7

Fuzzy Decision Making

7.1 Introduction

Decision making is essentially an important aspect in all aspects of life. Making decisions is the fundamental activity of human beings. In any decision process we consider the information about the outcome and choose among two or more alternatives for subsequent action. If good decisions are made, then we may get a good expected output.

Decision making is defined to include any choice or selection alternatives. A decision is said to be made under certainty, where the outcome for each action can be determined precisely. A decision is made under risk. When the only available knowledge concerning the outcomes consists of their conditional probability distributions. The uncertainty existing is the prime domain for fuzzy decision (FD) making.

There are various ways in which the FD can be made. They are discussed in detail in the following sections.

7.2 Fuzzy Ordering

Fuzzy ordering involves the decision made on rank basis. Which has first rank, second rank, etc. If $x_1 = 2, x_2 = 5$, then $x_2 > x_1$, here there is no uncertainty, which is called as crisp ordering. The case where the uncertainty or ambiguity arises, then it is called fuzzy ordering or rank ordering. If the uncertainty in the rank is random, then probability density function (pdf) may be used for the random case.

Consider a random variable x_1, defined using Gaussian pdf, with a mean of μ_1 and standard deviation σ_1, also x_2, another variable which is also defined by using Gaussian pdf with a mean μ_2 and standard deviation σ_2. If $\sigma_1 > \sigma_2$ and $\mu_1 > \mu_2$, then the density functions are plotted as shown in Fig. 7.1.

The frequency of probability that one variable is greater than the other is given by

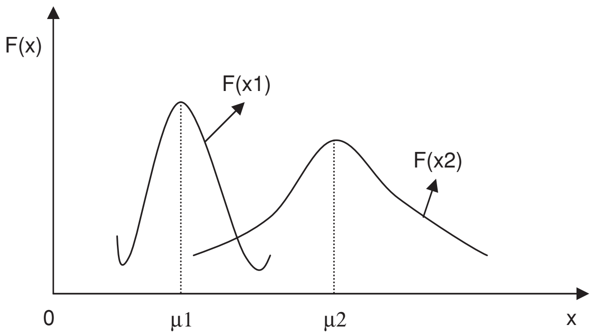

Fig. 7.1. Density function for two Gaussian random variables

$$P_{(x_1 \geq x_2)} = \int_{-\propto}^{+\propto} f_x \left(x_1 \, \mathrm{d}x_1 \right),$$

where F_1 is cumulative distribution function.

Also, if there are two fuzzy numbers $\underset{\sim}{P}$ and $\underset{\sim}{Q}$, the ranking that $\underset{\sim}{P}$ is greater than fuzzy number $\underset{\sim}{Q}$ is given by

$$R\left(\underset{\sim}{P} \geq \underset{\sim}{Q}\right) = \underset{x \geq y}{\mathrm{Sub}} \min\left(\mu_{\underset{\sim}{P}}(x), \mu_{\underset{\sim}{Q}}(y)\right).$$

Also

$$R\left(\underset{\sim}{p} \geq \underset{\sim}{Q}\right) = 1 \text{ if and only if } P \geq Q.$$

Example 7.1. Consider we have three fuzzy sets, given by

$$\underset{\sim}{A} = \left\{ \frac{1}{3} + \frac{0.8}{7} \right\}, \quad \underset{\sim}{B} = \left\{ \frac{0.6}{4} + \frac{1.0}{6} \right\}, \quad \underset{\sim}{C} = \left\{ \frac{0.8}{2} + \frac{1}{4} + \frac{0.4}{8} \right\}.$$

Make suitable decisions based on fuzzy ordering.

