Propagation of Light in the Gravitational Field of Binary Systems to Quadratic Order in Newton's Gravitational Constant

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Summary. The propagation of light is treated in the postlinear gravitational field of binary systems. The light deflection is calculated to quadratic order in Newton's gravitational constant and fourth order in the inverse power of the speed of light. Similarities and dissimilarities of linearized gravity and electrodynamics are discussed. A recent speed-of-gravity controversy is investigated.

1 Introduction

Today, technology has achieved a level at which the extremely high precision of current ground-based radio interferometric observations are approaching an accuracy of 1 μarcsec. Moreover the planned space-based astrometric telescope (GAIA)[1] and the space interferometric mission (SIM) are going to measure the positions and/or the parallaxes of celestial objects with uncertainties in the range 10^{-5}–10^{-6} arcsec. Furthermore the interferometer for the planned laser astrometric test of relativity mission (LATOR) will be able to measure light deflection angles of the order 10^{-8} arcsec.

To reach the desired accuracies of 10^{-6}–10^{-8} arcsec in the computation of light deflection in gravitational fields, corrections arising from the lack of spherical symmetry of the gravitating system, the motion of the gravitating masses, and the relativistic definition of the center of mass must be taken into account.

In this chapter, the light deflection in the postlinear gravitational field of two-bounded point-like masses is treated. However, to gain more insight into the interrelation between the Einstein field equations and the Maxwell equations, in the first part of the paper, the linear gravitational field is treated and its structural similarity with the electromagnetic field is discussed. Emphasis is put on the difference between linearized gravity and electrodynamics.

[1] The name GAIA derives from global astrometric interferometer for astrophysics, since GAIA was originally planned as a space-based interferometer.

In the second part of the paper, but still within the context of linearized gravity, a recent well-known speed-of-gravity controversy is discussed using a clear-cut approach for clarification. Finally, in the third part of the paper, in going over to the postlinear gravitational field, the light deflection is treated.

Notation

Let us summarize the notation and symbols used in this paper:

1. G is the Newtonian constant of gravitation
2. c is the speed of light
3. in Sect. 3, by c_{g} we denote the speed of gravity
4. The Greek indices α, β, γ, etc. are space–time indices and run from 0 to 3
5. The Latin indices i, j, k, etc. are spatial indices and run from 1 to 3
6. $\eta_{\mu\nu} = \eta^{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ is the Minkowski metric
7. $g_{\mu\nu}$ is a metric tensor of curved, four-dimensional space–time, depending on spatial coordinates and time
8. We suppose that space–time is covered by a harmonic coordinate system $(x^\mu) = (x^0, x^i)$, where $x^0 = ct$, t being the time coordinate
9. The three-dimensional quantities (3-vectors) are denoted by $\boldsymbol{a} = a^i$
10. The three-dimensional unit vector in the direction of $\boldsymbol{a}$ is denoted by $\boldsymbol{e}_a = e_a^i$
11. The Latin indices are lowered and raised by means of the unit matrix $\delta_{ij} = \delta^{ij} = \mathrm{diag}(1, 1, 1)$
12. By $_{,\sigma} = \partial_\sigma$ we denote the partial derivative with respect to the coordinate x^σ
13. The scalar product of any two 3-vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ with respect to the Euclidean metric δ_{ij} is denoted by $\boldsymbol{a} \cdot \boldsymbol{b}$ and can be computed as $\boldsymbol{a} \cdot \boldsymbol{b} = \delta_{ij} a^i b^j = a^i b^i$
14. The Euclidean norm of a 3-vector $\boldsymbol{a}$ is denoted by $a = |\boldsymbol{a}|$ and can be computed as $a = (\delta_{mn} a^m a^n)^{1/2}$
15. By $\boldsymbol{l}_{(0)}$ we denote the vector tangent to the unperturbed light ray and the unit vector $\boldsymbol{e}_{(0)}$ is defined by $\boldsymbol{e}_{(0)} = \boldsymbol{l}_{(0)}/|\boldsymbol{l}_{(0)}|$
16. ∇ denotes the vector operator $\boldsymbol{e}_x \partial_x + \boldsymbol{e}_y \partial_y + \boldsymbol{e}_z \partial_z$
17. Δ denotes the usual Laplace operator in flat space
18. By $\Box \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu = -\partial_0^2 + \Delta$ we denote the flat d'Alembertian operator

2 Analogies Between Electrodynamics and Einsteinian Gravity

In linearized approximation, the complicated Einstein theory with the group of general coordinate transformations as symmetry group simplifies to an abelian gauge theory. Electrodynamics is an abelian gauge theory too, if also with a single group parameter in contrast to linearized gravity theory which has four group parameters, so there are analogies between both theories to be expected.

2.1 Gauge-Invariant Electrodynamics

In vacuum space–time, the Maxwell equations have the form (Gaussian units)

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \qquad \boldsymbol{\nabla} \times \boldsymbol{E} + \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{B} = 0, \tag{1}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi\varrho, \qquad \boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{E} = \frac{4\pi}{c}\boldsymbol{j}. \tag{2}$$

Hereof the conservation equation for the charge follows:

$$\frac{\partial}{\partial t}\varrho + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0. \tag{3}$$

In covariant form, the Maxwell equations read,

$$\partial_\nu F^{\mu\nu} = \frac{4\pi}{c} j^\mu, \qquad \partial_\sigma F_{\mu\nu} + \partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} = 0, \tag{4}$$

and the conservation equation takes the form

$$\partial_\mu j^\mu = 0. \tag{5}$$

Here, the definitions hold,

$$F_{\mu\nu} = (\boldsymbol{E}, \boldsymbol{B}), \qquad j^\mu = (c\varrho, \boldsymbol{j}), \qquad F_{\mu\nu} = -F_{\nu\mu}. \tag{6}$$

The Lorentz force and power expressions are, respectively,

$$\boldsymbol{k} = \varrho\boldsymbol{E} + \frac{1}{c}\boldsymbol{j} \times \boldsymbol{B}, \qquad \boldsymbol{k} \cdot \boldsymbol{v} = \boldsymbol{E} \cdot \boldsymbol{j}, \tag{7}$$

where $\boldsymbol{j} = \varrho\boldsymbol{v}$. In covariant notation, the four-dimensional force density reads

$$k_\mu = \frac{1}{c} F_{\mu\nu} j^\nu = \left(-\frac{1}{c}\boldsymbol{E} \cdot \boldsymbol{j}, \boldsymbol{k}\right). \tag{8}$$

All the given expressions in this section have physical meaning, locally.

2.2 Electrodynamics in Gauge-Field Form

Introducing the gauge-field A_μ according to

$$\boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}, \qquad \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \tag{9}$$

or, in four-dimensional form,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad A_\mu = (-\phi, \boldsymbol{A}), \tag{10}$$

the field equations (1) and (2) transform into the equations

$$-\partial^\nu \partial_\nu A^\mu + \partial^\mu \partial_\nu A^\nu = \frac{4\pi}{c} j^\mu. \tag{11}$$

A gauge transformation is given by,

$$A'_\mu = A_\mu + \partial_\mu \Lambda, \qquad F'_{\mu\nu} = F_{\mu\nu}. \tag{12}$$

It includes one arbitrary function Λ. The Lorentz gauge condition is defined by

$$\frac{1}{c}\frac{\partial}{\partial t}\phi + \nabla \cdot \boldsymbol{A} = \partial_\mu A^\mu = 0. \tag{13}$$

Herewith, the field equations (11) result in

$$\partial^\nu \partial_\nu A^\mu = -\frac{4\pi}{c} j^\mu. \tag{14}$$

In gauge-field form, the Maxwell equations were put onto a footing which is close in form to the Einstein field equations in linearized approximation.

2.3 The Linearized Einstein Theory

In linearized approximation, applying the harmonic or Hilbert–Lorentz gauge condition, the Einstein field equations read, e.g., see [1]

$$\partial^\lambda \partial_\lambda \bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4} T^{\mu\nu}, \tag{15}$$

where the harmonic coordinate condition reads

$$\partial_\mu \bar{h}^{\mu\nu} = 0. \tag{16}$$

The field equations (15) together with the harmonic coordinate condition (16) imply the conservation law for the matter stress–energy tensor

$$\partial_\mu T^{\mu\nu} = 0. \tag{17}$$

The barred field $\bar{h}^{\mu\nu}$ is connected with the metric tensor $g_{\mu\nu}$ as follows

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}\bar{h}_{\alpha\beta}, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{18}$$

where raising and lowering of indices are with the Minkowski metric.
Introducing the notations, cf. [2]

$$T^{00} = \varrho c^2, \qquad T^{0i} = c j^i, \tag{19}$$

$$\bar{h}^{00} = 4\varphi/c^2, \qquad \bar{h}^{0i} = 4a^i/c^2, \qquad \bar{h}^{ij} = O(1/c^4), \tag{20}$$

and

$$\boldsymbol{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{a}, \qquad \boldsymbol{B} = \nabla \times \boldsymbol{a}, \tag{21}$$

the field equations (15) take the form,

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \qquad \boldsymbol{\nabla} \times \boldsymbol{E} + \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{B} = 0, \tag{22}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi\varrho, \qquad \boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{E} = \frac{4\pi}{c}\boldsymbol{j}. \tag{23}$$

Hereof the time component of the conservation equation (17) follows

$$\frac{\partial}{\partial t}\varrho + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0, \qquad (\partial_\nu T^{0\nu} = 0). \tag{24}$$

Obviously, whereas from the field equations (15), together with the gauge condition (16), four conservation equations follow, namely (17), from the field equations (22) and (23), only one conservation equation results (24).

