5

Defuzzification

5.1 Introduction

Defuzzification means the fuzzy to crisp conversions. The fuzzy results generated cannot be used as such to the applications, hence it is necessary to convert the fuzzy quantities into crisp quantities for further processing. This can be achieved by using defuzzification process. The defuzzification has the capability to reduce a fuzzy to a crisp single-valued quantity or as a set, or converting to the form in which fuzzy quantity is present. Defuzzification can also be called as "rounding off" method. Defuzzification reduces the collection of membership function values in to a single sealer quantity. In this chapter we will discuss on the various methods of obtaining the defuzzified values.

5.2 Lambda Cuts for Fuzzy Sets

Consider a fuzzy set $\underset{\sim}{A}$, then the lambda cut set can be denoted by A_λ, where λ ranges between 0 and 1 $(0 \leq \lambda \leq 1)$.

The set A_λ is going to be a crisp set. This crisp set is called the lambda cut set of the fuzzy set $\underset{\sim}{A}$, where

$$A_\lambda = \left\{ x/\mu_{\underset{\sim}{A}}(x) \geq \lambda \right\},$$

i.e., the value of lambda cut set is x, when the membership value corresponding to x is greater that or equal to the specified λ. This lambda cut set can also be called as alpha cut set. The λ cut set A_λ does not have title underscore, because it is derived from parent fuzzy set $\underset{\sim}{A}$. Since the lambda λ ranges in the interval [0, 1], the fuzzy set $\underset{\sim}{A}$ can be transformed to infinite number of λ cut sets.

Properties of Lambda Cut Sets:

There are four properties of the lambda cut sets, they are:

(1) $\left(\underset{\sim}{A} \cup \underset{\sim}{B}\right)_\lambda = A_\lambda \cup B_\lambda$

(2) $\left(\underset{\sim}{A} \cap \underset{\sim}{B}\right)_\lambda = A_\lambda \cap B_\lambda$

(3) $\left(\overline{\underset{\sim}{A}}\right)_\lambda \neq \left(\overline{A_\lambda}\right)$ except for a value of $\lambda = 0.5$

(4) For any $\lambda \leq \alpha$,where α varies between 0 and 1, it is true that, $A_\alpha \subseteq A_\lambda$, where the value of A_0 will be the universe defined.

From the properties it is understood that the standard set of operations or fuzzy sets is similar to the standard set operations on lambda cut sets.

5.3 Lambda Cuts for Fuzzy Relations

The lambda cut procedure for relations is similar to that for the lambda cut sets. Considering a fuzzy relation $\underset{\sim}{R}$, in which some of the relational matrix represents a fuzzy set. A fuzzy relation can be converted into a crisp relation by depending the lambda cut relation of the fuzzy relation as:

$$R_\lambda = \{x, y/\mu_R(x, y) \geq \lambda\}.$$

Properties of Lambda Cut Relations:

Lambda cut relations satisfy some of the properties similar to lambda cut sets.

(1) $\left(\underset{\sim}{R} \cup \underset{\sim}{S}\right)_\lambda = R_\lambda \cup S_\lambda$.

(2) $\left(\underset{\sim}{R} \cap \underset{\sim}{S}\right)_\lambda = R_\lambda \cap S_\lambda$.

(3) $\left(\overline{\underset{\sim}{R}}\right)_\lambda \neq \left(\overline{R}_\lambda\right)$.

(4) For $\lambda \leq \alpha$, where α between 0 and 1, then $R_\alpha \subseteq R_\lambda$.

5.4 Defuzzification Methods

Apart from the lambda cut sets and relations which convert fuzzy sets or relations into crisp sets or relations, there are other various defuzzification methods employed to convert the fuzzy quantities into crisp quantities. The output of an entire fuzzy process can be union of two or more fuzzy membership functions. To explain this in detail, consider a fuzzy output, which is formed by two parts, one part being triangular shape (Fig. 5.1a) and other part being trapezoidal (Fig. 5.1b). The union of these two forms (Fig. 5.1c) the outer envelop of the two shapes.

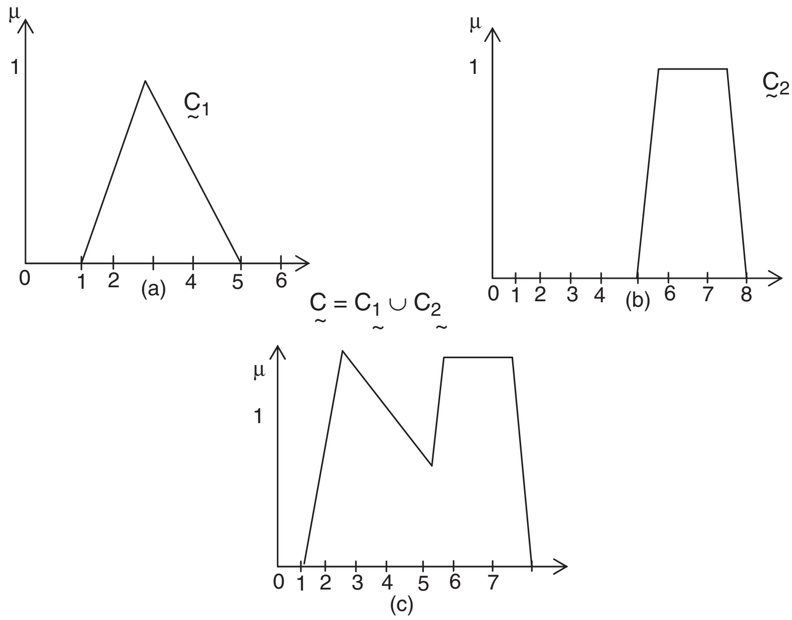


Fig. 5.1. Typical fuzzy output

Generally this can be given as:

