Chapter 7

Pseudostatic Limit Equilibrium Analysis

Turfan 高昌故城 in silk-road China was an independent trading city in early seventeenth Century. Being ruled by a Chinese royal family, Turfan warmly hosted a Chinese monk, Xuánzang, 唐之玄奘, who traveled to India on foot for studying true Buddhism. This city had been destroyed by war, however, when the monk visited it again 17 years later on his way back to China.

7.1 Seismic Coefficient

Strong earthquake motion used to destroy many brick structures and killed many people. Figure 7.1 illustrates an example in Tokyo in which a brick tower of 12 storeys was destroyed in the middle. In those days there was no clear idea to design structures against earthquake effects. The same mechanism of collapse is still seen widely in many nonengineered structures in the world and the number of victims is substantial.

Fig. 7.1 Destroyed tower in Tokyo during 1923 Kanto earthquake (photograph from JSCE report)

The method of seismic coefficient (震度法) is the first measure to design facilities against earthquake effects. This method statically applies a force to a designed facility (pseudostatic or quasi-static method). The magnitude of this force is specified to be $K \times W$ in which K is called the seismic coefficient and W is the weight of the facility. Before this idea, there was no design method against earthquake effects. Since this method works easily with static calculation, it is still widely used. Figure 7.2 is an example of slope stability analysis.

A theoretical background of seismic coefficient lies in the *d´Alembert*'s principle (ダランベールの原理) of mechanics. When a base of a structure has an acceleration of A, the effects of this shaking to the overlying structure is equivalent to a force of $(A/g)W$ in the opposite direction from the acceleration (Fig. 7.3); "g" stands for the gravity acceleration. Thus, the seismic coefficient of "K" appears to be equivalent to A/g.

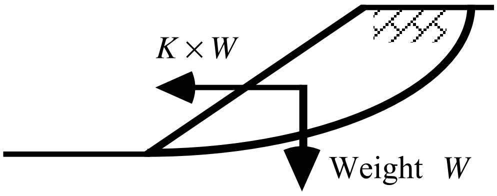


Fig. 7.2 Example of elementary seismic slope stability analysis

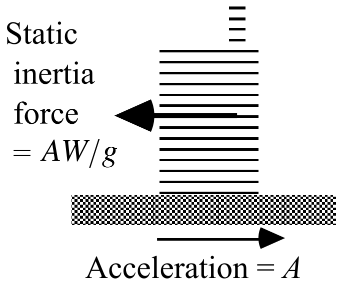


Fig. 7.3 d´Alembert´s principle of mechanics.

The value of K today in Japan is 0.15–0.2 or greater. There is a variation in K, depending upon the local seismic activity, the importance of facilities, and the local geology or soil conditions.

The method of seismic coefficient is good because it is simple and the factor of safety can be calculated by the same way as the conventional static stress calculation. No advanced analysis is therein necessary. It made a great contribution to the improvement of seismic safety.

Fig. 7.4 A small hut that survived the 1994 Northridge earthquake

Problems lying in the seismic coefficient are as follows:

1. The real seismic force is cyclic, changing direction with time, and its duration time is

limited. In contrast, the seismic coefficient method applies a force in a static manner. This seismic force overestimates the risk of earthquake failure.

2) At the time of 1994 Northridge earthquake near Los Angeles, the maximum horizontal acceleration of *1.8* G or possibly *1.9* G was recorded at Tarzana site (Fig. 6.25). Within tens of meters from the accelerometer here, a small hut did not suffer a damage (Fig. 7.4). Was this structure well designed against a horizontal static force as intense as 1.8 times its weight?

3) Thus, the relation between K and the maximum ground acceleration is not clear. 1.9 G acceleration does not mean $K = 1.9$. Study on seismic damage of quay walls led Noda et al. (1975) to propose

$$K = \left(A_{\max}/g\right)^{1/3}/3, \qquad (7.1)$$

in which $A_{\max}$ is the maximum horizontal acceleration (Sect. 12.3).