The force and power expressions have to be added to (22) and (23) from outside because $(c\varrho, j^i)$ is treated therein as 4-vector and not as components of a tensor

$$\boldsymbol{k} = -\left(\varrho\boldsymbol{E} + \frac{4}{c}\boldsymbol{j} \times \boldsymbol{B}\right), \qquad \boldsymbol{k} \cdot \boldsymbol{v} = -\boldsymbol{E} \cdot \boldsymbol{j}, \tag{25}$$

where $\boldsymbol{j} = \varrho\boldsymbol{v}$ and where a point-mass model has been assumed for the matter.

For point masses, some analogy between electrodynamics and the linearized Einstein theory has been achieved apart from a minus sign and a factor of 4. The first difference relates to the attraction of gravity for all masses and the second one to the tensorial structure of gravity. However, there is a much bigger difference present which also relates to the treatment of $(c\varrho, j^i)$ as 4-vector. The electromagnetic field equations (4) are gauge invariant against the transformation

$$A'^0 = A^0 - \partial_0 \Lambda, \tag{26}$$

$$A'^i = A^i + \partial_i \Lambda. \tag{27}$$

The linearized Einstein field equations in electrodynamic form, (22) and (23), however, are not invariant against the gauge transformations of linearized gravity which are given by, containing four arbitrary functions ϵ^μ,

$$\bar{h}'^{00} = \bar{h}^{00} - \partial_0\epsilon^0 + \partial_j\epsilon^j, \tag{28}$$

$$\bar{h}'^{0i} = \bar{h}^{0i} + \partial_i\epsilon^0 - \partial_0\epsilon^i, \tag{29}$$

$$\bar{h}'^{ij} = \bar{h}^{ij} + \partial_i\epsilon^j + \partial_j\epsilon^i - \delta_{ij}\partial_\mu\epsilon^\mu. \tag{30}$$

Only in the case of vanishing ϵ^i, the above field equations (22) and (23) are invariant. This means that the linearized Einstein field equations in the electrodynamic form, (22) and (23), have no physical meaning, locally, in contrast to the Maxwell equations (1) and (2).

2.4 The Linearized Einstein Theory in Gauge-Invariant Form

A locally gauge-invariant representation of the linearized Einstein theory can be achieved with the aid of the Riemann curvature tensor

$$R_{\mu\nu\sigma\tau} = \frac{1}{2}(\partial_\nu\partial_\sigma h_{\mu\tau} + \partial_\mu\partial_\tau h_{\nu\sigma} - \partial_\nu\partial_\tau h_{\mu\sigma} - \partial_\mu\partial_\sigma h_{\nu\tau}) \tag{31}$$

which is an invariant object under the gauge transformations (28)–(30)

$$R'_{\mu\nu\sigma\tau} = R_{\mu\nu\sigma\tau}. \tag{32}$$

Calling, respectively, e.g., see [3],

$$E_{ij} = R_{i0j0}, \qquad H_{ij} = \frac{1}{2}\epsilon_{ikl}R_{klj0}, \tag{33}$$

the electric and magnetic components of the curvature tensor, all its components can be recovered in the form:

$$R_{i0j0} = E_{ij}, \qquad R_{ijk0} = \epsilon_{ijl}H_{lk}, \tag{34}$$

$$R_{ijkl} = \epsilon_{ijm}\epsilon_{kln}\left(-E_{mn} + \frac{1}{2}J_{mn}\right), \tag{35}$$

$$J_{ij} = \frac{8\pi G}{c^4}\left(-T_{ij} + \frac{1}{2}\delta_{ij}(T_{00} + T_{kk})\right). \tag{36}$$

The fully gauge-invariant field equations for linearized Einstein theory read

$$\nabla\cdot\mathbf{H} = 0, \qquad \nabla\times\mathbf{E} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{H} = 0, \tag{37}$$

$$\nabla\cdot\mathbf{E} = \nabla\cdot\mathbf{J}, \qquad \nabla\times\mathbf{H}^T - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{J}, \tag{38}$$

where $\mathbf{H}^T$ denotes the transposed of the dyadic $\mathbf{H}$; $\mathbf{E}^T = \mathbf{E}$, $\mathbf{J}^T = \mathbf{J}$. (Notice the similarity of the inhomogeneous equations with the macroscopic Maxwell equations with polarization, i.e., dipole sources.) These equations do have local meaning as the expression

$$K_i = -\left(c^2E_{ij}X^j + 2c\epsilon_{ikl}V^kH_{lj}X^j\right) = -\left(E_i + \frac{2}{c}(\mathbf{V}\times\mathbf{H})_i\right) \tag{39}$$

does which describes the tidal force on two particles with unit mass, separated by the vector X^i ($V^i = \frac{dX^i}{dt}$), where

$$E_i = c^2E_{ij}X^j, \qquad H_i = c^2H_{ij}X^j. \tag{40}$$

The second-order field equations for components of the Riemann tensor read

$$\Box E_{ij} = \frac{8\pi G}{c^4}\left[\partial_0^2(T_{ij} - \frac{1}{2}\delta_{ij}T) + \partial_i\partial_j(T_{00} + \frac{1}{2}T) - \partial_0(\partial_iT_{j0} + \partial_jT_{i0})\right], \tag{41}$$

$$\Box H_{ij} = \frac{8\pi G}{c^4}\epsilon_{ilk}\left[\partial_0\partial_k(T_{jl} - \frac{1}{2}\delta_{jl}T) - \partial_k\partial_j T_{l0}\right]. \tag{42}$$

Under stationarity conditions, the field equations become,

$$\Delta E_{ij} = \frac{4\pi G}{c^4}\partial_i\partial_j(T_{00} + T_{kk}), \tag{43}$$

$$\Delta H_{ij} = \frac{8\pi G}{c^4}\epsilon_{ilk}\partial_l\partial_j T_{0k}. \tag{44}$$

In the Newtonian limit, the well-known tidal-force potential results,

$$E_{ij} = -\frac{1}{c^2}\partial_i\partial_j\varphi, \qquad \varphi = G\int d^3x'\,\frac{\varrho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}. \tag{45}$$

3 On the Speed-of-Gravity Controversy

Recently it has been claimed that the speed of gravity should be measurable by radio observations of a bright radio quasar J0842+1835, during the time of its line-of-sight close angular encounter with Jupiter by very long baseline interferometer (VLBI), predicted to occur on 8 September 2002 [4]. The theoretical basis of above erroneous conclusion rests upon interpreting relativistic corrections to the famous Shapiro delay. The first criticism of [4], raised by Asada, points out that the excess time delay is a light-cone effect only, hence should not involve the speed of gravity [5]. Recently, inaugurated by a new paper which supports Kopeikin's earlier interpretation of "the higher-order Shapiro delay" [6], a strong criticism was raised by Will, who presented a detailed calculation for the relativistic corrections to the Shapiro delay in the parametrized post-Newtonian framework and showed that the above-mentioned VLBI measurements are insensitive to the speed of propagation of gravity [7]. In the final version of his recent publication, Kopeikin strongly criticized the conclusions reached by Asada and Will by pointing out unsatisfactory aspects, both conceptual and calculational, of their treatments [6]. In this article, we will provide a firm mathematical footing to the analysis of Asada and Will, and convincingly show that the speed of gravity is not sensitive to the measurements of radio waves, emitted by the quasar J0842+1835 and deflected by the moving Jupiter, performed by VLBI. We shall also point out the conceptual error committed by Kopeikin which allowed him to interpret erroneously the above-mentioned VLBI observations [8].

3.1 The Approach by C.M. Will

Though the final expression for the relativistic time delay presented by Asada is consistent with that obtained by Kopeikin (compare (10) and (12) in [5] and [4], respectively), he pointed out that Asada's derivation assumed that the position of Jupiter be fixed at retarded light-cone time, which makes his derivation somewhat ad hoc. Kopeikin also raised few concerns over the higher-order time-delay computations in [7], especially the way time-delay integral was evaluated (refer Sect. B in [6]). Below, we will present an elegant integration of the relativistic time-delay equation, which is free of blemishes associated with Will's treatment, as indicated by Kopeikin. This will help us to justify mathematically Asada's result too.

We start with the time-delay equation, (16) of [7], but dropping the summation symbol there,

$$\Delta(t_r, t_e) = (1+\gamma)\frac{Gm_a}{c^2}\int_{t_e}^{t_r}\frac{(1-(2+\zeta)\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_a(s_a)/c)dt}{|\boldsymbol{z}-\boldsymbol{x}_a(s_a)|-\boldsymbol{v}_a(s_a)\cdot(\boldsymbol{z}-\boldsymbol{x}_a(s_a))/c_g}, \quad (46)$$

where $\boldsymbol{x}_a$, $\boldsymbol{v}_a$, and m_a are the position vector, the velocity vector, and the mass of the gravitational source, respectively. t_e and t_r denote the light ray (photon) emission and reception instances. The Newtonian gravitational constant and the speed of gravity are denoted by G and c_g. The constant unit vector along the incoming light ray $\boldsymbol{e}_{(0)}$ helps us to define the unperturbed photon trajectory as

$$\boldsymbol{z} \equiv \boldsymbol{z}(t) = \boldsymbol{e}_{(0)}c(t-t_e) + \boldsymbol{z}_e, \quad (47)$$

and the retarded time s_a is given by $s_a = t - |\boldsymbol{z}-\boldsymbol{x}_a(s_a)|/c_g$. The underlying reference frame is an inertial frame where as well the observer as the source of the light ray is treated to be at rest. The time-delay expression in the Einstein theory results from putting $\gamma = 1, \zeta = 0$, and $c_g = c$. The advantage of working within a well-posed generalized framework is the natural difference therein between the speed of gravity and the speed of light.

