Multi-objective Optimization of Problems with Epistemic Uncertainty

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Abstract. Multi-objective evolutionary algorithms (MOEAs) have proven to be a powerful tool for global optimization purposes of deterministic problem functions. Yet, in many real-world problems, uncertainty about the correctness of the system model and environmental factors does not allow to determine clear objective values. Stochastic sampling as applied in noisy EAs neglects that this so-called epistemic uncertainty is not an inherent property of the system and cannot be reduced by sampling methods. Therefore, some extensions for MOEAs to handle epistemic uncertainty in objective functions are proposed. The extensions are generic and applicable to most common MOEAs. A density measure for uncertain objectives is proposed to maintain diversity in the nondominated set. The approach is demonstrated to the reliability optimization problem, where uncertain component failure rates are usual and exhaustive tests are often not possible due to time and budget reasons.

1 Introduction

The traditional way to define optimization problems is to create a model of the system and state it to be exact and deterministic. Clearly defined decision values are mapped to likewise clearly defined, non-varying objective values.

Respecting the fact that nature doesn't adhere to determinism, stochastic optimization problems and their evolutionary solution methods emerged and gained importance [1]. Yet, the main part of this approaches still abide to the certainty of observed objectives. High sampling rates of a given decision value could simply reveal the underlying distribution of the random processes modelled by the system [2, 3]. MOEA approaches dealing with aleatory uncertainty are presented in [4] and [5].

Models of real systems are built without perfect knowledge of the system simulated. Often the objective values stay highly uncertain even if the real (aleatory) variance is minimal because of a fundamental lack of information about environmental factors or the system itself. In this case even infinitive sampling rates won't help as we simply don't know the distributions to sample from. This so-called epistemic uncertainty must not be ignored in the optimization process.

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Indeed, there is a trend in reliability science and other application areas to formulate models that incorporate and propagate epistemic uncertainties to the simulation outputs rather than generating sharp values. The results are very often only given as intervals or belief functions and thus a need for algorithms capable to handle this types of data is needed. Some approaches towards this issue can be found in [6] and [7].

This work is structured as followed. Section 2 introduces epistemic uncertainty modelling and its mathematical and computational representation. Section 3 discusses different possible extensions of common decision criteria for single and multi-objective evolutionary algorithms (MOEAs). Two ways of redefining the Pareto order over objective vectors and the order over one-dimensional objective functions are introduced. Section 4 shows a way to extend standard MOEAs to handle uncertain objectives in both selection and repository processes. The extension is generic and thus could be applied to most of the commonly used MOEAs. Section 5 proposes a niching strategy to prevent diversity among uncertain solutions. Section 6 shows the application of the proposed approach to the reliability design problem. Finally, some outlines and further research directions are proposed.

2 Belief, Plausibility and the Representation of Epistemic Uncertainty

2.1 Aleatory and Epistemic Uncertainty

There are at least two types of uncertainty that have to be distinguished because of their difference in origin, modelling and effects: Aleatory and epistemic uncertainty. Oberkampf et al. [8] defines aleatory uncertainty as the "inherent variation associated with the physical system or the environment under consideration". Aleatory uncertainty of a quantity can often be distinguished from other types of uncertainty by its characterization as a random value with known distribution. The exact value will change but is expected to follow the distribution. A simple example for aleatory uncertainty is the uncertainty about the outcome of a dice toss $X \in \{1, 2, 3, 4, 5, 6\}$. We are uncertain about the number we will receive, but we are sure that each of the numbers will occur with a probability $p(X = 1) \cdots p(X = 6) = 1/6$.

On the contrary, epistemic uncertainty describes not uncertainty about the outcome of some random event due to system variance but the uncertainty of the outcome due to "any lack of knowledge or information in any phase or activity of the modelling process"[8]. This shows the important difference between this two types of uncertainty. Epistemic uncertainty is not an inherent property of the system. A gain of information about the system or environmental factors can lead to a reduction of epistemic uncertainty. We now focus again on the dice example. Somebody told us that the dice is pronged and so we expect that the probability is limited as $p(X = 1) \dots p(X = 6) \in [1/12, 7/12]$. Of course, the dice would follow a distribution and if we would carry out an infinite number

of experiments, we would find out that it is $p(X = 1) \dots p(X = 5) = 1/9$, $p(X = 6) = 4/9$. Before we do this, we don't have enough information to assume any possible distribution without neglecting that reality may be anywhere else. Hence, epistemic uncertainty is our inability to model reality.

Epistemic uncertainty is often ignored and some arbitrary distribution over the uncertain value is stated as "the best/most realistic/most intuitive". Alternative approaches [9, 10] include epistemic uncertainty in the modelling process and apply frameworks of probability calculus that allow arithmetic with uncertain probability values.

2.2 The Dempster-Shafer-Framework of Evidence

The probabilistic calculus used in this work is the Dempster-Shafer-Framework of evidence first described by Dempster [11] and extended by Shafer [12]. It has proven to be a well-suited framework for representing both epistemic and aleatory uncertainty and has found application in various fields [13, 14]. Thus, a short introduction to the general concepts of this theory is given here. A more detailed overview can be found in [15].

