

Chapter 1

Location Modeling and Covering Metrics



1.1 Location Science

The field of location science is firmly rooted in several substantive developments, including the ground-breaking work of von Thunen (1826), Launhardt (1872), Weber (1909), Hotelling (1929), Hoover (1948, 1967), Christaller (1933), Lösch (1954), Weiszfeld (1937), Isard (1956), Moses (1958), Cooper (1963, 1964), Manne (1964), Hakimi (1964, 1965), Buffa et al. (1964) and Toregas et al. (1971). These authors may be considered founding fathers of location science, and they dealt with problems involving the competitive uses of land and land allocation, the location of industrial and communication facilities, the spatial arrangement of retail centers across a landscape, the location of competitors and competition through pricing, the layout of factory space, and the early use of computers in structuring and solving location problems. Since these early contributions, the field has expanded into new areas of application, new theoretical models, specialized solution approaches, and conceptual/technical forms of modeling location decisions and representing the spatial domain within Geographical Information Systems (GIS). Finally, as the field of location science has matured so too have the applications in both the public and private sectors.

The modern era of location science began with the development of quantitative models, in particular the optimal location of a production facility and associated transport investments (Launhardt 1872; Weber 1909) and explaining the allocation of land using economic and transport principles (von Thunen 1826). There are many branches of this field, and they are most often defined in terms of a unifying objective or construct. The central theme of this book involves the location of one or more facilities or objects in order to provide some type or level of coverage. Coverage is usually defined within the context of a performance standard. A number of examples of coverage standards are given in Table 1.1, but consider in particular the case of cellular phone service. A cell phone needs to be within a feasible communication distance of a service tower. This means that cell phone towers need to be placed so

Table 1.1 Coverage standards examples

Standard	Context
3–50 km	Cellular antenna
5–6 min	Emergency 911 call
70 dB audibility	Outdoor warning or message
400 m	Reasonable walking distance for bus access
70 miles	Essential air service access for rural communities
800 m	Suitable access for rail/subway
Species presence	Nature reserve design
1 day ride (horseback)	Mail delivery
120 miles	Doppler radar moisture detection
Same day service	Express package delivery
6 h	Search and rescue
1500 m	Visibility distance of camera mounted on tower
60 min	Aeromedical response for trauma care
3 min	Cardiac arrest response
1000 m	Wildlife road crossing
150 feet	Street light intensity

that subscribers can access towers. Since cell phone towers cost money to build and maintain, the system must be designed to provide needed coverage while using the fewest towers possible. That is, cover as much regional demand for service as possible with minimal operational (tower) costs.

Although many of the concepts used in location covering models are simple and straightforward, any application may require additional elements that will likely increase problem complexity. For example, in the location of cell towers, the main objective may be to keep costs low by minimizing the number of towers that are sited. But, there are three complicating factors in the design of a network of cell towers that “cover” customers. First, even though cell phone transmission seems simple, it may be disrupted by intervening buildings, trees, terrain, weather and other barriers. A second issue is that signal strength degrades with distance. Both factors complicate service estimation associated with which areas are within “communication reach” of a possible tower site. A third issue is that the capacity of a tower to handle simultaneous calls is limited to the number of frequencies that have been assigned to that tower. Although it may be possible for a cell phone to communicate with a tower over a distance of twenty to thirty miles, cell towers are often located much closer to each other because of capacity limitations in handling calls and/or data transmission. A fourth issue is that when towers are sited closer together, the strength of the transmission signal is often decreased to reduce the possibility of interference between nearby towers using the same frequencies. Collectively, providing cell phone coverage involves the placement of towers in a way that most customers can make calls (i.e., people are within communication range of a cell tower when they need service), ensuring that the customer base relying on a given

tower will not overwhelm the capacity of that tower, and assigning towers a set of frequencies that do no conflict with neighboring towers (see Erdemir et al. 2008).

Cellular tower placement is very representative of the complexities involved in coverage modeling. Coverage of a potential tower site is not trivial as it is predicated on spatial configuration and proximity. Service facilities may have a limited capacity. Beyond this, cellular tower siting is also a good example of defining and representing the problem domain. Most cell phone calls may be placed (or received) anywhere in a region, including along a highway connecting two populated areas, at a park or playground, on a boat floating in a lake, on a hiking trail, etc. This means that, conceptually, we want to potentially cover an entire region consisting of residential and commercial land parcels, schools, roadways, waterbodies, open space and the like. Potential cellular tower sites may consist of a finite set of points (e.g., on tall buildings, mountains and other high points on the landscape) or across continuous space that represent regions of feasible placement. Such detail means that the complexity of solving a coverage problem can be substantial.

The basic underlying motivation in this book is that we wish to locate one or more facilities (or objects/services) so as to provide service coverage of demand in an efficient manner. Covering is conceptually a very simple locational construct, however, there are many different ways in which to define and represent the provision of cover within a modeling and application domain. Further, most location covering models are NP-hard, and as such may be difficult to solve to optimality. A wide range of applications and model structures are addressed in this book. We present not only an introduction to this classic subject area of location science but provide as well an in-depth review of the main types of covering models and applications. Simply put, modeling coverage has expanded from its historical roots in the 1970s to a very large literature involving engineering, computer science, geography, management science, operations research, health sciences, as well as many other disciplines. The next section reviews standards-based covering constructs. Following this, site selection context and history is provided. The chapter then discusses a number of location design problems that have been addressed using a covering construct.

