Chapter 5
Anti-cover

5.1 Introduction

The anti-covering location problem (ACLP) is a well-recognized coverage-based dispersion model. Admittedly, reaching this conclusion requires a little work, but in fact this problem is related to the node packing, vertex packing, stable/independent set and *r*-separation problems, with considerable attention being devoted to each of these related problems (see Padberg 1973; Erkut 1990; Nemhauser and Sigismondi 1992; Murray 1995; Erkut et al. 1996; Murray and Kim 2008; Niblett 2014; Niblett and Church 2015). The name *anti-cover* can be attributed to Moon and Chaudhry (1984) who attempted to distinguish it from other well-known coverage problems. The name, therefore, reflects a sort of opposing goal compared to the set covering problem. The anti-covering location problem seeks to maximize the total weighted benefit of facilities sited in a region, doing so in a manner that ensures at least a minimum pre-specified distance or travel time between facilities and demand is maintained. If the benefit is the same for each potential facility location, then this is equivalent to maximizing the number of facilities that can be sited while maintaining minimum separation restrictions between all facilities and demand or between a sited facility and all other sited facilities. Of course, the goal of the location set covering problem detailed in Chap. 2 is to minimize the number of facilities needed for complete coverage of all demand, assuming the costs for selecting facilities is the same for every potential site. In this sense, then, the two problems have contrasting intents.

A host of practical planning contexts are recognized where the ACLP reflects goals and planning needs, including community impact assessment, service and trade area delineation, safety and security, environmental protection, military defense, forestry, water well design, habitat carrying capacity, etc. Few would argue the importance of siting nuclear facilities away from people and recreational spaces. A retail outlet would certainly not want another franchisee nearby that impinges on their consumer market area, possibly driving them out of business.

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Positioning two munitions depots next to each other would not be strategically sound as an accident, fire or attack at one would likely ignite the other. Excessive disturbance of a woodlands, forested area or watershed that harms flora and fauna is likely unsustainable. There are many other examples as well, all highlighting the significance of the ACLP across a range of application domains.

The literature associated with anti-covering location can be traced back to at least Berge (1957) who provides an early description of the stable set problem. Edmonds (1962) details the internally stable set or a packing problem, seeking the largest subset of vertices on a graph of vertices where no two vertices are joined by an edge. In this sense, the anti-covering location problem is nothing other than a node/vertex packing problem, or a stable/independent set problem, applied in a highly geographic context. This geographic context is highlighted in Moon and Chaudhry (1984), Murray (1995), Murray and Church (1996), Erkut et al. (1996) and Church and Murray (2009). The significance of the geographic context is that subsequent evaluation is necessary to transform place and distance into a network of nodes and arcs. With this, the node packing problem (or any other naming convention) arises. For convenience these inter-related problems are simply referred to as the anti-covering location problem.

5.2 Separation Context

The general problem of interest in this chapter involves the need to simultaneously site multiple facilities of the same type that provide some sort of service. Such a facility might be a power generation plant, like a nuclear reactor or coal fired operation. In this case the facility is often viewed as noxious, with many people and activities preferring not to be too close by. Moreover, if a facility is nearby, it is clearly undesirable to have other facilities nearby as well since such a possible concentration of facilities increases exposure and risk. Another example of a facility fitting the context here is one involving waste processing, like a dump, recycling center or transfer station. Some view these facilities, too, as being noxious or obnoxious. Alternatively, the facility might be a restaurant or outlet, one that is part of a regional or national chain. Of course, any outlet of the chain would not want their other restaurants too near as it would erode market share. The facility may also be part of military or defense services, such as a missile silo, fuel/munitions depot, etc. Siting these too close together could prove fatal, increasing system vulnerability to an attack or accident. As noted above, there are many, many more types of facilities possible where the common underlying goal is to locate many of these facilities to enhance service provision or access, but recognition that spatial separation is necessary for various reasons. Based on this, the anti-covering location problem can be stated as follows:

Select multiple facilities so as to maximize the total benefit of the facilities sited in a region while ensuring that there is a pre-specified minimum distance or travel time between facilities or between facilities and demand

Note that this problem definition includes two possibilities for enforcing separation: (1) keep each sited facility at least a minimum distance or travel time from all other facilities, or (2) keep each sited facility at least a minimum distance or travel time away from demands. It is also possible to consider the case where both types of separation standards must be met, that is maintain separations between facilities and between facilities and demand. Such a distinction has been highlighted in Francis et al. (1978) and Moon and Chaudhry (1984).

5.2.1 Separation to Avoid Concentration Around Demand

One interpretation of separation is that there is demand for a facility to provide service as long as there is no concentration in any one area (see Grubesic and Murray 2008; Church and Cohon 1976). To this end, one facility in or near a neighborhood is (reluctantly) deemed useful and needed in order to ensure access to the goods and services provided, but more than this would be considered excessive or even dangerous. There are many examples, many of which are NIMBY (not in my backyard) like. Perhaps most prominent are waste processing and recycling stations. Everyone producing waste needs the facility, implicitly and explicitly, but would prefer they be located in another nearby community. Certainly more than one would be considered unacceptable. Similar situations can be observed for rehabilitative and other social services. We need them, but only sparingly due to their localized impacts and/or negative externalities.

To illustrate this particular form of separation, consider Fig. 5.1. A neighborhood area is shown around one location, demand i. Of interest then in this situation is ensuring that no more than one facility be sited in the neighborhood. Imposing this in a model seeking to identify the best locations for facilities therefore requires stipulating conditions that prohibit multiple sited facilities in an area. This is generally done with constraints. The important point is that the constraints would be associated with each demand area, relating facility siting to the impacts on or in neighborhoods.

5.2.2 Separation Between Sited Facilities

In contrast to attention on demand, a second interpretation of separation is to focus solely on the facilities being located (see Zeller et al. 1980; Mealey et al. 1982; Downs et al. 2008; Ratick et al. 2008). Specifically, the intent is to ensure physical separation between sited facilities. This orientation too can be driven by the need to

