Chapter 13
Stimulating Mathematical Creativity through Constraints in Problem-Solving

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13.1 Introduction

In the project entitled *Mathematical Creativity Squared* (MC-squared; www.mc2-project.eu), we began exploring the premise that enabling teachers to be more creative in their design of tasks to be used in their lessons can help their learners in these teachers' classrooms be more creative (see, e.g. Bokhove, Jones, Mavrikis, Geraniou, & Charlton, 2014; Bokhove, Mavrikis, & Jones, 2015). In working on the design of mathematical tasks during the project, we pondered how the use of so-called open problems often appears to be emphasised in deliberations about mathematical problem-solving (Boaler, 1998; Pehkonen, 1997; Silver, 1995; Sullivan, Clarke, & Clarke, 2012; Sweller, Mawer, & Ward, 1983). The reason seems to be that problem-solving is deemed best supported by the provision of open-ended tasks. On top of that, it is further argued that the openness of problems is more conducive to students' mathematical creativity than closed tasks. Silver (1997, p. 77), for instance, argued that the development of learners' 'creative fluency' is 'likely to be encouraged through the classroom use of ill-structured, open-ended problems that are stated in a manner that permits the generation of multiple specific goals and possibly multiple correct solutions'. Similarly, Kwon et al. (2006, p. 51), in their study, concluded that the use of open-ended tasks 'may provide a possible arena for exploring the prospects and possibilities of improving mathematical creativity'.

In this chapter we problematise this idea that problem-solving and creativity are seen as best supported by providing open-ended tasks. In doing so we take a somewhat different, but related, approach to that proposed by Haught-Tromp and Stokes (2016) who describe what they call 'constraint pairs', where one constraint precludes something, while its 'pair' directs the search for a substitute (such as the next

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most efficient strategy) as a way to inform lessons that help students develop creativity. In contrast, we make the case for what might be called 'constraints-based' task design. In this latter approach, which we relate to research in economics on scarcity (and the American television series character *MacGyver*), we show how tasks that are 'moderately closed' (being neither fully 'open' nor fully 'closed') can provide for creative mathematical thinking and problem-solving. In some cases, the use of feedback can provide cues to students. Using examples from a range of topics, we explore examples of 'constraints-based' creativity such as producing geometry constructions solely with ruler and compass, ways of tackling number puzzles and solutions to sets of equations. We conclude that such examples demonstrate that classroom tasks for mathematical problem-solving and creativity need not be restricted solely to open-ended problems; rather, we argue that tasks with suitable constraints can serve as creativity-inducing problem-solving tasks as well.

13.2 Problem-Solving, Creativity and Task Constraints

13.2.1 Problem-Solving Tasks

While problem-solving is a key component of the school mathematics curriculum internationally, the nature of what constitutes the 'problem' (or task) to be solved during mathematical problem-solving is not necessarily straightforward. Borasi (1986), for example, attempted to clarify the notion of a mathematics 'problem' in terms of the following structural elements:

(a) The formulation of the problem; the explicit definition of the task to be performed.
(b) The context of the problem; the situation in which the problem itself is embedded.
(c) The set of solution(s) that could be considered acceptable for the problem given.
(d) The methods of approach that could be used to reach the solution.

Although this clarifies some aspects of the notion of a mathematics 'problem', it leaves much under-explored. For example, Christiansen and Walther (1986, p. 244) argue that even though classroom work on mathematical problems in the form of 'exercises' [which they describe as 'drill and practice in relation to previously described concepts and procedures'] is 'centrally placed at all levels of mathematics teaching', such use 'rests on an inadequate and insufficient differentiation with respect to the relationships between the concepts *task* and *activity*' (ibid., pp. 245–46), where the task is set by the teacher and the activity is what the learners engage in. In this, they argue (ibid., p. 275), the completion by learners of *tasks* that are 'exercises' *does not* 'contribute to a genuine development of knowledge' in terms of learner *activity*, while the use of nonroutine *tasks* provides 'optimal conditions' for cognitive development of new knowledge. Thus, as Boaler (1998, p. 42) explains, supporters of the use of nonroutine tasks contend that 'if students are given open-ended, practical and investigative work that requires them to make their own decisions, plan their own routes through tasks, choose methods and apply their mathematical knowledge, the students will benefit in a number of ways'.

Sullivan et al. (2012, p. 57) differentiate task goals into 'closed' or 'open', where a closed goal 'implies there is only one acceptable response', whereas an open goal 'has more than one (preferably many more than one) possible response'. Table 13.1 provides an illustrative example of what might be considered a 'closed' task (one with a goal that is likely to be closed) and an 'open' task (one with a goal that is rather open). Such different types of tasks reflect different qualities and priorities in task design. This can include the degree to which the task is constrained for the student (for more on this, see Stoyanova, 1998) or the role the task may play in relationship with problem-solving (for more see Silver, 1995). More recently, Yeo (2017) proposed a framework to characterise the 'openness' of tasks with five elements: goal, method, task complexity, answer and extension. All this suggests that 'closed-open' is one of the main distinctions that is made about the form of mathematical tasks that can be used in the mathematics classroom.