1.2 Standards Based Coverage

As suggested previously, location science is comprised of a number of formal constructs that represent general areas of application. These constructs are built around specific goals or objectives of interest. For example, the Classical Plant Location Problem involves siting a set of production facilities while minimizing the costs of product distribution, costs for building plants, and the costs of product manufacture (Manne 1964). The overarching objective is therefore one of maximizing system efficiency through the minimization of associated costs in production and distribution. This classic construct fits the needs of many private sector businesses

and is considered to be a fundamental, classic model of location science. Whereas, efficiency, return on investment, and profit often characterize the goals of private sector companies, the public sector operates with wider sweeping goals of providing good service and addressing equity. In that vein, models like the Classical Plant Location Problem do not fully address the concerns of the public. Hakimi (1964) in a ground breaking article defined two location constructs: (1) minimize total weighted distance in serving a set of demand points, and (2) minimize the maximum distance in serving anyone. Whereas the first characterizes a goal for efficiency and accessibility, the second involves a concern for the individual that receives the poorest level of service. Hakimi (1964) described the problem of locating a switching center for a local phone network, where one would want to minimize the total length of wire in connecting each customer to the switching center. This is equivalent to minimizing the total weighted distance. However, when locating something like a police station or a hospital among a set of communities, Hakimi (1964) suggested that we should locate the station or hospital such that the maximum distance to any community is minimized. He called such positioning strategies medians and centers, respectively. Whereas the median location problem seeks efficiency, the center problem seeks fairness. This is viewed as fairness in the sense that the objective addresses those who experience the worst degree of access (in terms of furthest distance).

Although positioning services like hospitals and police stations by minimizing the maximum service distance is an attempt to be as fair as possible, attention to this alone may have a significant impact on all of the other demand to be served. In addition, this farthest or maximum service distance is often defined by one demand or community and may not be a measure of the true value of system service when considering everyone. For example, in urban areas, fire services are graded in their ability to respond to most fires within a desired response time, say 5 min. To meet this design requirement, fire stations need to be located so that most neighborhoods are within a mile and a half of a fire station. This type of requirement, response within 1.5 miles, is a standards based service. A standards based service is often more meaningful than focusing only on those who receive the worst case of access, as defined by the maximum distance or time that anyone has to travel for service access. If the desired standard is service within 1.5 miles, how meaningful is it to focus on cases experiencing worst case access of 5 miles (or 15 min), as an example? Most would argue that although the worst case service level is of interest, what is important is that as many people as possible are provided a level of service that is meaningful, in this case service within 1.5 miles. For example, the quintessential problem might be to arrange fire stations in such a manner that all neighborhoods receive service within 1.5 miles (or 5 min response time). This problem is called the location set covering problem, and involves the search for a solution which “covers” or serves all within the standard while minimizing the resources required to provide coverage for all. This is the first true location covering problem and was first defined by Toregas et al. (1971). The underlying idea is that as long as neighborhoods and business districts are provided with “coverage”, whether it is a 2 min or a 5 min response, service will be considered acceptable. But what if resources do not exist to cover or serve everyone with the desired standard? Then, we might turn our attention

to providing as many people as possible with such a level of service. That is, maximize the demand that is provided coverage while spending no more than an allowable budget. This second construct is called the maximal covering location problem and was first posed by Church and ReVelle (1974). This does not mean that we are not interested in equity and do not have concern for those who are not well served, but that we want to provide what is deemed to be good service to as many as possible. There are ways we can address equity within a covering model, but the principal underlying element is that there is a standard that can be used to define if a suitable level of service has been provided and that the objective is to allocate resources in such a manner that everyone or as many as possible are provided such service coverage. Even though early covering models were suggested as a means for allocating and locating public services, their use and areas of application have expanded in both public and private sector contexts. Simply put, covering models address important standards-based problems, including siting cell phone towers, fire stations, surveillance cameras, and retail establishments.

Locating agents, stations, cameras, cell towers, etc. to cover demand is the central issue of this book. We address how coverage is modeled, the nuances of different problem settings, as well as how models have been expanded to address many different problems, even ones which do not appear to involve covering at first glance. One of the main themes in this book is that there can be a wide variety of problem domains. For example, is the region or object being covered defined by a continuous, bounded domain or is it defined by a finite set of discrete points? Are facilities, activities, or objects that are being located restricted to specific points or is their placement unrestricted within a continuous domain? Does demand for coverage vary spatially or is it homogeneously distributed? Is access in standards based service defined by a three dimensional view shed, are their limits to how sound travels that impacts audibility, or does travel only occur along road segments of a network? Are facilities logical objects, do they represent specific product types, or are they physical “bricks and mortar” entities? Are there a set of standards, defining perhaps good service, better service and best service rather than one single standard of coverage? Is the service extent defined in advance or is it to be determined endogenously? Can covering facilities be congested, altering whether a standard of service has been met? All of these nuances and more are addressed in the chapters included in this book.