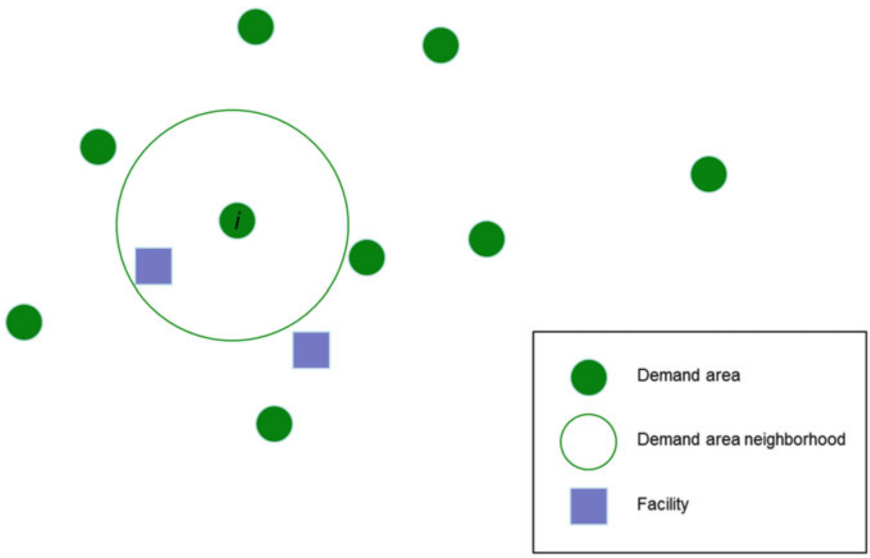

Fig. 5.1 Separation to avoid concentration around demand

minimize local area impacts. For example, environmental impacts associated with forest harvesting should be dispersed, with no two neighboring areas simultaneously scheduled for harvest (Thompson et al. 1973; Mealey et al. 1982; Murray 2007). Alternatively, for safety and security facilities may be prohibited from being too close to each other. Moon and Chaudhry (1984) discuss military installations separated in order to guard against simultaneous enemy/terrorist attack. Ratick et al. (2008) site backup facilities housing critical documents, data, emergency supplies, etc., providing protection in the event of a disaster at the main facility.

To illustrate this second situation, consider Fig. 5.2. An area is depicted around one of the sites, facility *j*, within which no other facility may be sited. In contrast with Fig. 5.1, the area of emphasis is now around the facility, not the demand. What is challenging in this case is that we do not know in advance which locations will be selected for facility placement. This means that restrictions must be structured on a conditional basis. That is, if a site is selected for facility placement, then no other facility may be sited that would be too close. Conditional restrictions are typically structured and imposed using constraints in the model. The important distinction in Fig. 5.2 is that the constraints are associated with potential facility sites, in contrast to constraints associated with demand sites in Fig. 5.1.

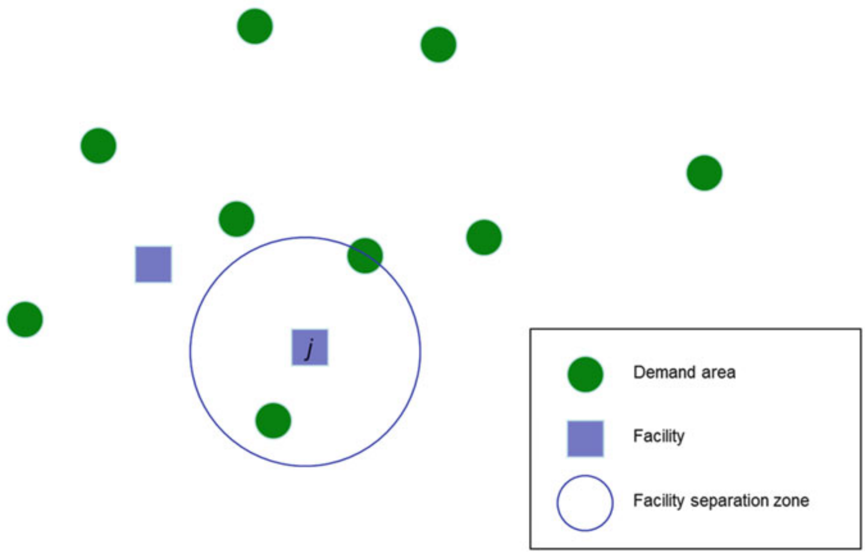

Fig. 5.2 Separation between facilities

5.3 Model Construct

An integer programming based formulation of the anti-covering location problem (as well as vertex packing, the r-separation problem, and other equivalent problems) is possible once spatial processing, analysis and evaluation is carried out. Without loss of generality, we limit the discussion to the case where separation between sited facilities is of concern (Fig. 5.2). Consider the following notation:

j (and k) = index of potential facility sites (entire set J)
α_j = benefit associated with locating a facility or allocating an activity at site j
d_{jk} = distance or travel time between potential facility sites j and k
M = large number
r = minimum required separation
$\Omega_j = \{k | d_{jk} \leq r\}$

$$X_j = \begin{cases} 1 & \text{if facility located at site } j \\ 0 & \text{otherwise} \end{cases}$$

This notation reflects geographic processing that is necessary, typically accomplished using GIS, in order to structure the ACLP. First, one must identify all potential facility sites that would be too close to a facility at site j, and they would be members of the set Ω_j. This is based on the separation requirement imposed, r, but also on the evaluation of proximity between potential facility sites, d_{jk}. Second, one must also derive the benefit associated with siting a facility at a potential site j, α_j.

This might be expected income or return on investment, but also could be safety, security, diversity, etc.

With this information and the above notation, a classic ACLP formulation can be specified (Moon and Chaudhry 1984, Erkut 1990):

$$\text{ACLP1}: \qquad \textit{Maximize} \ \sum_j \alpha_j X_j \tag{5.1}$$

Subject to:

$$MX_j + \sum_{k \in \Omega_j} X_k \le M \quad \forall j \tag{5.2}$$

$$X_j = \{0, 1\} \quad \forall j \tag{5.3}$$

The objective, (5.1), is to maximize the total weighted benefit of facilities located. Spatial limitations on facility configuration are imposed in constraints (5.2), as locating a facility at site j implies that no nearby or conflicting locations may be selected. The core of this model is the set of restriction constraints (5.2). Note that if a given $X_j = 1$ then the value $MX_j = M$, which means that for all members k of set Ω_j, their associated X_k must equal zero in value. Constraints (5.3) impose binary integer restrictions on decision variables.

One can depict this decision making problem graphically, using a network of nodes and arcs. The nodes represent the potential facility sites and the arcs connect those sites that are deemed too close. Figure 5.3 shows a hypothetical case of 29 land parcels in an area, among which we seek to select as many parcels as possible as long as they do not share an edge. Think of this as a land use problem found in the forest industry where the parcels that we select will be harvested. Standards prevent us from harvesting any two neighboring parcels. Figure 5.4 depicts the network interpretation, where parcels are represented as nodes and arcs are between any two nodes (parcels) that would violate intended spatial separation if simultaneously selected for harvesting. In this case there are 29 nodes and 63 arcs. The intent is to select the greatest total weighted collection of nodes (parcels in this case) without any two selected nodes sharing an arc.

