\Gamma_3^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_2 - \dot{\delta}_3|)\right)_{max} < 0 \\ \left(|\Gamma_4^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_4|)\right)_{max} < 0 \end{cases} \tag{20}$$

Function $\Gamma_{max}(*)$ can be deduced from a template, torque actuator/velocity, given by the actuator manufacturer.

- We must take into account constraints on the ground reaction $R_j = (R_{jx}\ R_{jy})^T$ in the tip of the stance leg j, $j = 1$ in single support and $j = 1, 2$ in double support. The ground reaction must be inside a friction cone defined by the friction coefficient μ. This is equivalent to write both inequalities

$$R_{jx} - \mu R_{jy} < 0$$
$$-R_{jx} - \mu R_{jy} < 0.$$

By summing these two inequalities, the condition of no take off is deduced

$$\Rightarrow R_{jy} > 0\ . \tag{21}$$

- There is also a constraint on the swing leg tip to avoid an impact with the ground during its transfer. This constraint is defined by a parabola function

$$\min_{\alpha \in [\alpha_{iss}, \alpha_{fss}]} \left[y(\alpha) - \left(\frac{x^2(\alpha)}{d^2} - 1 \right) y_{max} \right]$$

where (x, y) are the coordinates of the swing leg tip and y_{max} is the maximum height of the parabola.

- Optimal motions are defined for different velocities with the constraint

$$d = v(T_{ss} + T_{ds}) \tag{22}$$

where d is the distance between the tips of stance legs (see Fig. 2), v is the desired average velocity of the biped, and T_{ss} is the time of the single-support phase. The calculation of time T_{ss} of the single-support phase is given by $T_{ss} = \int_{\alpha_{iss}}^{\alpha_{fss}} \frac{1}{\dot{\alpha}} d\alpha$

4 Optimal Walk

Many values of parameters presented in Sect. 3 can give a periodic bipedal gait satisfying constraints (16)–(22).

Then a parametric optimization process, minimizing a criterion under nonlinear constraints, is possible to find a particular nominal motion. Let us define this optimization process

$$\min_p C(p) \tag{23}$$
$$g_i(p) \leq 0 \quad i = 1, 2, \ldots, n$$

where p is the vector of parameters, $C(p)$ is the criterion to minimize with n constraints $g_i(p) \leq 0$ to satisfy. We give now some details about the way to calculate the criterion during the single-support phase and the double-support phase, and about the optimization process.

4.1 Criterion

To find the nominal motion criterion C_Γ, which is a torque minimizing criterion, is considered

$$C_\Gamma = \frac{1}{d} \int_0^{T_{ss}+T_{ds}} \Gamma^T \Gamma dt = \frac{1}{d} \left(\int_{\alpha_{iss}}^{\alpha_{fss}} \frac{\Gamma^T \Gamma}{\dot{\alpha}} d\mu + \int_0^{T_{ds}} \Gamma^T \Gamma dt \right) \tag{24}$$

where T_{ss} and T_{ds} are the times of single support and double support. For electrical motors such as DC motors the torque is usually proportional to the induced current. Then the criterion C_Γ represents the losses by Joule effects to cover distance d, see [18, 19]. To consider an energy minimizing criterion, it would only be necessary to add the losses by friction in the joints.

4.2 Single-Support Phase

From calculation of the integral term (12) using the polynomial functions (7), we obtain $\Phi(\alpha) = \sigma^2 - \sigma_{iss}^2$. Velocity $\dot{\alpha}$ and acceleration $\ddot{\alpha}$ can be obtained with relations (13) and (14). We then have determined the dynamics of the under actuated biped in single support for a reference trajectory. The torques are determined from the four last equations of (1)

$$A_{25}(\delta)\ddot{q} + H_{25}(q, \dot{q}) + Q_{25}(q) = D_{\Gamma 25}\Gamma \tag{25}$$

where $A_{25}(4 \times 5)$, $H_{25}(4 \times 5)$ and $D_{\Gamma 25}(4 \times 4)$ are the submatrices of A, H and D_Γ, $Q_{25}(4 \times 1)$ is the subvector of Q. The invertible matrix $D_{\Gamma 25}$ allows to determine the torque vector Γ. The ground reaction $R_i = (R_{ix}, R_{iy})$ at the tip of the stance leg i are calculated using the equations (6).