$$\underset{r}{C_n} = \sum^{n} \underset{r}{C_i} = \underset{\sim}{C}.$$

There are seven methods used for defuzzifying the fuzzy output functions. They are:

(1) Max-membership principle,
(2) Centroid method,
(3) Weighted average method,
(4) Mean–max membership,
(5) Centre of sums,
(6) Centre of largest area, and
(7) First of maxima or last of maxima

(1) Max-membership-principle

This method is given by the expression,

$$\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z) \quad \text{for all} \quad z \in z.$$

This method is also referred as height method. This is shown in Fig. 5.2.

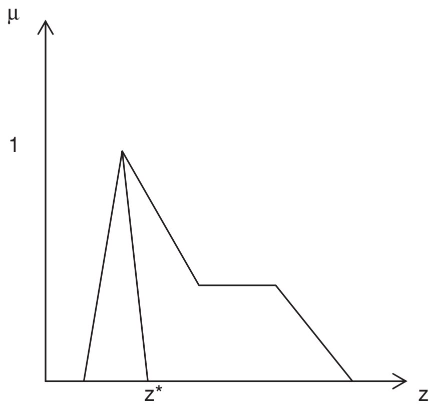


Fig. 5.2. Max-membership method

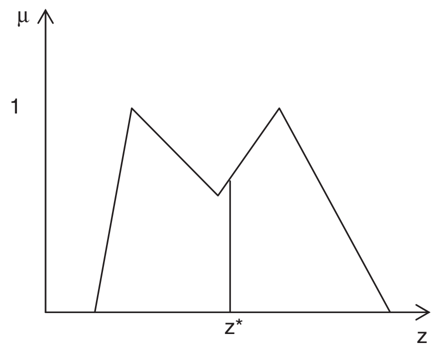


Fig. 5.3. Centroid method

(2) Centroid method

This is the most widely used method. This can be called as center of gravity or center of area method. It can be defined by the algebraic expression

$$z^* = \int \frac{\mu_{\underset{\sim}{C}}(z)\, z\, \mathrm{d}z}{\mu_{\underset{\sim}{C}}(z)\, \mathrm{d}z},$$

$\int$ is used for algebraic integration. Figure 5.3 represents this method graphically.

(3) Weighted average method

This method cannot be used for asymmetrical output membership functions, can be used only for symmetrical output membership functions. Weighting each membership function in the obtained output by its largest membership value forms this method. The evaluation expression for this method is

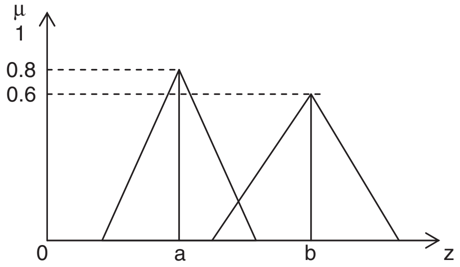


Fig. 5.4. Weighted average method

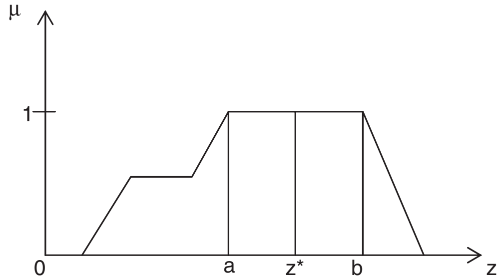


Fig. 5.5. Mean–max-membership

$$z^* = \frac{\sum \mu_{\underset{\sim}{C}}(\overline{z})\ \overline{z}}{\sum \mu_{\underset{\sim}{C}}(\overline{z})},$$

Σ is used for algebraic sum.

From Fig. 5.4

$$z^* = \frac{a\,(0.8) + b\,(0.6)}{0.8 + 0.6}.$$

(4) Mean–max-membership

This method is related to max-membership principle, but the present of the maximum membership need not be unique, i.e., the maximum membership need not be a single point, it can be a range. This method is also called as middle of maxima method the expression is given as

$$z^* = \frac{a + b}{2},$$

where a $\times$ b are the end point of the maximum membership range as shown in Fig. 5.5.