Even though location science is a relatively new sub-discipline, principally expanding within the era of computers and operations research methods, standards of need involving distance or time limits, as well as other metrics, have been a part of the human fabric of civilization for centuries. The next section presents a historical perspective, noting elements of development where coverage standards or limits defined the very notion of how specific systems evolved or how specific problems were addressed. This historical take is meant to firmly establish the fact that desired service standards are at the very nature of things. Coverage is an important concept that has promoted safety, ensured service provision and relates to other functions in many societies. After this selective, but historical venture, the remaining sections of

this chapter are devoted to describing a wide variety of problem application domains, all approached through the use of a location covering model.

1.3 Site Selection Context and History

It is likely that the first location problems were collectively solved by small groups of hunters and gatherers. They dealt with locating encampments and clearing needed trails. Important criteria considered was how easily an area could be cleared, whether resources could be efficiently accessed, and could sites be easily protected against intruders. There is no doubt that some location decisions were not very successful. For example, several of the California missions sited by Franciscan missionaries from Spain in the late 1700s and early 1800s had to be moved, including three because of flooding issues (Santa Cruz, San Gabriel, and Santa Clara), two because of unreliable water sources (San Diego and San Antonio), one after it was destroyed by an earthquake (Mission la Purisima), and another when the weather proved to be inhospitable (San Francisco). Mann (2005) describes issues with chosen locations for early cities in the Americas, from the Yucatan to Mississippian settlements. It is safe to assume that early failures heightened interest among societies and industries in making location decisions that promoted safety and security, enhanced livability and survivability, and eventually increased profits and productivity.

Military movements were examined in ReVelle and Rosing (2000), providing a post analysis of the Roman army deployment during the reign of Constantine as well as the British naval deployment in the early 1900s. The Legions of Rome had dwindled from 50 to 25 by the time Constantine came into power. The 25 legions were grouped into four field armies. The problem in deployment involved keeping a hedge on how far an army might be from the next uprising or enemy incursion. The British suffered a similar decline in naval resources and needed to consider a strategy in positioning only four battle fleets in order to protect their interests. ReVelle and Rosing (2000) demonstrated that the strategic positions and deployment of military forces could be structured using a covering framework, and further showed that the actual deployment solutions of Constantine and the British could have been improved.

The early western expansion of the United States involved strategic placement of military forts. These forts played a major role in providing protection and safe refuge to a surrounding area. For example, the City of Minneapolis, MN began with the development of Fort Snelling in 1819. Minneapolis is located at the only natural waterfall along the Mississippi River. This was a strategic location because the waterfall and flow volume provided energy for mills. In the latter 1800s virtually all wheat grown in the US was transported to Minneapolis for milling (most of it hauled by train). Figure 1.1 depicts the location of Texas forts in 1849. The systematic spacing of locations from north Texas to the Rio Grande River and the Gulf is evident, clearly suggesting the strategic focus of protection along the western front. Many of the settlement patterns of today in fact reflect a variety of site

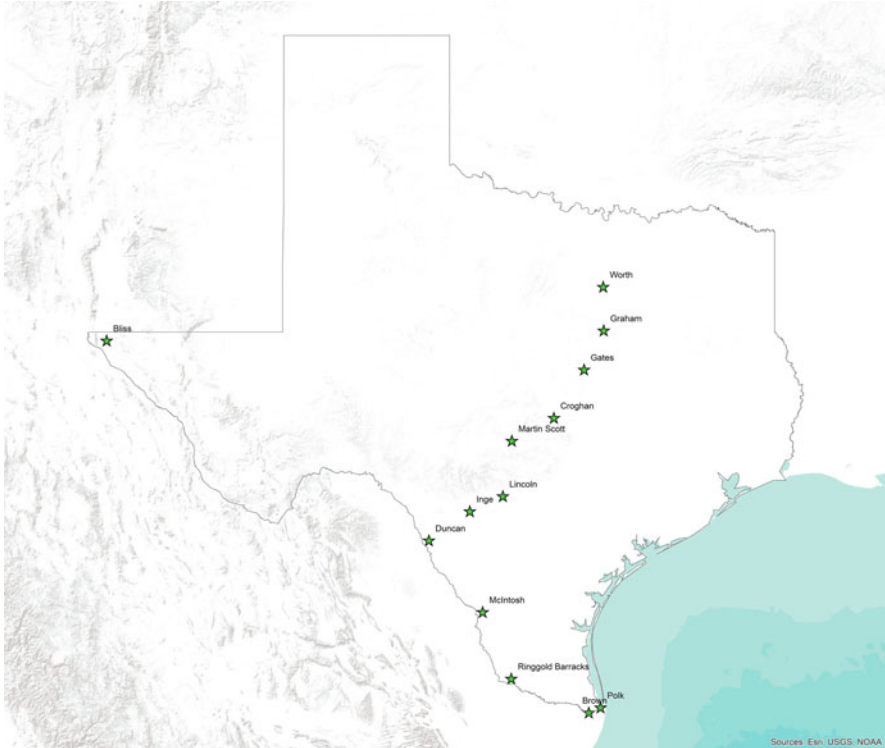


Fig. 1.1 Fort locations in Texas in 1849

selection objectives and metrics, include taking advantage of local resources (e.g., energy generation at St. Anthony Falls on the Mississippi River), minimizing costs of transportation (e.g., proximity along waterways), maximizing protection (e.g., the citadel at Quebec City, Quebec), and providing accessible services (e.g., Sutter’s Fort in Sacramento, CA that supplied miners during the gold rush of the 1850s).