In the classical discrete probability calculus, a probability mass $m(a)$ is defined for each possible value of X and $p(X = a) = m(a)$. Dempster-Shafer-Structures on the real line are similar to discrete distributions with one important difference. The probability mass function is not a mapping $\mathbb{R} \to [0, 1]$ but instead a mapping from $2^{\mathbb{R}} \to [0, 1]$, where probability masses are assigned to sets instead of discrete values. A Dempster-Shafer-Structure can be described by its basic probability assignment (bpa) or by its focal elements. For the problems modelled, it is adequate (but not necessary) to restrict the focal elements to intervals rather than more complicated sets.

Definition 1. *A basic probability assignment m over the real line is a mapping $m : 2^{\mathbb{R}} \to [0, 1]$ provided:*

$$m(\emptyset) = 0 \tag{1}$$

$$\sum_{B \subseteq \mathbb{R}} m(B) = 1 \tag{2}$$

Definition 2. *A focal element $A = [\underline{a}, \overline{a}] \subseteq \mathbb{R}$ is an interval with a nonzero mass $m(A) > 0$.*

Because of the uncertainty modelled it is not possible to give an exact probability $p(X \in B)$ for a value or interval B, yet upper and lower bounds can be calculated. Associated with each bpa are two functions $Bel, Pl : 2^{\mathbb{R}} \to [0, 1]$ which are referred to as belief and plausibility of an event.

Definition 3. *The belief and plausibility of an interval $B \subseteq \mathbb{R}$ are given by*

$$Bel(B) = \sum_{A \subseteq B} m(A) \tag{3}$$

$$Pl(B) = \sum_{A \cap B \neq \emptyset} m(A) \tag{4}$$

It is obvious that $Bel(B) \leq Pl(B)$ because $A \subseteq B \Rightarrow A \cap B \neq \emptyset$. In fact $Bel(B)$ and $Pl(B)$ can be interpreted as bounds on the probability $p(X \in B)$.

Informally the belief function represents the maximal value that we despite all epistemic uncertainty "believe" to be smaller than $p(X \in B)$, the plausibility function represents the highest "plausible" value of $p(X \in B)$. Belief and plausibility values are used in this work to describe the output of an objective function.

3 Decision Criteria Using Belief Functions

Classical optimization problem formulations are mappings from the decision space X which could be of any type to an objective space $Y \subseteq \mathbb{R}^n$ where the goal is to find a vector $\mathbf{x_{opt}} \in X$ which maximizes the objective function

$$\begin{aligned} &f : X \to Y \\ &f(\mathbf{x}) = \mathbf{y} = \begin{pmatrix} \mathrm{y}_1 \\ \vdots \\ \mathrm{y}_n \end{pmatrix} \end{aligned} \tag{5}$$

respective to a partial order relation $\succ$ over Y. If $n = 1$, then we deal with a single-objective optimization problem and $\succ$ is a total order relation (the "bigger than" relation $>$). If $n > 1$, then there is a need to define an order relation $\succ$ over vectors $\mathbf{y} \in \mathbb{R}^n$. In the following work, the order relation inside vector dimension $i \in 1...n$ of the objective vector are denoted as $\succ^i$ while relations between objective vectors are described as $\succ$. Two common approaches should be named here: The aggregation approach and the Pareto approach. The aggregation approach uses an aggregation function u which maps the objective space to the real line (or an arbitrary other space providing a total order).

$$\begin{aligned} &u : Y \to \mathbb{R} \\ &u(\mathbf{y}) = \mathrm{z} \in \mathbb{R} \end{aligned} \tag{6}$$

The resulting order relation is backpropagated to Y. The second approach that has gained much attention in evolutionary optimization during the last years is the Pareto approach using the Pareto dominance criterion [16].

Definition 4. *A vector* $\mathbf{y} \in \mathbb{R}^n$ *Pareto dominates another vector* $\mathbf{y}' \in \mathbb{R}^n$ *(*$\mathbf{y} \succ_p \mathbf{y}'$*) if:*

$$\forall i \in 1...n : \mathrm{y}_i \geq \mathrm{y}'_i \tag{7}$$

$$\exists i \in 1...n : \mathrm{y}_i > \mathrm{y}'_i \tag{8}$$

The Pareto relation relies on the total order inside the dimensions of the element vector. But what if the results of f are uncertain values, represented as intervals of belief and plausibility values. Formally the objective function changes to:

$$f : X \to Y \subseteq (\mathbb{R} \times \mathbb{R})^n$$
$$f(\mathbf{x}) = \mathrm{y} = \begin{pmatrix} [\underline{\mathrm{y}}_1, \overline{\mathrm{y}}_1] \\ \vdots \\ [\underline{\mathrm{y}}_n, \overline{\mathrm{y}}_n] \end{pmatrix} \tag{9}$$

There are several ways to define partial or total order relations over intervals that can make sense depending on the application.