(5) Centre of sums

It involves the algebraic sum of individual output fuzzy sets, say $\underset{\sim}{c_1}$ and $\underset{\sim}{c_2}$ instead of union. In this method, it is noted that the intersecting areas are

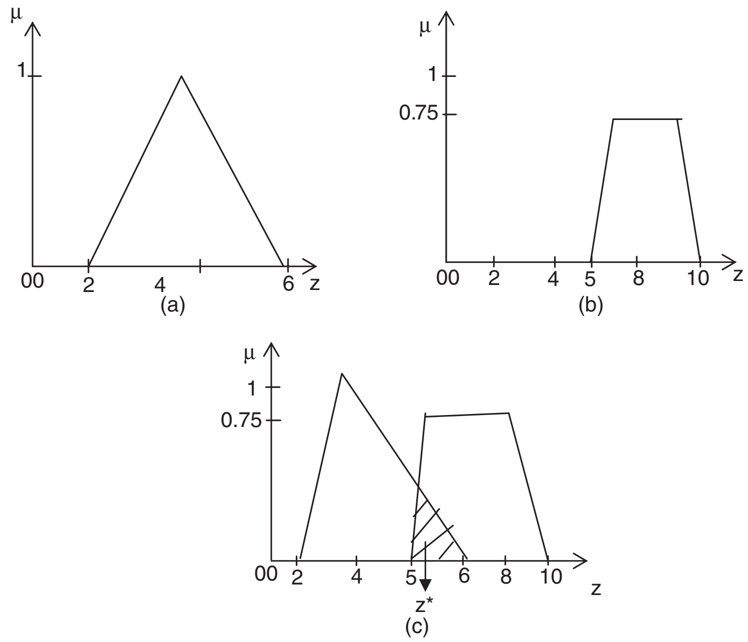


Fig. 5.6. (**a**) First membership, (**b**) second membership, and (**c**) defuzzification step

added twice. This method is similar to the weighted average method, but in center of sums, the weights are the areas of the respective membership functions whereas in the weighted average method, the weights are individual membership values.

The defuzzified value z^* is given as

$$z^* = \frac{\int_2 z \sum_{k=1}^{n} \mu_{\underset{\sim}{C_k}}(\underset{\sim}{z})\,\mathrm{d}z}{\int_2 z \sum_{k=1}^{n} \mu_{\underset{\sim}{C_k}}(\underset{\sim}{z})\,\mathrm{d}\underset{\sim}{z}}.$$

Figure 5.6 represents the center of sums method.

(6) Center of largest area

If the fuzzy set has two convex subregions, then the entire of gravity of the convex subregion with the largest area can be used to calculate the defuzzification value. The equation is given as

$$z^* = \frac{\int \mu_{\underset{\sim}{c_m}}(\underset{\sim}{z})\,\underset{\sim}{z}\mathrm{d}z}{\int \mu_{\underset{\sim}{c_m}}(\underset{\sim}{z})\,\mathrm{d}z},$$

where $\underset{\sim}{c_m}$ is the convex region with largest area. The value z^* is same as the value z^* obtained by centroid method. This can be done even for non-convex

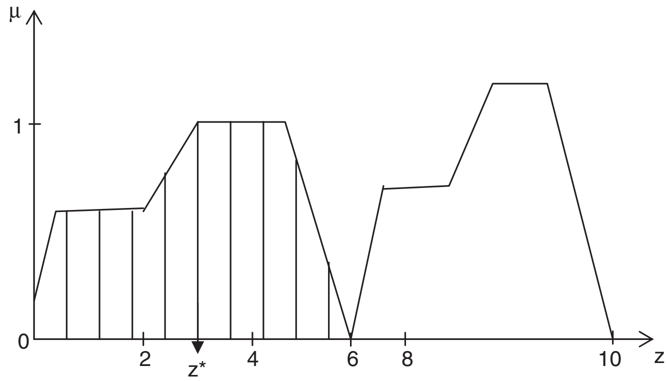


Fig. 5.7. Center of largest area

regions. Figure 5.7 represents the center of largest area method.

(7) First of maxima or last of maxima

Here, the compute output of all individual output fuzzy sets $\underset{\sim}{c_k}$ is used to determine the smallest value, with maximized membership degree in $\underset{\sim}{c_k}$.

The evaluation expressions are

Let largest height in the union is represents by $hgt\,(\underset{\sim}{c_k})$, then it is found by:

$$hgt\left(\underset{\sim}{c_k}\right) = \sup_{z \in z} \mu\, \underset{\sim}{c_k}\,(z)\,.$$

First of maxima is found by

$$z^* = \inf_{z \in z} \left\{ z \in z/\mu\, \underset{\sim}{c_k}\,(z) = hgt\left(\underset{\sim}{c_k}\right) \right\}.$$

Fast of maxima is found by,

$$z^* = \sup_{z \in z} \left\{ z \ \in z/\mu\, \underset{\sim}{c_k}\,(z) = hgt\ \left(\underset{\sim}{c_k}\right) \right\}.$$

The inf denotes infirm (greatest lower bound) and the sup denotes supremum (least upper bound). This method is shown in Fig. 5.8.