Embedded in this history of location is the fact that many site selections were periodic in position and reflected distance ranges of significance, a form of a covering or standards-based structure. For example, the 21 Spanish missions in California were placed approximately 30 miles apart along a 600 mile trail so that one could travel between one mission and the next in a “hard days ride”. This periodicity can be found in other transportation/location examples, even during the era of the Pharaohs in Egypt (Church and Bell 1988). As another example, stage coach stations along the Butterfield stage route of the late 1850s were positioned approximately every 20 miles (a total of 139 along a 2800 mile route from St. Louis, Missouri to San Francisco). For most of the route, a stage coach would be no further than approximately 10 miles from a station (less than a 2 h journey). The route itself was chosen so that it would be passable in both winter and summer, by leaving St. Louis in a south westerly direction until it reached El Paso, Texas, crossing the

Rio Grande, and then heading principally in a westerly direction to Southern California, where it then turned north to San Francisco. The distance range between stations also reflected a pragmatic need to change teams of horses every twenty miles or so. If there was a breakdown or a horse was injured, one could reasonably make it to the next station by nightfall.

In the early 1930s Walter Christaller developed a theory of settlement location defined upon a theoretical triangular lattice of hamlets/villages. This lattice represented an unbounded plain of uniformly fertile soil and plentiful water. Distance reflected what a team of horses could travel in 2 h. It was reasoned that farmsteads would be no further than 2 h away from their closest village using a horse-drawn wagon. This allowed farmers to leave in the morning, travel to their market/service town, purchase supplies, and return in time for the main noontime meal. Thus, Christaller (1933) built into his theory the notion that settlements would be periodic in position. He also argued that retailers, in order to be successful, also needed a minimum number of customers or sufficient market size, and these market thresholds varied by the type of business. For example, we know that in our modern society people frequently purchase groceries and gasoline, whereas jewelry shopping is less frequent. To stay in business, a grocery store needs a smaller market than what might be needed for a jewelry store. To account for different market sizes necessary to support retailers of different products, Christaller (1933) reasoned that there would be a hierarchy of retail centers. This means that low level centers provide basic services, such as groceries and gasoline noted above, and higher level centers provide more specialized goods and services, like automobile dealerships and jewelry stores in the context of our previous examples.

Christaller (1933) used the “central place” hierarchical patterns to explain the location of settlements in Southern Germany. Others have applied central place theory to other farming regions like Iowa (Berry et al. 1962). Even though some of the underlying assumptions made in the development of central place theory rarely hold true, two of the concepts remain particularly important and relevant today. First, there is the fact that a retailer requires at least a minimum number of customers (called a market threshold) to remain in business. Second, it is true that customers are willing to go only so far for a particular type of product or service on a regular basis (called the range of a good). Accordingly, one can boil down Christaller’s theory as based upon two principal concepts of proximity: (1) the distance (or travel time) at which a given center would generate enough customers to be economically viable; and, (2) the distance (or travel time) beyond which customers will no longer be willing to travel to a facility on a regular basis. Storbeck (1988, 1990) showed that a coverage framework could be used to structure a model of central place theory based upon threshold and range concepts.

Even though Christaller and others like the Spanish missionaries used a periodic distance in facility placement that reflected basic needs (e.g. like being no further than a days’ ride on horseback), such factors are still relevant today. For example, a retail trade zone can be defined as the region that would be served by a given store

location. Salvaneschi (2002) describes an approach used in branch store location: “. . . a market should be planned so that all its [retail trade zones] overlap by a certain percentage.” He added that “. . . all stores should be impacted slightly; otherwise there is no guarantee that all customers will be properly served.” When describing the McDonald’s location strategy for Los Angeles, Salvaneschi (2002) states that “. . . the plan was to form a constellation of stores . . . locating stores approximately 2–2 and a half miles away from each other.” In fact, McDonald’s specifies an address and not a territory for most of their franchisee participants. Of course, distances are not the only indicator of “reach” for a facility. Other factors or impedances may also be taken into account, like speed limit, level of congestion, etc. associated with deriving an effective distance. Nonetheless, the primary market of a store in densely populated areas is often based upon a predefined market size or proximity standard, just as distance and demand limits the effective coverage of a cell tower.

Beyond the notion that many facilities are located in periodic arrangements and reflect the effective reach of a facility or its services, like forts, market centers and stagecoach stations, one must understand the underlying standard for each problem, reflecting the desired range to be met. Examples of this abound. We wish to view as much of the forest as possible from a set of towers, so that fires can be detected. We want to place stations along a route so that horses can be relieved every few hours. Of course there are many others too. In the remaining sections we describe different areas of research and application that have involved the development and use of a covering model.

