# Mud Inflow Risk Assessment in Block Caving Operation Based on AHP Comprehensive Method



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## 1 Introduction

The mining industry is almost considered as a highly potential risk industry due to the complexity and various sources of uncertainty. Among the mining methods, underground caving operations are naturally affected by mud and debris inflow; since, these methods are connected to surface or previous mining area with a broken subsidence zone, where is potential to accumulate water (Jacubec and Clayton 2012). Although mud rushes can have different origins, almost always four elements are required for a mud rush to occur: water, potential mud-forming material, a disturbance of mud, and discharge point (Butcher et al. 2000). At the mines that are likely to mud rush occurrence, a range of mitigating and controlling measures is adopted. Butcher et al. (Butcher et al. 2000) suggested a mud rush prevent approach based on three aspects: Keeping fine material far enough from the mining operations<sup>1</sup>; prohibiting water ingress into muck pile and proper definition of draw strategy to inhibit the discharge of hang ups, air blasts, and mud pockets. Heslop

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<sup>&</sup>lt;sup>1</sup>Site selection of waste dumps and tailing dams should be performed with the objective of eliminating the risk of material flow into underground operations.

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(2000) indicated that to prevent mud inflow to the production level it is necessary to perform measures to control water ingress. Moreover, analyzing the draw strategies in order to prevent percolating mud into the draw column as well as extracting all draw-points continuously is the other approaches suggested by Helslop to avoid generating static impermeable mud barriers where more mud and water can accumulate (Heslop et al. 2000).

Due to the complex nature of cave dynamics and water distribution, risk assessment and management are likely the most effective approach to control and minimize the hazards of mud inflow. The risk assessment of mud entrance to working area includes the strategies to identify all possible sources of water or slurry and examine the possibility of finding their way into the operational zone. These strategies enable the project managers to avoid potential problems. In this research a safety risk assessment framework is presented based on analytic hierarchy process (AHP) to facilitate the risk evaluation operation. In the previous studies AHP method was mostly used as a decision-making process for the evaluation of the alternatives (Saaty 2003; Bascetin 2004; Yurdakul 2004; Dagdeviren and Yüksel 2008). While in this study, AHP is using to develop a decision-aid system to rank factors associated with the occurrence of mud inflow into the extraction points. Therefore, the most influenced parameters of mud inflow were considered as elements of the hierarchy tree. Then the pair-wise comparison between these elements was achieved based on the statistical analyses of mine data at El Teniente copper mine, Chile.

The proposed framework presents a method for prioritization of safety risks to create a rational budget for mud inflow prevention during planning and feasibility study of block caving mining projects.

#### 2 The Analytic Hierarchy Process (AHP)

AHP is one of the multiple criteria decision-making (MCDM) methods first developed by Saaty (1980). In AHP, the decision problem is usually divided into a hierarchy of sub-problems, which can be analyzed independently. Due to the nice mathematical properties and the fact that the required input data are easy to obtain, the AHP has found interesting by many researchers (Kousalya et al. 2012). Broad areas in which the AHP has been applied include alternative selection, resource allocation, forecasting, risk assessment, quality function deployment, balanced scorecard, benchmarking, public policy decision, health care, and many more (Bascetin 2004; Kousalya et al. 2012; Mustafa and Al-Bahar 1991; Hekmat et al. 2008; Aminbakhsh et al. 2013; Lee 2014).

The first step of the application of AHP is developing the decision hierarchy, in which a complex MCDM problem breaks into a hierarchy of interrelated decision elements (goal, criteria, sub-criteria, and alternatives). This is the most creative and important part of the process. Hierarchy indicates a relationship between elements of one level with those of the level immediately below. This relationship percolates

down to the lowest level of the hierarchy, and in this manner, every element is connected to every other one, at least in an indirect manner (Bhushan and Rai 2004). Once the hierarchy has been built, the properties of elements at each level are determined. Prioritization procedure starts in order to determine the relative importance of the criteria with each level; therefore, a set of comparison pair-wise matrices are constructed as shown in Eq. 1.

$$A = \begin{bmatrix} 1 & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & 1 & \cdots & \frac{w_1}{w_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$
(1)

where:  $w_1$ ,  $w_2$ , and  $w_n$  indicate the weights of the elements 1, 2, and *n*, respectively. The pair-wise comparisons are given in terms of how much one element is more important than the other one. The preferences are quantified using a nine-point scale that is shown in Table 1.