4.3 Double-Support Phase

From relations (15) $\alpha(t)$, $\dot{\alpha}(t)$ and $\ddot{\alpha}(t)$ are calculated as polynomial functions of time first at each time step, then $\delta_j(\alpha)$, $\dot{\delta}_j(\alpha)$ and $\ddot{\delta}_j(\alpha)$ $(j = 1, 2)$ are determined. There is an infinit set of solutions for the torques to realize the double support, because the biped is overactuated. Only three generalized coordinates, for example $\alpha(t)$, δ_1 and δ_2, are necessary to describe the motion completely. Then, we can parameterize the solution of torques as function of one variable. To find this variable we consider equation (6) and the equation of the angular momentum theorem applied at the leg tip 1. The equation of the angular momentum theorem in double support is equivalent to equation (10) but with the effect of ground reaction force of foot 2. It is also equivalent to the first line of model (1). This additional equation reads

$$A_1(\delta)\ddot{q} + H_1(q, \dot{q}) + Q_1(q) = -dR_{2y} \tag{26}$$

where $A_1(1 \times 5)$ and $H_1(1 \times 5)$ are the first line of A and H, $Q_1(1 \times 1)$ is the first element of Q. Term d is the distance between the two leg tips on the ground. Component R_{2x} does not appear in equation (26) because the ground is assumed to be horizontal and plane. From the second line of (6) and (26), for a given acceleration of the biped there is only one solution for R_{1y} and R_{2y}, independent of the torques. The torques only influence R_{1x} and R_{2x}. For this reason, a solution for the torques can be found as function of R_{1x} or R_{2x} as parameter. Let us choose R_{2x} and define the minimization problem with the associated constraint on component R_{2x}

$$\min_{R_{2x}} \Gamma^{*T} \Gamma^{*}$$

$$\begin{cases} -\mu R_{1y} - R_{1x} \leq 0 \\ -\mu R_{1y} + R_{1x} \leq 0 \\ -\mu R_{2y} - R_{2x} \leq 0 \\ -\mu R_{2y} + R_{2x} \leq 0\,. \end{cases} \tag{27}$$

The choice of the particular solution of this optimization problem is because it is also the solution that minimizes the criteria (24). With the four last lines of the vector-matrix equations (1) and (2) a relation between torques Γ^* and R_{2x} can be written

$$\Gamma^* = J - KR_{2x} \tag{28}$$

with $K = D_{\Gamma 25}^{-1} D_{2x\;25}$ and

$$J = D_{\Gamma 25}^{-1} \left(A_{25}\ddot{q} + H_{25}(q,\dot{q}) + Q_{25}(q) - D_{2y\;25} R_{2y} \right) + \Gamma_s \mathrm{sign}(D_\Gamma^T \dot{q}) + F_v D_\Gamma^T \dot{q}.$$

The solution $R_{2x\;opt\Gamma}$ which minimizes the square of the torques without constraints is given when $\Gamma^{*T} \frac{\partial \Gamma^*}{\partial R_{2x}} = 0$. Considering equation (28) $R_{2x\;opt\Gamma}$ is given by

$$R_{2x\;opt\Gamma} = \frac{K^T J}{K^T K}\,. \tag{29}$$

Defining a minimum value R_{2xinf} and a maximum value R_{2xsup}, the constraints on R_{2x} can be written under the simple form,

$$R_{2xinf} \leq R_{2x} \leq R_{2xsup} \tag{30}$$

Then a solution for the minimization problem (27) is given by three cases

- if $R_{2x\;inf} \leq R_{2x\;opt\Gamma} \leq R_{2x\;sup}$ then $R_{2x} = R_{2x\;opt\Gamma}$,
- if $R_{2x\;opt\Gamma} \leq R_{2x\;inf}$ then $R_{2x} = R_{2x\;inf}$,
- if $R_{2x\;sup} \leq R_{2x\;opt\Gamma}$ then $R_{2x} = R_{2x\;sup}$.

