Completeness Conditions of a Class of Pattern Recognition Algorithms Based on Image Equivalence*

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Abstract. The paper presents recent results in establishing existence conditions of a class of efficient algorithms for image recognition problem including the algorithm that correctly solves this problem. The proposed method for checking on satisfiability of these conditions is based on the new definition of image equivalence introduced for a special formulation of an image recognition problem. It is shown that the class of efficient algorithms based on estimate calculation contains the correct algorithm in its algebraic closure. The main result is an existence theorem. The obtained theoretical results will be applied to automation of lymphoid tumor diagnostics by the use of hematological specimens.

1 Introduction

During last several years in Scientific Council “Cybernetics” of the Russian Academy of Sciences the research was conducted in the field of development of mathematical techniques for image analysis and estimation. The theoretical base for this research is the Descriptive Theory of Image Analysis and Recognition [2-5,10,11]. This paper presents recent results in establishing existence conditions of a class of efficient algorithms for image recognition problem, including the algorithm, which solves this problem correctly. The proposed method for checking on satisfiability of these conditions is based on the new definition of image equivalence, introduced for the special formulation of an image recognition problem. It is shown that the class of efficient Algorithms Based on Estimate Calculation (AEC) [10,11] contains the correct algorithm in its algebraic closure. The main result is an existence theorem.

One of the issues of the day in the field of image recognition is the search of the algorithm, which correctly classifies an image by using its description. The approach to image recognition, developed by the authors, is a specialization of the algebraic approach to recognition and classification problems (Yu. Zhuravlev [10,11]). The main idea of this approach consists in the following. There are no accurate mathematical models for such poorly formalizable areas as geology, biology, medicine, and sociology. However, in many cases nonrigorous methods based on the

* The paper was partly supported by the grants of the Russian Foundation for Basic Research nos. 01-07-90016, 02-01-00182 and Russian Academy of Sciences (project no. 2.14 of the Complex Program of Scientific Research of the Presidium of RAS “Mathematical Modeling and Intellectual Systems”).

A. Sanfeliu and J. Ruiz-Shulcloper (Eds.): CIARP 2003, LNCS 2905, pp. 506–511, 2003.

heuristic considerations yield appreciable practical results. Therefore, it is enough to construct a family of such heuristic algorithms for solving appropriate problems, and then, to construct the algebraic closure of this family. The existence theorem is proved, which states that any problem from a set of problems concerning poorly formalized situations, is solvable in this algebraic closure [10].

An image recognition is a classical example of a problem with ill-formalized and partly contradictory information. This gives us good reasons to believe that the use of the algebraic approach to image recognition can lead to important results and, consequently, an “algebraization” of the field is the most promising way of development of desired mathematical techniques for image analysis and estimation.

Note, that the idea of creating the unified algebraic theory covering different approaches and procedures used in image and signal processing has a certain background beginning with the works of von Neumann and extended by S. Unger, U. Grenander, M. Duff, Yu. Zhuravlev, G. Matheron, G. Ritter, J. Serra, et al. [1,7-11]. Our research is carried out in the field of the Descriptive Approach to Image Analysis and Recognition, which differs from the studies in the field of algebra mentioned above, and is completely original.

Unfortunately, the algebraic approach developed by Yu. Zhuravlev cannot be applied directly to an image recognition problem. Mostly, it is due to the difficulty of an image as a recognition object and to considerable distinctions between the classical pattern recognition problem and the image recognition problem which consist in the following:

- a standard object in the classical pattern recognition theory is, as a rule, described by a set of features; whereas there is no such natural way for image description that does not lose an important information about image; common methods of image description are either too complicated and require much computational recourses (e.g., raster representation of an image), or semantically primitive (set of features);
- a single object (a scene) can correspond to several images differing in brightness, contrast, scale, and observer’s point of view; within the framework of a recognition problem, it means that different images of an object should be identically classified by the recognizing algorithm.

Thus, the problem of image equivalence provokes much interest, especially the use of this property in pattern recognition problem [3–5].