5.5 Solved Examples

Example 5.1. Two fuzzy sets $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ are defined on x as follows:

$\mu(x_1)$	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{P}$	0.1	0.2	0.7	0.5	0.4
$\underset{\sim}{Q}$	0.9	0.6	0.3	0.2	0.8

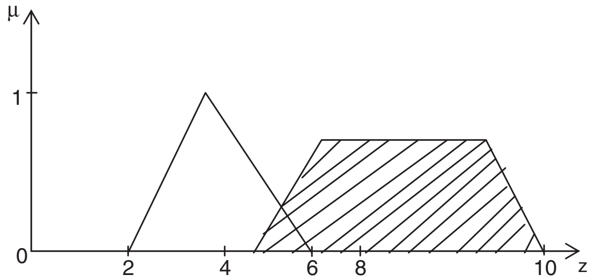


Fig. 5.8. First of or last of maxima

Find the following λ cut sets

(a) $\left(\overline{\underset{\sim}{P}}\right)_{0.2}$ (b) $(\underset{\sim}{Q})_{0.3}$ (c) $\left(\underset{\sim}{P} \cup \underset{\sim}{Q}\right)_{0.5}$ (d) $\left(\underset{\sim}{P} \cap \underset{\sim}{Q}\right)_{0.4}$

(e) $\left(\underset{\sim}{Q} \cup \overline{\underset{\sim}{P}}\right)_{0.8}$ (f) $\left(\underset{\sim}{P} \cup \overline{\underset{\sim}{P}}\right)_{0.2}$.

Solution. Given

$$\underset{\sim}{P} = \left\{\frac{0.1}{x_1} + \frac{0.2}{x_2} + \frac{0.7}{x_3} + \frac{0.5}{x_4} + \frac{0.4}{x_5}\right\},$$

$$\underset{\sim}{Q} = \left\{\frac{0.9}{x_1} + \frac{0.6}{x_2} + \frac{0.3}{x_3} + \frac{0.2}{x_4} + \frac{0.8}{x_5}\right\}.$$

Finding

$$\overline{\underset{\sim}{P}} = \left\{\frac{0.9}{x_1} + \frac{0.8}{x_2} + \frac{0.3}{x_3} + \frac{0.5}{x_4} + \frac{0.6}{x_5}\right\},$$

$$\overline{\underset{\sim}{Q}} = \left\{\frac{0.1}{x_1} + \frac{0.4}{x_2} + \frac{0.7}{x_3} + \frac{0.8}{x_4} + \frac{0.2}{x_5}\right\}.$$

(a) $\left(\overline{\underset{\sim}{P}}\right)_{0.2} = \left\{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} + \frac{1}{x_5}\right\},$

(b) $\left(\overline{\underset{\sim}{Q}}\right)_{0.3} = \left\{\frac{0}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} + \frac{0}{x_5}\right\},$

(c) $\left(\underset{\sim}{P} \cup \underset{\sim}{Q}\right) = \left\{\frac{0.9}{x_1} + \frac{0.6}{x_2} + \frac{0.7}{x_3} + \frac{0.5}{x_4} + \frac{0.8}{x_5}\right\},$

$\left(\underset{\sim}{P} \cup \underset{\sim}{Q}\right)_{0.6} = \left\{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{0}{x_4} + \frac{1}{x_5}\right\},$

(d) $\left(\underset{\sim}{P} \cup \overline{\underset{\sim}{P}}\right) = \left\{\frac{0.9}{x_1} + \frac{0.8}{x_2} + \frac{0.7}{x_3} + \frac{0.5}{x_4} + \frac{0.6}{x_5}\right\},$

$\left(\underset{\sim}{P} \cup \overline{\underset{\sim}{P}}\right)_{0.8} = \left\{\frac{1}{x_1} + \frac{1}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{0}{x_5}\right\}$

(e) $\left(\underset{\sim}{P} \cap \underset{\sim}{Q}\right) = \left\{\frac{0.9}{x_1} + \frac{0.8}{x_2} + \frac{0.7}{x_3} + \frac{0.5}{x_4} + \frac{0.6}{x_5}\right\},$

$\left(\underset{\sim}{P} \cap \underset{\sim}{Q}\right)_{0.4} = \left\{\frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{1}{x_5}\right\},$

(f) $\left(\underset{\sim}{P} \cap \overline{\underset{\sim}{P}}\right) = \left\{\frac{0.1}{x_1} + \frac{0.2}{x_2} + \frac{0.3}{x_3} + \frac{0.5}{x_4} + \frac{0.4}{x_5}\right\},$

$\left(\underset{\sim}{P} \cap \overline{\underset{\sim}{P}}\right)_{0.8} = \left\{\frac{0}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} + \frac{1}{x_5}\right\}.$

Example 5.2. Given three fuzzy sets:

$$\underset{\sim}{A} = \left\{\frac{0.9}{x_1} + \frac{0.5}{x_2} + \frac{0.2}{x_3} + \frac{0.3}{x_4}\right\},$$

$$\underset{\sim}{B} = \left\{\frac{0.2}{x_1} + \frac{1.0}{x_2} + \frac{0.8}{x_3} + \frac{0.4}{x_4}\right\},$$

$$\underset{\sim}{C} = \left\{\frac{0.1}{x_1} + \frac{0.7}{x_2} + \frac{0.5}{x_3} + \frac{0.6}{x_4}\right\}.$$

Find $A_{0.6}$, $B_{1.0}$, $C_{0.3}$, $\overline{\underset{\sim}{A}}_{0.2}$, $\overline{\underset{\sim}{B}}_{0.8}$, $\overline{\underset{\sim}{C}}_{0.5}$.