At the last step, the mathematical process is commenced to normalize and find the relative weight of each matrix. The process is summarized as follows:

(1) Normalized each row vector of A:

$$\bar{a}_{ij} = \frac{a_{ij}}{\sum_{k=1}^{n} a_{kj}} \quad (i = 1, 2, \cdots, n)$$
<sup>(2)</sup>

| Intensity of weight                  | Definition   | Explanation  |  |
|--------------------------------------|--|--|--|
| 1                                    | Equal importance   | Two activities contribute equally to the objective                             |  |
| 3                                    | Moderate importance  | Experience and judgment slightly favor one over another                        |  |
| 5                                    | Strong importance  | Experience and judgment strongly favor one over another                        |  |
| 7                                    | Very strong<br>importance  | An activity is strongly favored, and its dominance is demonstrated in practice |  |
| 9                                    | Absolute<br>importance   | The importance of one over another affirmed on the highest possible order      |  |
| 2, 4, 6, 8                           | Intermediate values  | Used to represent compromise between the priorities listed above               |  |
| Reciprocals of above nonzero numbers | If activity $i$ has one of the above nonzero numbers assigned to it<br>when compared to activity $j$ , then $j$ has the reciprocal value when<br>compared with $i$ |  |  |

 Table 1
 Saaty's 1–9 scale for pair-wise comparison (Saaty 1980)

**Table 2** Random consistency (RC) index [n = size of the reciprocal matrix]

| n  | 1 | 2 | 3    | 4   | 5    | 6    | 7    | 8    | 9    | 10   |
|----|---|---|------|-----|------|------|------|------|------|------|
| RC | 0 | 0 | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 |

(2) Summed each column vector of  $\overline{A}$ :

$$\overline{w_i} = \sum_{j=1}^n \bar{a}_{ij} \quad (i = 1, 2, \dots, n)$$
(3)

(3) Normalized each vector of  $\overline{W} = (\overline{w}_1, \overline{w}_2, \dots, \overline{w}_n)$ :

$$w_i = \frac{\bar{w}_i}{\sum_{i=1}^n \bar{w}_i} \tag{4}$$

Saaty (1990) suggested using the maximum eigenvalue method to determine the judgment matrix as:

$$A \times W = \lambda_{\max} \times W \tag{5}$$

where  $\lambda_{\text{max}}$  is the maximum eigenvalue of the matrix A.

For a reliable comparison, it is important to note that the inconsistency of the comparison matrix A must be less than 10%. The consistency is defined by relation between the entries of A:  $a_{ij} \times a_{jk} = a_{ik}$ . According to Saaty (1990), the consistency of judgments can also be evaluated using the Eq. (6):

Consistency ratio = 
$$CR = \frac{CI}{RC}$$
 (6)

CI is the consistency index and is defined as:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{7}$$

The random consistency index (RC in Eq. 6) can be obtained from Table 2. Since the column(s) of any  $1 \times 1$  or  $2 \times 2$  comparison matrices are dependent, RC is assumed to be 0. This means division by zero in Eq. (6) and causes CR to tend toward infinity; that is, matrices of sizes 1 and 2 are always consistent (Aminbakhsh et al. 2013).

#### 3 Case Study

This study aimed to evaluate the mud entrance risk in one sector of the world's largest copper-molybdenum undergrounds mine, El Teniente which is owned by Codelco, Chile. Respecting the reserve size, El Teniente is classified as the sixth largest copper deposit in the world. This mine is located at the 70 km south-southeast of Santiago in the Andes mountain range, Chile. The mine uses block and panel caving methods to extract ore with the daily production rate of about 140,000 tonne and the mean grade of 0.86% copper. Mining is carried out at different levels around a non-mineralized formation called the Braden Pipe that houses mining infrastructure of each level. The current mine level contains six mining blocks around the Braden Pipe at different elevations including the Esmeralda, Reservas Norte, Diablo Regimiento, and Pipa Norte mining blocks (Fig. 1).

Located in Ands mountain range, the main source of water inflow is surface water, especially winter snow melting during spring (Ferrada 2011). Additionally, underground water and the water which is used for hydraulic fracturing are the other sources of water appearance in the operating area. Drainage system is piping water along the main conveyor belt to the surface. Among the 14 operating sectors in 2011, five sectors have been faced with the problem of mud entrance of which Diablo Regimiento is one of them.