Definition 5. *An interval* $y = [\underline{\mathrm{y}}, \overline{\mathrm{y}}] \subseteq \mathbb{R}$ *dominates another interval* $y' = [\underline{\mathrm{y}}', \overline{\mathrm{y}}'] \subseteq \mathbb{R}$ *certain* ($y \succ_c y'$)*, if*

$$\underline{\mathrm{y}} > \overline{\mathrm{y}}' \tag{10}$$

Definition 6. *An interval* $y = [\underline{\mathrm{y}}, \overline{\mathrm{y}}] \subseteq \mathbb{R}$ *dominates another interval* $y' = [\underline{\mathrm{y}}', \overline{\mathrm{y}}'] \subseteq \mathbb{R}$ *uncertain* ($y \succ_{uc} y'$)*, if*

$$\underline{\mathrm{y}} \geq \underline{\mathrm{y}}' \wedge \overline{\mathrm{y}} \geq \overline{\mathrm{y}}' \wedge y \neq y' \tag{11}$$

The certain and uncertain domination criterion is only a partial order. If e.g. $y' \subset y$, then neither $y \succ y'$ nor $y' \succ y$ holds. $y \| y'$ will be used to denote this indifference. Certain dominance is a stronger criterion than uncertain dominance ($y \succ_c y' \Rightarrow y \succ_{uc} y'$). If the certain dominance criterion holds, we can be sure that an uncertain value is better than another. This cannot be inferred by uncertain dominance. Nevertheless it could be argued that uncertain dominance is also a reasonable relation to speed up the optimization process because it does not stay indifferent when there is high uncertainty.

Both certain and uncertain dominance can't help if $y \subset y'$. There is no straightforward relation without background knowledge that can give us a hint which value is superior. Many different interval aggregation functions of the form

$$\mathbb{R} \times \mathbb{R} \to \mathbb{R}$$
$$f_{agg}(y) = \mathrm{z} \in \mathbb{R} \tag{12}$$

have been proposed that map intervals to a total ordered space. We will denote this relation as $\succ_{agg}$. A good survey is given in [17]. Their big drawback is that they explicitly or implicitly assume a distribution on y like averaging or the maximum entropy approach [18].

In case of multi-objective problems we have to redefine the Pareto dominance relation if we deal with partial orders in the vector dimensions. Definition 4 can be extended for partial orders as follows.

Definition 7. *A vector* y *Pareto dominates another vector* y' *weak* ($\mathrm{y} >_w \mathrm{y}'$)*, if*

$$\forall i \in 1...n : y_i \succ^i y'_i \vee y_i \| y'_i \vee y_i = y'_i$$
$$\exists i \in 1...n : y_i \succ^i y'_i \tag{13}$$

Definition 8. *A vector* $\mathbf{y}$ *Pareto dominates another vector* $\mathbf{y}'$ *strong* ($\mathbf{y} >_s \mathbf{y}'$), *if*

$$\begin{array}{l} \forall i \in 1...n : y_i \succ^i y'_i \vee y_i = y'_i \\ \exists i \in 1...n : y_i \succ^i y'_i \end{array} \tag{14}$$

Both relations reduce to $\succ_p$ when $i = 1...n : y_i$ and y'_i are degenerated intervals ($\underline{y}_i = \overline{y}_i$) or $i = 1...n : \succ^i$ are total orders. The one to be applied depends again on the aims of the user. Strong dominance will only hold if $\mathbf{y}$ is at least equal to $\mathbf{y}'$ in all objectives and better in one while weak dominance holds if $\mathbf{y}$ is at least indifferent $\mathbf{y}'$ in all objectives and better in one.

4 Algorithmic Approach

In this section, an approach to integrate the introduced relations in a Pareto-based multi-objective evolutionary algorithm (PMOEA) is proposed. Various different PMOEAs are used amongst practitioners, possibly the most popular are NSGA2 [19] and SPEA2 [20]. Almost all PMOEAs follow an algorithmic scheme as given in Fig. 1. Binary tournament selection will be used because of its common use in PMOEAs and for the same reason one-point crossover. In two different parts, a relation over Y is used:

Selection relation R_{sel}**.** This relation is used by the tournament selection operator. It is applied each time two individuals are compared in the selection process. The winner survives and can generate offsprings.

Repository relation R_{rep}**.** This relation defines the nondominated solutions stored in the repository. Is is used only in PMOEAs which maintain a set of currently nondominated solutions found during the optimization process. Its task is to determine which solutions from both population and repository are nondominated and thus have to be kept.

If we deal with certain objective values, both R_{sel} and R_{rep} are standard Pareto relations $\succ_p$ while the relations inside the objective functions $\succ^{1...n}$ are "bigger than" operators over the real line. We thus denote $R_{sel} = R_{rep} = (\succ, \{i = 1...n : \succ^i\})$. Which types of relations can be meaningful for R_{sel} and R_{rep} in case

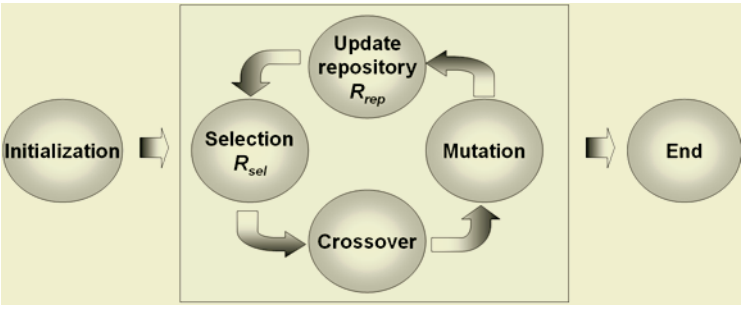

Fig. 1. Standard PMOEA algorithm

of uncertain values? In [5], an aggregation function for intervals is used. A known distribution over the intervals $[\underline{y}, \overline{y}]$ and $[\underline{y}', \overline{y}']$ is assumed and the expected values $E([\underline{y}, \overline{y}])$ and $E([\underline{y}', \overline{y}'])$ are compared mapping the problem to a standard multi-objective form. The relations used are $R_{sel} = R_{rep} = (\succ, \{i = 1...n : \succ^i_{agg}\})$. This approach is not feasible in our context, where the distribution could not be assumed without risk. If the model designers and parameter estimators are not able to come up with a distribution inside their uncertain results, how could the optimizer be able to assume the correct one. Before selecting among the proposed relations, desired characteristics of R_{sel} and R_{rep} will be outlined which lead to a choice use in the algorithm.