In what follows we examine the case for what might be called 'constraints-based' task design by exploring tasks that are 'moderately closed' (neither fully 'open' nor fully 'closed') and how these can provide for creative mathematical thinking and problem-solving. We begin by examining the notion of creativity (both broadly and in mathematics education) and from that go on to explore the role and nature of constraints in mathematical creativity and problem-solving.

13.2.2 Defining Creativity

Creativity is a frequently used term in education in general and in mathematics education in particular. Despite this, and the increasing interest in promoting creativity in all sectors and across all individuals and groups, the term remains a vague and difficult construct both to define and to operationalise (Mann, 2006; Sriraman, Haavold, & Lee, 2014). This is mainly because of the wide range of theoretical approaches and disciplines that have been used to address creativity (Cropley, 1999). There is currently an array of theories and perspectives on creativity that provides critical insights into a better understanding of the phenomenon. For example, in the field of psychology, many co-existing paradigms been developed, ranging from linear to integrative approaches to creativity. Sternberg (2003), for instance,

Table 13.1 Illustrative examples of 'closed' and 'open' tasks

Illustrative example of a 'closed' task	Illustrative example of an 'open' task
You have two dogs; one eats 1 can of dog food per day, while the other eats 3 cans. If a can of dog food costs 80p, what is the total of feed the two dogs each day?	How much does it cost to keep a pet?

identified at least eight frameworks for viewing creativity, from 'mystical' approaches to those entailing psychodynamic, cognitive, psychometric, pragmatic, social personality, evolutionary and confluence models of creativity.

A major distinction that is often made is whether creativity is 'Big-C' creativity (Creativity with a capital C) or 'everyday' or 'little-c' (creativity with a little c). The former, 'Big-C', has a tradition in creativity research that is typically concerned with the exceptionally creative activity of some really very talented but rare individuals (who might get referred to as 'geniuses') (Simonton, 2010). Yet creativity can also be seen as a potential that all people are capable of displaying and which can find expression in various situations of everyday life; this is 'everyday' or 'little-c' creativity (Simonton, 2013, 2017). Boden (1994) provides another term for 'little-c' creativity: 'psychological creativity'. This is when something is identified as creative at least by the creator themselves, without being necessarily an outstanding contribution to some specific domain.

Notwithstanding these distinctions, most definitions of creativity found in research literature (Runco & Albert, 1990; Runco & Pritzker, 1999; Kaufman & Sternberg, 2010) include two structural elements: (1) novelty (originality, unexpectedness) of the creative work and (2) its value (relevance, appropriateness, significance, usefulness, effectiveness). These elements are apparent in the *Handbook of Creativity* (Kaufman, Glăveanu, & Baer, 2017; Sternberg, 1999; Kaufman & Sternberg, 2010), which summarises contemporary creativity research. Some definitions that show this common pattern are:

1. 'Creativity is the ability to produce work that is both novel (i.e. original, unexpected) and appropriate (i.e. useful, adaptive concerning task constraints)' (Sternberg & Lubart, 1999, p. 3).
2. '[our definition] involves novelty and value: The creative product must be new and must be given value according to some external criteria' (Gruber & Wallace, 1999, p. 94).
3. 'A creative idea is one that is both original and appropriate for the situation in which it occurs' (Martindale, 1999, p. 137).
4. 'Creativity from the Western perspective can be defined as the ability to produce work that is novel and appropriate' (Lubart, 1999, p. 339).

The 1999 edition of the handbook contained a comparison table compiled by Mayer (1999, p. 450) which shows how these two elements are prevalent in definitions of creativity.

While these existing definitions of creativity can be used to help specify creativity in mathematics and mathematics education (Mann, 2006), no single and widely accepted definition of mathematical creativity exists. One distinction often made in the definition question is whether it is the *process* or the *product* that is in focus. Hadamard (1945), for example, refers to the mathematicians' creative process using the four-stage Gestalt model: preparation – incubation – illumination – verification. Liljedahl (2013) extended that model by also emphasising the inventive process of mathematical creativity by adding the phenomenon of the AHA! experience. In particular, Liljedahl and Sriraman (2006) describe mathematical creativity at the school level, as (1) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problem and/or (2) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle. The product approach to creativity focuses on the outcomes that result from creative processes. It starts from the premise that to identify a process or activity as creative, one has to discern the existence of some creative outcome. Sternberg and Lubart (1999) seem to support this view when saying creativity is the capability to produce unexpected, original, appropriate and useful pieces of work. Leikin and Pitta-Pantazi (2013) give a broad overview of the 'state of the art' regarding creativity and mathematics education, arguing that the focus can be on the creative person, the creative process, the creative product or the creative environment. Liljedahl and Sriraman (2006) make the case that the creative process and creative product are inseparable and indistinguishable.