A statistical study of humidity, size distribution, and production characteristics of 227 draw-points at Diablo Regimiento (D.R) sector was accomplished in this research. D.R sector is in the south part of the deposit. This sector is divided into five planning phases and is placed under three previous mined sectors, of which one of those (Regimiento) had been closed due to the existence of mud. Mud entrance study of this sector showed that the initial entry of mud into D.R was due to the connection of this sector to the upper mined sectors. Once mudflows into the caved column, it immigrated laterally into other parts of D.R based on draw control and mine planning strategies.

#### 4 Mud Inflow Risk Affecting Parameters

To evaluate the risk of mud inflow at D.R, a risk-based hierarchy was constructed considering the potential considering the potential risk items threatening the safety of mining operation in terms of mud inflow into the extraction points. The influence of each parameter in mud inflow was studied based on the historical database of the mine. The hierarchy was constructed comprising three criteria, each of which was then divided into different sub-criteria (Fig. 2). The results of statistical analyses of all these elements were used to make the pair-wise comparison among them.



Fig. 1 Location of different sectors at El Teniente mine (CODELCO 2009)

## 4.1 Fine Material

According to Butcher (2005), the percentage of fine material in extraction point is one of the four required factors to provoke mud rush in cave mining. Considering the caving circumstances, generation of fine material in extraction points is an inevitable process, due to extended friction between fragments, secondary fragmentation, geological conditions, and fine migration. The common size classification in the D.R sector is presented in Table 3. Generally, the material with the size less than 5 cm is considered as fine material. Historical review of several underground mines shows that the size less than 5 cm and/or less than 25 cm are the most perilous sizes in mud generation.

#### 4.2 Water Content

The other important parameter to produce mud, which was identified by Butcher (2005), is the existence of water. Water has adverse influence on the mud entrance development. Without water, there is no risk of mud rush, even if all of the other criteria are existing (Jacubec and Clayton 2012). Likewise, in the absence of fine material, the water will flow through the bulk material without any risk of mud rush occurrence. Thus, these two factors should be measured regularly at each extraction



Fig. 2 Hierarchy of mud inflow risk affecting parameters

Table 3 Size distribution classification of different status of draw-points (with mud and dry)

| Size classification   | No. of data |       | Average percentage |       | Standard deviation |       |
|-----------------------|-------------|-------|--------------------|-------|--------------------|-------|
|                       | With<br>mud | Dry   | With<br>mud        | Dry   | With<br>mud        | Dry   |
| Less than 5 cm        | 68          | 20182 | 60.88              | 30.51 | 21.94              | 24.95 |
| Between 5 and 25 cm   |             |       | 26.91              | 33.77 | 12.49              | 17.41 |
| Between 25 and 50 cm  |             |       | 9.63               | 21.75 | 12.20              | 16.70 |
| Between 50 and 100 cm |             |       | 2.43               | 9.49  | 6.94               | 15.37 |
| More than 100 cm      |             |       | 0.15               | 4.04  | 1.21               | 12.64 |

**Table 4** Humidityclassification in El Tenientemine

| Class code | Qualitative expression         | Moisture percentage |  |
|------------|--------------------------------|---------------------|--|
| 0          | Dry                            | 0%                  |  |
| 1          | Low humidity                   | Less than 4%        |  |
| 2          | Humid                          | 4–7%                |  |
| 3          | Mud incipient                  | 7–10%               |  |
| 4          | Mud                            | More than 10%       |  |
| Α          | Coarse material and water flow |                     |  |
|            |                                |                     |  |

| Material size (G) ≤ 25 cm    |                                     |  |  |  |
|------------------------------|-------------------------------------|--|--|--|
| G < 30%<br>(Coarse material) | $30\% \le G < 70\%$                 | G ≥ 70%<br>(Fine   |  |  |
|                              |                                     |  |  |  |
|                              |                                     |  |  |  |
|                              |                                     |  |  |  |
|                              |                                     |  |  |  |
|                              | Mat<br>G < 30%<br>(Coarse material) | Material size (G) $\leq$ 25 crG < 30%<br>(Coarse material) $30\% \leq$ G < 70% |  |  |

Table 5 Wet muck classification matrix at El Teniente copper mine

| Normal condition |  |
|------------------|--|
| Mud observation  |  |
| Critical risk    |  |

point. In El Teniente mine, draw-points are classified into five groups regarding the humidity percentage (Table 4).