Solution.

(a) $A_{0.6} = \left\{\frac{1}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4}\right\},$

(b) $B_{1.0} = \left\{\frac{0}{x_1} + \frac{1}{x_2} + \frac{0}{x_3} + \frac{0}{x_4}\right\},$

(c) $C_{0.3} = \left\{\frac{0}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right\},$

(d) $\overline{\underset{\sim}{A}} = \left\{\frac{0.1}{x_1} + \frac{0.5}{x_2} + \frac{0.8}{x_3} + \frac{0.7}{x_4}\right\},$

$\overline{\underset{\sim}{A}}_{0.2} = \left\{\frac{0}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right\},$

(e) $\overline{\underset{\sim}{B}} = \left\{\frac{0.8}{x_1} + \frac{0.0}{x_2} + \frac{0.2}{x_3} + \frac{0.6}{x_4}\right\},$

$\overline{\underset{\sim}{B}}_{0.8} = \left\{\frac{1}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0.9}{x_4}\right\},$

(f) $\overline{\underset{\sim}{C}} = \left\{\frac{0.9}{x_1} + \frac{0.3}{x_2} + \frac{0.5}{x_3} + \frac{0.4}{x_4}\right\},$

$\overline{\underset{\sim}{C}}_{0.5} = \left\{\frac{1}{x_1} + \frac{0}{x_2} + \frac{1}{x_3} + \frac{0}{x_4}\right\}.$

Example 5.3. The fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are defined as universe, $x = \{0, 1, 2, 3\}$, with the following membership fractions:

$$\mu_{\underset{\sim}{A}}(x) = \frac{2}{x+3},$$

$$\mu_{\underset{\sim}{B}}(x) = \frac{4x}{x+5}.$$

Define the intervals along x-axis corresponding to the λ cut sets for each fuzzy set $\underset{\sim}{A}$ and $\underset{\sim}{B}$ for following values of λ. $\lambda = 0.2, 0.5, 0.6$.

Solution.

$$x = \{0, 1, 2, 3\}$$

X	0	1	2	3
$\mu_{A\sim}(x) = 2/x+3$	2/3	2/4	2/5	2/6
$\mu_{B\sim}(x) = 4x/2(x+5)$	0	4/12	8/14	12/16

Therefore the values are:

X	0	1	2	3
$\mu_{A\sim}(x)$	0.67	0.5	0.4	0.33
$\mu_{B\sim}(x)$	0	0.33	0.57	0.75

(a) When $\lambda = 0.2$
$A_{0.2} = \{0, 1, 2, 3\}$,
$A_{0.2} = \{1, 2, 3\}$.
(b) When $\lambda = 0.5$
$A_{0.5} = \{0, 1\}$,
$B_{0.5} = \{2, 3\}$.
(c) When $\lambda = 0.6$
$A_{0.6} = \{0\}$,
$A_{0.6} = \{3\}$.

Example 5.4. For the fuzzy relation

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.5 & 0.9 & 0.6 \\ 0.4 & 0.8 & 0.7 \end{bmatrix},$$

find the λ cut relations for the following values of $\lambda = 0^+, 0.2, 0.9, 0.5$.

Solution. Given,

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.5 & 0.9 & 0.6 \\ 0.4 & 0.8 & 0.7 \end{bmatrix}.$$

(a) $\lambda = 0^+$,

$$R_{0^+} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(b) $\lambda = 0.2$,

$$R_{0.2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(c) $\lambda = 0.9$,

$$R_{0.9} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

(d) $\lambda = 0.5$,

$$R_{0.5} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Example 5.5. For the given fuzzy relation

$$R = \begin{bmatrix} 0.2 & 0.5 & 0.47 & 1 & 0.9 \\ 0.3 & 0.5 & 0.6 & 1 & 0.8 \\ 0.4 & 0.6 & 0.4 & 0.5 & 0.3 \\ 0.9 & 1 & 0.3 & 0.3 & 0.2 \end{bmatrix},$$

find the cut λ cut relation for the following values of $\lambda = 0.4, 0.7, 0.8$.

Solution. (a) $\lambda = 0.4$,

$$R_{0.4} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

(b) $\lambda = 0.7$,

$$R_{0.7} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

(c) $\lambda = 0.8$,

$$R_{0.8} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Example 5.6. For the given membership function as shown in Fig. 5.9 determines the defuzzified output value by seven methods.

Solution. (a) Centroid method

$A_{11}(0, 0)$, $(2 - 0.7)$

The straight line may be:

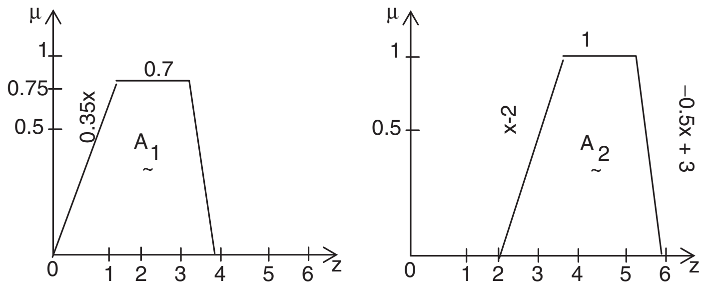


Fig. 5.9. Membership function

$$
\begin{aligned}
&y-0=\frac{0.7}{2}\,(x-0)\,,\\
&y=0.35x.\\
&A_{12}:y=0.7.\\
&A_{13}:\text{not needed.}\\
&A_{21}:(2,0)(3,1),\\
&y-0=\frac{1-0}{3-2}\,(x-2)\,,\\
&y=x-2.\\
&A_{22}:y=1.\\
&A_{23}:(4,1)(6,0),\\
&y-1=\frac{0-1}{6-4}\,(x-4)\,,\\
&y=\frac{-1}{2}\,(x-4)+1=-0.5x+3.\\
&\text{Solving } A_{12} \text{ and } A_{21},\\
&Y=0.7 \quad y=x-2,\\
&x-2=0.7,\\
&x=2.7,\\
&y=0.7.
\end{aligned}
$$

$$
\begin{aligned}
\text{Numerator} &= \int_0^2 0.35\ z^2\ \mathrm{d}z+\int_2^{2.7} 0.7K\mathrm{d}z+\int_{2.7}^3 \left(z^2-2z\right)\mathrm{d}z\\
&\quad+\int_3^4 z\ \mathrm{d}z+\int_4^6 \left(-0.5z^2+3z\right)\mathrm{d}z\\
&=10.98.\\
\text{Denominator} &= \int_0^2 0.35\ z^2\ \mathrm{d}z+\int_2^{2.7} 0.7K\ \mathrm{d}z+\int_{2.7}^3 \left(z^2-2\right)\mathrm{d}z\\
&\quad+\int_3^4 \mathrm{d}z+\int_4^6 \left(-0.5z^2+3z\right)\mathrm{d}z\\
&=3.445.
\end{aligned}
$$

$$z^* = \frac{\text{Numerator}}{\text{Demoninator}} = \frac{10.98}{3.445} = 3.187.$$

(b) Weighted average method

$$z^* = \frac{2 \times 0.7 + 4 \times 1}{1 + 0.7} = 3.176.$$

(c) Mean–max method

$$z^* = \frac{2.5 + 3.5}{2} = 3.$$

(d) Center of saws method

$$\begin{aligned} z^* &= \frac{\int_0^6 \left(\frac{1}{2} \times 0.7 \times (3+2) \times 2 + \frac{1}{2} \times 1 \times (2+4) \times 4\right)}{\int_0^6 \left(\frac{1}{2} \times 0.7 \times (3+2) + \frac{1}{2} \times 1 \times (2+4) \times 4\right)} \\ &= \frac{\int_0^b (3.5+12)\,\mathrm{d}z}{\int_0^b (1.75+3)\,\mathrm{d}z} = 2.84. \end{aligned}$$

(e) First of maximum
$z^* = 3.$
(f) Last of maxima
$z^* = 4.$
(g) Center of largest area

$$\begin{aligned} \text{Area of } I &= \frac{1}{2} \times 0.7 \times (2.7 + 0.7) = 1.19, \\ \text{Area of } II &= \frac{1}{2} \times 1 \times (2+3) \times \frac{1}{2} \times 0.7 \times = 2.255. \end{aligned}$$

Area of II is larger, So,

$$\begin{aligned} z^* &= \frac{\int_{2.7}^3 \frac{1}{2} \times 0.3 \times 0.3 \times 2.85\,\mathrm{d}z + \int_3^4 1 \times 1 \times 3.5\,\mathrm{d}z + \int_4^6 \frac{1}{2} \times 2 \times 1\,\mathrm{d}z}{\int_{2.7}^3 \frac{1}{2} \times 0.3 \times 0.3\,\mathrm{d}z + \int_4^3 1 \times 1\,\mathrm{d}z + \int_4^6 \frac{1}{2} \times 2 \times 1\,\mathrm{d}z} \\ &= \frac{\int_{2.7}^3 0.12825\,\mathrm{d}z + \int_3^4 3.5\,\mathrm{d}z + \int_4^6 5\,\mathrm{d}z}{\int_{2.7}^3 0.045\,\mathrm{d}z + \int_3^4 \mathrm{d}z + \int_4^6 \mathrm{d}z} \\ z^* &= 4.49. \end{aligned}$$

Example 5.7. Using Matlab program find the crisp lambda cut set relations for $\lambda = 0.2$, the fuzzy matrix is given by

$$\underset{\sim}{R} = \begin{bmatrix} 0.2 & 0.7 & 0.8 & 1 \\ 1 & 0.9 & 0.5 & 0.1 \\ 0 & 0.8 & 1 & 0.6 \\ 0. & 0.4 & 1 & 0.3 \end{bmatrix}.$$

Solution. The Matlab program is

Program

```
clear all
% Enter the matrix value
R=input('Enter the matrix value')
% Enter the lambda value
lambda=input('enter the lambda value')
[m,n]=size(R);
  for i=1:m
        for j=1:n
                if(R(i,j)<lambda)
                b(i,j)=0;
                else
                b(i,j)=1;
                end
        end
  end
% output value
display('the crisp value is')
display(b)
```