It may already be noted here that terms of the type $\boldsymbol{e}_{(0)}\cdot\mathbf{v}_a(s_a)/c$ in the numerator of (46) can be neglected for the interpretation of the data from the Jupiter VLBI experiment. Only the denominator in (46) is relevant. Throughout the rest of the chapter, we will assume that the source of the gravitational field is uniformly moving, making $\boldsymbol{v}_a$ a constant.

Following techniques used in the computation of electric and magnetic fields, using Liénard–Wiechert potentials (see [10], Sect. 63), we write the denominator in the integrand of (46) as

$$|\boldsymbol{z}-\boldsymbol{x}_a(s_a)| - \boldsymbol{v}_a\cdot\frac{\boldsymbol{z}-\boldsymbol{x}_a(s_a)}{c_g} = |\boldsymbol{z}-\boldsymbol{x}_a(t)|\left(1-\left(\frac{v_a}{c_g}\right)^2\sin^2\theta_t\right)^{1/2}, \quad (48)$$

where θ_t is the angle between $\boldsymbol{z}-\boldsymbol{x}_a(t)$ and $\boldsymbol{v}_a$. To elegantly integrate (46), we introduce the following expression, where the retardation is with respect to the speed of light c,

$$|\boldsymbol{z}-\boldsymbol{x}_a(u_a)| - \boldsymbol{v}_a\cdot\frac{\boldsymbol{z}-\boldsymbol{x}_a(u_a)}{c} = |\boldsymbol{z}-\boldsymbol{x}_a(t)|\left(1-\left(\frac{v_a}{c}\right)^2\sin^2\theta_t\right)^{1/2} \quad (49)$$

where $u_a = t - |\boldsymbol{z} - \boldsymbol{x}_a(u_a)|/c$. Using the above expression, we may write (48) as

$$|\boldsymbol{z} - \boldsymbol{x}_a(s_a)| - \boldsymbol{v}_a \cdot \frac{\boldsymbol{z} - \boldsymbol{x}_a(s_a)}{c_g}$$
$$= \left[|\boldsymbol{z} - \boldsymbol{x}_a(u_a)| - \boldsymbol{v}_a \cdot \frac{\boldsymbol{z} - \boldsymbol{x}_a(u_a)}{c} \right] \left(\frac{1 - (v_a/c_g)^2 \sin^2 \theta_t}{1 - (v_a/c)^2 \sin^2 \theta_t} \right)^{1/2} . \tag{50}$$

Restricting right-hand side of above equation to $O(v_a^2)$ and plugging it in (46) we obtain,

$$\Delta(t_r, t_e) = (1+\gamma) \frac{G m_a}{c^2} \int_{t_e}^{t_r} \frac{(1 - (2+\zeta)\boldsymbol{e}_{(0)} \cdot \mathbf{v}_a/c) dt}{|\boldsymbol{z} - \boldsymbol{x}_a(u_a)| - \boldsymbol{v}_a \cdot (\boldsymbol{z} - \boldsymbol{x}_a(u_a))/c} . \tag{51}$$

Using (25), (28), (45), and (50) in [9], which are quite the merit equations of that paper, it is straightforward to obtain, without any further approximation, the relativistic time-delay expression in the following form

$$\Delta(t_r, t_e) = -(1+\gamma) \frac{G m_a}{c^3} \left(1 - (1+\zeta) \boldsymbol{e}_{(0)} \cdot \frac{\boldsymbol{v}_a}{c} \right) \ln \frac{r_{ra}(u_r) - \boldsymbol{e}_{(0)} \cdot \boldsymbol{r}_{ra}(u_r)}{r_{ea}(u_e) - \boldsymbol{e}_{(0)} \cdot \boldsymbol{r}_{ea}(u_e)} , \tag{52}$$

where the retarded times u_r and u_e are given by

$$u_i = t_i - \frac{r_{ia}(u_i)}{c} , \qquad i = r, e \tag{53}$$

and $\boldsymbol{r}_{ia}(u_i) = \boldsymbol{z}_i - \boldsymbol{x}_a(u_i)$ with $r_{ia}(u_i) = |\boldsymbol{r}_{ia}(u_i)|$.

It is clear that above equation is not very useful, as it involves unknown constants like t_e, the instant of time when the photon was ejected and $\boldsymbol{z}_e$, a vector associated with its origin. To eliminate t_e and $\boldsymbol{z}_e$, we introduce a second observer and let t_1 and t_2 be the reception times at these receivers for a photon characterized by t_e and $\boldsymbol{z}_e$. The relativistic time delay, given by (52), becomes

$$\Delta(t_2, t_1) = -(1+\gamma) \frac{G m_J}{c^3} \left(1 - (1+\zeta) \boldsymbol{e}_{(0)} \cdot \frac{\boldsymbol{v}_J}{c} \right) \ln \frac{r_{2J}(u_2) - \boldsymbol{e}_{(0)} \cdot \boldsymbol{r}_{2J}(u_2)}{r_{1J}(u_1) - \boldsymbol{e}_{(0)} \cdot \boldsymbol{r}_{1J}(u_1)} , \tag{54}$$

where the index J stands for the Jupiter. The new retarded instances are

$$u_i = t_i - \frac{r_{iJ}(u_i)}{c} , \qquad i = 1, 2 \tag{55}$$

along with

$$\boldsymbol{r}_{iJ}(u_i) = \boldsymbol{z}_i - \boldsymbol{x}_J(u_i), \quad r_{iJ} = |\boldsymbol{r}_{iJ}(u_i)|, \qquad i = 1, 2. \tag{56}$$

It is clear that in the expression for the relativistic time delay, given by (54), the speed of gravity plays absolutely no role. The expression, which should be used to interpret astronomical observations like that made by VLBI on the fall of 2002, may be obtained by simply replacing $(1-(1+\zeta)\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_J/c)$ by 1 in (54). This is so as the effects associated with the g_{0i}-component of the gravitational field (see [7]), the so-called *gravitomagnetic field*, may be neglected during such observations. The final expression for the relativistic Shapiro delay reads

$$\Delta(t_2, t_1) = -(1+\gamma)\frac{Gm_J}{c^3}\ln\frac{r_{2J}(u_2) - \boldsymbol{e}_{(0)}\cdot\boldsymbol{r}_{2J}(u_2)}{r_{1J}(u_1) - \boldsymbol{e}_{(0)}\cdot\boldsymbol{r}_{1J}(u_1)}. \qquad (57)$$

This proves the correctness of the ansatz used in [5], when $\gamma = 1$. The above equation also agrees with (34) and (35) of [7]. We feel that it is important to stress again what (54) or (57) really implies. They demonstrate that, whenever measurements of the gravitational time delay for electromagnetic radiation passing by a moving massive object, similar to the VLBI observations of 8 September 2002, are interpreted, the only field velocity that enters the analysis is that of the light.

3.2 The Treatment by S.M. Kopeikin

In this section, we closely scrutinize Kopeikin's arguments to see how he reached his erroneous conclusion that the above-mentioned VLBI observations measure the speed of gravity. The time-delay equation employed by Kopeikin reads

$$\Delta(t_r, t_e) = \frac{2Gm_a}{c^2}\int_{t_e}^{t_r}\frac{\left(1 - 2\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_a(s_a)/c_g\right)dt}{|\boldsymbol{z}-\boldsymbol{x}_a(s_a)| - \boldsymbol{v}_a(s_a)\cdot(\boldsymbol{z}-\boldsymbol{x}_a(s_a))/c_g}, \qquad (58)$$

where

$$s_a = \tau - \frac{|\boldsymbol{z}-\boldsymbol{x}_a(s_a)|}{c_g} \quad \text{with} \quad \tau \equiv \frac{ct}{c_g}. \qquad (59)$$

The velocities $\boldsymbol{v}_a(s_a)$ are also defined with respect to the new time variable τ. However, for the light propagation he still uses (47), which is

$$\boldsymbol{z} \equiv \boldsymbol{z}(t) = \boldsymbol{e}_{(0)}c(t-t_e) + \boldsymbol{z}_e, \qquad (60)$$

Note that (58) is quite similar in form to (46), we employed in Sect. 3.1. We integrate (58) in the same manner as the time-delay integral was performed in Sect. 3.1. The final result, expressed in terms of τ, reads

$$\Delta(\tau_r, \tau_e) = -\frac{2Gm_a}{c^3}\left(1 - \boldsymbol{e}_{(0)}\cdot\frac{\boldsymbol{v}_a}{c_g}\right)\ln\frac{r_{ra}(s_r) - \boldsymbol{e}_{(0)}\cdot\boldsymbol{r}_{ra}(s_r)}{r_{ea}(s_e) - \boldsymbol{e}_{(0)}\cdot\boldsymbol{r}_{ea}(s_e)}, \qquad (61)$$

where the retarded times s_e and s_r, associated with the positions of emission and reception of the photon, are

$$s_i = \tau_i - \frac{r_{ia}(s_i)}{c} \,, \qquad i = e, r \tag{62}$$

In above equation, $r_{ea}(s_e)$ and $r_{ra}(s_r)$ are given by

$$r_{ia}(s_i) = |\boldsymbol{z}_i - \boldsymbol{x}_a(s_i)| \,, \quad \boldsymbol{r}_{ia}(s_i) = \boldsymbol{z}_i - \boldsymbol{x}_a(s_i) \,, \quad i = e, r. \tag{63}$$

Introducing a second observer and following exactly what have been done after (52) to get (57), we obtain, for $\gamma = 1$,

$$\Delta(\tau_2, \tau_1) = -\frac{2Gm_J}{c^3} \ln \frac{r_{2J}(s_2) - \boldsymbol{e}_{(0)} \cdot \boldsymbol{r}_{2J}(s_2)}{r_{1J}(s_1) - \boldsymbol{e}_{(0)} \cdot \boldsymbol{r}_{1J}(s_1)}, \tag{64}$$

where τ_1 and τ_2 are the fiducial reception times for the deflected photon at the positions of two VLBI observers.