While the above proximity definition of separation is based on sharing an edge, other forms are possible as well, such as a minimum distance of separation. But, a prominent interpretation is adjacency, that two facility sites/units sharing a common edge or point are considered too close and therefore cannot be simultaneously selected like in the forestry example. Using the above model notation, we can introduce the restriction of not being able to share a boundary between two selected parcels as follows. Let $d_{jk} = 0$ if sites j and k are adjacent and 1 if not. If $r = 0$, then proximity based on adjacency would be imposed given this definition. The practical interpretation can be illustrated by considering Fig. 5.5 that depicts nine sites, and the network that would result is shown in Fig. 5.6. In this case, there are 16 arcs that reflect spatial separation restrictions in the interpreted network. Again, the intent is to

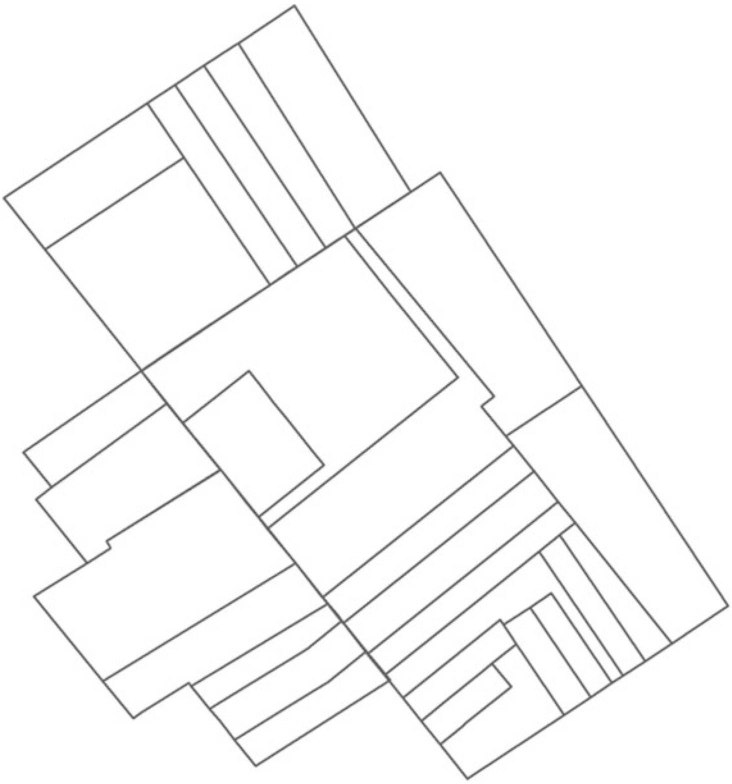

Fig. 5.3 Commercial parcels to select one or more facility sites

select the maximum total weighted collection of nodes without selected nodes sharing a common arc.

5.4 Mathematical Structure

There has been considerable interest in the structure of constraints (5.2) for imposing spatial limitations. Murray and Church (1997) and Church and Murray (2009) refer to (5.2) as a neighborhood adjacency constraint. The reason for this is somewhat intuitive as the set Ω_j represents those potential facilities in the spatial separation neighborhood of potential facility site j. Considerable interest in the structure of constraints (5.2) arises because the ACLP, (5.1–5.3), is recognized as being computationally challenging to solve. Moon and Chaudhry (1984), Nelson and Brodie (1990), Torres-Rojo and Brodie (1990), Yoshimoto and Brodie (1994), Murray (1995), Murray and Church (1995a, b, 1997) and Erkut et al. (1996) all discuss the difficulty of solving even small planning problem applications. This has everything

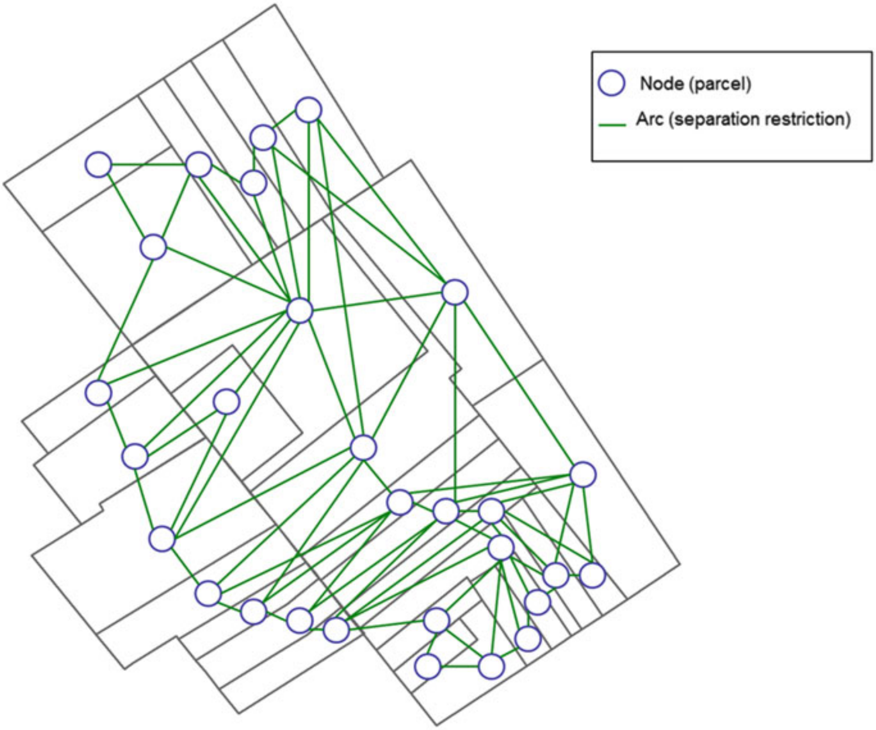

Fig. 5.4 Network representation of commercial parcels and separation restrictions

to do with the inherent mathematical structure associated with constraints (5.2) that impose spatial separation between sited facilities. Simply put, the structure of constraints (5.2) tends to require significant computational effort to solve to optimality, as this structure does not lend itself to natural integer-optimal solutions. The remainder of this section demonstrates that alternative formulations of the ACLP are possible. The significance is that such alternatives have very desirable mathematical properties that enhance their use, especially using commercial integer programming solvers. Thus, not only is it often possible to formulate a particular problem differently, but there may be good reason for considering such alternative models.