1.4 Surveillance, Sensors, and Warning Systems

A number of metrics can be used in determining the “effective service distance” of a facility. An interesting example is that developed by Agnetisa et al. (2009). Their problem involved the surveillance of a meandering river with the location of sensors (radar units) along the river. In monitoring traffic, or even an incursion along a river, they wanted to place sensors so that all segments of the river were within the field of view of at least one sensor. Each sensor had a maximum distance of view, or ability to detect an incursion. They represented this field of view as a disc with a given radius. Their intent was to locate a set of sensors (discs in this case) capable of monitoring all segments of the river while minimizing cost.

Surveillance resources are often positioned along a front (like a coastline), or to an area or region. A historical example of such a problem context is that the US Army deployed 11 observation towers along a 40 mile segment of the coastline of Delaware in order to detect enemy vessels during World War II in order to help protect key industries upstream on the Delaware River. Having towers located close together allowed observers at two neighboring towers to triangulate an accurate position for directing defensive fire. The distance of human observation has also

played a major role in a number of other defense and security problems. Perhaps the most well-known is the use of lookout towers for early detection of forest fires. In 1910 there was a 3 million acre fire in Washington, Idaho, and Montana. It is said that the smoke from this fire reached Washington, D.C. In response to this devastating event, the US Forest Service issued a rule making local communities responsible for the costs of firefighting. This led to a high priority being placed on early detection and fire suppression. In order to detect forest fires, a vast network of fire lookout towers was built across the forested regions of the western United States. In Idaho alone nearly 1000 lookout towers were deployed. Such towers were often 30–100 feet or more in height and were usually placed on high ridges and peaks in order to maximize the size of the viewshed of each tower. An obvious objective was to place towers so that as much of the landscape was within view (covered) by the network of towers. Goodchild and Lee (1989) were the first to propose this as an optimization model using a covering framework.

Surveillance systems can be quite massive, as the previous example suggests. Another example is the Ground Observer Corps of the U.S. Air Force in the 1950s during the height of the cold war. They used hundreds of thousands of volunteers to man over 16,000 posts scattered along the gaps in radar coverage to detect a possible bomber attack from the Soviet Union. Many other examples of sensor placement are based upon a covering construct, and in some cases enhanced security is provided when multiple sensors can detect an intrusion at a given location. This means that it may be necessary to deploy facilities in such a way as to provide backup cover (Hogan and ReVelle 1986), multiple cover (Church and Gerrard 2003), and expected cover (Daskin 1983).

Although these historical examples show that visual surveillance has often taken considerable resources to implement and was deployed for a variety of reasons, visual surveillance is even more commonplace today. Visual surveillance is used to enhance security, prevent theft, and recreate crime scenes. Modern systems rely less on people and more on a variety of equipment, including satellites, cameras, motion sensors, image processing software, and face recognition algorithms. These systems are often connected with a communications network to a central monitoring or recording center. For example, with a CCTV (closed circuit television) network (i.e., video system), British police in London were able to piece together the activities of four bombers in a major terrorist attack in London, UK in 2005. But even more important is the design of surveillance systems so they “cover” an area at least cost (Murray et al. 2007). In a very recent monitoring project, Bao et al. (2015) use a maximal covering model to deploy camera-based watchtowers for fire detection in China. Monitoring can also involve other types of sensors, beyond cameras and radar. Examples include the design of monitoring networks for weather (e.g., Doppler radar), underground water pollutant plumes (Meyer and Brill 1988; Hudak and Loaiciga 1992), air pollutants (Houglund and Stephens 1976), and rain gages (Hogan 1990).

Another critical need in many communities is the ability to warn inhabitants of an impending emergency, like a tornado or an industrial accident. This can often involve a network of warning sirens. Sirens are often placed on top of buildings

