Laser Ranging Delay in the Bimetric Theory of Gravity

Sergei M. Kopeikin[1] and Wei-Tou Ni[2]

[1] Department of Physics & Astronomy, University of Missouri-Columbia, Columbia, MO 65211, USA
[2] Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing, 210008, China

Summary. We introduce a linearized bimetric theory of gravity with two metrics. The metric $g_{\alpha\beta}$ describes null hypersurfaces of the gravitational field while light moves on null hypersurfaces of the *optical* metric $\bar{g}_{\alpha\beta}$. Bimetrism naturally arises in vector–tensor theories with matter being nonminimally coupled to gravity via long-range vector field. We derive explicit Lorentz-invariant solution for a light ray propagating in space–time of the bimetric theory and disentangle relativistic effects associated with the existence of the two metrics. This analysis may be valuable for future spaceborne laser missions ASTROD and LATOR dedicated to map various relativistic gravity parameters in the solar system to unparalleled degree of accuracy.

Recently Carlip [1] has introduced a bimetric theory of gravity with two metrics, $g_{\alpha\beta}$ and $\bar{g}_{\alpha\beta}$, and a unit vector field w^α coupled to matter via constant parameter ϵ. This theory is a variant of a vector–tensor theory [2] where the vector field w^α obeys the source-free field equations and is responsible for the spontaneous violation of the Lorentz invariance of gravity [3, 4] in the sense that it introduces a preferred frame which effects can be observed only in gravitational experiments conducted in nonnegligible gravitational field. One can show that in the case of a bimetric theory of gravity the Huges–Drever experiments and other precision experiments constrain the asymptotic (vanishing gravity) difference between these two metrics severely [5–7]. Here we consider more complicated (nonvanishing gravity) case of geometric optics of light rays (laser beams) in the bimetric theory. Carlip's bimetric theory [1] adopts specific values of parameters in Jackobson's theory [2] and adds one more parameter ϵ which is a coupling constant between the vector field w^α and the stress–energy tensor of matter. It is useful to understand what kind of relativistic effects one can expect in the bimetric theory in application to the spaceborne laser ranging experiments like ASTROD [8,9] and LATOR [10–12]. We shall investigate this problem by calculating the time delay of light propagating in the time-dependent gravitational field of an arbitrary moving body.

In what follows, we shall consider an isolated N-body system (a solar system) resided in an asymptotically flat space–time of the gravity metric $g_{\alpha\beta}$. According to Carlip [1] the optical metric

$$\bar{g}_{\alpha\beta} = g_{\alpha\beta} + \left(1 - \frac{1}{\epsilon^2}\right) w_\alpha w_\beta \ , \tag{1}$$

$$\bar{g}^{\alpha\beta} = g^{\alpha\beta} - \left(\epsilon^2 - 1\right) w^\alpha w^\beta \ , \tag{2}$$

where ϵ is a constant parameter defining the coupling of the vector field w^α with matter and describing the degree of violation of the Lorentz invariance for electromagnetic field (and other material fields). Following Carlip [1] we assume that the Greek indices are raised and lowered with the metric $g_{\alpha\beta}$.

Let us work in a global coordinate system $x^\alpha = (ct, x^i)$, where c in Carlip's theory [1] is the speed of gravity and x^i are spatial coordinates. In the global frame the linearized expansions for the metric and the vector field are

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \ , \tag{3}$$

$$g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} \ , \tag{4}$$

$$w^\alpha = V^\alpha + \zeta^\alpha \ , \tag{5}$$

$$w_\alpha = V_\alpha + \zeta_\alpha - h_{\alpha\beta}V^\beta \ , \tag{6}$$

where $\eta_{\alpha\beta} = \mathrm{diag}(-1, +1, +1, +1)$ is the Minkowski metric, $h_{\alpha\beta}$ is the perturbation of the gravity metric, and ζ^α is the perturbation of the vector field which unperturbed value in the global frame is V^α. We emphasize that V^α remains arbitrary and our analysis is not limited to the case of the preferred frame where $V^\alpha = (1, 0, 0, 0)$.