5.4.1 *Cliques*

An important concept in mathematical programming is a facet inducing construct known as a clique (Padberg 1973; Nemhauser and Trotter 1975; Nemhauser and Wolsey 1988). For our purposes here, members of a clique are mutually in conflict with each other. For our problem here, we want to ensure that selected facility sites

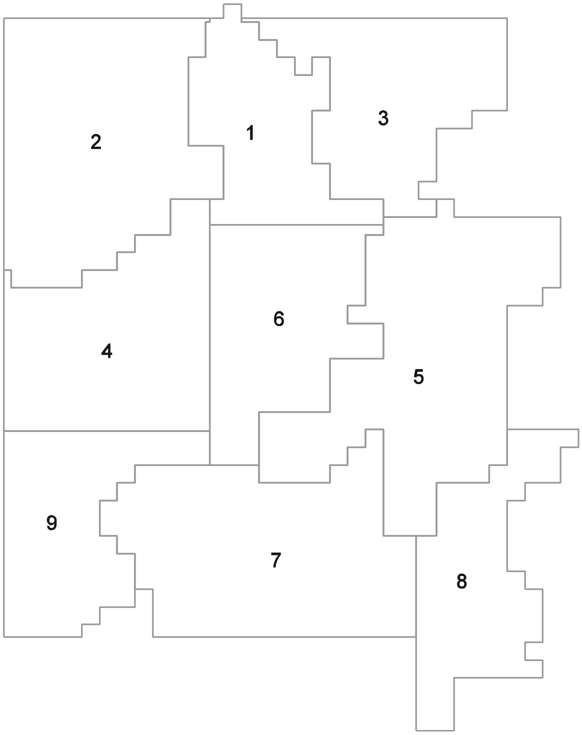

Fig. 5.5 Sites for activity selection

are spatially separated based on an established criteria or standard. Within this context, a clique is a set of two or more members, facility sites, where at most one member may be selected without violating some required facility separation. Relating this to the ACLP formulation above, the implication is that it is possible to identify groups or sets of potential facility sites that share mutual separation requirements. For example, suppose there are three potential facility sites, {1, 2, 3}, and they are all too close to each other, where the selection of one of these sites precludes the selection of any of the other two. More specifically, the decision variable for each potential facility site, X_1, X_2 and X_3, gives rise to the condition that at most one may be selected. Since the decisions are binary, a clique constraint for this set follows: $X_1 + X_2 + X_3 \leq 1$.

Let us suppose then that it is possible to identify all necessary clique constraints to impose all separation restrictions for a particular problem. This could be used to change the mathematical structure in the ACLP. Consider the following additional notation:

l = index of set of cliques (entire set L)
Φ_l = set of potential facility site members of clique l

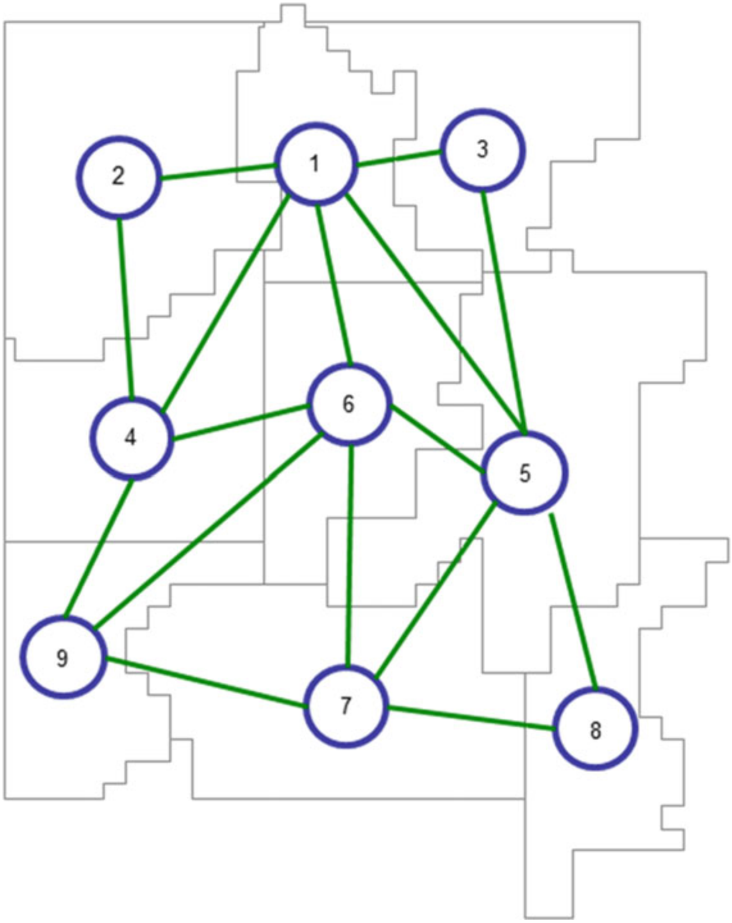

Fig. 5.6 Network resulting from adjacency separation

All that remains is to replace constraints (5.2) in the above formulation of the ACLP with clique constraints. The result is as follows:

$$\text{ACLP2}: \qquad \mathit{Maximize} \ \sum_j \alpha_j X_j \tag{5.4}$$

Subject to:

$$\sum_{k \in \Phi_l} X_k \leq 1 \quad \forall l \tag{5.5}$$

$$X_j = \{0, 1\} \quad \forall j \tag{5.6}$$

The objective, (5.4), remains to maximize the total weighted benefit of facilities located. The spatial separation between sited facilities is imposed in constraints (5.5) through the use of cliques. Constraints (5.6) impose binary integer restrictions on decision variables.

A special case of the clique is a pair of potential facilities that are too close to each other. Consider clique l consisting of potential sites 1 and 2, which means that $\Phi_l = \{1, 2\}$. The associated constraint is $X_1 + X_2 \leq 1$, a subset of the above clique involving potential facility sites 1, 2 and 3.

An entire set of pair based cliques can readily be specified using existing notation:

$$X_j + X_k \leq 1 \quad \forall j, k \in \Omega_j \tag{5.7}$$

This pairwise form is appealing because it is simple, and accounts for all separation conditions originally imposed using constraints (5.2). A potential negative is that the use of this constraint structure, pairs, will result in a relatively large number of constraints, which too has proven to increase computational difficulty when actual planning problems are solved. For superior mathematical structure, it is far more beneficial to have individual cliques with the largest possible number of members (Murray and Church 1997). That is, we would like $|\Phi_l|$ to be a big number, but ultimately the size of cliques is predicated on spatial separation requirements and associated structure that results.

Approaches to identify cliques can be found in Bron and Kerbosch (1973) and more recently Tomita et al. (2006) and Cazals and Karande (2008), among others. For use in the ACLP, the work of Jones et al. (1991), Murray and Church (1997), Goycoolea et al. (2005) and Murray and Kim (2008) offer practical approaches that have proven to be effective. The use of cliques in the ACLP or model extensions has proven to be beneficial in a number of ways, enabling problems to be solved faster as well as larger problem instances to be considered. A potential issue is that identifying a complete set of needed cliques may be computationally expensive. Theoretically, identifying a complete set of cliques involves enumerating all possible cliques. Given this, alternative constraint structures continue to be sought, offering the potential to provide good mathematical structure without excessive computational overhead to identify all cliques.