4) Many structures exhibit dynamic deformation during earthquake shaking. The intensity of shaking is normally greater in the upper portion than in the lower level. Therefore, a greater inertia force seems more appropriate near the top than near the bottom. This idea, called the modified seismic coefficient method, is already practiced in many situations; for example fill-type dams (Sect. 7.2).

The pioneer of seismic coefficient method of design in a modern sense was Prof. Toshikata Sano (1916 佐野利器博士; Fig. 7.5). He got an idea to apply horizontal force in design after his damage investigation on Great San Francisco earthquake (Sano, 1906). Since then, this method has been used at many places of the world. This method was further combined with the Coulomb active earth pressure theory (Appendix 1) to be the famous Mononobe-Okabe seismic (active) earth pressure theory (Sect. 12.5).

Fig. 7.5 Prof. T.Sano (from Memorial book of Dr. Toshikata Sano owned by Civil Engineering Library, University of Tokyo)

The idea of earthquake resistant design based on the seismic coefficient is written as

$$\text{Factor of safety} = \text{Resistance} / (\text{Static} + \text{seismic force}) > 1. \qquad (7.2)$$

Housner (1984) stated that the method of seismic coefficient was adopted in a design regulation in Italy after the 1908 Messina earthquake; Prof. M. Panetti proposed to design the first floor of a building with the seismic coefficient of 1/12, while upper stories with 1/8. The increased seismic coefficient in upper floors stands for the dynamic response of a building. This idea is equivalent with the modified seismic coefficient in Fig. 7.9.

Nakamura (2005) carried out dynamic centrifugal tests on distortion of a gravity retaining wall (refer to Fig. 12.25). He considered that $A_{\max}$ = 670 Gal of his seismic shaking was equivalent to the results of pseudostatic analysis in which $K=0.39$ made the factor of safety = 1. This is because $A_{\max}$ = 670 Gal triggered lateral translation of 1.5% of the wall height which seems to be equivalent with factor of safety = 1. Note that $A_{\max}$ = 670 Gal as substituted in (7.1) gives $K = 0.29$.

The method of seismic coefficient has drastically reduced the extent of damage and the number of casualties (victims) when it is "properly applied" to design and construction practice. It seems that this design principle was most effective for such brittle structures as the one in Fig. 7.1 which were made of bricks and a single big impact was enough to completely destroy them. See the highly brittle relationship between force and displacement in Fig. 7.6.

It seems that traditional (brick, adobe, and wooden) structures have had such a highly brittle nature (Fig. 7.6) that the resistance force drops significantly after the peak resistance. According to the method of seismic coefficient, seismic safety is achieved if the peak resistance is greater than the force (*static+seismic*). A catastrophic failure is possible, however, if the force level after the peak is lower than the static force. This was the case in the tower in Fig. 7.1. Another example of this type was Arg-e-Bam in Fig. 7.7. Being constructed before 500 BC, Arg-e-Bam was a miraculous ruin of an old fortress or citadel and a town where all the structures were made of adobe bricks. Upon the earthquake in 2003, however, those marvelous brick structures were destroyed instantaneously by strong shaking.

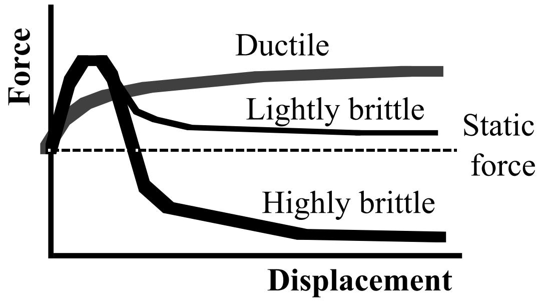


Fig. 7.6 Conceptual illustration of force-displacement relationships

Recent developments of reinforced concrete and steel structures as well as geotechnical structures have changed the force–displacement relationship from a highly brittle one to a lightly brittle or ductile one. Since the force level after large displacement (deformation) is still held greater than the static force, a catastrophic failure is not so likely. In such a situation, it may not be necessary to maintain the factor of safety greater than 1 by making very elaborate design and spending money on high resistance. For more economical construction, the design requirement may be relaxed to some extent by allowing for the seismic factor of safety < 1 and still keeping the resultant displacement small enough (within an allowable extent). This is the aim of recent performance-based seismic design (Sect. 14.7).