Selection Relation R_{sel}

High sensitivity. The selection relation should be able to decide between different solutions as often as possible. Therefore, a relation with a high degree of indifference could degrade the evolution process to a randomized search.

Lower accuracy. Too many "wrong" decisions between individuals have to be prevented. Yet, when dealing with uncertainty we have to make a compromise between the incommensurable goals sensitivity and accuracy. If we want a sensitive decision criterion, some decisions that would have been wrong knowing the certain values can't be prevented. As EAs have proven to be robust optimization methods, sensitivity is more important.

Repository Relation R_{rep}

High accuracy. In the repository, candidate solutions for a posteriori selection of the user are stored. Using uncertain objective functions, we cannot simply leave out a solution that occupies an interesting region of the objective space only because it is eventually worse than another solution. We must be certain that this solution is dominated.

Low sensitivity. The impact of solutions from the repository on the optimization process is low. The repository is often passive and there is no feedback to the current population. The optimization speed therefore can't be slowed down by a high amount of indifference. Yet if the repository is bounded in size, high indifference can rapidly lead to an overflow of nondominated solutions. This problem is well-known and many different methods to restrain the size while preserving diversity of the nondominated set have been proposed [21, 19, 20, 22]. A method handling uncertainty is shown in section 5. Therefore relations should be chosen that guarantee high accuracy.

Taking into account this demands, the following choice of relations is proposed:

$$R_{rep} = (\succ_s, \{i = 1...n : \succ^i_{ic}\}) \tag{15}$$

$$R_{sel} = (\succ_w, \{i = 1...n : \succ^i_{iuc}\}) \tag{16}$$

Table 1. Selection relations $TS1$ and $TS2$: Comparison results

	Bel	**Pl**
$m(TS1)$	1.520	3.415
$m(TS2)$	1.363	3.051
$v(TS1)$	0.021	0.097
$v(TS2$	0.026	0.101
$P(H_0)$	$2.31E-8$	$2.33E-9$

The strong Pareto dominance $\succ_s$ guarantees that only solutions are thrown out of the archive, if at least one solution is equal or better in all objectives. Indifference in only one objective prevents the solution from being eliminated. This fulfils the need for accuracy. For all objective dimensions, the certain domination $\succ_{ic}^{i}$ should be used as it is the only presented relation that provides maximal accuracy on this level.

The selection relation gains sensitivity through the uncertain dominance relation $\succ_{iuc}$ that is much less indifferent than $\succ_{ic}$. The vector relation is also relaxed to the weak Pareto dominance $\succ_w$, which allows us to make a decision between two alternatives even if there is indifference in some objectives. To prove this arguments, 100 runs on each of the test sets $TS1 : R_{sel} = (\succ_w, \{i = 1...n :\succ_{iuc}^{i}\})$ and $TS2 : R_{sel} = (\succ_s, \{i = 1...n :\succ_{ic}^{i}\})$ were carried out. Test problem and parameter settings are presented in detail in 6. The results were evaluated with a normalized hypervolume metric. Table 1 shows the median m and variance v of the hypervolume values (belief and plausibility). Furthermore both belief and plausibility values were tested on the equality of medians $P(H_0)$ by a Wilcoxon signed rank test. The highly significant results show that $TS1$ performs better and the selection relation choice is meaningful.

5 A Niching Strategy for Uncertain Objectives

In this section, a straightforward extension of the nearest neighbor method as described in [23] is introduced. This density estimate is used e.g. in SPEA2 [20] and has proven to be effective in both conserving diversity and promote variability among solutions in the repository. [5] extends the method to uncertain solutions with known distributions using expectation values. Each nondominated solution with uncertain objective vector $\mathtt{y}$ is assigned a fitness value $f_{del}(\mathtt{y})$ which depends on the distance of the k-nearest neighbor. As the scales of the objective dimensions may differ in orders of magnitude, the Euclidean distance is normalized by the maximal extension $\hat{d}$ of the nondominated repository $\mathtt{Rep} = \{\mathtt{y}', \mathtt{y}'', ...\}$:

$$\hat{d}_i = \frac{1}{\max\limits_{\mathtt{y}' \in \mathtt{Rep}} (y_i') - \min\limits_{\mathtt{y}'' \in \mathtt{Rep}} (y_i'')} \tag{17}$$

The normalized distance between two objective vectors $\mathtt{y}, \mathtt{y}'$ is then given as:

$$d(\mathtt{y}, \mathtt{y}') = \sqrt{\sum_{i=1\ldots n} \hat{d}_i^{\,2} \cdot (y_i - y_i')} \tag{18}$$