5.4.2 Hybrid

Recognizing that cliques, (5.5), may result in too many constraints or require too much time to identify, and that the structure of the neighborhood constraint, (5.2), has poor mathematical properties for successful solution using a commercial solver, Murray and Church (1997) explored an alternative constraint structure for imposing spatial separation (see also Erkut et al. 1996). They proposed a hybrid form that combined a maximal clique constraint associated with each potential facility site along with a modified neighborhood constraint structure. Consider the following additional notation:

$\widehat{L}$ = reduced set of cliques $\left(\widehat{L} \subset L\right)$
$\widehat{\Phi}_l$ = set of potential facility site members in clique l
$\widehat{\Omega}_j$ = reduced set of neighbors, $\left\{k \in \Omega_j \middle| k\&j \notin \widehat{\Phi}_l \text{ for any } l \in \widehat{L}\right\}$
$\widehat{\lambda}_j$ = smallest coefficient necessary to impose restriction associated with $\widehat{\Omega}_j$

Effectively, this reflects an attempt to avoid an excessive number and/or excessive search times in clique identification. This approach also adds important mathematical structure to the model formulation and improves the success in solving a problem using standard techniques. Structure is added by adding the best cliques (largest in size) to the original model (5.1)–(5.3). Then, whatever spatial separation conditions are not imposed in the identified sub-set of clique constraints will be imposed using neighborhood like constraints. The tradeoff here is that mathematical structure is being sacrificed somewhat in order to reduce pre-processing effort, while at the same time producing a tighter formulation. The ACLP model formulation with hybrid constraints, a combination of clique and neighborhood, is as follows:

$$\text{ACLP3}: \qquad Maximize \sum_j \alpha_j X_j \tag{5.8}$$

Subject to:

$$\sum_{k \in \widehat{\Phi}_l} X_k \leq 1 \quad \forall l \tag{5.9}$$

$$\widehat{\lambda}_j X_j + \sum_{k \in \widehat{\Omega}_j} X_k \leq \widehat{\lambda}_j \quad \forall j \left(\widehat{\Omega}_j = \varnothing\right) \tag{5.10}$$

$$X_j = \{0, 1\} \quad \forall j \tag{5.11}$$

The objective, (5.8), remains as before, to maximize the total weighted benefit of facilities located. The spatial separation between sited facilities is now imposed using two sets of constraints, (5.9) and (5.10). Constraints (5.9) are based on cliques, but do not likely constitute a complete set of cliques. To supplement constraints (5.9), whatever conditions that are not imposed by the subset of all clique constraints are enforced by the neighborhood constraints (5.10). Finally, constraints (5.11) impose binary integer restrictions on decision variables.

Murray and Church (1997) offered an approach for identifying hybrid constraints in forest planning. In the forestry context, the size of the largest cliques tends to be quite small. Murray and Kim (2008) developed a more general hybrid constraint set identification approach based on the use of GIS. Thus, a complete set of hybrid constraints is computationally efficient to identify, and their associated mathematical structure in the ACLP has proven to be effective for problem solution.

5.4.3 Theoretical Bounds

Up to this point, an argument has been made that constraint structure can impact the solvability of a model when applied to address a particular planning problem/situation. Why does this prove to be true? Taking as an example the ACLP, it is an integer programming (IP) problem due to the integer requirements on decision variables. In fact, it is a special case of an IP as it is has specific integer requirements that values be binary (0 or 1). The ACLP can also be considered a linear program (LP) with added restrictions on decision variables. The objective and major constraints are linear functions. It is of little surprise then that a common approach for solving an IP, and the ACLP, is based on LP. Specifically, LP with branch and bound is often used to solve an IP (Nemhauser and Wolsey 1988). Avoiding too much detail, this approach solves a series of LP relaxations of the IP (integer requirements temporarily ignored). The hope is that all decision variables in the LP turn out to satisfy all integer requirements. If not, which is likely, then the process of branching and bounding proceeds. At each branch linear constraints are added that serve as a cut, attempting to resolve each fractional decision variable in the associated LP relaxation. The process continues, branching off on a fractional decision variable present in a solution to a LP relaxation, until the optimal IP solution can be inferred.

With this in mind, we can now establish the significance of alternative formulations of the ACLP in a more formal manner. Assuming that the objective function of the ACLP is denoted using Z, the total weighted benefit, we can understand the relationships between the linear relaxations of the various ACLP formulations. Empirical evidence based on Moon and Chaudhry (1984), Erkut et al. (1996), Murray and Church (1997) and Murray and Kim (2008) suggests the following:

$$Z^* \leq Z^{LP}(\text{ACLP2}) \leq Z^{LP}(\text{ACLP3}) \leq Z^{LP}(\text{ACLP3}') \leq Z^{LP}(\text{ACLP1}) \tag{5.12}$$

where Z^* is the optimal IP objective value, $Z^{LP}(\text{ACLP2})$ is the optimal LP solution to the relaxed model using clique constraints, (5.5), $Z^{LP}(\text{ACLP3})$ is the optimal LP solution to the relaxed model using hybrid constraints, (5.9) and (5.10), $Z^{LP}(\text{ACLP3}')$ is the optimal LP solution to the relaxed model using pair based clique constraints, (5.7), and $Z^{LP}(\text{ACLP1})$ is the optimal LP solution to the relaxed model using neighbor based constraints, (5.2).

An optimal solution to a relaxed LP provides a bound on the best possible solution meeting the IP conditions (Nemhauser and Wolsey 1988). In this case, the ACLP is a maximization problem, so the optimal IP solution, Z^*, will be less than or equal to any LP relaxation of the model formulation. Generally speaking, the tighter the LP bound, the more likely an all (or nearly all) integer solution will result. The implication in solving an IP using LP with branch and bound is that less computational effort will be required for solving an IP when the best possible LP relaxation is utilized. Thus, a facet inducing construct is one or more constraints that enhance model solvability by making the feasible LP region closer to the feasible IP region, thereby decreasing (or eliminating) branching and bounding.

Returning to relationships established in (5.12), it should be observed that Z^{LP} (ACLP1) provides the worst LP bound, explaining why in practice it has proven to be more difficult to solve an ACLP with neighborhood based spatial separation constraints. Similarly, the comparatively better Z^{LP}(ACLP2) bound supports why in practice the ACLP with clique based spatial separation constraints has proven very good for successful solution with the least amount of computational effort.