Fig. 7.7 Damage of brittle structure (Arg-e-Bam Castle after 2003 Bam earthquake, Iran)

Since the performance-based design focuses mainly residual displacement, a large value of acceleration, A_{max}, is not necessarily taken seriously. In case the duration of A_{max} is short (Fig. 5.11), an equation of motion does not give large displacement. The nature of earthquake motion will be more reasonably considered by performance-based design than the conventional seismic coefficient method, which is influenced unduly by the magnitude of A_{max}. Since displacement analysis is conducted, on the other hand, the performance-based design requires more detailed understanding of soil behavior (more than strength) and hence more precise soil investigation.

Note that well-designed structures may be ductile during a strong earthquake, but furniture in rooms may fall down (highly brittle behavior) to injure residents.

7.2 Modified Method of Seismic Coefficient

The idea of uniform acceleration from the top to the bottom of a structure (Fig. 7.2) is not necessarily correct. It is often the case that the top exhibits a greater magnitude of motion than the bottom; amplification in flexible structures. One of the examples of this situation is found in an earth dam of which the trapezoidal shape increases the top motion significantly (see Sect. 6.13).

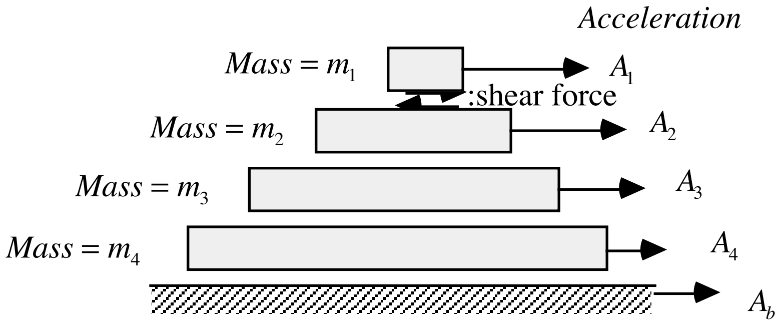
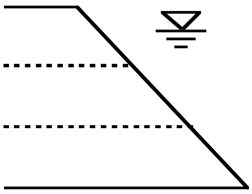


Fig. 7.8 Modified method of seismic coefficient

Figure 7.8 illustrates a situation in which a fill is subjected to an amplified shaking. The shear force between the top and the second blocks is given by $\tau_1 = m_1 A_1$. Since the acceleration varies in the vertical direction, the shear force at lower elevations is calculated by summation

$$(\text{Shear force})_k = \sum_{i=1}^{k} m_i A_i = \sum_{i=1}^{k} (m_i g) \frac{A_i}{g},$$

where g stands for the gravitational acceleration, $m_i g$ is the weight of a block and $A_i / g = K_i$ is the seismic coefficient relevant for the ith block. It is evident that A_i is different from the base acceleration, A_b. The use of different values of $K_i = A_i / g$ in the vertical direction is called the modified method of seismic coefficient (修正震度法).



$K_1 = K_b \times 1.8$

$K_2 = K_b \times 1.4$

$K_3 = K_b \times 1.0$

K_b : seismic coefficient at base

Local seismic activity	K_b
High	0.18
Intermediate	0.16
Low	0.13

Fig. 7.9 Modified seismic coefficient in rockfill dam design

Figure 7.9 is an example idea of the modified seismic coefficients which is currently practiced for a seismic design of rockfill dams. Figure 7.10 is an example analysis on seismic limit equilibrium in which the critical slip plane is detected. In addition to this, consideration on a surface slip is necessary (Fig. 7.11).