Due to uncertainty of $\mathtt{y}$ and $\mathtt{y}'$, $d(\mathtt{y}, \mathtt{y}')$ is also an uncertain value with belief and plausibility. Interval calculus defines that

$$Bel(d(\mathtt{y}, \mathtt{y}')) = \min_{\mathbf{a}\in\mathtt{y}, \mathbf{a}'\in\mathtt{y}'} (d(\mathbf{a}, \mathbf{a}')) \tag{19}$$

$$Pl(d(\mathtt{y}, \mathtt{y}')) = \max_{\mathbf{a}\in\mathtt{y}, \mathbf{a}'\in\mathtt{y}'} (d(\mathbf{a}, \mathbf{a}')) \tag{20}$$

and thus require twice the solution of an optimization problem. As $\sqrt{\ }$ and $\sum$ are monotonically growing functions, the problems reduce to:

$$\min_{\mathrm{a}_i\in y_i, \mathrm{a}_i'\in y_i'} (\mathrm{a}_i - \mathrm{a}_i')^2 \tag{21}$$

$$\max_{b_i\in y_i, b_i'\in y_i'} (b_i - b_i')^2 \tag{22}$$

Both problems can be analytically solved as:

$$\min_{\mathrm{a}_i\in y_i, \mathrm{a}_i'\in y_i'} (\mathrm{a}_i - \mathrm{a}_i')^2 = \begin{cases} 0, \text{ if } bel(y_i) - pl(y_i') < 0 \vee bel(y_i') - pl(y_i) < 0 \\ \min((bel(y_i) - pl(y_i'))^2, (bel(y_i') - pl(y_i))^2) \text{ else} \end{cases} \tag{23}$$

$$\max_{\mathrm{a}_i\in y_i, \mathrm{a}_i'\in y_i'} (\mathrm{a}_i - \mathrm{a}_i')^2 = \max((bel(y_i) - pl(y_i'))^2, (bel(y_i') - pl(y_i))^2) \tag{24}$$

The distance of the k-nearest neighbour $d_k(\mathtt{y})$ is then defined as the kth normalized distance sorted in ascending order:

$$d_1(\mathtt{y}) \leq d_2(\mathtt{y}) \ldots \leq d_n(\mathtt{y}) \tag{25}$$

$Bel(d_k(\mathtt{y}))$ and $Pl(d_k(\mathtt{y}))$ are given as the kth distance of the sorted Belief/Plausibility values $Bel(d(\mathtt{y}, \mathtt{y}'))$, $Pl(d(\mathtt{y}, \mathtt{y}'))$. The deletion fitness function $f_{del}(\mathtt{y})$ is then defined as:

$$f_{del}(\mathtt{y}) = -d_k(\mathtt{y}) \tag{26}$$

If the repository grows above the constraining size, individuals are selected by binary tournament selection and their deletion fitness f_{del} is compared by the $\succ_{uc}$ relation. The dominated individual is deleted from the repository. If the comparison stays indifferent, one of the individuals is chosen at random. This approach differs from [5] where a distribution inside the intervals is known or at least assumed. To preserve solutions at the edges of the nondominated set, the solutions with best plausibility or belief values in one objective obtain $f_{del}(\mathtt{y}) = -\infty$. The right choice of the parameter k is also a critical problem. Small values prevent small clusters while large values lead to a more global diversity but do not prevent small clusters. [23] suggests to the square root of the considered point number, so $k = \sqrt{|\mathtt{Rep}|}$ is chosen in the examples.

6 Application - The Optimal Reliability Design Problem

A practical application of uncertain data is the reliability design problem as recently presented e.g. in [24, 25], and in a comprehensive overview of older approaches in [26]. In the time where a wide range of "Commercial-of-the-Shelf" (COTS) components with different characteristics is available, it becomes a big problem to choose the best component combination for a technical system that is both reliable and cheap in production. Zafiropoulos and Dialynas [27] describe this task as a multi-objective optimization problem where they regard only components with constant failure rates. Their chosen objectives were cost and system failure rate (95% confidence interval) that was obtained through Monte Carlo simulation of the system. In this work, a different formulation which aims to find an optimal design regarding cost and system mean time to failure (MTTF) is defined:

1. Maximize the system mean time to failure $MTTF(\mathbf{x})$. $MTTF(\mathbf{x})$ is defined as the expected time until the system consisting of components $x_1 ... x_n$ fails.
2. Minimize the system costs $CS(\mathbf{x})$.

The system (Fig. 2) is given as a reliability block diagram $G = (\{C_{1...n}\}, E)$ [28], a special case of a stochastic flow network, where each block symbolizes a component placeholder that can be replaced by one of several component alternatives (Table 2). The system is functional if and only if there is a path of working components from the source node A to the sink node B. The calculation of system reliability from component reliability is performed by the minimal cut set method [28] that can be extended to handle uncertain probabilities. The system reliability was obtained as a Dempster-Shafer-Structure and the MTTF given as the uncertain expected value. Component failure probability were defined in one of three possible ways.