5.5 Relaxations and Extensions

One of the things that has been noted throughout the book, and in this chapter in particular, is that models can be formulated, reformulated, extended and modified in various ways. This is most certainly true of the ACLP, where its underlying mathematical structure is part of larger, more detailed planning model formulations. An example is forest harvest scheduling models where the decisions involve not only where but when as well (Thompson et al. 1973; Kirby et al. 1986; Vielma et al. 2007). This adds a temporal dimension to the ACLP as well as other supporting considerations, like limits on associated outputs in any given time period. Recognition of the mathematical relationship to the ACLP (and node packing) has ultimately resulted in improved methods for solving more detailed forest planning problems. There are many other examples as well, with the general point being that making the connection can be important in many ways, such as a more efficient formulation, a model that is easier to solve, enabling larger planning problems to be considered, etc.

While a detailed and lengthy review will be not be provided here, it is worthwhile to observe aspects of how the ACLP could be modified to support problem variants. The remainder of this section reviews a basic relaxation as well as an extension.

5.5.1 Relaxation

The suggestion raised in Hochbaum and Pathria (1997) is of interest in weighing the relative merits of spatial configuration against the economic returns associated with harvest activity. That is, certain restrictions between neighboring sites might be better to relax, or not imposed, if the overall economic return is significantly enhanced. The question of course is which restrictions and how to assess relative tradeoffs. To address this, Hochbaum and Pathria (1997) formulated what they termed the generalized independent set problem. This can be conceived of as a relaxation of the ACLP, or independent set problem, because it explicitly structures a way to allow sites in conflict to be selected simultaneously but accounting for benefits gained by doing so. Consider the following additional notation:

β_{jk} = penalty for not imposing separation between facilities sited at j and k

$$Y_{jk} = \begin{cases} 1, \text{if restriction between potential facility sites } j\&k \text{ is relaxed} \\ 0, \text{otherwise} \end{cases}$$

A penalty, β_{jk}, is introduced to quantify the significance or impact of relaxing the spatial conflict that exists between two selected facility sites j and k. Further, the variable Y_{jk} is used to track if this restriction has been relaxed or not. Using this notation, the generalized independent set problem (GISP) is formulated as follows:

$$\text{GISP}: \qquad \mathit{Maximize}\ \ Z_1 = \sum_j \alpha_j X_j \tag{5.13}$$

$$\mathit{Minimize}\ \ Z_2 = \sum_j \sum_{k \in \Omega_j} \beta_{jk} Y_{jk} \tag{5.14}$$

Subject to:

$$X_j + X_k - Y_{jk} \leq 1 \quad \forall j, k \in \Omega_j \tag{5.15}$$

$$X_j = \{0, 1\} \quad \forall j, \ \ Y_{jk} = \{0, 1\} \quad \forall j, k \in \Omega_j \tag{5.16}$$

The first objective, (5.13), remains the same as that of the ACLP, to maximize total benefit of selected facility sites. A second objective, (5.14), is introduced to track the total penalty incurred by relaxed spatial restrictions. Constraints (5.15) are similar to the pair based clique given in (5.7), but with an additional variable added to enable relaxing a specific separation restriction. Specifically, if $Y_{jk} = 1$, then both X_j and X_k can equal one and still satisfy the right hand side. That is, both could simultaneously be selected. Alternatively, if $Y_{jk} = 0$, then the original intention of the spatial separation condition is imposed where only one or the other site could be selected. Binary integer restrictions are imposed in constraints (5.16).

A visual interpretation of the GISP is offered in Fig. 5.7. The depicted network arcs correspond to the relationships associated with site 6 in order to simply the graphic. In contrast to the relationships Fig. 5.6, the dashed arcs shown in Fig. 5.7 may or may not be imposed. If not imposed, those dashed arcs would incur a penalty. For example, if the condition between sites 6 and 7 is relaxed, that would mean that $Y_{67} = 1$, with a penalty cost of β_{67} added to objective (5.14).

It should be noted that the original formulation presented in Hochbaum and Pathria (1997) integrated objectives (5.13) and (5.14) as a weighted combination (equivalent to the weighting method in multi-objective optimization, see Cohon 1978), but is presented here in a more general form. Though not recognized as related, the map labeling models detailed in Ribeiro and Lorena (2008), Cravo et al.

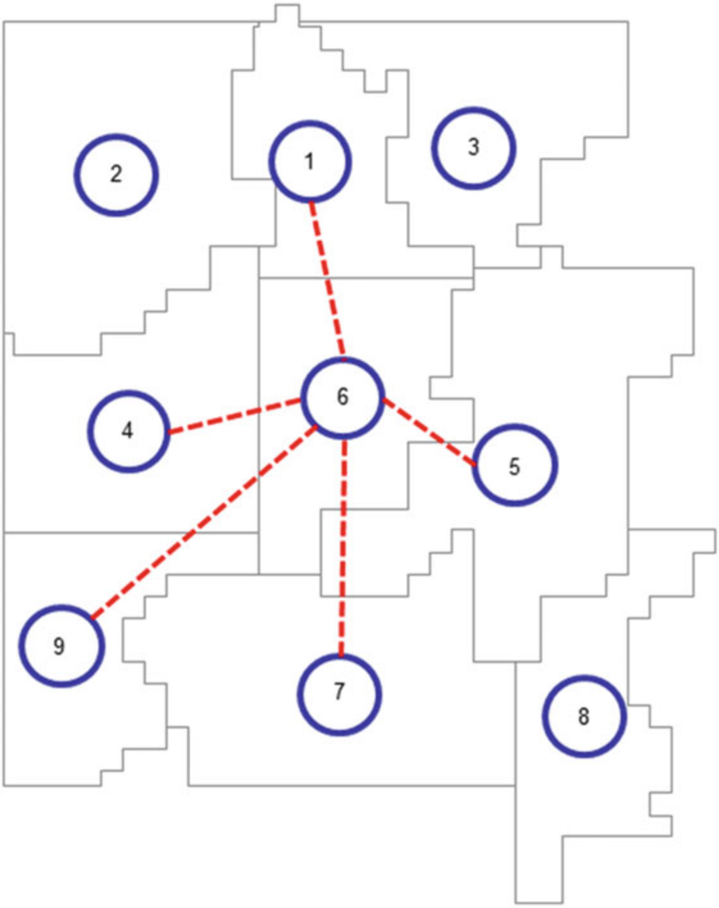

Fig. 5.7 Spatial conditions for site 6 in the generalized independent set problem

(2008) and Mauri et al. (2010) are effectively generalized independent set problems similar to that of Hochbaum and Pathria (1997).