Silver (1997), among others, sees creativity as an orientation or disposition towards mathematical activity that can be broadly fostered in the general school population. This approach is more associated with long periods of work and reflection rather than rapid and exceptional insight. This has consequences for how one might approach creativity as it suggests that creativity-enriched teaching and other support might be conducive for a broad range of students. Here, creativity is not merely something for a few exceptional individuals. This view is often accompanied by thoughts about problem-posing and problem-solving (Bonotto & Dal Santo, 2015; Singer & Voica, 2016).

More specifically, problem-posing and problem-solving are closely related (Singer, Ellerton, & Cai, 2015). It is in the interplay of formulating, attempting to solve, reformulating and eventually solving, a problem that is seen as creative activity. Features of creativity that could be detected within this frame are novelty in the problem formulation or solution, shift in direction during the problem-solving process and number of different solution paths or different solutions.

In his assessments of 'divergent thinking', Guilford's (1967) *Alternative Uses Task* distinguished four components of creativity that could be applied to the domain of creative mathematical thinking:

- Fluency as it relates to the continuity of ideas, flow of associations and use of basic and universal knowledge. This entails the student's ability to pose or come up with as many responses as possible (ideas or configurations).
- Flexibility as associated with changing ideas, approaching a problem in various ways and producing a variety of solutions. It describes the student's ability to vary the approach or suggest a variety of different methods towards a problem or situation.
- Originality/novelty as seeking for unique ways of thinking and unique products of a mental or artistic activity. The criterion pertains to the student's ability to try novel or unusual approaches towards a problem.
- Elaboration as relating to the capability to describe, illuminate and generalise ideas. Such a criterion should assess the person's capacity to redefine a problem or situation to create others.

Sternberg and Kaufman (2010) emphasise another important criterion, namely, ‘quality’ or ‘usefulness’. They describe how, under the criteria of novelty, drawing random lines on a piece of paper could be deemed creative as ‘in all likelihood, no one has ever before created quite the same pattern of lines’ (p. 467). At the same time, it is not a given that random lines on a piece of paper would necessarily pass the test of quality or usefulness. Sternberg and Kaufman assert that this is a definitional *constraint*, if only because novelty is ‘in the eye of the beholder’. In general, there are many constraints on creativity, something which some see as an impediment for creativity. The issue for us in this chapter is the role and impact of constraints in creativity and problem-solving. It is to this issue that we turn next.

13.2.3 Constraints in Creativity and Problem-Solving

Sternberg and Kaufman (2010) contend that the concept of creativity is subject to many constraints. These constraints are, in a sense, already embedded in the definition of creativity itself – work can be creative only if it is both novel and useful in some way. Apart from the constraints of being novel and useful in some way, there also are constraints internal to the problem-solver such as risk-taking or motivation. Society also imposes external constraints. Such internal and external constraints interact. Sternberg and Kaufman (2010, p. 481) give the following example: ‘someone who is constantly beaten down as a result of being creative may give up and simply decide not to be creative again’.

This chapter runs with the idea that constraints do not necessarily harm creative potential. As constraints are part of the construct of creativity itself, we rather see them as a mechanism by which creativity can be stimulated. Here, the words of Sternberg and Kaufman (2010) are pertinent: ‘Many consider the haiku [a traditional form of Japanese poetry that consists of three lines with the first and last lines having five syllables each and the middle line seven syllables] to be an ultimate creative expression precisely because only a handful of words is allowed. What makes a person or product creative is the flair of originality constrained by usefulness, and the benefit of usefulness constrained by originality’ (p. 481).

There are numerous examples where constraints have played a positive role regarding creativity. In an informal way, one can think of how in economics the ‘scarcity’ of certain goods pushes individuals and organisations to be creative (e.g. Legros, Newman, & Proto, 2014). The ‘MacGyver’ meme (humorous illustrations or pieces of text spread rapidly by Internet users) shows how television personality *MacGyver* (from the American television series of the same name) always manages to escape from a serious predicament by ‘just using a paperclip’; the message being that MacGyver’s inventive character can always find a creative solution for any challenge. Yet Amabile and Kramer (2011a & b) would likely not call this ‘scarcity’; rather, they would call it ‘necessity’ because, they say, constraints can ‘stoke the innovation fire’ (Amabile & Kramer, 2011b). A similar view was expressed by Yahoo! CEO Marissa Mayer (2006) writing for *Businessweek*:

'Constraints shape and focus problems, and provide clear challenges to overcome as well as inspiration. Creativity loves constraints, but they must be balanced with a healthy disregard for the impossible'.