Solution. Using the truth value of inequality, $\underset{\sim}{A} \geq \underset{\sim}{B}$, as follows:

$$\begin{aligned} T(\underset{\sim}{A} \geq \underset{\sim}{B}) &= \max_{x_1 \geq x_2} \left\{ \min(\mu_{\underset{\sim}{A}}(x_1), \mu_{\underset{\sim}{B}}(x_2)) \right\} \\ &= \max\{\min(0.8, 0.6), \min(0.8, 1.0)\} \\ &= \max\{0.6, 0.8\} \\ &= 0.8. \end{aligned}$$

Similarly,

$$T(\underset{\sim}{A} \geq \underset{\sim}{C}) = 0.8, \quad T(\underset{\sim}{B} \geq \underset{\sim}{A}) = 1.0, \quad T(\underset{\sim}{B} \geq \underset{\sim}{C}) = 1.0, \quad T(\underset{\sim}{C} \geq \underset{\sim}{A}) = 1.0,$$
$$T(\underset{\sim}{C} \geq \underset{\sim}{B}) = 0.6.$$

Then,

$$T(\underset{\sim}{A} \geq \underset{\sim}{B}, \underset{\sim}{C}) = 0.8,$$
$$T(\underset{\sim}{B} \geq \underset{\sim}{A}, \underset{\sim}{C}) = 1.0,$$
$$T(\underset{\sim}{C} \geq \underset{\sim}{A}, \underset{\sim}{B}) = 0.6.$$

From this calculation, the overall ordering of the three fuzzy sets would be $\underset{\sim}{B}$ first $\underset{\sim}{A}$ second, and $\underset{\sim}{C}$ third.

Thus the fuzzy ordering is performed

7.3 Individual Decision Making

A decision situation in this model is characterized by:

- Set of possible actions
- Set of goals $p_i(i \in x_n)$, expressed in terms of fuzzy set
- Set of constraints $Q_i(j \in x_m)$, expressed in terms of fuzzy sets.

It is common that the fuzzy sets impressing goals and constraints in this formulation are not defined directly on the set of actions, but through the other sets that characterize relevant states of nature.

For the set A, then

$$\begin{gathered} P_i(a) = \text{Composition } [P_i(a)] = {P_i}^1(P_i(a)) \\ \text{with } P_i^1, \\ Q_j(a) = \text{Composition of } \mathrm{Q_i(a)} = {Q_j}^1(q_j(a)) \\ \text{with } Q_i^1, \end{gathered}$$

for $a \in A$.

Then the FD is given by

$$\mathrm{FD}(a) = \min\left[\inf_{i \in N_n} P_i(a), \inf_{i \in N_m} Q_j(a)\right].$$

7.4 Multi-Person Decision Making

When decision are made by many persons, the difference of it from the individual decision maker is:

1. The goals of single decision makers differ, such that each places a different ordering arrangements.
2. The individual decision makers have access to different information upon which to base their decision.

In this case, each member of a group of n single decision makers has a preference ordering $P_k, K \in N_n$, which totally or partially orders a set x.

Then a function called "Social Choice" is to be found, given the individual preference ordering. The social choice preference function is defined by fuzzy relation as

$$S : X \times X \to [0, 1].$$

which has membership of $S(x_i, x_j)$ which indicates the preference of alternative x_i over x_j.

If number of persons preferring x_i to $x_j = N(x_i, x_j)$,

Total number of decision makers $= N$.

Then,

$$S\left(x_i, x_j\right) = \frac{N\left(x_i, x_j\right)}{n}.$$

This defines the multi-person decision making also

$$S\left(x_i, x_j\right) = \begin{cases} 1 & \text{if} \quad x_i \underset{K}{>} x_j \text{ for some } k, \\ 0 & \text{other wise.} \end{cases}$$

7.5 Multi-Objective Decision Making

The process involves the selection of one alternative a_i, from many alternatives A, given a collection or set, say $\{0\}$ objectives which is important for a decision maker.