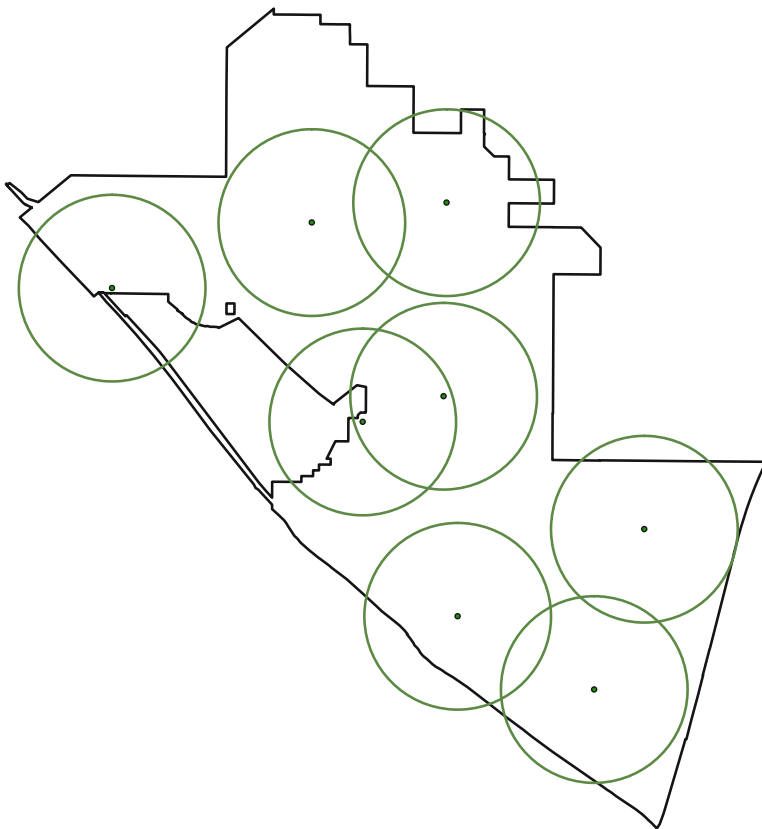


Fig. 1.2 Siren coverage for Huntington Beach, CA

and poles. Depending on their design, the distance at which they can be heard at a given decibel rating can be determined, as audibility decreases with distance from the siren. In this way a coverage pattern of a siren can be determined. In many cases coverage is circular, reflecting the distance which just meets a minimum decibel warning level. Some modern sirens are designed to follow the siren blast with a spoken announcement. For example, the City of San Francisco has recently replaced and updated an antiquated air raid siren system with a new Outdoor Public Warning System over the last 10 years. In all, the new system has 109 digital sirens and speakers. A number of these broadcast bilingual messages, some in English and Spanish and some in English and Cantonese. Figure 1.2 depicts the warning system of Huntington Beach, California, where the audible area for each siren is shown as circular. Modeling the efficient deployment of siren systems using a covering model started with the work of Current and O’Kelly (1992) and has been an area of considerable continuing research (Murray and Tong 2007; Murray et al. 2008; Wei and Murray 2014; Wei et al. 2014).

1.5 Emergency Response

Urban areas have experienced many significant fire catastrophes on a massive scale. In the United States, there were sizable city fire disasters in the latter 1800s and early 1900s. The Chicago fire of 1871 destroyed 18,000 buildings. The 1904 Baltimore fire consumed 1500 buildings in the central downtown. The great Atlanta fire of 1917 burned 1900 structures. The San Francisco earthquake and fire of 1906 wiped out 80% of the city. Much of what was not damaged by the earthquake was lost to fire. Such fires and destruction led to the development of better building standards, fire protection, and enhanced water availability through fire hydrant networks.

After a fire is ignited, it often takes only a few minutes to reach the point of flashover, which occurs when thermal conditions in a room are such that all combustible materials will ignite. This is when a small fire turns into a large fire, and begins to spread to other rooms. Besides heat, flashover is also associated with high levels of lethal gasses. Such events can easily occur within 4–6 min and makes the time to respond a critical issue. Once an alarm or call is made, a dispatcher must determine which crews are to respond. Notified crews often take a minute or so to assemble and take their places on their vehicles before travelling to a fire. Once on scene, it can also take time to assess the situation, set-up hoses and begin their rescue and attack. The response time model of Rand Corporation (Kolesar et al. 1975) estimates that it takes on average 3.1 min to travel to a fire when the travel distance is 1.5 miles. In general, it has been recommended that built up areas should have their closest station within this distance. For example, the City of Elk Grove, California established a maximal distance standard of 1.18 miles, where they estimate it will take no more than 5 min to respond (presumably including the time to assemble at the station, called the turn-out time, and the time to handle the call and send a dispatch request to a station) (Murray et al. 2012). Using such standards, cities today seek to locate fire stations in such a manner that as many structures and people are within their adopted time/distance response standards (Plane and Hendrick 1977; Schilling et al. 1980; Murray 2013). The majority of the yearly service costs are closely related to the number of stations used, as each requires a complement of fire fighters and equipment. In Los Angeles County alone the cost of fire service involves approximately 340 stations and costs more than \$2 billion per year. Across the US, fire services are dispatched from more than 38,000 stations. Optimal deployment of stations in providing coverage can save money. In addition, coverage models can be used to optimally expand a system in growing communities as demonstrated in Murray et al. (2012).

Fire protection is but one element of public safety services. Another is the provision of police. Police departments are often organized by divisions or precincts, each a well-defined geographical area that includes a police station. For example, the Island of Manhattan is divided into 22 precincts. Stations usually contain holding cells, a booking area, changing rooms, offices, etc. Precincts are organized so that a

police response from the station or defined patrol areas is kept within a maximal response time for the majority of emergency calls. Curtin et al. (2010) demonstrates how this deployment problem can be defined as a covering model, where they developed a decision support system to locate patrol activity centers (sectors) within police divisions in Dallas, Texas in order to improve response times to emergency calls.

Another major public service is that of emergency medical response. There are two general problems, one that involves responding to trauma cases and the second for other types of emergency calls. One of the first major applications of a coverage model for EMS occurred in Austin, Texas (Eaton et al. 1985). When locating EMS units, a major issue is calculating how often specific units are available to respond to a call and how often they are busy on another call. Daskin (1983) was one of the first to model the expected level of coverage for a system, relying on the use of a system-wide probability of busyness. This concept has been expanded in a number of ways, including the use of simulation, localized busyness estimates, queuing theory, and the development of a spatially defined queuing model called hypercube queuing. These approaches are explained in detail in Chap. 4.