Such views are not only apparent in the entertainment and business realm. A study by Marquc, Förster, and Van Kleef, (2011) showed that tough obstacles can prompt people to open their minds, look at the 'big picture' and make connections between things that are not obviously connected. Participants in the study were asked to play a computer maze game. For one group the maze contained an obstacle, severely constraining the number of possible routes to escape. The other group had no obstacle. Using the *Remote Associates Test* as the creativity measure, the group *with* the obstacle performed 40% better. It was hypothesised that the constraint had forced members of the obstacle group into a more creative mindset. Stokes (2001) reports that related experimental research suggests two things. One is that along with learning how to do something, people learn how to do it in different and variable ways, so they can continue doing it. A second thing Stokes shows is that constraints play a role because high variability is caused by constraints. Stokes gives the example of Claude Monet's painting: his high level of variability in painting was acquired during the first part of his life and was maintained throughout his adult career by a continuous series of task constraints imposed by the artist on his own work.

In her book, Stokes (2005) lists different types of constraints. The first set of constraints is domain constraints. Individuals in any field can only be creative if they first acquire expertise in the field. Acquiring expertise requires some agreed-upon performance criteria of a field, criteria that are seen as goal, subject and task constraints. Goal constraints specify a particular style, subject constraints involve content and task constraints refer to the particular materials that are used in a domain. It is seen as the foundation upon which variations can be produced. A second set of constraints concerns cognitive constraints. These pertain to the limitations of the human mind. It is here where developing expertise is relevant as this involves overcoming cognitive limitations. A third set of constraints is about variability constraints. These specify how differently something must or should be done. This suggests that maintaining a high-variability level is likely to be conducive for divergent thinking. Finally, the fourth set involves talent constraints. A domain might require special talents and capabilities. If someone does not have such talents and capabilities, this is likely to constrain their capacity to achieve, whereas if someone does have the talents and capabilities, this is likely to help.

Rosso (2014) notes there is a paradox in the tension between freedom and constraint in the creative process. Although, in the abstract sense, there is the view that the ideal creative process is unstructured, open-ended and free of external limitations, there is evidence that, in some cases, creative individuals and teams can benefit from constraints. Here we have space to elaborate on only some of the many different types, with one of the most researched ones being *time constraints*. Typically, the presence of deadlines has been seen as a negative influence on creativity as it is not considered conducive to exploration and tends to reinforce fixed ways of working (Amabile, 1996). Nevertheless, there also are indications to the contrary that suggest that time constraints can have a positive impact on creativity, subject to conditions (e.g. Amabile, Hadley, & Kramer, 2002; Baer & Oldham, 2006). One prerequisite though, according to Hennessey and Amabile (2010), is that creators are protected from distractions and the feeling that they are on a mission. The same ambiguous findings are reported for *resource constraints*. While sufficient material resources need to be available to be maximally creative (e.g. Amabile, 1988, 1996), researchers such as Csikszentmihalyi (1997, p. 321) warn that excess resources can also make people too comfortable and that this has a 'deadening' effect on creativity. Another resource aspect that has been studied is the impact of standardised routines and processes. Here again some research indicates that standardised routines and processes might hurt creative efforts, while other studies – especially in team work – indicate that standardised processes can be conducive to creativity. Groups might structure or bound their work in ways that enhance their creativity (e.g. Hargadon & Sutton, 1996; Stokes, 2005).

In sum, empirical research on the impact of constraints on creativity suggests that constraints do not necessarily impede creativity. In fact, subject to conditions, constraints may even be conducive to creativity. In the next sections, we give concrete examples for mathematics education of ways in which this might work. For these examples, we adopt the view, adapting Sternberg, that creative mathematical thinking is a combination of fluency, flexibility, originality, elaboration and usefulness.

13.3 Cases Illustrating Task Constraints

13.3.1 Case Example 1: Restricting Geometry Tools and Operations

Our first case example is about restricting geometry tools and operations. There are three ancient geometry problems that are considered to be extremely influential not only in the development of geometry but also on mathematics as a field (Coxeter & Greitzer, 1967; Sibley, 2015). These problems, each of which could be tackled using only a compass and straightedge, are the following: squaring the circle, doubling the cube and trisecting an angle.

The problem of 'squaring the circle' was to construct a square with the same area as a given circle; see Fig. 13.1 for an illustration. While the ancient Greeks did find exact constructions for this problem (and the other problems listed above) using methods beyond a straightedge and compass (it is less clear that in ancient times such constraints were imposed), nowadays, these classic problems such as 'squaring the circle' are formulated as problems which have to be solved using solely a compass and straightedge (for more on these classic problems from a computer-orientated position, see Meskens & Tytgat, 2017).

In tackling geometrical construction problems solely with compass and straightedge, the argument is that this provides a rich environment for problem-solving

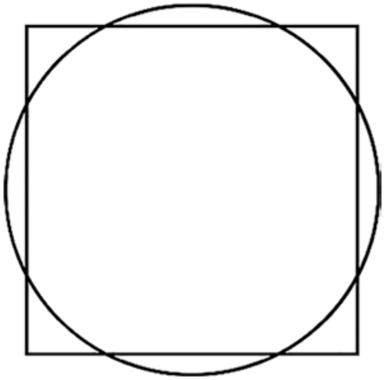

Fig 13.1 'Squaring the circle'; construct a square with the same area as a given circle using only a compass and straightedge

because 'the restriction to using compass and straightedge only, 'forces' the solver to exercise higher-order thinking skills such as analysis, evaluation, hypothesising, [and] organising' (Lim, 1997, p. 144). This remains the case even though in the nineteenth century it was proved that the three constructions listed above are actually impossible to solve with only a compass and unmarked straightedge.