Define universe of n alternatives, i.e.,

$$A = \{a_1, a_2, \ldots, a_n\} \text{ and}$$

set of "r" objectives

$$O = \{0_1, 0_2, \ldots, 0_r\}.$$

The decision function (DF) here is given as intersection of all objectives

$$\mathrm{DF} = 0_1 \wedge 0_2 \wedge 0_3 \wedge \cdots \wedge 0_r.$$

The membership for the alternative is given by,

$$\mu_{\mathrm{DF}}(a^*) = \max_{a \in A}(\mu_{\mathrm{DF}}(a)).$$

Let $\{P\} = \{b_1, b_2, \ldots, b_r\} = b_i, \quad i = 1$ to r,

then, $\mathrm{DF} = \mathrm{DM}(0_1, b_1) \wedge \mathrm{DM}(0_2, b_2) \wedge \cdots \wedge (\mathrm{DM}(0_r, b_r))$ where $\mathrm{DM}(0_n, b_n)$ is called decision measure (DM).

The DM for a particular alternative is

$$\mathrm{DM}(0_i(a) b_i) = b_i \to 0_i(a) = \overline{b_i} \cup 0_i(a).$$

Thus $b_i \to 0_i$ indicates a unique relationship between preference and objective.

Thus, the DF may be given by

$$\mathrm{DF} = \bigcap_{i=1}^{r} (\bar{b}_i \cup 0_i(a)).$$

and a^* is the alternative that maximizes D.

Let $p_i = \bar{b}_i\ u\ 0_i(a)$.

So,

$$\mu p_i(a) = \max\lfloor \mu_{b_i} - (a), \mu_{0_i}(a)1 \rfloor.$$

The membership form of the optimal solution is:

$$\mu_{\mathrm{DF}}(a*) = \max_{a \in A} \lfloor \min\{\mu_{p_1}(a), \mu_{p_2}(a), \ldots, \mu_{p_r}(a)\} \rfloor.$$

Thus the decision is made as discussed.

7.6 Fuzzy Bayesian Decision Method

Classical Bayesian decision methods preassumes that the future states of the nature can be characterized as probability events. The problem here in fuzzy Bayesian method is that the events are vague and ambiguous and uncertain. This is solved by the following method:

Consider the formation of the probabilistic decision method.

Assuming the set of state of nature as:

$$S = \{S_1, S_2, \ldots, S_n\}.$$

So, the probabilities that these states occur are given by

$$P = \{P(S_1), P(S_2), \ldots, P(S_n)\}$$

and

$$\sum_{i=1}^{n} p(s_i) = 1.$$

These are called as prior probabilities. If decision maker chooses m alternatives, then

$$A = \{a_1, a_2, \ldots, a_{\mathrm{m}}\}$$

and for an alternative a_j, the utility value is u_{ji}, if the future state is the state S_i.

The utility values are to be found by the decision maker for each $a_j - S_i$ combination.

The expected utility with jth alternative would be

$$E(u_j) = \sum_{i=1}^{n} U_{jI} p(s_i).$$

The common decision criterion is the maximum expected utility among all alternatives

$$E(u^*) = \max_j \ E(u_j).$$

Which selects a_x if $u^* = E(x_k)$.

Summary

Fuzzy decision making involves various methods, which were described in this chapter. The fuzzy ordering involves the ordering formed on the rank basis. The decision situation is found to vary between the individual decision making and multi-person decision making. Fuzzy Bayesian decision making is one of the most important decision making process discussed. In the case of multi-objective decision making one alternative is found to be selected from many alternatives. Thus the various decision making process are described in this chapter.

Review Questions

1. What is meant by fuzzy decision making process?
2. What are the various methods used for fuzzy decision making?
3. Write short note on fuzzy ordering. State an example for fuzzy ordering.
4. How is the decisions made individually?
5. What are the characteristics of decision situations in individual decision making?
6. Discuss in detail on the multi-person decision making.
7. Compare the accuracy rate of individual decision making and multi-person decision making
8. What is the main aim of multi-objective decision making?
9. Derive an expression for the membership for optimal solution using multi-objective decision making.
10. What is the importance of fuzzy Bayesian decision making?
11. Define prior probabilities in fuzzy Bayesian method.