In the case where there is no solution, i.e $R_{2xinf} \geq R_{2xsup}$, we choose R_{2x} to minimize the violation of constraints such as

$$R_{2x} = \frac{R_{2x\ inf} + R_{2x\ sup}}{2} \ .$$

In this last situation, the constraints are not satisfied. However, the optimization process will tend to satisfy the constraints of the motion, and the final solution will always satisfy $R_{2xinf} \leq R_{2xsup}$. This violation will only occur during the optimization process.

4.4 Optimization Algorithm

The algorithm *NPSOL*, see [20] is used to solve this optimization problem with its nonlinear constraints. The sequence of treatment of constraints according to their importance is described Fig. 3. From level 0 to level 4, the constraints must be satisfied to simulate one step. Other constraints as the maximum velocity of the biped, the torques limits are considered in level 5.

Sometimes, while solving the problem (23), the optimization process can ask a value of the criterion or the constraints in a point p_0 where they are not defined. Therefore an intermediate optimization process is started to find another point p_M, the closest from p_0. For example if constraints $g_i(p_0) \leq 0$, $i = 1, 2, \ldots, m_0$ are not satisfied, p_M is determined as the solution of the problem

$$\begin{gathered} \min_p \|p_0 - p\| \\ g_i(p) \leq 0 \quad i = 1, 2, \ldots, m_0 \ . \end{gathered} \tag{31}$$

Then the constraints not defined at the point p_0 will be computed at the point p_M. And using gradient information at p_M, an interpolated value will be determined at p_0. This interpolation ensures that constraints and criteria are continuous and differentiable functions, even at the boundary of their space of definition. This is a necessary condition for the optimization program to solve this modified problem.

During the optimization process the constraints can be violated. But it tends to satisfy the constraints at the end of the optimization. Since we add in the problem the constraints specifying the sub-space where all constraints and criterion are defined, at the end of the optimization the walking motion will be defined and satisfy all the constraints. The only situation where the algorithm will not find a solution that satisfies constraints is if there is no such solution (if we ask for a walk too fast and the actuators are not sufficient to do it, for example) or if the problem is not convex. Indeed the algorithm used is a local optimization algorithm. For a non convex problem, it will probably find only a local non feasible solution, whereas other feasible solution exists. However, we have tried many random initial conditions for the optimization process and always found the same optimal solution that satisfied constraints. We can therefore assume that our problem is convex.

To solve the intermediate optimization problem (31) and the general optimization problem (23), the gradient in function of the vector of parameters p of the criterion and constraints is necessary. To obtain an efficient algorithm, these gradients were calculated analytically.