The last two classes in Table 4 (classes 3 and 4) are defined as the highest risk of mud entrance into the operation area. According to Tables 3 and 4, a risk matrix was developed by the risk evaluation center to manage the mud rush risk at the mine (Table 5). Based on Table 5, the draw-points with a high risk of mud entrance are closed to prevent the hazards of mud rush.

## 4.3 Topography

Surface crater monitoring is one of the preventive measurements in the operational mud rush risk assessment. In the case of caving operation, the internal and external mud rushes are likely to occur because they connect to the surface with a caved zone which provides a potential point of entry for water and mud. On the positive side, when mines go dipper, correlation between topography and external mud rush inflows diminished. However, the connection of caved column with previous mined sectors would be influential. Figure 3 shows the surface topography and the mud advance zone in D.R. It is obvious that mudflow appears in draw-points where located beneath the subsidence zone with the less distance from the potential area of water accumulation in the surface.



Fig. 3 Surface topography and mud advance zone in D.R

# 4.4 In Situ Height

Analyzing different conditions of the first mud observation in a draw-point shows the significant influence of extraction strategy and height of broken column on mud entrance into the working area. Figure 4 illustrates the mud presence date and extraction height of draw-points in D.R. from 2009 to 2013. It is obvious in this chart that the mud is discovered at the height of more than 100 m which is the height of in situ column, between D.R sector and its upper sector, Regimienro (R). Thus, it can be concluded that the risk of mud entrance when extraction height is less than the in situ height is almost zero.



Fig. 4 Extraction height of the first mud appearance in draw-points at D.R



Fig. 5 North-south section of extraction height profiles along the first mud "mud-water" status draw-point at D.R from 2005 to 2013

## 4.5 Uniformity of Draw

To characterize the mudflow behavior, it is required to consider drawing manner of each draw-point over time. Based on the extraction height profile, it is possible to qualitatively evaluate the uniformity of draw by determining how a profile is equidistance comparing to its previous profile at a certain time.

Figure 5 shows the extraction height profile along north–south sections of the first "mud-water" status draw-point at D.R during 2005–2013. This draw-point (with the ID number of 23–27-H) has been closed on March 2009, due to the high risk of mud rush. In this figure it is obvious from the annually extraction heights that a specific draw control discipline were applied a no uniform profile with the objective of providing arch failure to insure progressive caving. However, implementing this type of drawing strategy yields preferential flow of mud, which caused mud inflow from the zones with maximum profile height toward the lowest points in the profile (Fig. 5).

#### 4.6 Caving Initiation Strategy

Production planning of D.R was carried out with the objective of creating a continuous propagation of caving by generating the active volume in the center of the sector. Therefore, extraction was started from the specific draw-points in the center of the first production phase of the sector, and the broken "arch" reaches the former sector as shown in Fig. 6. The historical data analyses showed that mud entrance appeared in the center of the sector when the active volume reached the overlying mine sector.



Fig. 6 Status of extraction zone at the time of caved zone's connection to the former mining sector

#### 4.7 Status of Extraction Point

According to Table 4, draw-points with the most probability of mud rush occurrence are classified as "mud-water status" and have been closed permanently to inhibit the entrance of mud to the working area. Preventing mud entrance, to the working area, results in mud distribution into the adjacent extraction points. Under those circumstances, the neighborhood draw-points would have the hazards of lateral immigration of mud. The extraction rate of these points should be controlled to avoid mud migration into the operative extraction points. For this reason, these draw-points are allocated as "limited status." Furthermore, some draw-points are used as barrier to control the entry of mud. These draw-points are labeled as "barrier status." In this status, the content of moisture and fine material are not essentiality critical. However, they are considered as high-risk draw-points regarding the flow direction of mud.

Owing to the dynamic condition of caving procedure, the status of extraction point changes continuously during extraction. Figure 7 illustrates mud distribution to different draw-points during five-year operation. It is obvious in Fig. 7 that the status of draw-points at the vicinity of "mud-water" draw-points changed and mud distribute into other points and change their status during operating years.



