The DSC Algorithm for Edge Detection*

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Abstract. Edge detection is one of the fundamental operations in computer vision with numerous approaches to it. In nowadays, many algorithms for edge detection have been proposed. However, most conventional techniques have assumed clear images or Gaussian noise images, thus their performance could decrease with the impulse noise. In this paper, we present an edge detection approach using Discrete Singular Convolution algorithm. The DSC algorithm efficiently detects edges not only original images but also noisy images which are added by Gaussian and impulse noise. Therefore, we evaluate that the performance of the DSC algorithm is compared with other algorithms such as the Canny, Bergholm, and Rothwell algorithm.

1 Introduction

Edge detection is a front-end processing step in most computer vision and image understanding systems such as the AI research field. The accuracy and reliability of edge detection is critical to the overall performance of these systems. Among the edge detection methods proposed so far, the Canny edge detector is the most rigorously defined operator and is widely used. We select the Canny algorithm to compare with the DSC algorithm.

In recently, a discrete singular convolution (DSC) algorithm was proposed as a potential approach for computer realization of singular integrations [1]. The mathematical foundation of the algorithm is the theory of distributions [2] and wavelet analysis. Sequences of approximations to the singular kernels of Hilbert type, Abel type and Delta type were constructed. In solving differential equations, the DSC approach exhibits the accuracy of a global method for integration and the flexibility of a local method for handling complex geometry and boundary conditions. In the context of image processing, DSC kernels were used to facilitate a new anisotropic diffusion operator for image restoration from noise [3]. Most recently, DSC kernels were used to generate a new class of wavelets, which include the Mexican hat wavelet as a special case.

The purpose of this paper is to propose a new approach based on the DSC algorithm for edge detection. We illustrate this approach by using a special

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class of DSC kernels, the DSC kernels of delta type. In particular, DSC kernels constructed from functions of the Schwartz class are easy to use. Comparison is made between the DSC detection algorithm and the existing algorithms for edge detection such as the Canny, Bergholm, and Rothwell. Experiments indicate that the new approach is effective for image edge detection under severe Gaussian white noise and impulse noise.

2 The Overview of the Previous Algorithms

The Canny edge detection algorithm is considered a standard method used by many researchers. The Bergholm edge focusing algorithm was selected because it represented an approach that used a scale space representation to try to find edges that are significant. [4]. The last algorithm included in the experiment was unique in that it employed dynamic thresholding that varied the edge strength threshold across the image. The implementation of this algorithm was performed by combining pieces of the Canny edge detector code and pieces of C++ code obtained from the authors of the paper[5].

3 Discrete Singular Convolution

3.1 The DSC Algorithm

It is most convenient to discuss singular convolution in the context of the theory of distributions. A singular convolution is defined as equation(1). Let T be a distribution and $\eta(x)$ be an element of the space of test functions

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\,\eta(x)\,dx \tag{1}$$

Where $T(t-x)$ is a singular kernel. Of particular relevance to the present study is the singular kernels of the delta type in equation(2)

$$T(x) = \delta^n(x), \qquad n = 0, 1, 2, \ldots \tag{2}$$

Where δ is the delta distribution.With a sufficiently smooth approximation, it is useful to consider a discrete singular convolution (DSC)

$$F_\alpha(t) = \sum_k T_\alpha(t-x_k)\,f(x_k) \tag{3}$$

Where $F_\alpha(t)$ is an approximation to $F(t)$ and x_k is an appropriate set of discrete points on which the DSC is well defined. Here, in general, $f(x)$ is not required to be a test function.

An important example of the DSC kernels is Shannon's delta kernel

$$\delta_\alpha(x) = \frac{sin(\alpha x)}{\pi x} \tag{4}$$

3.2 The DSC Filters

From the point of view of signal processing, Shannon's delta kernel $\delta_\alpha(x)$ corresponds to a family of ideal low pass filters, each with a different bandwidth.

$$\psi_\alpha(x) = \frac{sin2\alpha x - sin\alpha x}{\pi x} \tag{5}$$

Their corresponding wavelet expressions are band pass filters. Both $\delta_\alpha(x)$ and its associated wavelet play a crucial role in information theory and theory of signal processing. However, their usefulness is limited by the fact that $\delta_\alpha(x)$ and $\psi_\alpha(x)$ are infinite impulse response (IIR) filters and their Fourier transforms $\hat{\delta}_\alpha(x)$ and $\hat{\psi}_\alpha(x)$ are not differentiable. Computationally, $\phi(x)$ and $\psi(x)$ do not have finite moments in the coordinate space; in other words, they are de-localized. This non-local feature in coordinate is related to the band limited character in the Fourier representation according to the Heisenberg uncertainty principle. To improve the asymptotic behavior of Shannon's delta kernel in the coordinate representation, a regularization procedure can be used and the resulting DSC kernel in its discretized form can be expressed as

$$\delta_{\sigma,\alpha}(x - x_k) = \frac{sin(\pi/\Delta)(x - x_k)}{(\pi/\Delta)(x - x_k)}\ e^{-(x-x_k)^2/2\sigma^2} \qquad \sigma > 0 \tag{6}$$