ReVelle et al. (1976) were the first to propose a dual location problem where both hospitals and ambulances could be located so that their coordinated action could keep total service time within a standard. Their approach has been expanded to the location of trauma units and helicopters so that service can be provided within what has been called the golden hour of service (Branas et al. 2000). Issues of coordination, contingencies, and coordinating units (like ambulances and helicopters) are discussed in Chap. 3.

1.6 Nature Reserve Protection

Radar, lookout towers, and surveillance cameras all have physical limits to what can be detected. These limits of detection can be defined in terms of proximity, lighting, weather, presence of barriers, etc. Retail/service location strategies can involve time and distance limits as do public safety systems like EMS, police and fire response. But, there are other areas of application for covering models that are important and relevant in today's society. One of these is the protection of threatened and endangered species. From the 1980s, conservation biologists have been concerned with the location and selection of reserve sites. Such sites, which when taken together, are intended to protect as many species as possible (Csuti et al. 1997). With the exception of the model of Cocks and Baird (1989), many of the early attempts to develop site selection approaches relied on selection heuristics. For example, Kirkpatrick and Harwood (1983) suggested the following site selection strategy: select as the first site for the reserve that site which contains the greatest number of

species of concern; for the second site in the reserve choose that site which contains the most species of concern that are not represented by the first site; for the third site pick the site which represents the most species not represented by the first two selected sites; continue this selection process until all species are protected. Many will recognize this as a greedy heuristic, which was first proposed for covering problems by Church (1974) and Chvatal (1979). But more important, the underlying concern was to select as few sites as possible while at the same time representing all species in the reserve system. Underhill (1994) was the first to recognize this to be a covering problem. Church et al. (1996) suggested that when resources did not exist to protect all species in a reserve system, then one could use a maximal covering framework. Since the mid-1990s, a relatively large number of covering models have been structured for the selection of a portfolio of sites as a biological preserve. These include stochastic elements such as uncertainty of species presence (Cocks and Baird 1989), site quality (Church et al. 2000), and threat of development and site degradation (O’Hanley et al. 2007). A review of such models can be found in Snyder and Haight (2016).

Many of the biological reserve site selection models do not consider the nearness of selected sites, but it might be important to not only represent species more than once within a reserve system, but also provide corridors of connectedness and nearby sites to support meta-populations (Williams 1998). This may mean that selection of one site for a reserve system may be contingent on selecting at least another nearby site as well. This type of contingent selection has been used in reserve design as well in other covering contexts (Williams et al. 2003). These topics are discussed at greater length in Chap. 3.

The simple reserve selection models, like that suggested by Underhill (1994) and Church et al. (1996), are logic-based covering models. This is because the intent focuses on what is contained within the site itself that is being covered, rather than those elements that are within some distance of the site, like the service area of a fire station. There are a number of other such logic-based covering models that have been defined in the literature. Each of these models involve the selection of a portfolio of elements such that altogether the portfolio contains all of the desired properties or as many of the desired properties as possible. We can think of the portfolio as covering a property when an item in the portfolio contains that property. For example, for reserve design, the portfolio represents the sites that have been selected and the items that are being represented are the species of concern. Other settings exist where the use of a logic-based approach for covering is applicable. For example, Klimberg et al. (1991) developed a model for scheduling inspections of possible violations of industrial facilities by the US FDA so that each possible violation would be inspected once in a 2-year planning horizon. In another logical design problem, Serra (2013) describes how to select a set of products with specific properties so that customers will find within this set a product matching many of their needs. This analysis technique is called TURF (Total Unduplicated Reach and Frequency analysis) and is approached as a covering problem. Cocking et al. (2009)

used a covering model to select dental color shade guides for teeth replacement, using real teeth as targets that dental shade guides should represent within a specific degree of accuracy. They found that a covering model identified a superior shade matching. Finally, covering models have been suggested as an aid in media selection for advertisers (Dwyer and Evans 1981).

1.7 Spatial Separation

While coverage has generally been viewed in positive terms thus far, there are situations where a standard may reflect an incompatibility, violation, danger, etc. That is, the notion of spatial proximity can indicate when facility siting, as an example, is unacceptable. From a modeling perspective, this can be viewed as a requirement for spatial separation. Moon and Chaudhry (1984) proposed a framework for models where the objective is to maximize the number of facilities being placed while maintaining that any two sited facilities are separated by at least a minimum prescribed distance. They called this the anti-cover location model. At first one might wonder whether such a problem construct is more theoretical rather than practical. One early suggested application of this approach involved the location of missile silos (keeping them apart would mean that a successful strike on one missile silo might not damage any nearby silos). Another possible application involved the location of hazardous materials, suggesting that they should be kept apart within a manufacturing site. Practically speaking, however, many applications for this covering construct exist. Recent applications of the anti-cover construct have evolved in many different ways and new innovative techniques have been developed to solve this type of problem. Recent applications of the anti-cover model have included habitat analysis (Downs et al. 2008; Church 2013; Church et al. 2015), examining liquor store market penetration (Grubestic et al. 2012), and assessing housing policy impacts (Grubestic and Murray 2008; Murray and Kim 2008) to name a few.