In contemporary mathematics curricula, it is ruler (a marked straightedge) and compass constructions that continue to be specified for classroom teaching. So-called 'standard' constructions include constructing the perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point and bisecting a given angle. This continues to be the case even though forms of dynamic geometry software (DGS) (such as Cabri, Cinderella, GeoGebra, Geometer's Sketchpad and others) have been available since the mid-1990s. In such software, there is the decision of the designer about which menu items to provide in the software (such as a perpendicular line or a circle through three points), and then there is the decision of the task designer (or the teacher) about whether or not to constrain the choice of menu items that is available to learners (Jones, Mackrell, & Stevenson, 2010; Mackrell, 2011).

For example, Mariotti (2001) reports on a research project where the decision was made to start the teaching with the software having a menu of tools that was empty. Menu items were subsequently introduced only after having been the subject of classroom discussion. Mariotti gives the example of the menu item giving a perpendicular line. This, says Mariotti, 'incorporates the piece of theory concerning the definition of perpendicularity and the theorem validating the existence of a perpendicular line', which 'may too complex [for learners] to grasp all at once' (p. 715). The approach of beginning with an empty menu, and only adding menu tools when these had been explored and understood, is one way of designing geometrical construction tasks that have constraints on what learners do. The reason for having constraints, as Kortenkamp and Dohrmann (2010, p. 59) describe, is the tensions within problem-solving: 'if students learn to combine several steps into a more complex operation (say, instead of using the compass twice and connecting the intersections to find the midpoint between two points, one can use the midpoint operation provided by the software) this reduces their workload when doing constructions, but it increases the complexity of their software use'.

The issue of constraints in geometry problems is not restricted to DGS. Figure 13.2 provides an example of a project created using the software *Desmos*, while Fig. 13.3 shows a task from our enGasia project (http://engasia.soton.ac.uk). In the latter, the number of 'stars' indicates the number of different proofs that are valid. Returning to the issue of the provision of menu items in DGS, there are parallels that can be drawn between this issue and the notions of procedures and sub-procedures in the 'turtle geometry' microworld available in *Logo* (for more on procedures, and sub-procedures, in *Logo*, see Papert, 1972).

In each of the examples in this section, tasks are set in a constrained domain, with constrained operations. Nevertheless, in each case there are multiple ways to approach and solve the tasks. The latter is the space for creativity.

13.3.2 Case Example 2: Operations and 'Countdown'

Our second case example focuses on how elements of a popular TV game show illustrate elements of constraints in creativity and also how similar elements are apparent in two online digital applications (the WisWeb application 'number factory' and a digital book from the MC-squared project). In addition, for the latter application, we show how the constraints can be operationalised for creativity.

Countdown is a British game show involving word and number puzzles that is based on the French game show *Des chiffres et des lettres* (Numbers and Letters), created by Armand Jammot. The game show has a so-called numbers round in which a contestant is asked to select 6 of 24 shuffled tiles. There are two groups of numbers: one group with four 'large numbers' (25, 50, 75 and 100) and one group with 'small numbers' with two of each of the numbers 1–10. The contestant first

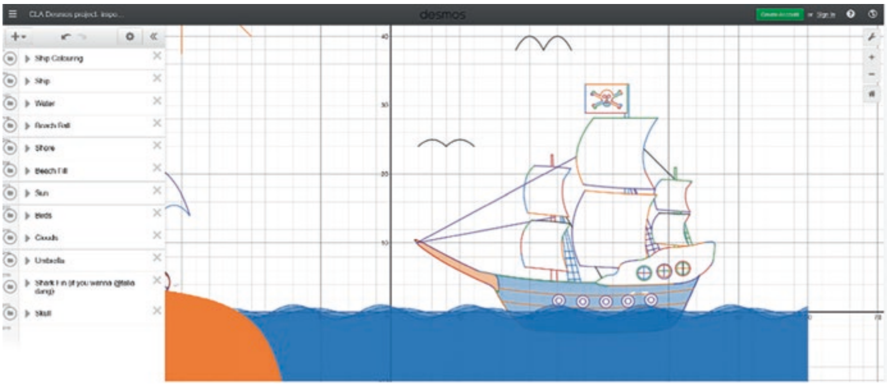

Fig. 13.2 An example of a project created with *Desmos* (and used with permission); see https://www.desmos.com/calculator/umgyltokag