1. Exponential failure distribution $f_{\exp}(t) = \frac{1}{\lambda} e^{-\frac{t}{\lambda}}$ with uncertain parameter λ (Exp).
2. Weibull failure distribution $f_{weib}(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}$ with uncertain parameters α, β (Weib).
3. Estimates of an arbitrary failure distribution defined by a bpa (Raw).

In system reliability analysis, two distribution functions have gained high popularity for modelling component failure functions. Electronic components neither

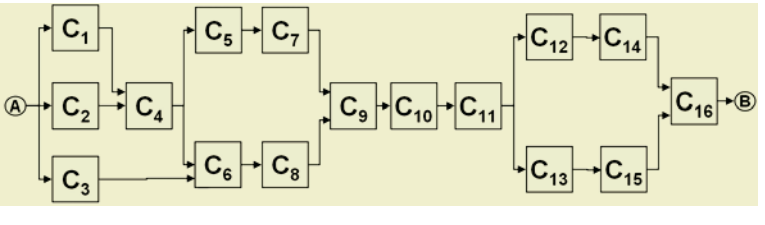

Fig. 2. Example system in block diagram structure

Table 2. Exemplary component failure and cost data

Components	Choice	Type	Parameters	Costs
1	1	Exp.	λ=[33000,43000]	[1000,1010]
	2	Exp.	λ=[51000,61000]	[1100,1105]
	3	Exp.	λ=[75000,82000]	[1200,1210]
	4	Exp.	λ=[88000,96000]	[1280,1290]
2	1	Weib.	α=[15000,20000], β=[0.75,0.85]	[970,1000]
	2	Weib.	α=[29000,35000], β=[0.7,0.8]	[1210,1220]
	3	Weib.	α=[45000,54000], β=[0.8,0.9]	[1120,1140]
	4	Weib.	α=[70000,90000], β=[0.7,0.85]	[1150,1160]
3	1	Raw	[0,4000,m=0.1], [3000,15000,m=0.1], [12000,19000,m=0.1], [18000,30000,m=0.1], [25000,50000,m=0.15], [45000,80000,m=0.15], [60000,90000,m=0.15], [80000,128000,m=0.15]	[635,640]
	2	Raw	[0,8000,m=0.1], [5000,12000,m=0.1], [12000,25000,m=0.1], [22000,40000,m=0.1], [35000,60000,m=0.15], [40000,90000,m=0.15], [60000,100000,m=0.15], [80000,180000,m=0.15]	[710,725]
	3	Raw	[0,9000,m=0.05], [6000,15000,m=0.1], [10000,30000,m=0.1], [28000,49000,m=0.05], [45000,87000,m=0.25], [70000,120000,m=0.25]	[780,785]
...	...	...	...	...

suffer heavily from wear out nor from teething problems if tested before. Because of its property of being memoryless (constant failure rate) the exponential distribution has proven to be adequate for modelling such parts.

Mechanical systems normally tend to degrade over time and therefore require more complex distribution types. This is modelled through a two-parameter Weibull distribution which is often used to estimate component failures from field failure and accelerated lifetime test data [29].

A third possibility is the specification of an arbitrary failure function by expert estimates which is represented through focal elements of a Dempster-Shafer-Structure.

The component costs are considered with low uncertainty as they are normally available at a stipulated price. Yet, there are sources of uncertainty, e.g. unknown integration costs. This costs can be especially high for mechanical parts where interchanging a component can lead to substantial design changes. In a commonly used cost function [30], $CS(x)$ is calculated as the sum of all component costs:

$$CS(\mathbf{x}) = -\sum_{i=1...n} CS(c_i) \tag{27}$$

Using this system, exemplary runs with repository size 25 and 100 were carried out. Mutation probability was set to 0.1, crossover probability to 0.9. The runs over 100 generations were carried out with a population size of 20. The results (Fig. 3-5) show a sample front of nondominated solutions regarding the proposed repository relations $R_{rep} = (\succ_s, \{i = 1...n : \succ_{ic}^{i}\})$ after 100 generations. The nondominated solutions of repository size 25 (shown as rectangles to reflect uncertain values in Fig. 3) are forming a diverse front. Repository size 100 spreads even better and achieves better results. Due to the clarity of visualization, only belief and plausibility were plotted (Fig. 5). The overlapping intervals could be seen in a contour plot of the