Since the time t, associated with the photon propagation, is related to the fiducial time τ by $t = (c_g/c)\,\tau$, we are free to introduce another retardation $u = (c_g/c)\,s$. This indicates that we have the freedom to replace Kopeikin's arbitrarily defined retardations s_1 and s_2 with u_1 and u_2, where

$$u_a = t - \frac{|\boldsymbol{z} - \boldsymbol{x}_a(u_a)|}{c}, \quad a = 1, 2. \tag{65}$$

In terms of u_1 and u_2, (64) completely agrees with (57) when $\gamma = 1$. We emphasize that it is the time t, associated with the propagation of light, that is involved in the true measurements of velocities and hence to be used in the interpretation of astronomical observations. Kopeikin, however, used fiducial τ to interpret the VLBI observations of 8 September 2002.

4 Light Deflection in the Gravitational Field of a Compact Binary System

In this section, we shall recapitulate the computations of the angle of light deflection in the gravitational field of a compact binary system in the linear and postlinear approximations, which were presented in [9,11]. Both the light source and the observer are assumed to be located at infinity in an asymptotically flat space. The equations of light propagation are explicitly integrated to the second order in G/c^2. We assume that the impact parameter $|\boldsymbol{\xi}|$ is much larger (five times or more) than the distance r_{12} between the two components of the binary system.

4.1 Light Propagation and Light Deflection in the Gravitational Field of Compact Binary System

Since the light ray is propagating in a weak gravitational field, we can assume that the light propagation is very well governed by the laws of geometric optics, whereby light rays (photons) move in curved space–time along null geodesics. The equations of null geodesics with the time coordinate as parameter are given by (e.g., see [12])

$$\frac{dl^i}{dt} + \Gamma^i_{\alpha\beta} l^\alpha l^\beta = c^{-1} \Gamma^0_{\nu\sigma} l^\nu l^\sigma l^i, \tag{66}$$

where $\Gamma^\mu_{\rho\sigma}$ are the Christoffel symbols of the second kind and $l^\mu = \frac{dz^\mu}{dt}$ denotes the 4-vector $l^\mu = (c, l^i)$. Here, it is important to point out that l^μ is not exactly a 4-vector since we differentiate with respect to the time coordinate t. So l^μ is a 4-vector up to a factor. The spatial part of l^μ given by $l^i = dz^i/dt$ is the 3-vector tangent to the light ray $z^i(t)$. In the present case of null geodesics, l^μ has to fulfill the condition

$$l^2 \equiv g_{\mu\nu}[z^0, z^i(t), G] l^\mu l^\nu = 0. \tag{67}$$

Now we consider a light ray $z^i(t)$ that is propagating in a curved space–time $g_{\mu\nu}[z^0, z^i(t), G]$. If the gravitational field is weak, we can write the fundamental metric tensor as a power series in the gravitational constant G

$$g_{\mu\nu}[z^0, z^i(t), G] \equiv \eta_{\mu\nu} + \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}[z^0, z^i(t), G], \tag{68}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h^{(n)}_{\mu\nu}[z^0, z^i(t), G]$ is a perturbation of the order n in the gravitational constant G (physically, this means an expansion in the dimensionless parameter $Gm/c^2 d$ which is very small, d being the characteristic length of the problem and m a characteristic mass).

To obtain from (66) the equations of light propagation for the metric given in (68), we substitute the Christoffel symbols into (66). To save writing we denote the metric coefficients $h^{(1)}_{pq}[z^0, z^i(t), G]$ and $h^{(2)}_{pq}[z^0, z^i(t), G]$ by $h^{(1)}_{pq}$ and $h^{(2)}_{pq}$. Then the resulting equation of light propagation to the second order in G/c^2 is

$$\begin{aligned}
\frac{dl^i}{dt} = & \frac{1}{2} c^2 h^{(1)}_{00,i} - c^2 h^{(1)}_{0i,0} - c\, h^{(1)}_{0i,m} l^m + c\, h^{(1)}_{0m,i} l^m - c\, h^{(1)}_{mi,0} l^m \\
& - h^{(1)}_{mi,n} l^m l^n + \frac{1}{2} h^{(1)}_{mn,i} l^m l^n - \frac{1}{2} c\, h^{(1)}_{00,0} l^i - h^{(1)}_{00,k} l^k l^i \\
& + \left(\frac{1}{2} c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m} \right) l^m l^p l^i + \frac{1}{2} c^2 h^{(2)}_{00,i} - \frac{1}{2} c^2 h^{(1)ik} h^{(1)}_{00,k} \\
& - h^{(2)}_{00,k} l^k l^i - \left(h^{(2)}_{mi,n} - \frac{1}{2} h^{(2)}_{mn,i} \right) l^m l^n + h^{(1)ik} \left(h^{(1)}_{mk,n} - \frac{1}{2} h^{(1)}_{mn,k} \right) l^m l^n \\
& - h^{(1)}_{00} h^{(1)}_{00,k} l^k l^i,
\end{aligned} \tag{69}$$

where by $,0$ and $,i$ we denote $\partial/\partial z^0$ and $\partial/\partial z^i$, respectively. To calculate the light deflection we need to solve (69) for l^i. To solve this complicated nonlinear differential equation, we turn to approximation techniques.

The Approximation Scheme

We can write the 3-vector $l^i(t)$ as

$$l^i(t) = l^i_{(0)} + \sum_{n=1}^{\infty} \delta l^i_{(n)}(t), \tag{70}$$

where $l^i_{(0)}$ denotes the constant incoming tangent vector $l^i(-\infty)$ and $\delta l^i_{(n)}(t)$ the perturbation of the constant tangent vector $l^i_{(0)}$ of order n in G. After introducing the expression for $l^i(t)$ given by (70) into (69), we obtain differential equations for the perturbations $\delta l^i_{(1)}$ and $\delta l^i_{(2)}$

$$\begin{aligned}\frac{d\delta l^i_{(1)}}{dt} &= \frac{1}{2}c^2 h^{(1)}_{00,i} - c^2 h^{(1)}_{0i,0} - c\, h^{(1)}_{0i,m} l^m_{(0)} + c\, h^{(1)}_{0m,i} l^m_{(0)} - c\, h^{(1)}_{mi,0} l^m_{(0)} \\ &\quad - h^{(1)}_{mi,n} l^m_{(0)} l^n_{(0)} + \frac{1}{2} h^{(1)}_{mn,i} l^m_{(0)} l^n_{(0)} - \frac{1}{2} c\, h^{(1)}_{00,0} l^i_{(0)} - h^{(1)}_{00,k} l^k_{(0)} l^i_{(0)} \\ &\quad + \left(\frac{1}{2} c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m}\right) l^m_{(0)} l^p_{(0)} l^i_{(0)}\end{aligned} \tag{71}$$

and

$$\begin{aligned}\frac{d\delta l^i_{(2)}}{dt} &= \frac{1}{2}c^2 h^{(2)}_{00,i} - \frac{1}{2}c^2 h^{(1)ik} h^{(1)}_{00,k} - h^{(2)}_{00,k} l^k_{(0)} l^i_{(0)} - \left(h^{(2)}_{mi,n} - \frac{1}{2} h^{(2)}_{mn,i}\right) l^m_{(0)} l^n_{(0)} \\ &\quad + h^{(1)ik}\left(h^{(1)}_{mk,n} - \frac{1}{2} h^{(1)}_{mn,k}\right) l^m_{(0)} l^n_{(0)} - h^{(1)}_{00} h^{(1)}_{00,k} l^k_{(0)} l^i_{(0)} \\ &\quad - c\, h^{(1)}_{0i,m} \delta l^m_{(1)} + c\, h^{(1)}_{0m,i} \delta l^m_{(1)} - c\, h^{(1)}_{mi,0} \delta l^m_{(1)} \\ &\quad - h^{(1)}_{mi,n} \delta l^m_{(1)} l^n_{(0)} - h^{(1)}_{mi,n} l^m_{(0)} \delta l^n_{(1)} + h^{(1)}_{mn,i} \delta l^m_{(1)} l^n_{(0)} \\ &\quad - \frac{1}{2} c h^{(1)}_{00,0} \delta l^i_{(1)} - h^{(1)}_{00,k} \delta l^k_{(1)} l^i_{(0)} - h^{(1)}_{00,k} l^k_{(0)} \delta l^i_{(1)} \\ &\quad + c^{-1} h^{(1)}_{mp,0} \delta l^m_{(1)} l^p_{(0)} l^i_{(0)} - c^{-1} h^{(1)}_{0p,m} \delta l^m_{(1)} l^p_{(0)} l^i_{(0)} - c^{-1} h^{(1)}_{0p,m} l^m_{(0)} \delta l^p_{(1)} l^i_{(0)} \\ &\quad + \left(\frac{1}{2} c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m}\right) l^m_{(0)} l^p_{(0)} \delta l^i_{(1)}.\end{aligned} \tag{72}$$

To calculate the perturbations $\delta l^i_{(1)}(t)$ and $\delta l^i_{(2)}(t)$, we have to integrate (71) and (72) along the light ray trajectory to the appropriate order.