Output

```
Enter the matrix value
   [0.2 0.7 0.8 1;1 0.9 0.5 0.1;0 0.8 1 0.6;0.2 0.4 1 0.3]
   R =
        0.2000   0.7000   0.8000   1.0000
        1.0000   0.9000   0.5000   0.1000
             0   0.8000   1.0000   0.6000
        0.2000   0.4000   1.0000   0.3000
   Enter the lambda value 0.2
   lambda = 0.2000
   The crisp value is
   b =
          1   1   1   1
          1   1   1   0
          0   1   1   1
          1   1   1   1
```

Summary

Defuzzification is thus a natural and necessary process. Because the output to any practical system cannot be given using the linguistic variables like "moderately high," "medium," "very positive," etc., it has to be given only in crisp quantities. These crisp quantities are thus obtained from the fuzzy quantities using the various defuzzification methods discussed in this chapter.

Review Questions

1. Define defuzzification process.
2. What is the necessity to convert the fuzzy quantities into crisp quantities?
3. State the method lambda cuts employed for the conversion of the fuzzy set into crisp.
4. Discuss in detail on the special properties of lambda cut sets.
5. How is lambda cut method employed for a fuzzy relation?
6. List some of the methods to perform defuzzification process.
7. How does the max-membership method convert the fuzzy quantity to crisp quantity?
8. Centroid method is very efficient method for defuzzification, Justify. Give suitable example.
9. In what way does the weighted average method perform the defuzzification process?
10. Explain about the mean–max-membership method for converting the fuzzy quantity to crisp quantity. Give some details on the accuracy of the output obtained.
11. Compare the methods center of sums and center of largest area with necessary examples.
12. What is difference between first and last of maxima? Explain the process of conversion in each case with example.
13. Compare and contrast the methods employed for defuzzification process on the basis of accuracy and time consumption.
14. What are the four important criteria on which the defuzzification method is defined?

Exercise Problems

1. Determine crisp λ cut relation for $\lambda = 0.2$; for $j = 0, 1, \ldots, 10$ for the following fuzzy relation matrix $\underset{\sim}{\mathrm{R}}$:

$$\underset{\sim}{R} = \begin{bmatrix} 0.3 & 0.8 & 0.7 & 0.9 \\ 1 & 0.7 & 0.6 & 0.2 \\ 0.1 & 0.7 & 1 & 0.9 \\ 0.5 & 0.6 & 0.2 & 0.5 \end{bmatrix}.$$

2. The fuzzy set $\underset{\sim}{A}$, $\underset{\sim}{B}$, $\underset{\sim}{C}$ are all defined on the universe $X = [0, 5]$ with the following membership functions:

$$\begin{aligned} \mu_{\underset{\sim}{A}}(x) &= \frac{1}{1 + 5\,(x-5)^2}, \\ \mu_{\underset{\sim}{B}}(x) &= 2^{-x}, \\ \mu_{\underset{\sim}{C}}(x) &= \frac{2x}{x+5}. \end{aligned}$$

(a) Sketch the membership functions
(b) Define the intervals along x-axis corresponding to the λ cut sets for each of the fuzzy sets $\underset{\sim}{A}$, $\underset{\sim}{B}$, $\underset{\sim}{C}$ for the following values of λ.
$\lambda = 0.2, \lambda = 0.4, \lambda = 0.7, \lambda = 0.9.$

3. Two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ both defined on x are as follows:

$$\underset{\sim}{A} = \left\{ \frac{0.1}{x_1} + \frac{0.6}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.5}{x_5} + \frac{0.2}{x_6} \right\},$$

$$\underset{\sim}{B} = \left\{ \frac{0.8}{x_1} + \frac{0.6}{x_2} + \frac{0.3}{x_3} + \frac{0.2}{x_4} + \frac{0.6}{x_5} + \frac{0}{x_6} \right\}.$$

Find
(a) $\left(\underset{\sim}{\bar{A}}\right)_{0.5}$
(b) $\left(\underset{\sim}{B}\right)_{0.3}$
(c) $\left(\underset{\sim}{A} \cup \underset{\sim}{\bar{A}}\right)_{0.5}$
(d) $\left(\underset{\sim}{A} \cup \underset{\sim}{B}\right)_{0.4}$
(e) $\left(\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}\right)_{0.6}$
(f) $\left(\underset{\sim}{A} \cap \underset{\sim}{B}\right)_{0.64}$

4. For fuzzy relation R find λ cut relations for the following values of λ

$$R = \begin{bmatrix} 0.4 & 0.3 & 0.7 & 0.5 \\ 0.6 & 0.2 & 0.1 & 1 \\ 0.9 & 0.8 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0.3 & 0.2 \end{bmatrix}.$$

(a) $\lambda = 0^+$ (c) $\lambda = 0.4$ (e) $\lambda = 0.3$
(b) $\lambda = 0.2$ (d) $\lambda = 0.7$ (f) $\lambda = 0.6$

5. Show that any λ cut relation of fuzzy tolerance relation results in a crisp tolerance relation.

Show that any λ cut relation of a fuzzy equivalence relation results in a crisp equivalence relation.