The optical metric $\bar{g}_{\alpha\beta}$ is decomposed as follows

$$\bar{g}_{\alpha\beta} = \bar{\eta}_{\alpha\beta} + \bar{h}_{\alpha\beta} \ , \tag{7}$$

$$\bar{g}^{\alpha\beta} = \bar{\eta}^{\alpha\beta} - \bar{h}^{\alpha\beta} \ , \tag{8}$$

where the unperturbed part of the optical metric is defined by

$$\bar{\eta}^{\alpha\beta} = \eta^{\alpha\beta} - \left(\epsilon^2 - 1\right) V^\alpha V^\beta \ , \tag{9}$$

$$\bar{\eta}_{\alpha\beta} = \eta_{\alpha\beta} + \left(1 - \frac{1}{\epsilon^2}\right) V_\alpha V_\beta \ , \tag{10}$$

and the perturbation

$$\bar{h}^{\alpha\beta} = h^{\alpha\beta} + \left(\epsilon^2 - 1\right) \left(V^\alpha \zeta^\beta + V^\beta \zeta^\alpha\right) . \tag{11}$$

According to Carlip [1] light propagates in the bimetric theory along light geodesics of the optical metric $\bar{g}_{\alpha\beta}$. In geometric optics limit, the light rays are defined by a covariant equation for electromagnetic phase (eikonal) φ which reads [1, 13]

$$\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi = 0 \ . \tag{12}$$

This equation is formally equivalent to the equation of light propagating through dispersive medium with refraction index ϵ moving with respect to a preferred frame with velocity w^α. Theory of light propagation through the dispersive medium has been worked out by Synge [13] and we shall use his theory to integrate (12) and to interpret its solution.

As follows from (12), we do not need to know solutions for the metric perturbation $h^{\alpha\beta}$ and that of the vector field ζ^α separately. What we need to perform the integration of (12) is solution for the perturbation $\bar{h}^{\alpha\beta}$ of the optical metric (8). In the linearized approximation the metric perturbation $\bar{h}^{\alpha\beta}$ obeys the following gravity field equations [1]

$$\Box\bar{h}^{\alpha\beta} = -1/2\frac{16\pi G}{(1-4\varpi)\epsilon c^2}\left(S^{(\alpha}_\mu T^{\beta)\mu} - \eta^{\alpha\beta}T^\lambda_\lambda\right) \ , \tag{13}$$

where $\Box \equiv -c^{-2}\partial^2/\partial t^2 + \nabla^2$ is the wave operator in flat space–time defining null characteristics of the gravitational field, the constant tensor $S^{\alpha\beta} = \eta^{\alpha\beta} + 2\varpi V^\alpha V^\beta$, ϖ is another constant parameter of the bimetric theory ($\varpi = 0$ in general relativity), and $T^{\mu\nu}$ is the stress–energy tensor of the point-like bodies composing of the N-body system. Equation (13) is valid under imposing the following gauge conditions

$$\partial_\beta\left(h^{\alpha\beta} - \frac{1}{2}\eta^{\alpha\beta}h\right) + \left(1 - \frac{1}{\epsilon^2}\right)V_\alpha\partial_\beta\zeta^\beta = 0 \ . \tag{14}$$

In the linearized approximation the stress–energy tensor reads [1]

$$T^{\alpha\beta}(t,\boldsymbol{x}) = \sum_{a=1}^N \frac{M_a u_a^\alpha u_a^\beta \delta^{(3)}\big(\boldsymbol{x} - \boldsymbol{x}_a(t)\big)}{\gamma_a\sqrt{1 - \left(1 - \epsilon^{-2}\right)\left(u^\mu V_\mu\right)^2}} \ , \tag{15}$$

where the index $a = 1, 2, \ldots, N$ enumerates gravitating bodies of the solar system, M_a is the (constant) rest mass of the ath body, $\boldsymbol{x}_a(t)$ is time-dependent spatial coordinate of the ath body, $\boldsymbol{v}_a(t) = d\boldsymbol{x}_a(t)/dt$ is velocity of the ath body, $u_a^\alpha = \gamma_a(1, \boldsymbol{v}_a/c)$ is the 4-velocity of the ath body, $\gamma_a = \left(1 - v_a^2/c^2\right)^{-1/2}$ is the Lorentz factor, and $\delta^{(3)}(\boldsymbol{x})$ is the three-dimensional Dirac's delta function.