Angle of Light Deflection

The dimensionless vector $\alpha^i_{(n)}$ of order n in G, describing the angle of total deflection of the light ray measured at the point of observation and computed with respect to the vector $l^i_{(0)}$, is given by

$$\alpha^i_{(n)}(t) = P^i_q \frac{\delta l^q_{(n)}(t)}{|\boldsymbol{l}_{(0)}|}, \tag{73}$$

where $\delta l^i_{(n)}$ is the perturbation of the constant tangent vector of order n in G. Here,

$$P^i_q = \delta^i_q - e^i_{(0)} e_{(0)q}, \tag{74}$$

with $e^i_{(0)} = l^i_{(0)}/|\boldsymbol{l}_{(0)}|$, is the projection tensor onto the plane orthogonal to the vector $l^i_{(0)}$. In the case of light rays (photons) $|\boldsymbol{l}_{(0)}| = c$.

4.2 The Gravitational Field of a Compact Binary in the Linear Approximation

In the linear approximation (68) reduces to

$$g_{\mu\nu}(t,\boldsymbol{x}) = \eta_{\mu\nu} + h^{(1)}_{\mu\nu}(t,\boldsymbol{x}). \tag{75}$$

The metric perturbation $h^{(1)}_{\mu\nu}(t,\boldsymbol{x})$ can be found by solving the Einstein field equations which in the linear approximation and in the harmonic gauge (see [12]) are given by

$$\Box h^{(1)}_{\mu\nu}(t,\boldsymbol{x}) = -16\pi \frac{G}{c^4} S_{\mu\nu}(t,\boldsymbol{x}), \tag{76}$$

where

$$S_{\mu\nu}(t,\boldsymbol{x}) = T_{\mu\nu}(t,\boldsymbol{x}) - \tfrac{1}{2}\eta_{\mu\nu} T^\lambda{}_\lambda(t,\boldsymbol{x}). \tag{77}$$

As is well known, the solution of these equations has the form of a Liénard–Wiechert potential (e.g., see [13]).

For a binary system the matter stress–energy tensor reads

$$T^{\mu\nu}(t,\boldsymbol{x}) = \sum_{a=1}^{2} \mu_a(t) v^\mu_a v^\nu_a \delta(\boldsymbol{x} - \boldsymbol{x}_a), \tag{78}$$

where the trajectory of the mass m_a (in harmonic coordinates) is denoted by $\boldsymbol{x}_a(t)$; the coordinate velocity is $\boldsymbol{v}_a(t) = d\boldsymbol{x}_a(t)/dt$ and $v^\mu \equiv (c, \boldsymbol{v}_a)$; $\mu_a(t)$ represents a time-dependent mass of the body a defined by

$$\mu_a(t) = \frac{m_a}{\sqrt{1 - v^2_a(t)/c^2}}, \tag{79}$$

where m_a is the (constant) Schwarzschild mass.

After performing the integration of (76) with the help of the flat-retarded propagator, we finally get

$$h^{(1)}_{\mu\nu}(t,\boldsymbol{x}) = 4\frac{G}{c^4}\sum_{a=1}^{2}\frac{\mu_a(s_a)v_{a\mu}(s_a)v_{a\nu}(s_a) - (1/2)\eta_{\mu\nu}\mu_a(s_a)v_a^{\lambda}(s_a)v_{a\lambda}(s_a)}{r_a(s_a) - (1/c)(\boldsymbol{v}_a(s_a)\cdot\boldsymbol{r}_a(s_a))}, \tag{80}$$

where $\boldsymbol{r}_a(s_a)$ is given by $\boldsymbol{r}_a(s_a) = \boldsymbol{x} - \boldsymbol{x}_a(s_a)$, and $r_a(s_a)$ is the Euclidean norm of $\boldsymbol{r}_a(s_a)$. In (80) s_a denotes the retarded time $s_a = s_a(t,\boldsymbol{x})$ for the ath body which is a solution of the light-cone equation

$$s_a + \frac{1}{c}r_a(s_a) = t. \tag{81}$$

4.3 The Angle of Light Deflection in the Linear Approximation

By virtue of (71), (73), and considering that the metric coefficients $h^{(1)}_{\mu\nu}$ in (71) are smooth functions of t and $\boldsymbol{z}$, it can be shown that the expression for the angle of light deflection is given by (e.g., see [9])

$$\alpha^i_{(1)}(\tau) = \frac{1}{2c}\int_{-\infty}^{\tau} d\sigma l^{\alpha}_{(0)}l^{\beta}_{(0)}\hat{\partial}_i h^{(1)}_{\alpha\beta}(\tau,\boldsymbol{z}(\tau)) - \frac{1}{c}P^i_q l_{(0)\delta}h^{(1)\delta q}(\tau,\boldsymbol{z}(\tau)), \tag{82}$$

where $\hat{\partial}_i \equiv P^q_i\partial/\partial\xi^q$. Here, τ is an independent parameter defined by

$$\tau = t - t^*, \tag{83}$$

where t^* is the time of closest approach of the unperturbed light ray to the origin of an asymptotically flat harmonic coordinate system. Then the equation of the unperturbed light ray can be represented by

$$\boldsymbol{z}(\tau)_{\text{unpert.}} = \tau\boldsymbol{l}_{(0)} + \boldsymbol{\xi}, \tag{84}$$

where $\boldsymbol{\xi}$ is a vector directed from the origin of the coordinate system toward the point of closest approach (i.e., the impact parameter).

The integral in (82) can be calculated by applying the method developed by Kopeikin and Schäfer in [9]. After inserting the metric coefficients (80) into (82) and computing the integral, we finally obtain

$$\alpha^i_{(1)}(\tau) = \sum_a \frac{4(G/c^3)m_a\left[1 - \frac{\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_a(s_a)}{c}\right]}{\sqrt{1-\frac{v_a^2(s_a)}{c^2}}\left[r_a(\tau,s_a) - \frac{\boldsymbol{v}_a(s_a)\cdot\boldsymbol{r}_a(\tau,s_a)}{c}\right]}P^i_q v^q_a(s_a)$$
$$-\sum_a \frac{2(G/c^2)m_a\left[1 - \frac{\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_a(s_a)}{c}\right]^2\left[r_a(\tau,s_a) + (\boldsymbol{e}_{(0)}\cdot\boldsymbol{r}_a(\tau,s_a))\right]P^i_q r^q_a(\tau,s_a)}{\sqrt{1-\frac{v_a^2(s_a)}{c^2}}\left[r_a^2(\tau,s_a) - (\boldsymbol{e}_{(0)}\cdot\boldsymbol{r}_a(\tau,s_a))^2\right]\left[r_a(\tau,s_a) - \frac{\boldsymbol{v}_a(s_a)\cdot\boldsymbol{r}_a(\tau,s_a)}{c}\right]}. \tag{85}$$

For an observer located at infinity, we find

$$\begin{aligned}\alpha^i_{(1)} &= \lim_{\tau\to\infty} \alpha^i_{(1)}(\tau) \\ &= -4\frac{G}{c^2}\sum_{a=1}^{2} \frac{m_a\left[1-\frac{\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_a(s_a)}{c}\right]}{\sqrt{1-\frac{v_a^2(s_a)}{c^2}}R_a(s_a)}\left[\xi^i - P^i_q x^q_a(s_a)\right], \end{aligned} \tag{86}$$

where the quantity $R_a(s_a)$ is defined by

$$R_a(s_a) = r_a^2(0,s_a) - (\boldsymbol{e}_{(0)}\cdot\boldsymbol{x}_a(s_a))^2. \tag{87}$$

4.4 The Postlinear Gravitational Field of a Compact Binary

In [14,15], it was shown that leading order terms for the effect of light deflection in the linear gravitational field in the case of a small impact parameter $|\boldsymbol{\xi}|$ (i.e., an impact parameter small with respect to the distance between the deflector and the observer) depend neither on the radiative part ($\sim 1/|\boldsymbol{\xi}|$) of the gravitational field nor on the intermediate ($\sim 1/|\boldsymbol{\xi}|^2$) zone terms. The main effect rather comes from the near zone ($\sim 1/|\boldsymbol{\xi}|^3$) terms. Taking into account this property of strong suppression of the influence of gravitational waves on the light propagation, we can assume that the light deflection in the postlinear gravitational field of a compact binary is mainly determined by the near zone metric.

The Metric in the Near Zone

In [16], Blanchet et al. calculated the conservative 2PN harmonic coordinate metric for the near zone of a system of two-bounded point-like masses as function of the distance $\boldsymbol{z}$ and of the positions and velocities of the masses $\boldsymbol{x}_a(t)$ and $\boldsymbol{v}_a(t)$, respectively, with $a = 1, 2$. For the sake of simplicity we split the 2PN metric into two parts: the G-2PN and GG-2PN parts.