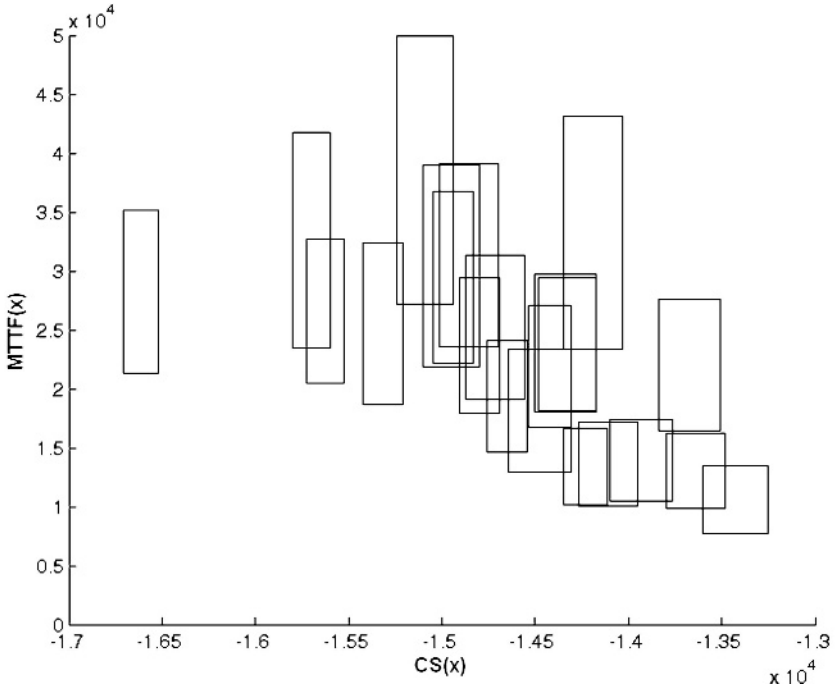

Fig. 3. Nondominated repository of size 25 after 100 generations

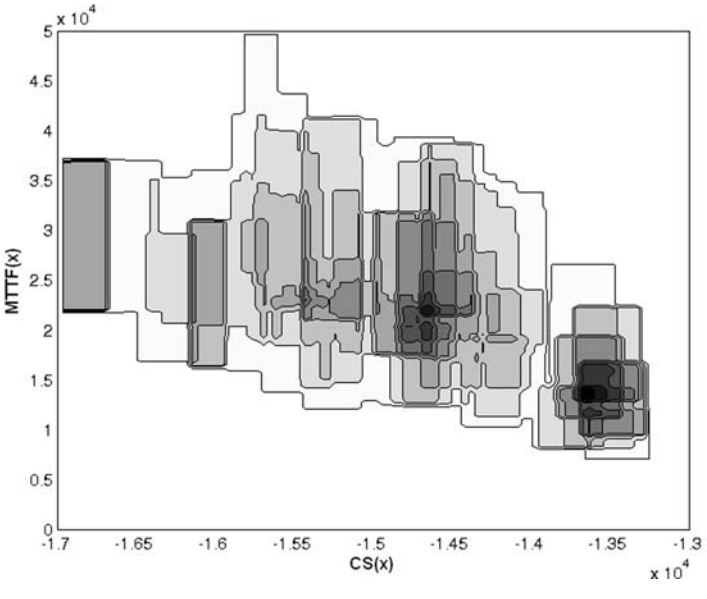

Fig. 4. Nondominated repository of size 100 after 100 generations, density plot

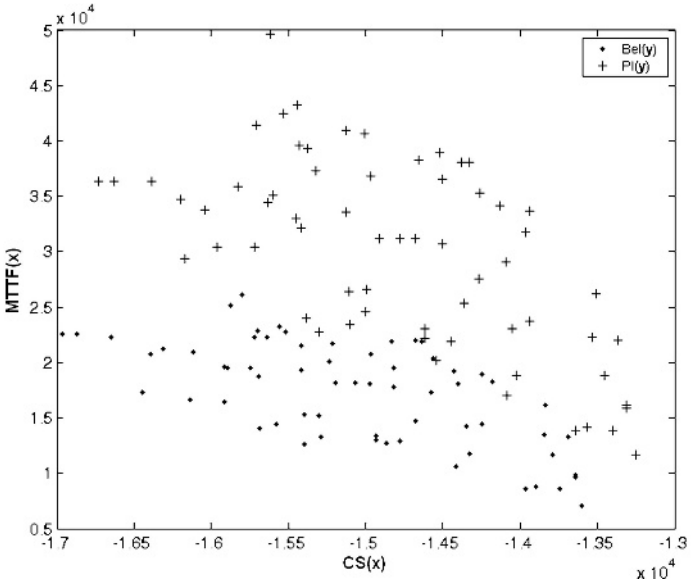

Fig. 5. Nondominated repository of size 100 after 100 generations (belief and plausibility values)

repository assuming uniform distribution inside the uncertain values (Fig. 4). As said before, this distribution is not used as density estimation and serves only for visualization purposes. From this nondominated set of uncertain results, the decision maker could a posteriori choose a solution to realize.

7 Conclusions and Further Research Directions

Problems incorporating epistemic uncertainty are of great practical importance as parametric and modelling knowledge is never perfect. The initial approach proposed shows that multi-objective evolutionary optimization is possible even if the objective function is disrupted by uncertainty resulting from a lack of knowledge. Some general and easy-to-implement extensions to the repository, selection and diversity measure are necessary to enable standard MOEAs to handle such problems. The reliability design problem well-known in the field of reliability analysis and engineering as introduced. In early development stages, when design changes are still inexpensive, parametric uncertainties are sometimes in the order of magnitudes. In an example it was shown how MOEAs can be applied even in this cloudy and blurred modelling phase.

Much work can be done in optimizing and specializing MOEAs towards this problem field. A first step to this empirical approach must be the extension of multi-objective performance metrics to uncertain solutions which leads to even more difficulties in assessing the quality of an algorithm than currently exist. Then different density estimates, selection methods and other algorithmic features can be analyzed and compared to the proposed approach.