Because the field equations (13) are linear, we can consider their solution as a linear superposition of the solutions for each body. It allows us to focus on the relativistic effects caused by one body (Sun, planet, etc.) only. Solving (13) by making use of the retarded Liénard–Wiechert tensor potentials [14], one obtains the metric tensor perturbation

$$\bar{h}^{\alpha\beta}(t,\boldsymbol{x}) = \frac{2GM}{(1-4\varpi)c^2}\frac{2u^\alpha u^\beta + \eta^{\alpha\beta} + 2\varpi\left(u^\alpha V^\beta + u^\beta V^\alpha\right)\left(u^\mu V_\mu\right)}{\sqrt{\epsilon^2 - \left(\epsilon^2 - 1\right)\left(u^\mu V_\mu\right)^2}}\frac{1}{r_R}, \tag{16}$$

where $r_R \equiv -u_\alpha r^\alpha$, $r^\alpha = x^\alpha - z^\alpha(s)$, $z^\alpha(t) = (ct, \boldsymbol{z}(t))$ is the world line of the light ray deflecting body parametrized by the coordinate time t, $u^\alpha(t) = c^{-1} dz^\alpha(t)/dt$.

Because we solved the field equations (13) in terms of the retarded Liénard–Wiechert potentials, the distance $r^\alpha = x^\alpha - z^\alpha(s)$, the body's world line $z^\alpha(s) = (cs, \boldsymbol{z}(s))$, and the body's 4-velocity $u^\alpha(s)$ in (16) are functions of the *retarded* time s. The retarded time s is found in the linearized approximation of the bimetric theory as a solution of the gravity null cone equation

$$\eta_{\mu\nu} r^\mu r^\nu \equiv \eta_{\mu\nu} \left(x^\mu - z^\mu(s) \right) \left(x^\nu - z^\nu(s) \right) = 0 \ , \tag{17}$$

i.e.,

$$s = t - \frac{1}{c} |\boldsymbol{x} - \boldsymbol{z}(s)| \ , \tag{18}$$

where the fundamental constant c in (18) is the fundamental speed of propagation of gravity.

Light rays are defined by a covariant equation (12) for electromagnetic phase (eikonal) φ. Assuming that unperturbed solution of (12) is a plane wave, we can write a general solution of this equations as follows

$$\varphi(x^\alpha) = \varphi_0 + k_\alpha x^\alpha + \psi(x^\alpha) \ , \tag{19}$$

where k_α is an unperturbed (constant) wave covector of the electromagnetic wave, and $\psi(x)$ is a relativistic perturbation of the eikonal generated by the metric tensor perturbation $\bar{h}_{\alpha\beta}$ defined in (16). Substitution of (19) to (12) yields

$$\bar{\eta}^{\alpha\beta} k_\alpha k_\beta = 0 \ , \tag{20}$$

$$\bar{\eta}^{\alpha\beta} k_\alpha \frac{\partial \psi}{\partial x^\beta} = \frac{1}{2} h^{\alpha\beta} k_\alpha k_\beta \ . \tag{21}$$