The intent of the GISP is to expressly examine the impacts of restrictions on attainable benefits associated with the selection of facility sites. Clearly more total economic return is possible, larger objective (5.13) value, as more spatial restrictions are relaxed. However, this is at the expense of a greater total penalty, higher objective (5.14) value, associated with relaxing spatial restrictions. Thus the GISP is a multi-objective model with competing objectives, where gains are possible in objective (5.13) only by increasing objective (5.14). Conversely, decreases in objective (5.14) are only possible by reducing objective (5.13). Through the use of multi-objective solution techniques (see Cohon 1978), it is possible to generate and examine the tradeoff solutions associated with a particular planning problem, providing insights on the relative impacts of emphasizing whether spatial restrictions are to be imposed or not.

The significance of the GISP is that it is effectively a relaxation of the ACLP. Such a relaxation is accomplished through the addition of a variable that allows a constraint to not be imposed, but doing so incurs a penalty in the objective.

5.5.2 Extension

In addressing problems requiring the use of the ACLP, Wei and Murray (2012) noted that the positional inaccuracy of site boundaries creates various challenges, both in applying the ACLP and interpreting results. Effectively, there are limitations in using the ACLP as it based on the assumption that spatial information is accurate. To address this issue, Wei and Murray (2012) formulated what amounts to an extension of the ACLP, where boundary uncertainty can be explicitly considered in the model (see also Murray et al. 2014).

After detailed analysis of spatial information, Wei and Murray (2012) concluded that it is possible to characterize some relationships as known and others as uncertain. For example, even taking into account positional uncertainty of boundaries, some sites are clearly beyond intended separation standards. That is, $d_{jk} > r$ for sites j and k, where d_{jk} is the distance between sites j and k and r is the established separation standard (as defined previously). Additionally, some sites remain clearly within the separation standard under conditions of positional uncertainty. That is, $d_{jk} < r$ for sites j and k. However, there is the in between case where some sites may or may not be within the separation standard. In practice, all spatial information is subject to at least some degree of error or uncertainty. To account for these different cases, Wei and Murray (2012) introduced notation along these lines:

ρ_{jk} = probability that sites j and k would violate spatial separation requirement if both selected
Γ_j = set of a sites that would violate separation requirement if selected along with site j
Δ_j = set of sites that might violate separation requirement if selected along with site j

Given the probability measures, ρ_{jk}, it is possible to characterize the relationship between two sites j and k. If $\rho_{jk} = 1$ (or some other threshold value that is interpreted to be equivalent to one), then the two sites are deemed too close, even taking into account positional uncertainty. This means that they should not be allowed to simultaneously be selected as they would clearly be in violation of separation standards. Thus, site k would be in the set Γ_j as they are too close irrespective of spatial uncertainty issues. If $\rho_{jk} = 0$ (or some other threshold value that is interpreted to be equivalent to zero), then no separation constraints are needed as sites j and k have no chance of ever being in violation if both are simultaneously selected. The final case is when $0 < \rho_{jk} < 1$, where there is a chance that the two sites could be too close when positional uncertainty is considered. Of course, there is also a chance that the two sites would not be too close. In this case, site k would be in the set Δ_j, reflecting that possibility that the two sites may or may not be too close. With these sets and notation, the extension proposed in Wei and Murray (2012) referred to as the error ACLP (EACLP) can be formulated as follows:

$$\text{EACLP}: \quad \mathit{Maximize}\ Z_1 = \sum_j \alpha_j X_j \tag{5.17}$$

$$\mathit{Minimize}\ Z_2 = \sum_j \sum_{k \in \Delta_j} \rho_{jk} Y_{jk} \tag{5.18}$$

Subject to:

$$X_j + X_k \leq 1 \quad \forall j, k \in \Gamma_j \tag{5.19}$$

$$X_j + X_k - Y_{jk} \leq 1 \quad \forall j, k \in \Delta_j \tag{5.20}$$

$$X_j = \{0, 1\} \quad \forall j, \;\; Y_{jk} = \{0, 1\} \quad \forall j, k \in \Delta_j \tag{5.21}$$

The first objective, (5.17), is the same as that of the ACLP, to maximize total benefit of selected facility sites. A second objective, (5.18), is introduced to track the total probability of conditions that are considered uncertain. Constraints (5.19) impose certain spatial separation restrictions using the pair based clique in constraint (5.7). Constraints (5.20) represent the uncertain conditions where it may or may not be necessary to impose separation restrictions. An additional variable is added to enable relaxation of the restriction. Specifically, if $Y_{jk} = 1$, then the condition is not imposed. This means that both X_j and X_k can equal one. Alternatively, if $Y_{jk} = 0$, then the spatial separation condition is imposed. Binary integer restrictions are imposed in constraints (5.21).

To illustrate the uniqueness of the EACLP, consider the network shown in Fig. 5.8 for site 6. This may be contrasted with Figs. 5.6 and 5.7. The solid arcs represent those spatial separation conditions that must be imposed because they are certain, constraints (5.19). Alternatively, the dashed arcs represent those spatial separation conditions that may or may not exist when boundary uncertainty is taken into account, constraints (5.20). Unlike what is possible between sites 6 and 7 in Figs. 5.7 and 5.8 highlights that the EACLP requires separation between sites 6 and 7. However, other relationships may be viewed as uncertain, e.g., sites 6 and 2, sites 6 and 3 and sites 6 and 9, and if not imposed would incur the probability of violation, ρ_{62}, ρ_{63}, and ρ_{69}, respectively, in objective (5.18).

There is clearly a relationship between the ACLP, the GISP, and the EACLP to account for spatial error. Further, the EACLP is similar to the GISP. In fact, the two are equivalent if $\Gamma_j = \varnothing$ for all sites j and spatial separation restrictions are considered uncertain. Worth mentioning as well is that alternative types of extension are possible, such as the site based approach detailed in Wei and Murray (2015).

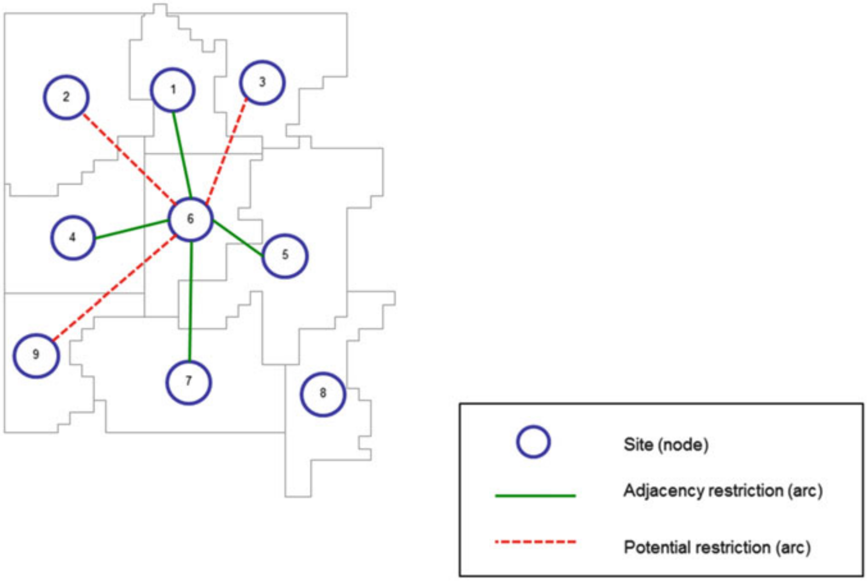

Fig. 5.8 Spatial conditions for site 6 in the uncertainty model of Wei and Murray (2012)