G-2PN Metric

The G-2PN part is given by

$$\begin{aligned} h^{(1)}_{00} &= 2\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{r_a} + \frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{r_a}\Big[-(\boldsymbol{n}_a\cdot\boldsymbol{v}_a)^2 + 4v_a^2\Big], \\ h^{(1)}_{0p} &= -4\frac{G}{c^3}\sum_{a=1}^{2}\frac{m_a}{r_a}v_a^p, \\ h^{(1)}_{pq} &= 2\frac{G}{c^2}\sum_{a=1}^{2}\frac{m_a}{r_a}\delta^{pq} + \frac{G}{c^4}\sum_{a=1}^{2}\frac{m_a}{r_a}\Big[-(\boldsymbol{n}_a\cdot\boldsymbol{v}_a)^2\,\delta^{pq} + 4v_a^p v_a^q\Big], \end{aligned} \tag{88}$$

where v_a^p denotes the velocity of the mass m_a, and n_a^p is the unit vector defined by $n_a^p = r_a^p/r_a$. By r_a^p we denote the vector $r_a^p = z^p - x_a^p(t)$ and by r_a we denote its Euclidean norm $r_a = |\boldsymbol{z} - \boldsymbol{x}_a(t)|$.

Here, it is worthwhile to point out that the parts of the G-2PN metric which contain the accelerations of the masses were introduced into the part of the GG-2PN metric after substituting the accelerations by explicit functions of the coordinate positions of the masses by means of the Newtonian equations of motion.

GG-2PN Metric

The GG-2PN part is given by

$$\begin{aligned} h_{00}^{(2)} &= \frac{G^2}{c^4}\left\{-2\frac{m_1^2}{r_1^2} + m_1 m_2\left(-\frac{2}{r_1 r_2} - \frac{r_1}{2r_{12}^3} + \frac{r_1^2}{2r_2 r_{12}^3} - \frac{5}{2r_2 r_{12}}\right)\right\} \\ &\quad + \frac{G^2}{c^4}(1 \leftrightarrow 2), \\ h_{pq}^{(2)} &= \frac{G^2}{c^4}\left\{\delta^{pq}\left[\frac{m_1^2}{r_1^2} + m_1 m_2\left(\frac{2}{r_1 r_2} - \frac{r_1}{2r_{12}^3} + \frac{r_1^2}{2r_2 r_{12}^3} - \frac{5}{2r_1 r_{12}} + \frac{4}{r_{12}S}\right)\right]\right. \\ &\quad \left. + \frac{m_1^2}{r_1^2} n_1^p n_1^q - 4 m_1 m_2 n_{12}^p n_{12}^q \left(\frac{1}{S^2} + \frac{1}{r_{12}S}\right)\right\} \\ &\quad + \frac{4G^2 m_1 m_2}{c^4 S^2}\left(n_1^{(p} n_2^{q)} + 2 n_1^{(p} n_{12}^{q)}\right) + \frac{G^2}{c^4}(1 \leftrightarrow 2), \end{aligned} \tag{89}$$

where the symbol $(1 \leftrightarrow 2)$ refers to the preceding term in braces but with the labels 1 and 2 exchanged; by S we denote $S = r_1 + r_2 + r_{12}$, where $r_1 = |\boldsymbol{z} - \boldsymbol{x}_1(t)|$, $r_2 = |\boldsymbol{z} - \boldsymbol{x}_2(t)|$, and $r_{12} = |\boldsymbol{x}_1(t) - \boldsymbol{x}_2(t)|$. The vectors n_1^p, n_2^p, and n_{12}^p are unit vectors defined by $n_1^p = r_1^p/r_1$, $n_2^p = r_2^p/r_2$, and $n_{12}^p = r_{12}^p/r_{12}$.

The Barycentric Coordinate System

We use a harmonic coordinate system, the origin of which coincides with the 1PN-center of mass. Using the 1PN-accurate center of mass theorem of [17], we can express the individual center of mass frame positions of the two masses in terms of the relative position $\boldsymbol{r}_{12} \equiv \boldsymbol{x}_1 - \boldsymbol{x}_2$ and the relative velocity $\boldsymbol{v}_{12} \equiv \boldsymbol{v}_1 - \boldsymbol{v}_2$ as

$$\boldsymbol{x}_1 = \left[X_2 + \frac{1}{c^2}\epsilon_{1\mathrm{PN}}\right]\boldsymbol{r}_{12}, \tag{90}$$

$$\boldsymbol{x}_2 = \left[-X_1 + \frac{1}{c^2}\epsilon_{1\mathrm{PN}}\right]\boldsymbol{r}_{12}, \tag{91}$$

where X_1, X_2, and ϵ_{1PN} are given by

$$X_1 \equiv \frac{m_1}{M}, \tag{92}$$

$$X_2 \equiv \frac{m_2}{M}, \tag{93}$$

$$\epsilon_{\rm 1PN} = \frac{\nu(m_1 - m_2)}{2M}\left[v_{12}^2 - \frac{GM}{r_{12}}\right]. \tag{94}$$

Here, we have introduced

$$M \equiv m_1 + m_2,\ v_{12} = |\boldsymbol{v}_{12}| \tag{95}$$

and

$$\nu \equiv \frac{m_1 m_2}{M^2}. \tag{96}$$

It is important to remark that, in our computation of the postlinear light deflection up to the order G^2/c^4, we need only to consider the 1PN corrections to the Newtonian center of mass, because, as we shall see in Sect. 4.5, the 2PN corrections to the Newtonian center of mass are related to postlinear light deflection terms of order higher than G^2/c^4.

4.5 The Postlinear Angle of Light Deflection

From (72) and (73), we see that the postlinear angle of light deflection $\alpha^i_{(2)}$ is a function of the GG-2PN metric coefficients $h^{(2)}_{\mu\nu}$, the G-2PN metric coefficients $h^{(1)}_{\mu\nu}$, and the linear perturbation $\delta l^i_{(1)}(\tau)$. To facilitate the computations, we separate the light deflection terms that are functions of the GG-2PN metric coefficients from the terms that are functions of the G-2PN metric coefficients and the linear perturbations.

The Linear Perturbation $\delta l^i_{(1)}(\tau)$

From (71) it follows that the perturbation $\delta l^i_{(1)}(\tau)$ is given by

$$\begin{aligned}\delta l^i_{(1)}(\tau) = & \frac{1}{2}\int_{-\infty}^{\tau} d\sigma\, l^\alpha_{(0)} l^\beta_{(0)} h^{(1)}_{\alpha\beta,i}\Big|_{(\rightarrow)} - c\, h^{(1)}_{0i} - h^{(1)}_{mi} l^m_{(0)} - h^{(1)}_{00}\, l^i_{(0)} \\ & + \frac{1}{2}\, c \int_{-\infty}^{\tau} d\sigma\, h^{(1)}_{00,0} l^i_{(0)}\Big|_{(\rightarrow)} \\ & + \int_{-\infty}^{\tau} d\sigma\, l^m_{(0)} l^p_{(0)} \left[\frac{1}{2} c^{-1} h^{(1)}_{mp,0} - c^{-1} h^{(1)}_{0p,m}\right] l^i_{(0)}\Big|_{(\rightarrow)}. \end{aligned} \tag{97}$$

On the right-hand side of (97) after evaluating the partial derivatives of the metric coefficients with respect to the photon's coordinates (i.e., $(z^0, z^i(t))$), we replace in the integrals the photon trajectory by its unperturbed approximation $z^i(\sigma)_{\rm unpert.} = \sigma l^i_{(0)} + \xi^i$ and the time coordinate z^0 by $\sigma + t^*$. In this chapter, we denote this operation by the symbol $|_{(\rightarrow)}$. After introducing the G-2PN metric coefficients (88) into (97), we obtain the explicit expression for $\delta l^i_{(1)}(\tau)$ which we have to integrate. Since the G-2PN metric coefficients are functions of the positions and velocities of the masses $\boldsymbol{x}_a(t)$ and $\boldsymbol{v}_a(t)$, respectively, the expression for $\delta l^i_{(1)}(\tau)$ is a function of these quantities. This means that we have to take into account the motion of the masses when we are going to compute the integrals. Considering that the influence of the gravitational field on the light propagation is strongest near the barycenter of the binary and that the velocities of the masses are small with respect to the velocity of light, we are allowed to make the following approximations:

1. We may assume that the linear gravitational field is determined by the positions and velocities of the masses taken at the time of closest approach ($t = t^*$) of the unperturbed light ray to the barycenter of the binary (i.e., to the origin of the asymptotically flat harmonic coordinate system). The expression, resulting from (97) after introducing the G-2PN metric coefficients and setting $t = t^*$ for the positions and velocities and computing the integrals, is denoted by $\delta l^i_{(1)\rm I}(\tau)$.
2. We treat the effect of the motion of the masses on light propagation as a correction to the expression of $\delta l^i_{(1)\rm I}(\tau)$, which we denote by $\delta l^i_{(1)\rm II}(\tau)$. We shall compute this correction in Sect. 4.6.

The total linear perturbation $\delta l^i_{(1)}(\tau)$ is then given by

$$\delta l^i_{(1)}(\tau) = \delta l^i_{(1)\rm I}(\tau) + \delta l^i_{(1)\rm II}(\tau). \tag{98}$$

Consequently, the corresponding angle of light deflection reads

$$\alpha^i_{(1)}(\tau) = \frac{1}{c} P^i_q \left[\delta l^i_{(1)\rm I}(\tau) + \delta l^i_{(1)\rm II}(\tau)\right], \tag{99}$$

where P^i_q is defined by (74).