Let us define a vector (see Fig. 1)

$$\sigma^\alpha = \bar{\eta}^{\alpha\beta} k_\beta = k^\alpha - \left(\epsilon^2 - 1 \right) \left(V^\beta k_\beta \right) V^\alpha \ , \tag{22}$$
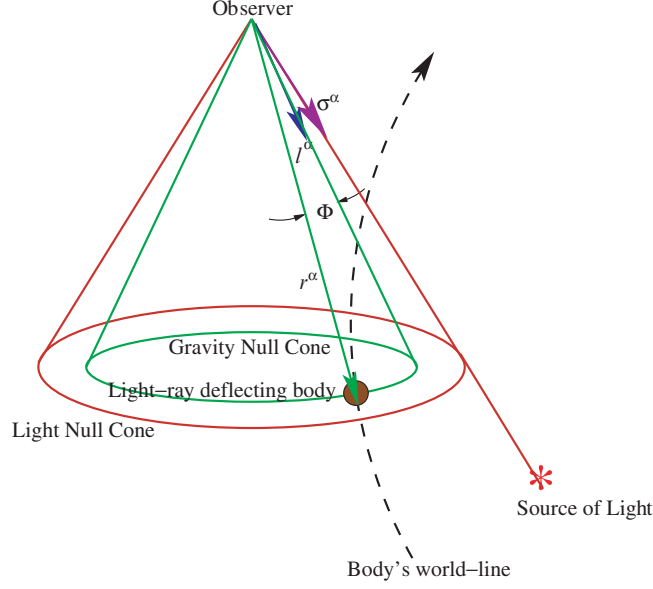
such that

$$\bar{\eta}_{\alpha\beta} \sigma^\alpha \sigma^\beta = 0 \ , \qquad \text{and} \qquad k_\alpha \sigma^\alpha = 0 \ . \tag{23}$$

Vector σ^α defines the direction of propagation of light ray from a source of light (laser, star) to observer (see Fig. 1). Making use of vector σ^α simplifies (21) and reduces it to the following form

$$\sigma^\alpha \frac{\partial \psi}{\partial x^\alpha} = \frac{1}{2} h_{\alpha\beta} \sigma^\alpha \sigma^\beta \ . \tag{24}$$

Unperturbed characteristics of the eikonal equation (24) are straight lines (light rays) parametrized by the affine parameter λ in such a way that

Fig. 1. The light and gravity null cones of the bimetric theory are shown. Gravity propagates from the source of light (laser) to observer along the gravity null cone defined by the metric $g_{\alpha\beta}$. Gravitationally unperturbed direction to the source of light is defined by vector σ^{α} lying on the null hypersurface of the optical metric $\bar{g}_{\alpha\beta}$. Gravitationally perturbed direction of the light propagation is l^{α} and this vector resides on the gravity null cone. Each light ray deflecting body (Sun, planet) deflects light from its retarded position defined with respect to observer by a null vector $r^{\alpha} = x^{\alpha} - x_{J}^{\alpha}(s)$ which also resides on the gravity null cone. The eikonal gravitational perturbation is $\psi = -(2GM/c^2)\chi \ln\Phi$, where $\Phi = -l_{\alpha}r^{\alpha}$. Gravitational light ray deflection experiments measure the range χ and shape Φ of the relativistic time delay of light. The range measurement allows us to pin down the parameter χ while measuring the shape Φ of the time delay gives us the components of the vector l^{α} under assumption that vector r^{α} is known. Deviation of l^{α} from the gravity-unperturbed direction σ^{α} of the light ray measures the degree of violation of the Lorentz invariance of the gravitational field.

$$\frac{d}{d\lambda} = \sigma^{\alpha}\frac{\partial}{\partial x^{\alpha}} \ . \tag{25}$$

Integration of (25) by making use of the unperturbed characteristics is straightforward (see, for example, [15]) and can be written as follows

$$\psi(x^{\alpha}) = -\frac{2GM}{c^2}\chi \ln\left(-l_{\alpha}r^{\alpha}\right) \ , \tag{26}$$

where

$$\chi = \frac{(\sigma_{\alpha}u^{\alpha})^2 + (1/2)(\sigma_{\alpha}\sigma^{\alpha}) + 2\varpi(\sigma_{\alpha}u^{\alpha})(\sigma_{\alpha}V^{\alpha})(u_{\alpha}V^{\alpha})}{(1-4\varpi)(\epsilon^2 - (\epsilon^{-2}-1)(u^{\mu}\zeta_{\mu})^2)^{1/2}(P_{\alpha\beta}\sigma^{\alpha}\sigma^{\beta})^{1/2}} \ , \tag{27}$$