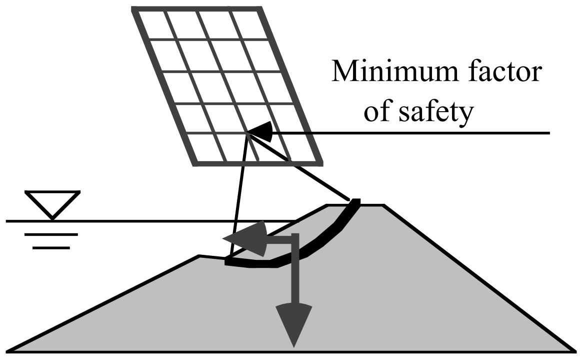


Fig. 7.10 Seismic analysis on earthdam by using limit equilibrium analysis

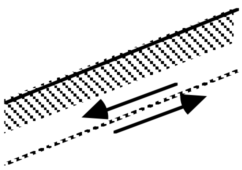


Fig. 7.11 Shallow slip failure

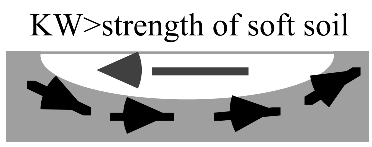


Fig. 7.12 Unrealistically predicted failure of level soft ground

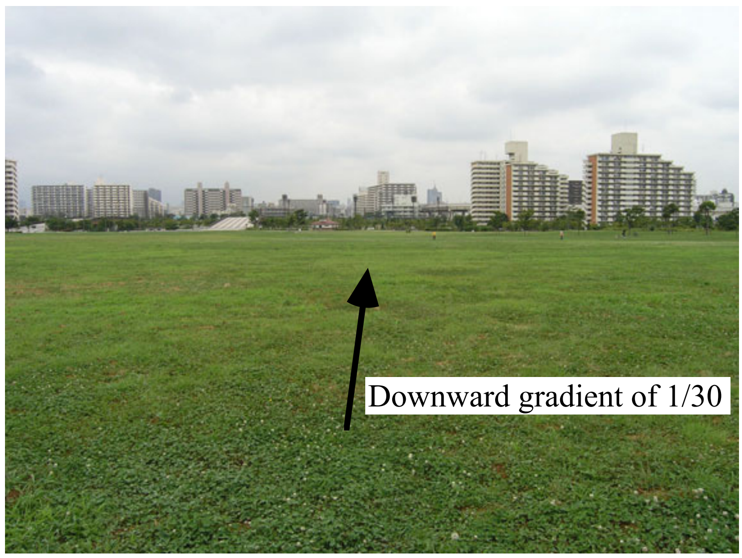


Fig. 7.13 Gentle slope of super river dike in Tokyo

The upstream slope of a fill dam is more gentle than the downstream slope. This is because the upstream soil is submerged in water and is heavier, generating a greater seismic inertia force. Moreover, the buoyancy force reduces the effective stress in the upstream side and makes the shear strength smaller. Possible development of excess pore water pressure and decrease in effective stress are important as well (Chap. 17).

One of the most ironical examples of the seismic coefficient method of analysis is that it predicts an overall failure of soft level subsoil (Fig. 7.12), although a level ground is unlikely to fail. This problem occurs because the method assumes a static one-way earthquake load despite that it is cyclic in reality. A symmetric loading in positive and negative directions does not accumulate deformation in a level subsoil. This shortcoming became a problem when a super river dike was designed in Tokyo (Fig. 7.13). The super rever dike has a slope gradient of merely 1/30 and buildings were placed on it. Hence, seismic stability of the dike slope was considered essentially important and a stability analysis was conducted. Since the dike was underlain by soft alluvial clay, the calculated factor of safety was less than unity in spite of the gentle slope. This case implies the importance of assessment of residual displacement by using, for example, the method in Sect. 12.1.

7.3 Vertical Motion

Conventionally, the horizontal motion has been attracting more attention than the vertical component. There are two reasons for this. First, any facility has some resistance against the vertical motion. The inertia force in the vertical downward direction increases the static force by, for instance, 20–50%. This increased load is often still within the static safety margin. Most failures in masonry structures are caused by the horizontal inertia force (Fig. 7.14).