2 Image Equivalence

Image equivalence relation on the set of images may be introduced in different ways:

a) we may consider equivalence as a closeness of image descriptions with respect to a metric in a metric space, for example, a metric in the Euclidean space E^n, where image is described by its n-dimensional feature vector;

b) if the set of allowable image transformations is given in the image recognition problem (for example, image rotations by angle $2\pi k/n$, $k=0,1,\ldots,n-1$), the two images are considered to be equivalent, if the first image is obtained from the second one by applying a certain transformation from the given set of allowable transformations.

In this work, we propose a definition of image equivalence, based on a special setup of an image recognition problem. Consider a set of allowable images, described by their n-dimensional feature vectors, and the recognizing algorithm A, which constructs l-dimensional information vector by using n-dimensional description vector. We remind, that information vector is a vector of object's membership of classes, where the values of vector components $0,1,\Delta$ mean "object does not belong to a class", "object belongs to a class", "algorithm fails to determine, whether object belongs to a class or not", respectively [10].

Definition 1. Two images are *equivalent* with respect to a recognizing algorithm A, if their information vectors, obtained by the recognizing algorithm A, coincide.

A simple way of constructing an image equivalence class is introduced, based on this definition. The idea consists in applying a certain set of transforms to generating images.

Let a binary image I of a plane polygon be given. We call it *generating image.* Let the transforms of plane rotation group C_n be given as a set of transforms: each n-order group C_n consists of all rotations by angles $2\pi k/n,\ k=0,1,\dots,n-1$, around a fixed point, and it is essentially, that given transforms form a group. Applying each transform from C_n to generating image I, we obtain a set of images.

The equivalence of the obtained images is established by the identity of their information vectors. Here, it is reasonable to describe images by vectors of invariant features and the simplest way is to exploit the invariants with respect to the given group of transformations C_n. As a result, all images from the obtained set have the same feature vectors, and recognizing algorithm constructs the same information vectors for these images. Consequently, all obtained images are equivalent. It is essential that, in case when transformations form a group, mathematical methods are developed for constructing invariants with respect to this group of transforms [6].

3 Mathematical Formulation of a Recognition Problem

In order to prove the existence theorem for AEC that correctly solves the image recognition problem, it is necessary to introduce a new formulation of an image recognition problem, differing either from classical formulation [10,11], or from the formulation of the Descriptive Approach [2]. Let us have a look at these formulations.

3.1 Classical Mathematical Formulation of Pattern Recognition Problem *Z*

$Z(I_0,S_1,\dots,S_q,P_1,\dots,P_l)$ is a pattern recognition problem, where I_0 is allowable initial information, $S_1,\dots,S_q$ is a set of allowable objects, described by feature vectors, $K_1,\dots,K_l$ is a set of classes, $P_1,\dots,P_l$ is a set of predicates on allowable objects, $P_i=P_i(S),\ i=1,2,\dots,l$. The problem Z is to find the values of predicates $P_1 \dots,P_l$.

Definition 2. Algorithm is *correct* for a problem Z [10], if the following equation is satisfied:

$$A\left(I,S_1,\dots,S_q,P_1,\dots,P_l\right)=\left\|\alpha_{ij}\right\|_{q\times l},\ \text{where}\ \alpha_{ij}=P_j\left(S_i\right).$$

3.2 Mathematical Formulation of Image Recognition Problem Z^1

Taking into account the introduced notion of image equivalence an image recognition problem may be formulated in the following way:

$Z^1\left(\left\{I_i^{j_i}\right\}_{i=1,2,\dots,q}^{j_i=1,2,\dots,p_i},\left\{M_i\right\}_{i=1,2,\dots,q},\left\{K_t\right\}_{t=1,2,\dots,l},\left\{P_t^{i\,j_i}\right\}_{t=1,2,\dots,l}^{i=1,2,\dots q;\,j_i=1,2,\dots,p_i}\right)$ is an image recognition problem Z^1, where $\left\{I_i^{j_i}\right\}$ are images, $i=1,2,\dots,q$, j_i is a number of an image within the i-th equivalence class, p_i is a quantity of images in the i-th equivalence class, $j_i=1,2,\dots,p_i$; $M_i=\{I_i^1,I_i^2,\dots,I_i^{pi}\}$, $i=1, 2,\dots,q$, is an equivalence classes on the set $\left\{I_i^{j_i}\right\}$; $K_1,K_2,\dots,K_l$ are classes in the image recognition problem; $P_t^{i\,j_i}:"I_i^{j_i}\in K_t"$, $t=1,2,\dots,l$, $i=1,2,\dots,q$, $j_i=1,2,\dots,p_i$, are predicates. The problem Z^1 is to find the values of predicates $P_t^{i\,j_i}$.

3.3 Mathematical Formulation of Image Recognition Problem Z^2

The distinction between the problem Z^2 and the problem Z^1 is that each equivalence class is replaced by a single image, a representative of a class, with number n_i , $1\le n_i \le p_i$,, where i is a number of an equivalence class. This replacement is realized with a help of a definition of an allowable transform.

Definition 3. An arbitrary transform f: $\{I_i^{\,j}\} \rightarrow \{I_i^{\,j}\}$ is an allowable transform, if $f(I_i^{\,j})$ and $I_i^{\,j}$ belong to the same equivalence class for each $I_i^{\,j}$.

$Z^2\left(\left\{I_i^{n_i}\right\}_{i=1,2,\dots,q},\left\{K_t\right\}_{t=1,2,\dots,l},\left\{P_t^{i}\right\}_{t=1,2,\dots,l}^{i=1,2,\dots q}\right)$ is an image recognition problem Z^2,

where $I_i^{n_i}$, $i=1,2,\dots, q$ are images, $I_i^{n_i}\in M_i$; $K_1,K_2,\dots,K_l$, are classes in the image recognition problem; $P_t^{i}:"I_i^{n_i}\in K_t"$, $t=1,2,\dots,l$, $i=1,2,\dots,q$, are predicates. The problem Z^2 is to find the values of predicates $P_t^{\,i}$.

4 Conditions of Completeness of the Class of Algorithms Based on Estimate Calculation for Image Recognition Problem

The main result of the paper is obtained for a class of efficient recognizing algorithms – AEC [10,11]. These algorithms are based on formalization of the idea of precedence or partial precedence: the "proximity" between parts of descriptions of the objects classified previously and object to be classified is analyzed. Suppose we have the standard descriptions of the objects $\{\tilde{S}\}$, $\tilde{S}\in K_j$ and $\{S'\}$, $S'\notin K_j$, and the method of the "closeness" evaluating for parts of the description of S and the corresponding parts of descriptions of $\{I(\tilde{S})\},\{I(S')\}$; S is an object presented for recognition, $j=1,2,\dots,l$.

By evaluating the “proximity” between the parts of descriptions of $\{I(\tilde{S})\}$ and $\{I(S')\}$, and between $I(S)$ and $I(S')$, respectively, it is possible to evaluate a generalized “proximity” between S and the sets of objects $\{\tilde{S}\}$, $\{S'\}$ (in the simplest case, the value of a generalized “proximity” is a sum of values of “proximity” between the parts of descriptions). Then, the total estimate for an object of a class is formed by the set of estimates, which is the value of object’s membership function of a class.

The existence theorem for AEC that solves recognition problem Z correctly is proved for the algebraic closure of the class of AEC.

Theorem 1 [10]. Let the conditions of non-identity of descriptions of classes and of objects in pattern recognition problem Z with given vectors of features be satisfied. Then the algebraic closure of the class of AEC-algorithms is correct for Z.

Note, that the definition of image equivalence is not used in the classical formulation of a recognition problem, therefore, the Theorem 1 of a correct algorithm existence cannot be applied directly to an image recognition problem.