Fig. 7 Mud appearance sequence in D.R from 2009 to 2013

## 4.8 Extraction Rate

Controlling the tonnage drawn from individual draw-points could avoid creating the conditions that could lead to mud rushes (Laubscher 2000). Analyzing the extraction rate of different draw-points showed that the frequency distribution function of extraction rate at draw-points with late mud entrance was almost uniform. Moreover, the cumulative distribution function (CDF) of extraction rate for different draw-points revealed that the extraction rate of early mud entrance draw-points. According to this study, increasing the extraction rate in a uniform manner will significantly decrease the mud inflow risk.

#### 4.9 Draw-Point Location

The historical study of mud entrance into the draw-points at D.R sector shows that these draw-points are mostly located beneath the former mining sectors with mud occurrence. Figure 8 shows a plan view of D.R in which the position of muddy draw-points is compared with the muddy draw-points in its upper sector (Regimiento-Pink bullets). It is obvious in Fig. 8 that the draw-points which are placed under the muddy former levels have a higher risk of mud entrance than others.



Fig. 8 Location of muddy draw-points in D.R and Regimiento sectors





# 5 AHP in Mud Inflow Risk Assessment

The AHP framework was illustrated in Fig. 2 which arranged the most influenced parameters on mud inflow into three main categories. The objective of this framework is to provide a decision tool to determine the adequate investments for

| Criteria          | Sub-criteria                  | Odd<br>ratio |
|-------------------|-------------------------------|--------------|
| Operational       | Extraction rate               | 13.4         |
| parameters        | Uniformity of draw            | 1.5          |
|                   | Caving initiation<br>strategy | 3.7          |
|                   | Status of draw-point          | 2.8          |
| Design parameters | In situ height                | 1            |
|                   | Draw-point location           | 2.3          |
| Geological        | Fine material                 | 6.3          |
| parameters        | Water content                 | 6.5          |
|                   | Topography                    | 0.98         |



Fig. 10 Priority weights of mud inflow risk influenced parameters

mud inflow prevention. Based on the hierarchy shown in Fig. 2, four reciprocal matrices were constructed to making the pair-wise comparisons among the elements of the hierarchy (Fig. 9).

The first pair-wise comparison in Fig. 9 was made among the parameters on the top level of the hierarchy (criteria). In this comparison, the highest weight was assigned to the operational parameters since these parameters are the most controllable ones, while the lowest weight was allocated to the geological parameters since they are less likely to be able to change. The odd ratio (OR) of the logistic regression between each sub-criterion and mud inflow was considered to make the pair-wise comparison of these elements (Table 6). The odd ratio is one way to

**Table 6**Odd ratio of mudinflow influenced parameters

quantify how strongly the presence or absences of elements in the inflow risk hierarchy are associated with the mud inflow into the working area. If the OR is greater than one, then having that element is considered to be associated with having mud inflow.

In order to insure the consistency of the judgments in all the reciprocal matrices, consistency ratios (CR) were calculated using the largest eigenvalues of the eigenvectors (Eqs. 5–7). Figure 10 shows the normalization weights for each of the elements in the hierarchy as well as the overall weightings. As it is illustrated in Fig. 10, the overall prioritization revealed that the "extraction rate" is the item with the most significant impact. Hence, to prevent mudflow risk, extraction rate requires the most significant investment and consideration among the other elements.

#### 6 Conclusion

Project safety risk assessment is the fundamental component of the project management since the efficiency of mining projects is highly influenced by safety problems such as mud rush, rock burst, air blast. In this paper, a framework was proposed to compare the most effective parameters on mud inflow into the extraction points at El Teniente copper mine, Chile. This method provides a tool for the mine managers to define the priority of influenced items to create a rational budget for mud rush prevention during the planning and feasibility study of block caving mining projects. The proposed framework was applied to a real mining project to illustrate how it can guide the decision makers in mud inflow risk assessment.

According to the overall weight of different elements, it is concluded that extraction rate has the first priority to be taken into consideration to avoid the risk of mud inflow occurrence. The second order belongs to the draw-points which are located beneath the former mining sectors with mud existence. The third preference of caving initiation strategy shows that the mud rush risk should be considered when applying an undercutting strategy. Even though the fine material and water content have a significant effect on mud inflow, these parameters are located in the fifth rank, since the geological parameters had the lowest subordinate weights. It is important to realize that even by applying drainage systems and draw strategies, the persistence of water and fine material is unavoidable in mining operation. However, controlling the extraction rate and uniform draw would decrease the probability of fine percolation and water inflow.

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