Here, it is important to remark that to obtain the total linear light deflection we have to add to (99) terms arising from the 1PN corrections in the positions of the masses, which we shall compute in Sect. 4.6. Since these terms are proportional to v^2_{12}/c^2, it is easy to see by virtue of the virial theorem that they are of the same order as the terms in G^2/c^4.

The Postlinear Light Deflection Terms That Depend on the *GG*-2PN Metric

It follows from (72) and (73) that a part of the postlinear light deflection is given by:

$$\alpha^i_{(2)\rm I} = \frac{1}{c} P^i_q \left[\frac{1}{2} c^2 \int_{-\infty}^{\infty} d\tau h^{(2)}_{00,q}\Big|_{(\rightarrow)} + \int_{-\infty}^{\infty} d\tau \left[\frac{1}{2} h^{(2)}_{mn,q} - h^{(2)}_{qm,n}\right] l^m_{(0)} l^n_{(0)}\Big|_{(\rightarrow)}\right]. \tag{100}$$

Upon introducing the GG-2PN metric coefficients given by (89) into (100), we obtain integrals whose integrands are functions of the distances r_1, r_2, S, and their inverses. Through the distances r_1, r_2, and S, the resulting integrals from (100) are functions of the positions of the masses $\boldsymbol{x}_a(t)$.

For the same reason as in the case of the linear perturbation, we are here allowed to fix the values of the positions of the masses $\boldsymbol{x}_a(t)$ to their values at the time t^* before performing the integration.

To evaluate the integrals that cannot be represented by elementary functions, we resort as usual to a series expansion of the integrands. The order of the expansion should be chosen in a consistent manner with the expansion in terms of G/c^2.

The Postlinear Light Deflection Terms That Depend on the *G*-2PN Metric

We denote the postlinear light deflection terms, which are functions of the G-2PN metric coefficients and the linear perturbations $\delta l^i_{(1)}(\tau)$, by $\alpha^i_{(2)\mathrm{II}}$. It follows from (72) and (73) that the resulting expression for the postlinear light deflection $\alpha^i_{(2)\mathrm{II}}$ is given by

$$\begin{aligned}
\alpha^i_{(2)\mathrm{II}} = \frac{1}{c} P^i_q \Bigg[&- \frac{1}{2} c^2 \int_{-\infty}^{\infty} d\tau\, h^{(1)qm} h^{(1)}_{00,m}|_{(\rightarrow)} \\
&+ \int_{-\infty}^{\infty} d\tau \left[h^{(1)qp} \big(h^{(1)}_{mp,n} - \frac{1}{2} h^{(1)}_{mn,p} \big) \right] l^m_{(0)} l^n_{(0)} \Big|_{(\rightarrow)} \\
&+ c \int_{-\infty}^{\infty} d\tau \left[h^{(1)}_{0m,q} - h^{(1)}_{0q,m} \right] \delta l^m_{(1)}(\tau) \Big|_{(\rightarrow)} \\
&+ \int_{-\infty}^{\infty} d\tau \left[h^{(1)}_{mn,q} \delta l^m_{(1)}(\tau) l^n_{(0)} - h^{(1)}_{mq,n} \delta l^m_{(1)}(\tau) l^n_{(0)} - h^{(1)}_{mq,n} l^m_{(0)} \delta l^n_{(1)}(\tau) \right] \Big|_{(\rightarrow)} \\
&- \int_{-\infty}^{\infty} d\tau\, h^{(1)}_{00,k} l^k_{(0)} \delta l^q_{(1)}(\tau) \Big|_{(\rightarrow)} \\
&- \frac{1}{c} \int_{-\infty}^{\infty} d\tau\, h^{(1)}_{0p,m} l^m_{(0)} l^p_{(0)} \delta l^q_{(1)}(\tau) \Big|_{(\rightarrow)} \Bigg] . \qquad (101)
\end{aligned}$$

To compute $\alpha^i_{(2)\mathrm{II}}$, we introduce the expressions for the perturbations $\delta l^i_{(1)}(\tau)$ given by (98) and the G-2PN metric coefficients given by (88) into the expression for $\alpha^i_{(2)\mathrm{II}}$. Here, we may use the same approximations as before, i.e., we can fix the values of the positions and velocities of the masses to their values at the time t^* before performing the integrals. As explained in the preceding section, with the help of a Taylor expansion of the integrands we can evaluate the integrals, which cannot be represented by elementary functions.

4.6 Relativistic Corrections

In this section, we give a brief account of the corrections that we have to consider in the calculation of the linear and postlinear light deflection. Further details are given in [11]:

- *Light deflection and the motion of the masses*
  As we mentioned before, the general expression for the linear perturbation $\delta l^i_{(1)}(\tau)$ is through the G-2PN metric coefficients, a function of the positions and velocities of the components of the binary.
  To find the correction terms to the linear perturbation $\delta l^i_{(1)\mathrm{I}}(\tau)$ and postlinear light deflection, we perform the Taylor expansion of the general expression for $\delta l^i_{(1)}(\tau)$ (i.e., of the expression resulting from the introduction of the G-2PN metric coefficients (88) into (97)) in which the coefficients depend on the sources' coordinates and their successive derivatives with respect to t, namely
  $$\frac{dx^i_a}{dt} = v^i_a(t); \quad \frac{d^2x^i_a}{dt^2} = \frac{dv^i_a}{dt} = a^i_a(t); \quad \ldots$$
  The corrections arising from the motion of the binary's components are denoted by $\delta l^i_{(1)\mathrm{II}}$ and $\alpha^i_{(2)\mathrm{III}}$.
- *The postlinear light deflection and the perturbed light trajectory*
  The linear perturbation of the light trajectory reads
  $$\delta z^i_{(1)}(\tau) = \int d\tau \Big[\delta l^i_{(1)\mathrm{I}}(\tau) + \delta l^i_{(1)\mathrm{II}}(\tau)\Big] + K^i, \tag{102}$$
  where K^i is a vectorial integration constant. After introducing the perturbation $\delta z^i_{(1)}(\tau)$ into (99), we obtain additional postlinear light deflection terms, which are denoted by $\alpha^i_{(2)\mathrm{IV}}$.
- *Light deflection and the center of mass*
  After introducing the 1PN corrections in the positions given by
  $$\delta \boldsymbol{x}_1 = \delta \boldsymbol{x}_2 = \frac{1}{c^2}\epsilon_{1\mathrm{PN}}\boldsymbol{r}_{12} \tag{103}$$
  into (99), we get additional terms to the linear and postlinear light deflection, which we denote by $\tilde{\alpha}^i_{(1)(2)}$. From (94) and (103), it is easy to see that the corrections vanish for equal masses and circular orbits.

4.7 The Total Linear and Postlinear Light Deflection

The total linear light deflection results from summing up (99) with the correction terms arising from the part of $\tilde{\alpha}^i_{(1)(2)}$ that is linear in G. Consequently the total linear light deflection reads

$$\alpha^i_{(1)}(\tau)_{\mathrm{tot.}} = \frac{1}{c}P^i_q\left[\delta l^i_{(1)\mathrm{I}}(\tau) + \delta l^i_{(1)\mathrm{II}}(\tau)\right] + \tilde{\alpha}^i_{(1)(2)}(G). \tag{104}$$

From Sects. 4.5 and 4.6, it follows that the total postlinear light deflection up to the order G^2/c^4 is given by

$$\alpha^i_{(2)} = \alpha^i_{(2)\mathrm{I}} + \alpha^i_{(2)\mathrm{II}} + \alpha^i_{(2)\mathrm{III}} + \alpha^i_{(2)\mathrm{IV}} + \tilde{\alpha}^i_{(1)(2)}(G^2), \tag{105}$$

where $\tilde{\alpha}^i_{(1)(2)}(G^2)$ denotes the part of $\tilde{\alpha}^i_{(1)(2)}$ that is quadratic in G.

4.8 Results

From (104) and (105), we obtain the general formulas for the angle of light deflection linear and quadratic in G. These formulas are given in an explicit form in [11]. Here, to study the important features of the derived formulas and in view of an application of the obtained formulas to the double pulsar PSR J0737-3039, we shall consider only the special case when the light ray is originally parallel to the orbital plane of a binary with equal masses (see [18]). In this case the resulting expressions for the angle of light deflection linear and quadratic in G (see [11]) read

$$\begin{aligned}
\alpha^i_{(1)\|} = \frac{GM_{\mathrm{ADM}}}{c^2\,\xi}\Bigg\{ &-4 + \left[1 - (\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2 \\
&+ \left[-\frac{1}{4} + \frac{1}{2}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2 - \frac{1}{4}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^4\right]\left(\frac{r_{12}}{\xi}\right)^4 \\
&+ \left[\frac{1}{16} - \frac{3}{16}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2 + \frac{3}{16}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^4 - \frac{1}{16}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^6\right]\left(\frac{r_{12}}{\xi}\right)^6 \\
&+ \Bigg[-\frac{1}{64} + \frac{1}{16}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2 - \frac{3}{32}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^4 + \frac{1}{16}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^6 \\
&\quad -\frac{1}{64}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^8\Bigg]\left(\frac{r_{12}}{\xi}\right)^8 \\
&+ \Bigg[\frac{1}{256} - \frac{5}{256}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2 + \frac{5}{128}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^4 - \frac{5}{128}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^6 \\
&\quad +\frac{5}{256}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^8 - \frac{1}{256}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^{10}\Bigg]\left(\frac{r_{12}}{\xi}\right)^{10} \\
&+ \Bigg[-\frac{1}{1024} + \frac{3}{512}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2 - \frac{15}{1024}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^4 + \frac{5}{256}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^6 \\
&\quad -\frac{15}{1024}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^8 + \frac{3}{512}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^{10} \\
&\quad -\frac{1}{1024}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^{12}\Bigg]\left(\frac{r_{12}}{\xi}\right)^{12} + \mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^{14}\right]\Bigg\} e^i_{\xi}
\end{aligned}$$