Fig. 7.14 Collapse of adobe house in Bam, Iran, in 2003

When a slope stability is maintained by the frictional law (Fig. 7.15), the normal and the tangential reactions, N and S, are

$$N = (1 \pm K_\mathrm{v}) W \cos\theta - K_\mathrm{h} \mathrm{W} \sin\theta \quad \text{and}$$
$$S = (1 \pm K_\mathrm{v}) W \sin\theta + K_\mathrm{h} \mathrm{W} \cos\theta \ ,$$

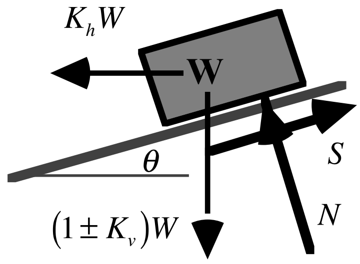


Fig. 7.15 Stability of block resting on frictional slope

where K_v and K_h stand for the vertical and horizontal seismic coefficients, respectively. Accordingly, the factor of safety, F_s, is derived as

$$F_\mathrm{s} = \frac{\mu(1 \pm K_\mathrm{v} - K_\mathrm{h} \tan\theta)}{(1 \pm K_\mathrm{v}) \tan\theta + K_\mathrm{h}},$$

in which μ denotes the coefficient of friction. Although $1 \pm K_\mathrm{v}$ does not vary substantially with K_v within a realistic range of variation, an increase in K_h directly reduces the factor of safety. Once the stability is lost, the block in Fig. 7.15 starts to slide down-slope and its displacement is (may be?) calculated by solving its equation of motion.

In the example above, the increase in the normal force (N) directly increases the frictional resistance (μN). From the soil-mechanic viewpoint, this means that the slope is dry or under drained conditions. Conversely when the slope is undrained, which is more realistic under rapid loading, the increase in N is transferred to excess pore water pressure. Hence, the effective stress and consequently the frictional resistance do not change. The factor of safety is then given by

$$F_\mathrm{s} = \frac{\mu N_\mathrm{initial}}{S} = \frac{\mu}{(1 \pm K_\mathrm{v}) \tan\theta + K_\mathrm{h}}.$$

Again the horizontal inertia force, K_h, reduces the factor of safety.

Table 7.1 Maximum earthquake motion data (National Research Institute of Earth Science and Disaster Prevention, 1995)

Sites	Acceleration (Gal=cm/s^2) NS	EW	Up-Down
Kobe meteorological observatory [a]	818	617	332
Kobe port const. office	502	205	283
NTT Kobe building (B3F)	331	153	169
New Kobe station	530	267	344
Kobe port 8th pier [b]	683	394	334
Takatori station	635	553	175
Nishi Akashi station	397	381	319
Factory in Amagasaki	321	472	311
	Velocity (kine=cm/s)		
Univ. Kobe	55.1	31.0	33.2
Fukushima, Osaka	31.0	29.8	9.6
Chihaya-Akasaka	5.2	4.9	2.5

a Recorded at the top of a small hill.
b Recorded upon a quay structure, not on soil.

Empirically it is known that the vertical acceleration is weaker than the horizontal acceleration. Table 7.1 compares the maximum acceleration in vertical and horizontal directions recorded during the major earthquake in Kobe (1995). Generally, the vertical acceleration is half of the horizontal acceleration.

7.4 Direction of Seismic Inertia Force in Design

In many cases the seismic inertia force for design has been applied in the horizontal direction because the vertical acceleration in the observed records is weaker than the horizontal component. It might be interesting, however, to make a brief discussion on the appropriate direction of the design inertia force. It should be borne in mind that the following discussion is not very practical because the existing design values of seismic coefficient have been determined on the basis of the idea of horizontal inertia force, whether the idea is appropriate or not.

Figure 7.16 illustrates a situation in which a rigid body of weight = W is resting on a frictional slope. The inclination of the slope is θ, while the frictional angle between the slope floor and the rigid body is ϕ. Note that the inertia force of KW is inclined by an angle of α from the horizontal direction. It is aimed at in what follows to detect a particular α that minimizes the calculated factor of safety.