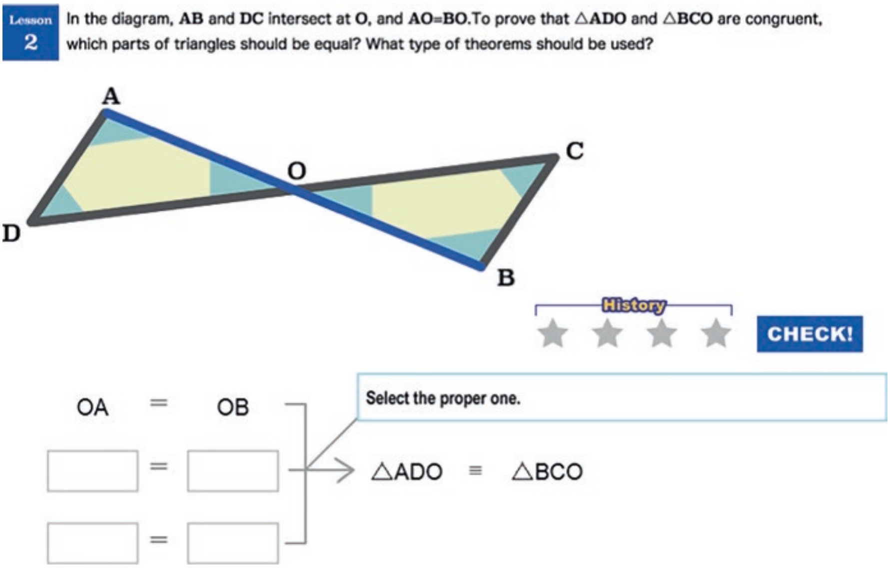

Fig. 13.3 An example of a task from the enGasia project (http://engasia.soton.ac.uk/); used with permission

chooses how many large numbers he or she wants, ranging from zero to four; the rest will be a small number. Then a random three-digit target is generated, and the contestant is asked to find a calculation with the six numbers that gets as close as possible to the target number using addition, subtraction, multiplication and division. Not all numbers need to be used; fractions are not allowed, only positive integers. In some games, there are multiple solutions; other games are unsolvable. This aspect demonstrates how constraints can lead to creativity, after all there are severe constraints regarding numbers, operations and time (in the game show a contestant has 30 s to come up with an answer). Yet this does not hold back the possibility of creative answers, as this actual example of a solution demonstrates (for the TV episode, see https://www.youtube.com/watch?v=pfa3MHLLSWI):

Numbers given: 25 50 75,100 3 6
Target: 952

The contestant came up with:
100 + 6 = 106
3*106 = 318
318*75 = 23,850
23,850–50 = 23,800
23,800/25 = 952

In the 'numbers round' game, Alliot (2015) shows the intricacies by providing a complexity analysis of the game, an analysis of solution algorithms and the presentation of a new algorithm that increases resolution speed by a factor of 20.

The core idea behind the numbers round is also the basis for a WisWeb application of ‘number factory’, which has been used in Dutch mathematics education. As with *Countdown*, there are numbers and operations, and a student has to make the target number.

Figures 13.4 and 13.5 show screenshots of a digital book, a so-called c-book, from the MC-squared project (www.mc2-project.eu). Figure 13.5 shows a task that

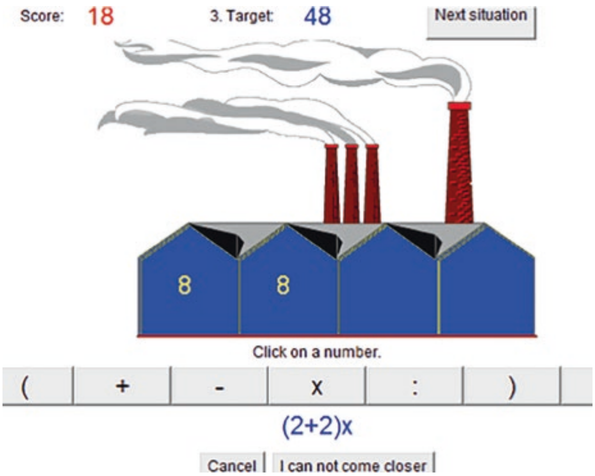

Fig. 13.4 Screenshot of a digital book (‘c-book’) with the 'number factory' application. (From WisWeb, www.wisweb.nl and the MC-squared project; used with permission)