3.3 The DSC Detectors

To design the edge detectors, we consider a one dimensional, *nth* order DSC kernel of the delta type

$$\delta^n_{\sigma,\alpha}(x - x_k), \qquad n = 0, 1, 2\ldots \tag{7}$$

Here $\delta^{(0)}_{\sigma,\alpha}(x - x_k) = \delta_{\sigma,\alpha}(x - x_k)$ is a DSC filter.

$$\delta^{(n)}_{\sigma,\alpha}(x_m - x_k) = \left[\left(\frac{d}{dx}\right)^n \delta_{\sigma,\alpha}(x - x_k)\right]_{x=x_m} \tag{8}$$

It is the impact of parameter σ on the filters in the time-frequency domain. The DSC parameter α can be utilized to achieve an optimal frequency selection in a practical application. For example, in many problems, the object to be processed may be corrupted by noise whose frequency distribution mainly concentrates in the high frequency region. Therefore, a small α value can be used to avoid the noise corruption.

In the present work, the *nth* order DSC edge detector for Noisy Image, or the nth order coarse-scale DSC edge detector, is proposed as

$$\begin{aligned} DSCNI^n(x_i, y_j) = &\left|\sum_{k=-Wn}^{Wn}\sum_{l=Wo}^{Wo} \delta^{(n)}_{\sigma_n,\alpha_n}(x_i - x_k)\delta^{(0)}_{\sigma_0,\alpha_0}(y_j - y_l I(X_k, y_l))\right| \\ &+\left|\sum_{k=-Wo}^{Wo}\sum_{l=Wn}^{Wn} \delta^{((0))}_{\sigma_0,\alpha_0}(x_i - x_k)\delta^{(n)}_{\sigma_n,\alpha_n}(y_j - y_l I(X_k, y_l))\right| \qquad n = 1, 2, \ldots \end{aligned} \tag{9}$$

Here I is a digital image. For simplicity, the details of this procedure are not presented in this paper.

4 Experimental Methods and Results

To demonstrate the efficiency of the proposed approach, we carry out several computer experiments on gray-level images. We select standard images, which are both real and synthetic images. Fig. 1 shows representative images. The resolution of all images is 8-bit per pixel. The size of all images is 256×256. The computation is carried out in a single computer. For definiteness and simplicity, we set the parameter W =3 for all experiments in this section. In the present work, the edge detection consists of two steps: edge magnitude calculation, and thresholding. For simplicity, a fixed threshold is used in the experiments.

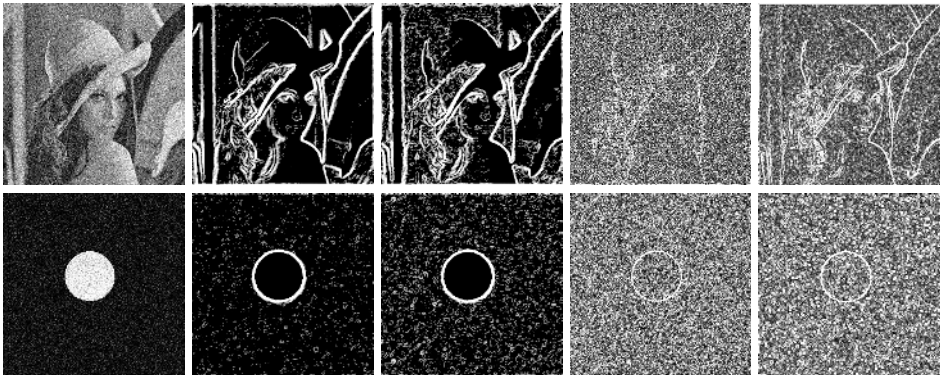
4.1 Noisy Images

To investigate the performance of the DSC algorithm under noisy environment, we consider a number of low grade images. The Fig. 1 (a) present the noisy images, which are generated by adding I.I.D. Gaussian noise and Impulse Noise, and the peak-signal-noise-ratio (PSNR) for each image is 15 dB. Fig. 1 illustrates the resulting edge images detected from noisy environment, obtained by DSC, the Canny detector, Bergholm, and Rothwell detectors.