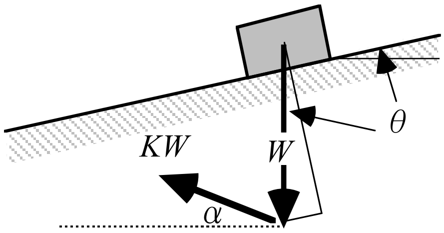


Fig. 7.16 Inclined direction of inertia force

The factor of safety, F_s, is calculated as the ratio of the frictional resistance and the driving force

$$F_s = \frac{(\text{Normal foce}) \times \tan\phi}{\text{Driving force}} = \frac{\{W\cos\theta - KW\sin(\theta+\alpha)\}\tan\phi}{W\sin\theta + KW\cos(\theta+\alpha)}$$
$$= \frac{\{\cos\theta - K\sin(\theta+\alpha)\}\tan\phi}{\sin\theta + K\cos(\theta+\alpha)}. \tag{7.3}$$

The minimum factor of safety for varying α is detected by

$$\frac{\partial F_s}{\partial\alpha} = \frac{-K\cos(\theta+\alpha)\{\sin\theta + K\cos(\theta+\alpha)\} + K\sin(\theta+\alpha)\{\cos\theta - K\sin(\theta+\alpha)\}}{\{\sin\theta + K\cos(\theta+\alpha)\}^2}\tan\phi = 0.$$

Accordingly,

$$\sin\alpha = K \text{ and } \cos\alpha = \sqrt{1-K^2} \text{ for } K<1$$

as is practiced commonly.

The minimum factor of safety, $F_{s,\min}$, is obtained by substituting this special α in (7.3)

$$F_{s,\min} = \frac{\cos\theta - K\{\sin\theta\cos\alpha + \cos\theta\sin\alpha\}}{\sin\theta + K\{\cos\theta\cos\alpha - \sin\theta\sin\alpha\}}\tan\phi$$
$$= \frac{\cos\theta - K\left\{\sqrt{1-\mathrm{K}^2}\sin\theta + K\cos\theta\right\}}{\sin\theta + K\left\{\sqrt{1-\mathrm{K}^2}\cos\theta - K\sin\theta\right\}}\tan\phi$$
$$= \frac{\sqrt{1-\mathrm{K}^2}\cos\theta - K\sin\theta}{\sqrt{1-\mathrm{K}^2}\sin\theta + K\cos\theta}\tan\phi. \tag{7.4}$$

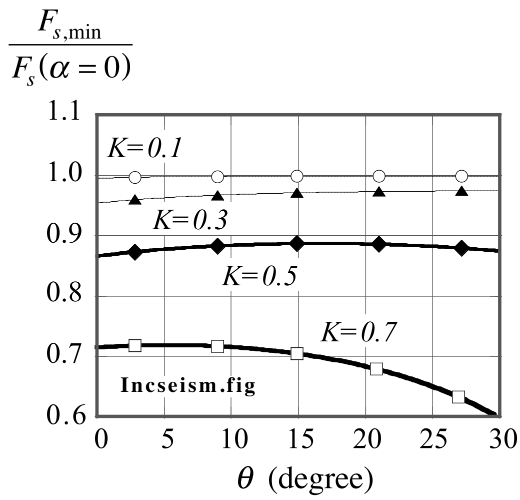


Fig. 7.17 Significance of inclined seismic inertia force

The conventional factor of safety, on the other hand, is derived by substituting $\alpha = 0$ in (7.3);

$$F_s = \frac{\cos\theta - K \sin\theta}{\sin\theta + K \cos\theta} \tan\phi. \tag{7.5}$$

Finally, the significance of the inclined inertia force is illustrated by using the ratio of (7.4) and (7.5), see Fig. 7.17. It is found that the inclined direction of the inertia force reduces the calculated factor of safety to some extent. It is not very important, however, unless the employed seismic coefficient, K, is very large.

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