6. For fuzzy relation $\underset{\sim}{A}$ and $\underset{\sim}{E}$ determine λ cut relations for the following values of λ:
(a) $\lambda = 0^+$ (b) $\lambda = 0.5$ (c) $\lambda = 0.9$

$$\underset{\sim}{A} = \begin{bmatrix} 0.8 & 1 & 0.5 & 0.3 & 0.1 & 0 \\ 0.2 & 0.3 & 0.5 & 0.7 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.4 & 0.8 & 0.7 & 0.1 \\ 0.2 & 0.1 & 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix},$$

$$\underset{\sim}{E} = \begin{bmatrix} 0.8 & 0.7 & 0.4 & 0.1 & 0 \\ 0.6 & 0.5 & 0.3 & 0.2 & 0.1 \\ 0.9 & 0.6 & 0.7 & 0.4 & 0.3 \\ 0.2 & 0.4 & 0.5 & 0.9 & 0.6 \\ 0.1 & 0.4 & 0.3 & 0.6 & 0.9 \\ 0.1 & 0 & 1 & 0.8 & 0.7 \end{bmatrix}.$$

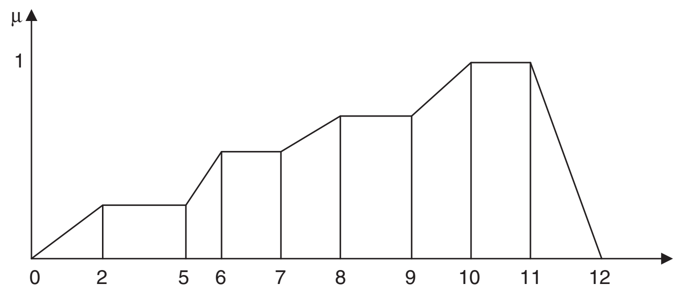
7. Determine the λ cut sets for the six set operation for two fuzzy set $\underset{\sim}{R}$ and $\underset{\sim}{S}$ using $\lambda = 0.2$ and 0.8:

$$\underset{\sim}{A} = \left\{\frac{0.1}{20} + \frac{0.6}{40} + \frac{0.4}{60} + \frac{0.3}{80} + \frac{0.9}{100}\right\},$$

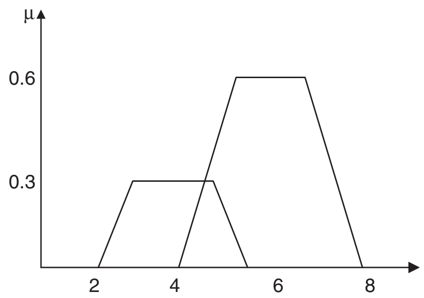
$$\underset{\sim}{B} = \left\{\frac{0.3}{20} + \frac{0.4}{40} + \frac{0.7}{60}\frac{0.4}{80}\frac{0.2}{100}\right\}.$$

For the fuzzy sets operation:
(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$ (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$ (c) $\bar{\underset{\sim}{A}}$ (d) $\underset{\sim}{A} / \underset{\sim}{B}$ (e) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$ (f) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$

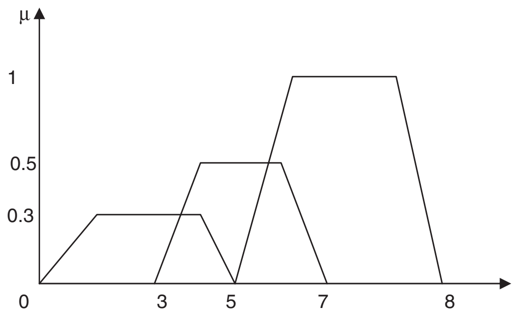
8. By using centroid method of defuzzification convert fuzzy value z to precise value Z^* for the following graph.



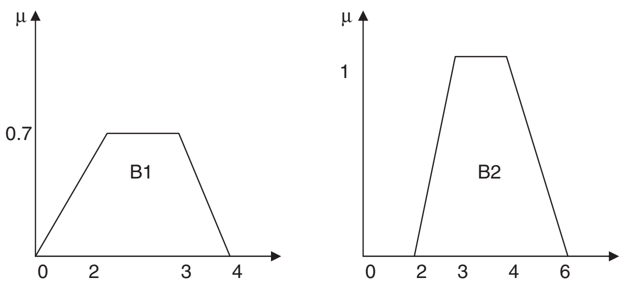
9. Find the defuzzified value by weighted average method shown in figure.



10. Find the defuzzified values using (a) center of sums methods and (b) center of largest area for the figure shown.



11. Find the defuzzified values for the figure shown above using first of maxima and last of maxima.
12. Two companies bid for a contract. The fuzzy set of two companies $\underset{\sim}{B_1}$ and $\underset{\sim}{B_2}$ is shown in the following figure. Find the defuzzified value z^* using different methods.



13. Using Matlab program find the crisp lambda cut set relations for $\lambda = 0.4$, the fuzzy matrix is given by:

$$\underset{\approx}{R} = \begin{bmatrix} 0.3 & 0.2 & 0.8 & 0 \\ 1 & 0.1 & 0.5 & 0.1 \\ 0 & 0.8 & 1 & 0.5 \\ 0.7 & 0.6 & 1 & 0.3 \end{bmatrix}.$$