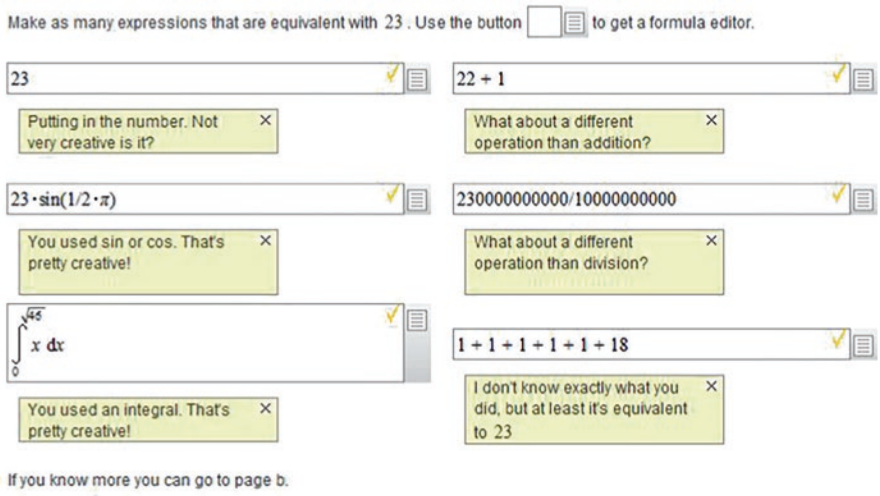

Fig. 13.5 Screenshot of a digital book (‘c-book’). (From the MC-squared project; used with permission)

is proposed, in this case ‘make 23’, along with prompts and responses from the c-book system. The chosen number can be randomised, so it shows a different number, or even expression, every time a student visits the page. The student is presented with several boxes and asked to give as many expressions that are equivalent to the target number. The example answers, not from a real student, by the way, but proof of concept, show that the solution strategies can be more varied. This is the case because there are fewer constraints on the operations. The figure shows answers with numbers, basic operations, trigonometric functions and even integrals. If desired, the generation of these answers could also be subjected to time constraints. In this particular case, feedback can be provided as well, to further stimulate students to think more widely than just the current answers. In our view, this again gives an example of how constraints might be used to trigger creativity. A decent amount of ‘closedness’ (and a balancing amount of openness) seems helpful.

The fact that creativity is there can also be seen by applying the creativity criteria described previously: fluency, flexibility, originality/novelty and elaboration. A lot of fluency (i.e. the student’s capacity to pose or come up with as many responses as possible) is required. The whole point of the key activity in the c-book is to provide as many equivalent expressions to a given expression. With regard to flexibility (i.e. the student’s capacity to vary the approach or suggest a variety of different methods towards a problem or situation), the task provides the opportunity for students to provide many alternative answers. This is demonstrated in the figure as well: integral signs, trigonometric functions and many solutions are acceptable. Of course, this does not mean that students necessarily do this, and this is where feedback could help to trigger more creative thinking. This aspect also touches on originality/novelty, as the student is allowed to try novel or unusual solutions (in this case, equivalent expressions) towards a problem or situation. Elaboration, in this task, has not been catered for yet but could be addressed by making variations of this page with a progression from simple numbers towards more generalised algebraic expressions.

13.3.3 Case Example 3: Diophantine Equations

Our third case example of creativity through constraints is the mathematics game *Wuzzit Trouble* from the company *InnerTube Games*, a venture initiated by Keith Devlin (from Stanford University and known as *The Math Guy* on US National Public Radio). With this game, Devlin builds on work from his book *Mathematics Education for a New Era* (Devlin, 2011). The free game follows the structure that is well-known from many modern ‘apps’ such as *Angry Birds*: there are levels (75 in total in the case of *Wuzzit Trouble*) that require the user to solve a puzzle. While the storyline is about Wuzzits, ‘cute characters’ that have to be saved, the better the user solves each puzzle (in the case of *Wuzzit Trouble*, this relates to the minimum number of moves), the more ‘stars’ are earned. The differing numbers of stars to be collected can be seen as a way to encourage multiple solution strategies.

Fig. 13.6 The mathematics game Wuzzit Trouble. (From the company InnerTube Games; used with permission)

The levels themselves consist of target numbers that need to be constructed by turning a cog. In the case shown in Fig. 13.6, the first target number is 5. The cog can be turned to the left and to the right. The keys have to be collected by making the numbers by turning the cog.

The topic of the game, through using cogs, entails integer partitions (though not, of course, in the formal sense). Actually the game relates to Diophantine equations, polynomial equations of the form that seek whole number solutions – the most famous example being the problem in Fermat's Last Theorem (does $a^n + b^n = c^n$ have any integer solutions for $n>2$?). Players are unlikely to feel that they are solving these equations; rather it is a puzzle. The tension is immediately apparent: there is a constrained domain, namely, integer partitions, with constrained operations, namely, turning cogs. Nevertheless, there are multiple ways to solve the tasks.

13.4 Discussion

In this chapter we examined a number of mathematical problem-solving scenarios, ranging from producing geometrical constructions to ways of tackling number puzzles and solving sets of equations. Through our analysis of these forms of tasks, we argue that classroom tasks for mathematical problem-solving and creativity need not be restricted solely to open-ended problems but that tasks with suitable constraints can serve as creativity-inducing problem-solving tasks as well.