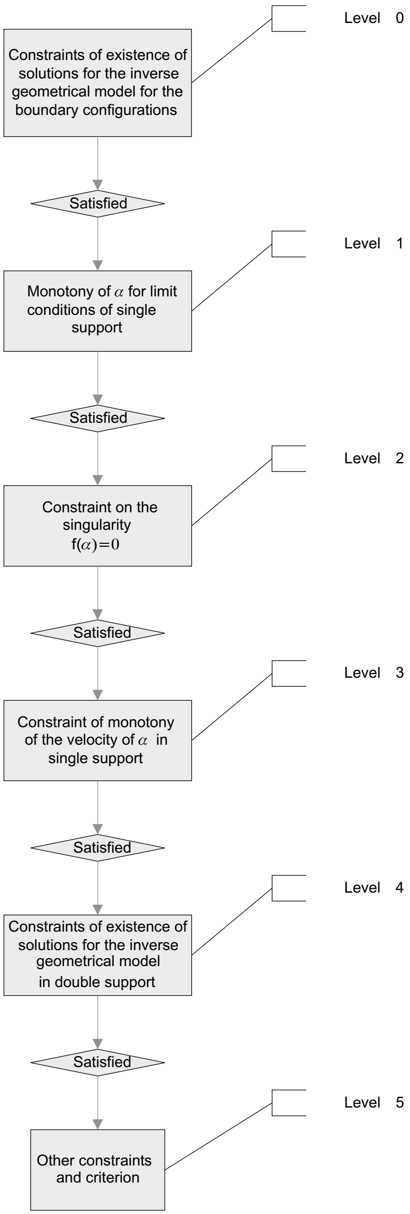

Fig. 3. Sequence of constraints to satisfy before the step can be defined

5 Simulation Results

Figures 4–7 are devoted to a chosen motion velocity for a biped which equals 0.3 m/s. Figure 4 shows that the needed torques for this trajectory are inside the template, motor torque/velocity, given by the manufacturer. The normal components of the ground reactions as functions of time, during one step are presented in Fig. 5. The constraint of unilateral contact on the leg tip 2 is active because the fixed limit 20 N is reached in the tip of leg 2 during the double-support phase. The double-support phase begins after time 0.93 s.

Figure 6 shows as functions of time the evolutions of joint variables δ_1, δ_2, δ_3 and δ_4 in single-support phase and double-support phase. Let us remark that the discontinuities in the graphes mark the limit between the single-support phase and the double-support phase. These discontinuities are not due to an impact (only an impactless motion is considered). These discontinuities appear in the graphes of Figs. 5–7 because the role of both legs are exchanged at the beginning of the double-support phase. Figure 7 shows the behavior of the variable α, which is monotone as expected. The discontinuity at the end of the single-support phase (time 0.93 s) is due to the exchange of the role of both legs.

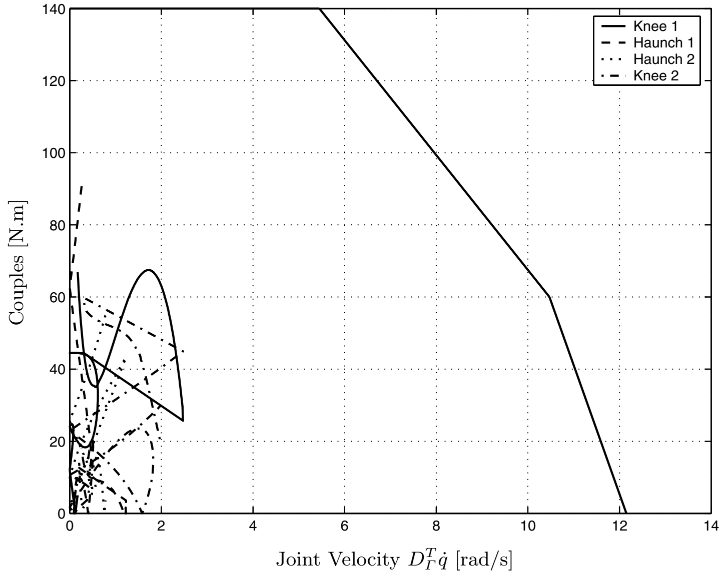

Fig. 4. Velocity versus torque for knee i and haunch i, $(i = 1, 2)$ are inside the template, motor torque/velocity, defined by the limit values 140 N.m and 12 rad/s