$$l^\alpha = \sigma_\perp^\alpha + \sigma_\perp u^\alpha \ , \tag{28}$$

$$\sigma_\perp^\alpha = P_\beta^\alpha \sigma^\beta \ , \tag{29}$$

$$\sigma_\perp = (\sigma_{\perp\alpha}\sigma_\perp^\alpha)^{1/2} = \left(P_{\alpha\beta}\sigma^\alpha\sigma^\beta\right)^{1/2} \ , \tag{30}$$

and

$$P_{\alpha\beta} = \eta_{\alpha\beta} + u_\alpha u_\beta \ , \tag{31}$$

is the operator of projection on the hyperplane orthogonal to the 4-velocity u^α of the light ray deflecting body ($P_{\alpha\beta}P_\gamma^\beta = P_{\alpha\gamma}$). It is easy to confirm that solution (26) is valid by observing that

$$\frac{d}{d\lambda}\ln\left(-l_\alpha r^\alpha\right) = -\frac{\sigma_\perp}{r_R} \ , \tag{32}$$

where $r_R = -u_\alpha r^\alpha$, and equations

$$\partial_\alpha r^\mu = \delta_\alpha^\mu - \frac{u^\mu}{\gamma}\partial_\alpha s \ , \tag{33}$$

$$\partial_\alpha s = -\gamma\frac{r_\alpha}{r_R} \ , \tag{34}$$

where $\gamma = (1-\beta^2)^{-1/2}$, have been used. Equation (28) allows us to recast the argument of the logarithm in (26) as

$$l_\alpha r^\alpha = \sigma_\perp^\alpha r_\alpha - \sigma_\perp r_R \ . \tag{35}$$

It is remarkable that both vectors l^α and r^α are null vectors of the gravity metric $g_{\alpha\beta}$ (see Fig. 1). Indeed, in the linearized approximation $g_{\alpha\beta} = \eta_{\alpha\beta}$ and one can easily prove by inspection that

$$\eta_{\alpha\beta}l^\alpha l^\beta = 0 \ , \tag{36}$$

$$\eta_{\alpha\beta}r^\alpha r^\beta = 0 \ , \tag{37}$$

which are consequences of the definitions given by (17) and (28). Thus, neither l^α nor r^α belong to the null cone of the optical metric $\bar{g}_{\alpha\beta}$ but characterize the null hypersurfaces of the gravity metric $g_{\alpha\beta}$.

Solutions (19) and (26) for the electromagnetic eikonal in the bimetric theory should be compared with a similar solution for the case of propagation of light in general relativity where the gravity and light null cones coincide [16]. The reader can see that the null characteristics of the gravity metric $g_{\alpha\beta}$ enter the gravitationally perturbed part of the eikonal (26) in the bimetric theory in the form of the dot product $l_\alpha r^\alpha$ which is the argument of the logarithm, where r^α is the null distance of the metric $g_{\alpha\beta}$ between the observer and the light ray deflecting body. A remarkable fact is that both l^α and r^α are null vectors of the metric $g_{\alpha\beta}$ describing null hypersurfaces of the gravitational field. Consequently, gravitational light ray deflection experiments in the field of moving bodies are sensitive to, and can measure, the divergence between