The distinction between the problem Z and the problem Z^1 is that, in the latter, the image equivalence classes are explicitly considered. In order to reduce the image recognition problem Z^1 to the standard recognition problem Z, it is necessary to proceed from classification of a group of objects to classification of a single object. The problem Z^2, differing from Z^1 by the presence of allowable transforms and by the lack of image equivalence classes, allows us to operate with a single image – a representative of a corresponding equivalence class – for each equivalence class under certain restrictions on the set of allowable transforms.

Direct generalization of Theorem 1 for the image recognition problem Z^2 is Theorem 2.

Theorem 2. Let the allowable transforms $\{f_1, f_2, \ldots\}$ form a transitive group. Then, the image recognition problem Z^1 may be reduced to the problem Z^2 and the algebraic closure of the class of AEC-algorithms is correct for Z^2.

Theorem 2 establishes the conditions of existence of a correct algorithm for image recognition problem and proves that such algorithm can be found in the algebraic closure of AEC-algorithms.

5 Conclusion

The task of searching for the correct algorithm for image recognition problem was investigated. The definition of image equivalence was introduced, and the formulation of image recognition problem was modified. It was proved that, under certain restrictions on the image transforms, an image recognition problem may be reduced; and the correct algorithm for the reduced problem can be found in the algebraic closure of AEC.

The future research will be devoted to detailed analysis of image equivalence and establishment of relationship between image equivalence and image invariance. The obtained theoretical results will be applied to automation of lymphoid tumor diagnostics by using hematological specimens.

References

1. Grenander, U.: General Pattern Theory. A Mathematical Study of Regular Structures, Oxford: Clarendon Press (1993)
2. Gurevich, I.B.: The Descriptive Framework for an Image Recognition Problem, Proc. of the 6th Scandinavian Conf. on Image Analysis, Oulu, Finland, Vol.1 (Pattern Recognition Society of Finland) (1989) 220–227
3. Gurevich, I.B., Jernova, I.A., Smetanin, Y.G.: A Method of Image Recognition Based on the Fusion of Reduced Invariant Representations: Mathematical Substantiation // Proceedings of the 16th International Conference on Pattern Recognition (ICPR2002), Quebec, Canada, August 11–15, 2002 [R.Kasturi, D.Laurendeau, C.Suen (Eds.)]: in 4 volumes. Vol.3 The Institute of Electrical and Electronics Engineers, Inc. (2002) 391–394
4. Gurevich, I.B., Smetanin, Y.G.: On the Equivalence of Images in Pattern Recognition Problems // Proceedings of the 12th Scandinavian Conference on Image Analysis, Bergen, June 11th – 14th, 2001. [Ed. I.Austvoll]. Norwegian Society for Image Processing and Pattern Recognition (2001) 679–685
5. Gurevich, I.B., Zhuravlev Y.I.: An Image Algebra Accepting Image Models and Image Transforms // Proceedings of the 7th International Workshop "Vision, Modeling, and Visualization 2002" (VMV2002), November 20–22, 2002, Erlangen, Germany [G.Greiner, H.Niemann, T.Ertl, B.Girod, H.-P.Seidel (Eds.)], IOS Press, B.V.Amsterdam, Infix, Akademische Verlagsgesellschaft, Aka GMBH, Berlin (2002) 21–26
6. Mundy, J.L., Zisserman, A.: Towards a New Framework for Vision, Geometric invariance in computer vision/ edited by J. Mundy, A. Zisserman. (1992) 1–39
7. Ritter, G.X., Wilson, J.N.: Handbook of Computer Vision Algorithms in Image Algebra, 2-d Edition. CRC Press Inc. (2001)
8. Serra, J.: Image Analysis and Mathematical Morphology. L., Academic Press (1982)
9. Sternberg, S.R.: Language and Architecture for Parallel Image Processing, Proceedings of the Conference on Pattern Recognition in Practice. Amsterdam (1980)
10. Zhuravlev, Yu.I.: An Algebraic Approach to Recognition or Classification Problems, Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications, V. 8, No. 1 (1998) 59–100
11. Zhuravlev, Yu.I., Gurevitch, I.B.: Pattern Recognition and Image Recognition, Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications in the USSR, V. 1, No. 2 (1991) 149–181