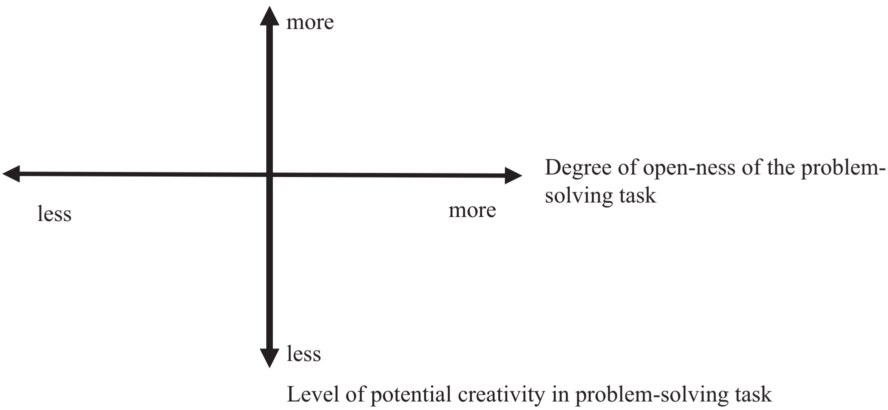

Fig. 13.7 Task openness and level of potential creativity

Notwithstanding the categorisation of Sullivan et al. (2012, p. 57) that a task 'goal' (taken as 'the result that students seek as a product of their activity in response to the task statement') can be open or closed, perhaps tasks can be positioned on a two-way grid, as shown in Fig. 13.7, in terms of the degree of 'openness' (on one axis) and the level of potential creativity in problem-solving task (on the other axis). This entails a more nuanced analysis of task design than an 'open' versus 'closed' opposition, or an opposition of creativity versus problem-solving, or creative versus noncreative. This would likely imply something more sophisticated than 'more open is more creative', or 'less open (so more closed) is less creative'. Of course it is not necessarily the case that tasks are more or less open; it might be preferable to talk about being less or more creative with less or more open tasks, in other words, moderately closed (or moderately open) tasks. As such, it may not be a case of placing the tasks discussed above at specific locations on the graph in Fig. 13.7; it may be more that the axes in Fig. 13.7 could be helpful in considering learner activities with any one of the tasks (and with other moderately closed, or moderately open, tasks).

This approach of relating task openness and level of potential creativity aligns with Lubart and Mouchiroud (2003, p. 142) who show how 'all problem-solving is not creative problem-solving' and that 'there exists a continuum' between the two extremes of open-closed in that 'some problem-solving relies heavily on existing [known] procedures but [also] requires some novelty, some enhancements to existing structure'. Mason (1991, p. 14) suggested that task qualities such as interesting or relevant, open or closed, belong 'not to questions, but arise only in the presence of people', while Guilford (1967) proposed that 'real' problem-solving involved actively seeking and constructing new ideas that fit with constraints imposed by a task or more generally by the environment. Nevertheless, there are also dangers; Wu (1994, p. 122-123), for example, argued that while the use of open-ended problems started off as 'a well-intentioned pedagogical device', the issue of mathematical substance 'got lost somewhere' such that there is a 'very real possibility of [open-ended problems] being an educational liability instead'. Another parallel is with the notion of ‘moderately challenging’ tasks developed by Carreira, Jones, Amado, Jacinto, and Nobre, (2016) in their project on mathematical problem-solving with technology. Also related is the proposal of Yeo (2017) to characterise the ‘openness’ of tasks.

All this raises a question regarding the way in which ‘creativity’ is often measured using the pioneering work of Torrance (1962) and the notions of fluency, flexibility, originality and elaboration. Our reconceptualization of creativity through constraints might indicate that such a measure of creativity may need some revision. Here the work of Ohlsson (2011) might be useful as he identifies four issues that he suggests need to be addressed by a successful theory of creativity: how are novel ideas possible; what are the key features that distinguish creative processes and justify calling them creative; and what gives direction to the creative process, what are the limiting factors, and why is it difficult to create.

13.5 Conclusion

In this chapter we have subjected to critical scrutiny the idea that problem-solving and creativity are seen as best supported by providing open-ended tasks. In doing so we have made the case for what might be called ‘constraints-based’ task design. In this latter approach, which we have related to research in economics on scarcity (and the American television series character *MacGyver)*, we have examined how tasks that are moderately closed (being neither fully open nor fully closed) can provide for creative mathematical thinking and problem-solving. In some cases, the use of feedback can provide cues to students.

Based on a range of examples from across a number of mathematical topics, our argument is that such examples demonstrate that classroom tasks for mathematical problem-solving and creativity need not be restricted solely to open-ended problems; rather, we argue that tasks with suitable constraints can serve as creativity-inducing problem-solving tasks as well. As Sullivan et al. (2012, p.14) argue, this entails classroom tasks that ‘provide appropriate contexts and complexity; that stimulate construction of cognitive networks, thinking, creativity, and reflection; and that address significant mathematical topics explicitly’. As we have shown in this chapter, based on our experience of designing task on the MC-squared project (www.mc2-project.eu), and on other projects such as enGasia (engasia.soton.ac.uk), moderately closed (or moderately open) tasks meet these requirements and can serve as creativity-inducing problem-solving tasks.

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