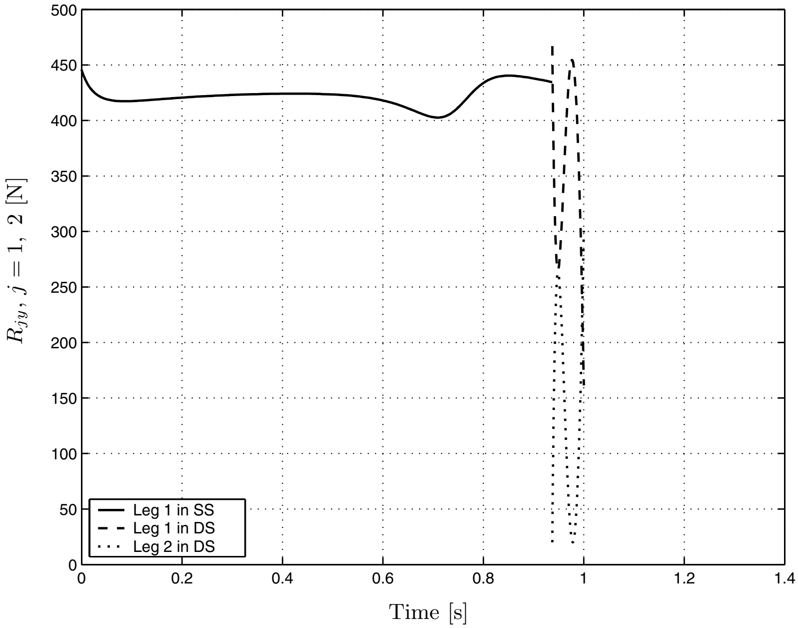

Fig. 5. Normal components in the stance leg tips

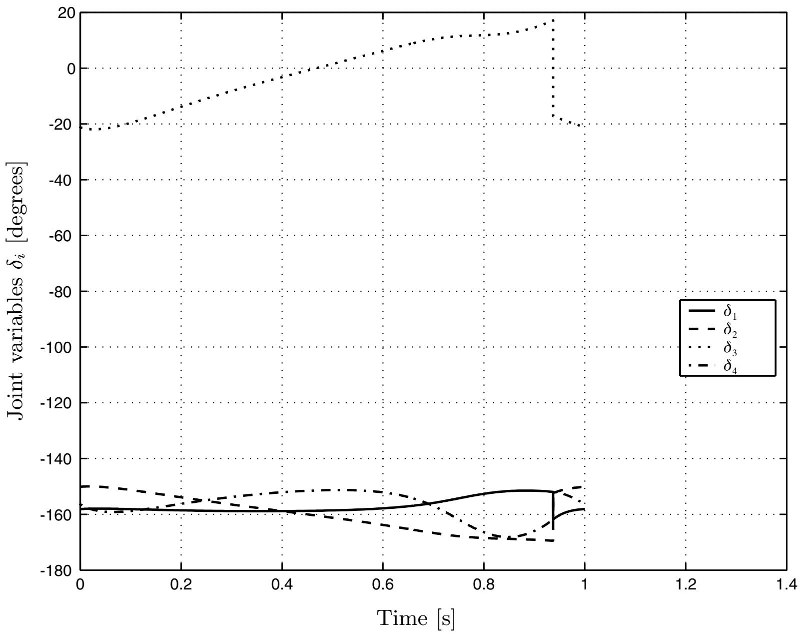

Fig. 6. Evolution of joint variables δ_1, δ_2, δ_3 et δ_4

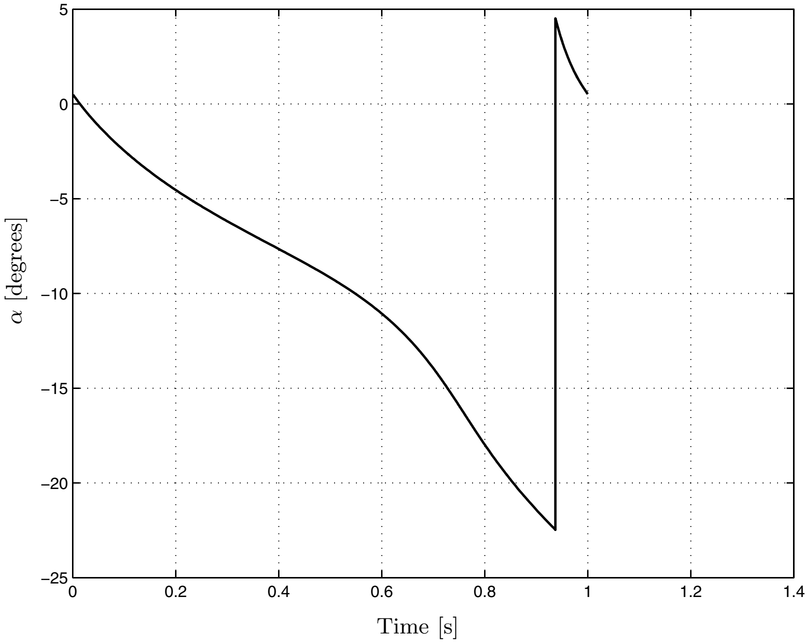

Fig. 7. Evolution of α in function of time

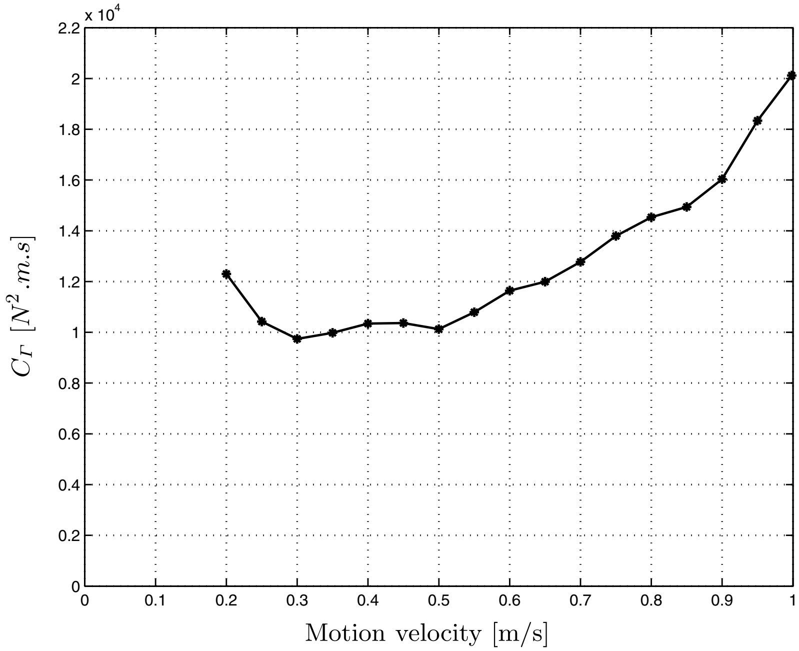

Fig. 8. C_Γ in function of several motion velocities for the biped

In conclusion, for the velocity 0.3 m/s of the biped an optimal motion is feasible according to the constraints. Other velocities of walk for the biped have been tested with success. In Fig. 8 discrete values of criterion C_Γ are presented versus the velocity of motion. The evolution of discrete criterion C_Γ versus velocity of motion is more regular if the optimal walks are obtained without taking into account Coulomb friction. This is due to the fact that the convergence for the case with friction is not very good, since torques are not smooth. For superior velocities a running gait is more appropriate, (see for example numerical experiments in the paper [18]).

6 Conclusion

An optimization process is proposed to design optimal bipedal gaits for a five-link biped. The walking gaits are composed of single-support phases and double-support phases, but with no impact. The criterion minimized is the integral of the square of the torques. A sequential procedure is done, taking into account the constraints according to their importance realizing a walk step. Coulomb frictions, which are nonlinear and discontinuous functions, are taken into account because their contribution cannot be neglected. A possible improvement would be to do a piecewise linear approximation of the Coulomb friction around the discontinuity point of the friction force for a null joint velocity. Currently the main drawback of the optimization method we used is that it is not exactly adapted to our problem. Our problem is a semi-infinite problem, that is an optimization problem with constraints that must be satisfied over an interval. We have then adapted our problem by considering the constraints over an interval only at their most constraining point. The optimization problem we then solve is with non-smooth constraints. But we obtained convergence even if *NPSOL* was not designed to cope with such non-smooth problems. To solve our problem, we would like to consider an optimization algorithm that can take into account a variable number of constraints in the future. Indeed, the number of maximum and minimum where we considered the semi-infinite constraints can change during the optimization process. We hope also to experiment on prototype *Rabbit* these reference trajectories and to extend also this work to a walking biped with more degrees of freedom.