the null characteristics of the gravity metric $g_{\alpha\beta}$ and the optical metric $\bar{g}_{\alpha\beta}$ in the case of a nonstationary gravity field in contrast to other relativistic experiments limiting the PPN-preferred frame parameters $\alpha_1, \alpha_2, \alpha_3$ [7]. This allows us to measure the spontaneous violation of the Lorentz invariance of the gravitational field predicted by the vector–tensor theories of gravity that admit existence of a vector field w^α coupled to matter via parameter ϵ parametrizing the difference between the gravity and optical metrics. In conventional type of the gravitational light ray deflection experiments conducted with VLBI [16, 17], one needs the angle Φ (see Fig. 1) to be as small as possible to magnify the Lorentz-invariance violation effects driven by gravity field. Currently, VLBI can measure gravitomagnetic effects of order v/c beyond the static Shapiro effect [18,19]. Further progress in measuring more subtle effects of the bimetric theory of gravity of order v^2/c^2 and higher beyond the static Shapiro time delay can be achieved with laser ranging technique in the experiments like LATOR [10–12] and/or ASTROD [8, 9]. In case of laser ranging between spacecrafts with a gravitating body (Sun) located near the direction of the laser beam, the angle Φ (see Fig. 1) can vary in a large dynamical range so that relativistic effects of the bimetric theory of gravity could be explored with much better precision than in VLBI experiments. We will study this situation for laser ranging experiment in various theories, e.g., in the more general vector–metric theories [20] and in the axion electrodynamics [21].

References

1. Carlip, S. *Clas. Quant. Grav.* **21**, 3803 (2004).
2. Jacobson, T. & Mattingly D. *Phys. Rev.* **D 64**, 024028 (2001).
3. Kostelecky, A. *Phys. Rev.* **D 69**, 105009 (2004).
4. Lämmerzahl, C. *Ann. Phys.* (Leipzig) **14**, 71 (2005).
5. Ni, W.-T. "Implications of Huges-Drever Experiments", In: *Proceedings of the 1983 International School and Symposium on Precision Measurement and Gravity Experiment*, pp. 519–529, ed. W.-T. Ni (National Tsing Hua University, 1983).
6. Ni, W.-T. *Int. J. Mod. Phys.* **D 14**, 901 (2005).
7. Mattingly, D. *Living Rev Relativity* **8**, 5 (2005).
8. Ni, W.-T. *Int. J. Mod. Phys.* **D 14**, 901 (2005).
9. Ni, W.-T., Shiomi, S. & Liao, A.-C. *Clas. Quant. Grav.* **22**, S269 (2004); and references therein.
10. Yu, J.W., Shao, M., Gursel, Y. & Hellings, R. "LATOR: laser astrometric test of relativity", In: *Amplitude and Intensity Spatial Interferometry II*, Proc. SPIE, Vol. 2200, pp. 325–334, ed. J.B. Breckinridge (1994).
11. Turyshev, S.G., Shao, M. & Nordtvedt, K. *Class. Quant. Grav.* **21**, 2773 (2004).
12. Turyshev, S.G., Shao, M. & Nordtvedt, K. "Science, Technology and Mission Design for the Laser Astrometric Test Of Relativity", this volume, page 479.
13. Synge J.L. *Relativity: The General Theory* (North-Holland: Amsterdam 1960).
14. Landau, L.D. & Lifshitz, E.M. *The Classical Theory of Fields* (Oxford: Pergamon) (1971).

15. Zeldovich, Ya.B. & Myskis, A.D. *Elements of Mathematical Phsycics* (Nauka: Moscow) (1973) (in Russian).
16. Kopeikin, S.M. & Schäfer, G. *Phys. Rev.* **D 60** 124002 (1999).
17. Kopeikin, S.M. & Mashhoon, B. *Phys. Rev.* **D 65** 064025 (2002).
18. Fomalont, E.B. & Kopeikin, S. M. *Astrophys. J.* **598** 704 (2003).
19. Kopeikin, S.M. "Gravitomagnetism and the Speed of Gravity", *Int. J. Mod. Phys.* **D 15**, 3005 (2006). [arXiv:gr-qc/0507001]
20. Luo, X., Xie, Y. and Huang, T. "A Metric–Vector Gravitational Theory and the Constraints on Its Parameters" *General Relativity and Gravitation* **37** in press (2006).
21. Cheng, S.L., Geng, C.Q. & Ni, W.-T. *Phys. Rev.* **D 52**, 3132 (1995).