In general, the detected edges are blurred due to the presence of noise. The three conventional detectors, the Canny, Bergholm, and Rothwell, detect not only spatially extended edges, but also many spurious features due to noise. As a result, the contrast of their edge images is poor. Whereas, much sharper edge images are successfully attained by the DSC detector, as shown in Fig. 1(b). The difference in contrast stems from the fact that the DSC detects edges at a coarse scale, in which the high frequency noise has been remarkably smoothed out. As mentioned in the introduction, the Canny detector [6]was formulated as an optimization problem for being used under noise environment. The parameter is taken as $\sigma = 1.5$ as suggested by other researchers. Obviously, there is a visual difference between those obtained by using the DSC detector and the Canny detector. These experiments indicate the performance of the DSC based edge detector is better than that of the Canny detector.

4.2 Objective Performances

To obviously validate the DSC detector further, we present an alternative evaluation in this subsection. Edge detection systems could be compared in many ways. For synthetic images, where the exact location of edges is known, Abdou and Pratt [7] proposed a figure of merit to objectively evaluate the performance of edge detectors. It is a common practice to evaluate the performance of an edge detector for synthetic images by introducing noise in the images. A plot of F against the PSNR gives the degradation in the performance of the detector. The value of F is less than or equal to 1. The larger the value, the better the performance.



(a) (b) (c) (d) (e)

Fig. 1. Sample images(a). Edge images of the real(first row) and synthetic(second row) images with Gaussian and Impulse noise were obtained by (b) DSC detector(column1) (c) Canny detector(column2) (d) Bergholm detector(column3) (e) Rothwell detector(column4)

In Fig. 2, when the noise level is low, the F values are very close to 1 and the performances of all the four detectors are very satisfactory. With the increase of the noise level, the F value of two difference detectors which are the Bergholm and the Rothwell detectors is dramatically decreased. The F value of difference between DSC and those of two detectors is almost 0.6 when PSNR is 15 dB. In contrast, the Canny detector and the DSC detector achieve large F values over the domain of interest, suggesting their superiority to other two detectors. It is noted that the performance of an DSC is better than that of the Canny detector for small PSNR values.

4.3 Discussion

The Rothwell detector obtains better performance than the Bergholm detector because of dynamic threshold method. However, these two detectors do not carry out excellent results at noisy images. It is well-known that the performance of the Canny detector depends on the computational bandwidth W and standard deviation σ. These parameters can be utilized to obtain edges which are optimized with respect to the space of parameters for each given image. In particular, the parameter σ gives rise to excellent time-frequency localization. However, the Canny filter does not provide much freedom for frequency selection. In contrast to the Canny detector, the DSC detector has one more parameter α. Thus, DSC detector should perform at least as well as the Canny detector.

The DSC detector has an extra parameter, α_n, which controls DSC filter frequency selection . Experiments indicated that, when α_n decreases, fine details are smoothed out and main edge structures appear significant. This property can be utilized to deal with images corrupted with color noise, for which the Canny

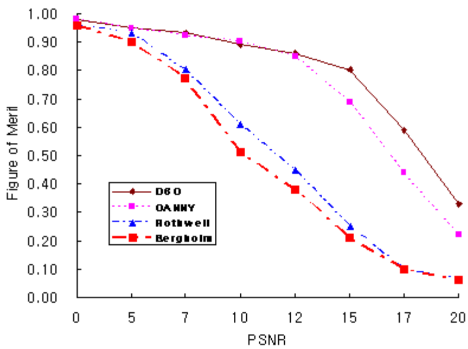


Fig. 2. The Figure of Merit of the synthetic image with noise

detector is not the best choice. The ability of frequency selection is important to many practical applications, for instance, AI research.

5 Conclusion

This paper introduces the DSC algorithm for edge detection. A number of DSC filters, low-pass filters, are proposed in the context of distribution theory. A family of regularized DSC kernels is constructed for denoising and data interpolation. The performance of the proposed algorithm is compared with other existing methods, such as the Canny, Bergholm, and Rothwell. The Canny detector can be optimized with respect to the filter length and time-frequency localization, whereas, the DSC detector can be optimized with respect to one more parameter,α, which plays the role of frequency selection. Experiments on a two kinds of images have been carried out with some selected DSC parameters, and the performance of DSC detectors is better than that of the Canny detector.

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