References

[1] M. Vukobratovic and D. Juricic. Contribution to the synthesis of biped gait. In *Proc. of the IFAC Symp. Technical and Biological on Control*, Erevan, USSR, 1968.

[2] V. V. Beletskii and P. S. Chudinov. Parametric optimization in the problem of bipedal locomotion. *Izv. An SSSR. Mekhanika Tverdogo Tela [Mechanics of Solids]*, (1):25–35, 1977.

[3] A. M. Formal'sky. *Locomotion of Anthropomorphic Mechanisms.* Nauka, Moscow [In Russian], 1982.
[4] I. E. Sutherland and M. H. Raibert. Machines that walk. *Scientific American*, 248:44–53, 1983.
[5] K. Hirai, M. Hirose, and T. Haikawa. The development of honda humanoid robot. In *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 1321–1326, 1998.
[6] K. Kaneko, F. Kanehiro, S. Kajita, H. Hirukawa, T. Kawasaki, M. Hirata, K. Akachi, and T. Isozumi. Humanoid robot hrp-2. In *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 1083–1090, 2004.
[7] A. A. Grishin, A. M. Formal'sky, A. V. Lensky, and S. V. Zhitomirsky. Dynamic walking of a vehicle with two telescopic legs controlled by two drives. *Int. J. of Robotics Research*, 13(2):137–147, 1994.
[8] F. Zonfrilli, M. Oriolo, and T. Nardi. A biped locomotion strategy for the quadruped robot sony ers-210. In *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 2768–2774, 2002.
[9] J. Pratt, C. M. Chew, A. Torres, P. Dilworth, and G. Pratt. Virtual model control: an intuitive approach for bipedal locomotion. *International Journal of Robotics Research*, 20(2):129–143, 2001.
[10] C. Chevallereau, G. Abba, Y. Aoustin, F. Plestan, E.R. Westervelt, C. Canudas de Wit, and J. W. Grizzle. Rabbit: a testbed for advanced control theory. *IEEE Control Systems Magazine*, 23(5):57–79, 2003.
[11] S. Miossec and Y. Aoustin. A simplified stability study for a biped walk with under and over actuated phases. *International Journal of Robotics Research*, 24(7):537–551, 2005.
[12] M. W. Spong and M. Vidyasagar. *Robots Dynamics and Control.* John Wiley, 1989.
[13] S. Miossec. *Contribution à l'Étude de la Marche d'un bipède.* PhD thesis, Université Nantes France, 2004.
[14] A. M. Formal'sky. *Ballistic Locomotion of a Biped. Design and Control of Two Biped Machines.* Human and Machine Locomotion, Ed. A. Morecki and K. Waldron. Springer-Verlag, 1997.
[15] Y. Aoustin and A. M. Formal`sky. Control design for a biped: Reference trajectory based on driven angles as functions of the undriven angle. *Journal of Computer and Systems Sciences International*, 42(4):159–176, 2003.
[16] P. H. Channon, S. H. Hopkins, and D. T. Pham. Derivation of optimal walking motions for a bipedal walking robot. *Robotica*, 10(3):165–172, 1992.
[17] C. Chevallereau, A. Formal'sky, and D. Djoudi. Tracking of a joint path for the walking of an under actuated biped. *Robotica*, 22(1):15–28, 2003.
[18] C. Chevallereau and Y. Aoustin. Optimal reference trajectories for walking and running of a biped. *Robotica*, 19(5):557–569, 2001.
[19] L. Roussel, C. Canudas de Wit, and A. Goswami. Generation of energy optimal complete gait cycles for biped. In *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 2036–2042, 2003.
[20] P. E. Gill, W. Murray, M. A. Saunders, and M. H. Wright. *User's guide for NPSOL 5.0.*