$$+\frac{GM_{\rm ADM}}{c^3\,\xi}\Bigg\{-\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_{12}\right)\left(\frac{r_{12}}{\xi}\right)$$
$$+\left[\frac{1}{4}\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_{12}\right)\left[1+\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^2\right]-\frac{1}{2}\left(\boldsymbol{n}_{12}\cdot\boldsymbol{v}_{12}\right)\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)\right]\left(\frac{r_{12}}{\xi}\right)^3$$
$$+\Bigg[-\frac{1}{16}\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_{12}\right)\left[1+2\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^2-3\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^4\right]$$
$$+\frac{1}{4}\left(\boldsymbol{n}_{12}\cdot\boldsymbol{v}_{12}\right)\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)\left[1-\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^2\right]\Bigg]\left(\frac{r_{12}}{\xi}\right)^5$$
$$+\Bigg[\frac{1}{64}\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_{12}\right)\left[1+3\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^2-9\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^4+5\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^6\right]$$
$$-\frac{3}{32}(\boldsymbol{n}_{12}\cdot\boldsymbol{v}_{12})(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})\left[1-2(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2+(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^4\right]\Bigg]\left(\frac{r_{12}}{\xi}\right)^7$$
$$+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^9\right]\Bigg\}P^i_q n^q_{12}$$
$$+\frac{GM_{\rm ADM}}{c^3\,\xi}\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)\Bigg\{\frac{1}{2}\left(\frac{r_{12}}{\xi}\right)-\frac{1}{8}\left[1-\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^2\right]\left(\frac{r_{12}}{\xi}\right)^3$$
$$+\frac{1}{32}\left[1-2\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^2+\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^4\right]\left(\frac{r_{12}}{\xi}\right)^5$$
$$-\frac{1}{128}\left[1-3\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^2+3\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^4-\left(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12}\right)^6\right]\left(\frac{r_{12}}{\xi}\right)^7$$
$$+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^9\right]\Bigg\}P^i_q v^q_{12}$$
$$+\frac{GM_{\rm ADM}}{c^4\,\xi}(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2\left\{\frac{1}{4}\left[v^2_{12}-(\boldsymbol{e}_{(0)}\cdot\boldsymbol{v}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^4\right]\right\}e^i_\xi \tag{106}$$

and

$$\alpha^i_{(2)\|}=\frac{G^2M^2_{\rm ADM}}{c^4\,\xi^2}\Bigg\{-\frac{15}{4}\,\pi-\frac{1}{24}\left[4+(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2+(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^4\right]\left(\frac{r_{12}}{\xi}\right)$$
$$+\frac{3}{256}\,\pi\left[250-797\,(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})^2\right]\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\Bigg\}e^i_\xi$$
$$+\frac{G^2M^2_{\rm ADM}}{c^4\,\xi^2}\left\{4\,(\boldsymbol{e}_{(0)}\cdot\boldsymbol{n}_{12})\left(\frac{r_{12}}{\xi}\right)^2+\mathcal{O}\left[\left(\frac{r_{12}}{\xi}\right)^3\right]\right\}P^i_q n^q_{12}, \tag{107}$$

where in this case the ADM mass is given by

$$M_{\mathrm{ADM}} = M \left[1 + \frac{1}{4} \left(\frac{v_{12}^2}{2c^2} - \frac{GM}{c^2 r_{12}} \right) \right] . \tag{108}$$

Here, we have assumed that the mass of each component of the binary is equal to $M/2$. In (106) the components e_ξ^i, $P_q^i n_{12}^q$, and $P_q^i v_{12}^q$ of the linear light deflection were expanded to the order $(r_{12}/\xi)^{12}$, $(r_{12}/\xi)^7$, and $(r_{12}/\xi)^7$, respectively, to reach the accuracy of the postlinear light deflection (107).

In (106)–(108), the quantities $\boldsymbol{n}_{12}$, v_{12}, and r_{12} are taken at the time t^*. Note that in this case the correction arising from the shift of the 1PN-center of mass with respect to the Newtonian center of mass (see (103)) vanishes.

In the limit $r_{12} \to 0$ (106) and (107) reduce to

$$\alpha^i_{(1)(\mathrm{E})} = -4 \frac{G M_{ADM}}{c^2 \xi} e_\xi^i \tag{109}$$

and

$$\alpha^i_{(2)(\mathrm{E-S})} = -\frac{15}{4} \pi \frac{G^2 M_{ADM}^2}{c^4 \xi^2} e_\xi^i , \tag{110}$$

which are the Einstein and Epstein–Shapiro light deflection angles, respectively [19].

Application of the formulas for the deflection angle given by (106) and (107) to the double pulsar PSR J0737-3039 for an impact parameter five times greater than the relative separation distance of the binary's components shows that the absolute corrections to an Epstein–Shapiro angle of about 10^{-6} arcsec lie between 10^{-7} and 10^{-8} arcsec.

5 Concluding Remarks

The main steps in the computations of the angle of light deflection in the gravitational field of a compact binary in the linear and postlinear approximations were recapitulated.

The equations of light propagation were explicitly integrated to the second order in G/c^2.

The expressions for the angle of light deflection in the event that the light ray is originally parallel to the orbital of a binary with equal masses were given in an explicit form. In the limit $r_{12} \to 0$ the Einstein angle and the Epstein–Shapiro light deflection angle were obtained from the expressions for the linear and postlinear light deflection, respectively.

Application of the derived formulas for the deflection angle to the double pulsar PSR J0737-3039 for an impact parameter five times greater than the relative separation distance of the binary's components has shown that the absolute corrections to an Epstein–Shapiro angle of about 10^{-6} arcsec lie between 10^{-7} and 10^{-8} arcsec.

We conclude that the corrections to the Epstein–Shapiro light deflection angle are beyond the sensitivity of the current astronomical interferometers. Nevertheless, taking into account that the interferometer for the planned mission LATOR (see [20]) will be able to measure light deflection angles of the order 10^{-8} arcsec, we believe that the corrections to the Epstein–Shapiro light deflection could well be measured by space-borne interferometers in the foreseeable future.

On the level of the light propagation in linear gravitational fields, the controversy on the speed-of-gravity measurement by the radio observations of the bright radio quasar J0842+1835 has been investigated. The conclusion has been drawn that, in that measurement, no speed-of-gravity effect was included.

Finally, a comparison of linearized Einstein's field equations with electrodynamics has been undertaken to clearly show the similarities and dissimilarities between both theories. We feel that this comparison should be useful for those researchers who like to think about linearized Einstein theory in terms of electrodynamics because on this route errors may enter easily when ignoring the different invariance groups of the both theories.

References

1. C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (W.H. Freeman, San Francisco, 1973)
2. B. Mashhoon, in *Reference Frames and Gravitomagnetism*, Eds. J.F. Pascual-Sánchez, L. Floría, A. San Miguel, and F. Vicente (World Scientific, Singapore, 2001), p. 121; also see, *Gravitoelectromagnetism: A Brief Review*, gr-qc/0311030
3. B.S. DeWitt, in *Gravitation: An Introduction to Current Research*, Ed. L. Witten (Wiley, New York, 1962), p. 266; in *Relativity, Groups and Topology*, Eds. C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1965), p. 585
4. S.M. Kopeikin, ApJ Letters **556**, L1 (2001)
5. H. Asada, ApJ Letters **572**, L69 (2002)
6. S.M. Kopeikin, Phys. Lett. **A312**, 147 (2003)
7. C.M. Will, ApJ **590**, 683 (2003)
8. E.B. Fomalont and S.M. Kopeikin, ApJ **598**, 704 (2003)
9. S.M. Kopeikin and G. Schäfer, Phys. Rev. D **60**, 124002 (1999)
10. L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Oxford, 1971)
11. M.H. Brügmann, Phys. Rev. D **72**, 024012 (2005)
12. S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972)
13. J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999)
14. T. Damour and G. Esposito-Farèse, Phys. Rev. D **58**, 044003 (1998)
15. S.M. Kopeikin, G. Schäfer, C.R. Gwinn, and T.M. Eubanks, Phys. Rev. D **59**, 084023 (1999)
16. L. Blanchet, G. Faye, and B. Ponsot, Phys. Rev. D **58**, 124002 (1998)

17. T. Damour and N. Deruelle, C. R. Acad. Sci. Paris **293**, 877 (1981)
18. A.G. Lyne, M. Burgay, M. Kramer, A. Possenti, R.N. Manchester, F. Camilo, M.A. McLaughlin, D.R. Lorimer, N. D'Amico, B.C. Joshi, J. Reynolds, and P.C.C. Freire, Science **303**, 1153 (2004)
19. R. Epstein and I.I. Shapiro, Phys. Rev. D **22**, 2947 (1980)
20. S.G. Turyshev, M. Shao, and K. Nordtvedt, Jr., Class. Quantum Grav, **21**, 2773 (2004); Int. J. Mod. Phys. D **13**, 2035 (2004)