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References

1. Arnold, D.V.: Noisy Optimization with Evolution Strategies. Volume 8 of Genetic Algorithms and Evolutionary Computation. Kluwer, Boston (2002)
2. Fetz, T., Oberguggenberger, M.: Propagation of uncertainty through multivariate functions in the framework of sets of probability measures. Reliability Engineering & System Safety **85** (2004) 73–87
3. Agarwal, H., Renaud, J.E., Preston, E.L., Padmanabhan, D.: Uncertainty quantification using evidence theory in multidisciplinary design optimization. Reliability Engineering & System Safety **85** (2004) 281–294
4. Hughes, E.J.: Multi-Objective Probabilistic Selection Evolutionary Algorithm. Technical Report DAPS/EJH/56/2000, Department of Aerospace, Power & Sensors, Cranfield University (2000)
5. Teich, J.: Pareto-Front Exploration with Uncertain Objectives. In: First International Conf. on Evolutionary Multi-Criterion Optimization. Volume 1993 of Lecture Notes in Computer Science., Springer-Verlag (2001) 314–328
6. Rudolph, G.: A partial order approach to noisy fitness functions. In: Proc. 2001 Congress on Evolutionary Computation (CEC'01), Seoul, Piscataway NJ, IEEE Press (2001) 318–325
7. Rudolph, G.: Evolutionary search under partially ordered fitness sets. In: Proc. Int'l NAISO Congress on Information Science Innovations (ISI 2001), Millet, Kanada, ICSC Academic Press (2001) 818–822
8. Oberkampf, W.L., Helton, J.C., Joslyn, C.A., Wojtkiewicz, S.F., Ferson, S.: Challenge problems: uncertainty in system response given uncertain parameters. Reliability Eng. & System Safety **85** (2004) 11–19
9. Walley, P.: Statistical Reasoning with Imprecise Probabilities. Chapman and Hall, London (1991)
10. Yager, R.R.: Uncertainty modeling and decision support. Reliability Eng. & System Safety **85** (2004) 341–354
11. Dempster, A.: Upper and lower probabilities induced by a multivalued mapping. Annals of Math. Statistics **38** (1967) 325–339
12. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press, Princeton, NJ, USA (1976)
13. Hégarat-Mascle, S.L., Richard, D., Ottlé, C.: Multiscale data fusion using demspter-shafer evidence theory. Integrated Comp. Aided Eng. **10** (2003) 9–22
14. Tonon, F.: Using random set theory to propagate epistemic uncertainty through a mechanical system. Reliability Eng. & System Safety **85** (2004) 169–181
15. Yager, R.R., Kacprzyk, J., Fedrizzi, M.: Advances in the Dempster-Shafer theory of evidence. Wiley, New York (1994)
16. Pareto, V.: Manual of political Economy. A.M. Kelley, New York (1971)
17. Yager, R.R., Kreinovich, V.: Decision Making under Interval Probabilities. International Journal of Approximate Reasoning **22** (1999) 195–215
18. Kreinovich, V.: Maximum entropy and interval computations. Reliable Computing **2** (1996) 63–79

19. Deb, K., Agrawal, S., Pratab, A., Meyarivan, T.: A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II. In: Proceedings of the Parallel Problem Solving from Nature VI Conf., Paris, France (2000) 849–858
20. Zitzler, E., Laumanns, M., Thiele, L.: SPEA2: Improving the Strength Pareto Evolutionary Algorithm. In: Evolutionary Methods for Design, Optimisation, and Control, Barcelona, Spain (2001)
21. Knowles, J.D., Corne, D.W.: The pareto archived evolution strategy: A new baseline algorithm for pareto multiobjective optimisation. In: Congress on Evolutionary Computation. Volume 1., IEEE Press (1999) 98–105
22. Bartz-Beielstein, T., Limbourg, P., Mehnen, J., Schmitt, K., Parsopoulos, K.E., Vrahatis, M.N.: Particle swarm optimizers for Pareto optimization with enhanced archiving techniques. In: Congress on Evolutionary Computation (CEC'03). Volume 3., Canberra, IEEE Press (2003) 1780–1787
23. Silverman, B.W.: Density estimation for statistics and data analysis. Chapman and Hall, London (1986)
24. Coit, D., Jin, T.: Multi-criteria optimization: Maximization of a system reliability estimate and minimization of the estimate variance. In: Proceedings of the 2001 European Safety & Reliability International Conf. (ESREL), Turin, Italy, (2001)
25. Elegbede, C., Adjallah, K.: Availability allocation to repairable systems with genetic algorithms: a multi-objective formulation. Reliability Eng. & System Safety **82** (2003) 319–330
26. Kuo, W., Prasad, V.: An annotated overview of system-reliability optimization. IEEE Transaction on Reliability **49** (2000) 176–187
27. Zafiropoulos, E.P., Dialynas, E.N.: Reliability and cost optimization of electronic devices considering the component failure rate uncertainty. Reliability Eng. & System Safety **84** (2004) 271 – 284
28. Billinton, R., Allan, R.N.: Reliability evaluation of engineering systems: concepts and techniques. Plenum, New York (1992)
29. Pozsgai, P., Krolo, A., Bertsche, B., Fritz, A.: SYSLEB-a tool for the calculation of the system reliability from raw failure data. In: Reliability and Maintainability Symposium, 2002. (2002) 542–549
30. Coit, D.W., Smith, A.E.: Reliability optimization of series-parallel systems using a genetic algorithm. IEEE Transaction on Reliability **45** (1996) 254–260