5.6 Inefficiency

The underlying premise of the ACLP is to site facilities in a manner that the greatest total benefit is derived, but in doing so ensure that service/separation ranges do not overlap. That is, the separation requirement between facilities and/or demand should not be violated. When the benefit for each site is the same (e.g., $\alpha_j = \alpha_{j'} \;\; j \neq j'$), this is equivalent to simply maximizing the total number of facilities sited in a region. Under these conditions then, the objective can be structured as:

$$Maximize \;\; Z = \sum_j X_j \tag{5.22}$$

The difference from the more general ACLP is that $\alpha_j = 1$ for each potential facility site in objective (5.22), reflecting the indistinguishability in benefits between potential facility sites. We can refer to the optimal solution to this ACLP special case as $Z^{\max}$. What can we say about a feasible, but sub-optimal solution? Well, the associated objective value will be less, or more precisely $Z < Z^{\max}$. But what if we were interested in knowing or identifying a feasible solution to the ACLP that is the most sub-optimal, yet there is no possibility of adding an additional facility to the region without violating spatial separation requirements. This is the problem examined in Niblett (2014) and Niblett and Church (2015). Identifying the worst case spatial configuration of facilities with respect to spatial restrictions provides important and meaningful context for an ACLP solution. This is particularly true when the use of the ACLP is oriented towards description and understanding of the spatial distribution of a good or service, something discussed in detail in Grubesic and Murray (2008), as an example.

Niblett (2014) and Niblett and Church (2015) refer to the worst case objective orientation as the disruptive ACLP (DACLP), because of the intent to configure facilities in such a manner as to disrupt the maximum potential that would be identified using the ACLP. The formulation is:

$$\text{DACLP}: \qquad \textit{Minimize} \sum_{j} \alpha_j X_j \tag{5.23}$$

Subject to:

$$MX_j + \sum_{k \in \Omega_j} X_k \le M \qquad \forall j \tag{5.24}$$

$$X_j + \sum_{k \in \Omega_j} X_k \ge 1 \qquad \forall j \tag{5.25}$$

$$X_j = \{0, 1\} \quad \forall j \tag{5.26}$$

The objective, (5.23), now minimizes the total weighted benefit of facilities located. This is in contrast to that of the ACLP, where the objective is maximized. Spatial limitations on facility configurations are imposed in constraints (5.24). Constraints (5.25) require that the resulting solution cannot leave some site unselected, if the configuration allows that site to be selected while maintaining all separation restrictions. That is, the solution must eliminate all possibility of adding an additional facility without violating a separation constraint. Either a site j has a facility or one or more sites within the spatial restriction area of site j must be selected as a facility. Accordingly, as many facilities are sited as needed, ensuring what Niblett and Church (2015) refer to as a proper solution, while minimizing the benefits generated by the selected facilities. Constraints (5.3) impose binary integer restrictions on decision variables.

An optimal solution for the DACLP would produce a minimal objective value for the ACLP. For the unitary α_j values discussed above, then we can think of this objective as $Z^{\min}$ for a particular problem application. Accordingly, the solutions would maintain the relationship that $Z^{\min} \le Z^{\max}$. Further, the difference would be very telling and informative in evaluating laws or restrictions that can be reflected in the use of the ACLP, or in the assessment of a franchise system as discussed in Chap. 7.

5.7 Facets and More

Throughout the chapter we have indicated that solving the ACLP has been of great interest because it remains a challenging problem to solve. As a result, there have been a wide range of exact and heuristic solution approaches proposed to solve the ACLP, and extensions of the ACLP.

On the exact side, Nemhauser and Trotter (1975) were among the first to report computational experience using cliques (5.5), along with other facet inducing constraints. This was done within a branch and bound process. Moon and Chaudhry (1984) examined solution using linear programming based on formulation ACLP1, (5.1–5.3), where neighborhood constraints are relied upon. Nemhauser and Sigismondi (1992) build upon the work of Nemhauser and Trotter (1975), detailing results based on a cutting plane approach. Erkut et al. (1996) and Murray and Church (1997) explored a number of different formulations, similar to those detailed above, reporting computational experience. A Lagrangian relaxation approach within branch and bound was detailed in Murray and Church (1996). More recently, the use of column generation is discussed in Warrier et al. (2005). Murray and Kim (2008) detailed an approach to identify effective hybrid constraints and examine their performance in solving the ACLP. Finally, a branch and cut scheme is presented in Giandomenico et al. (2013). Clearly there has been continued interest and improvement in solving the ACLP using exact methods that guarantee an optimal solution.

Similar interest and improved capabilities can be seen in heuristic solution development for the ACLP as well. Early work included greedy approaches by Chaudhry et al. (1986), but also more recently by Gamarnik and Goldberg (2010). Lagrangian relaxation was applied in Zoraster (1990). A genetic algorithm was proposed by Chaudhry (2006) and a greedy randomized adaptive search procedure (GRASP) was applied in Cravo et al. (2008). Recent work by Wei and Murray (2017) reports a multi-objective genetic algorithm for solving the EACLP, developed to provide an approximation of the non-inferior tradeoff curve in order to support planning and decision making.

5.8 Summary and Concluding Comments

The anti-covering location problem (ACLP), also referred to and/or related to the r-separation, node packing, vertex pack, maximum independent set and stable set problems, is an interesting and important planning problem in location coverage modeling. It has been considered and applied to a number of practical planning situations, including harvest scheduling, military defense, water well drilling, urban services, map labeling and others. Because of the broad application and appeal of this problem, there continues to be interest in the formulation, solution and use of this model as it has proven to be challenging to solve.

It should be of little surprise that the ACLP has been extended in various ways. For example, Zeller et al. (1980) structure a retail franchise design model that seeks to impose territorial exclusivity through the use of spatial separation constraints. Murray and Church (1999) developed a formulation based on integrating the MCLP with separation requirements (such an approach is also detailed in Berman and Huang 2008), discussing monitoring station and water well contexts. Williams (2008) presented a reserve design model with distance separation requirements where the intent is to ensure species survivability. Ratick et al. (2008) report a backup storage facility location model where facilities must be spaced apart for security reasons, accomplished using separation restrictions.

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