The definition of separation between facilities has been modified to suit the problem being addressed. An example of this involves the problem of scheduling harvesting units across a land holding (Murray and Church 1996). In one form of the problem harvest units are delineated in advance. Each harvest unit is usually less than a maximum allowable size and has a revenue value assigned to it based upon the inventory within the unit. Environmental restrictions often require that no two adjacent units be harvested in the same decade. Assume for simplicity that we are interested in one-time period, or decade, where we want to maximize revenue generated by determining which units to harvest such that no two harvested units are nearby to each other. Here the two units can be considered too close when they share a portion of their boundary, as an example. Other separation problems may allow spatial units to touch, but require that there be no overlap. For example, Grinde

and Daniels (1999) place fabric cutting patterns on a roll of fabric using a covering construct. They have a set of patterns which are needed and are sewn together for a piece of clothing. Each pattern object can be oriented in only certain ways on the fabric so that when a set of pieces all are sewn together, it makes a presentable piece of clothing. The objective is to place as many sets of patterns on the fabric without overlap. Maximizing coverage of the fabric for this problem minimizes fabric waste.

Another problem that involves placing elements on a background (like fabric) involves automatic label placement on a map. Here, name labels, like the name of a river, must be placed along the object, but separated from other labels by a distance of separation. Such separation ensures legibility. Placing and sizing pivot irrigation systems is another example where spatial separation is important. The footprint of one pivot irrigation unit (map label, fabric cutout pattern, etc.) must not overlap with any others, but must be accomplished in such a manner that as much of the land is irrigated (or as many of the labels are placed on the map as possible, or the remaining fabric waste is minimized, etc.). All of these are examples of spatial separation requirements, where coverage models have been important. This topic is addressed in Chap. 5.

1.8 Expanded Locational Constructs

With the exception of the early research work on facility layout (see Buffa et al. 1964), most researchers have assumed that facility sites are points in a Cartesian plane (Launhardt 1872) or as points on a network (Hakimi 1964). Morgan and Slater (1980) suggested that a facility may be a structure on a network, like a path or a tree. They proposed several problems that involved finding a path of a network that maximized accessibility to all nodes of the network. Current (1981) expanded on this concept when he proposed the shortest covering path problem. He assumed that a path origin and a path destination were already known in advance. The shortest covering path is the shortest path on the network which starts at the origin and ends at the destination while traveling within a preset coverage distance of all network nodes. Current et al. (1985) proposed another construct called the maximal covering shortest path problem. These constructs have formed the basis for a wide variety of models formulated for transit system design (see for example: Curtin and Biba 2011; Murray 2003; Wu and Murray 2005; Matisziw et al. 2006; Laporte et al. 2011).

1.9 General Form of Coverage

For many applications, coverage can be defined as being provided or not. For example, is a neighborhood within the desired service range of a fire station or does a camera network provide complete area surveillance? There are circumstances,

however, where coverage is not so easily defined. In fact, there may be degrees of coverage. For example, one may view service quality as a function of distance and quality level of the coverage provider. For EMS, units have been classified as Basic Life Support (BLS) and Advanced Life support (ALS), where coverage models allocate two vehicle types in order to provide everyone with at least one type of service level while maximizing the number of people provided ALS services (Eaton et al. 1985). Church and Roberts (1983) suggested that coverage should be defined over distance ranges to reflect defined standards like high, medium, and low service coverage. One of the reasons to suggest this construct is that even when a maximal time standard of 8 min is established for good EMS service, what about service within 8.10 min? Should that be counted as zero coverage, or an intermediate value of coverage (higher than zero and less than the desired, high-valued standard)? Church and Bell (1981) and Berman et al. (2003) have suggested that service values and coverage may degrade as a continuous function of increasing distance. Thus, the concept of coverage has been expanded to include problems where a discrete distance, time, visibility, or other metric is not so crisply defined with one simple cutoff. These issues are discussed in greater detail in Chap. 6.

1.10 Summary and Concluding Remarks

Location coverage problems can be found in a wide variety of activities and problem domains. These include public service delivery, transit system design, surveillance and monitoring systems, as well as biological reserve protection. Covering problems were addressed long before computational models were ever developed, and involved problems like protection (e.g., fort location), warning (e.g., fire lookout placement), administration, and retail location theory. Today, many of these problems are approached through the solution of a formal location covering model, using state of the art heuristics and algorithms. As outlined in Table 1.2, the goal of this book is to present representative examples of different types of covering models, problem domains, and to some extent solution approaches. This will involve probabilistic and stochastic coverage, logical coverage, quality of service coverage, and multiple-level coverage. Each chapter addresses a general area of coverage modeling or problem domain (e.g., discrete or continuous). We begin in Chap. 2 with the definition of the two classic models that underpin this subject of location science: the location set covering problem and the maximal covering location problem.

Table 1.2 Book coverage at a glance

Chapter	Topic	Major themes
1	Location modeling and covering metrics	<ul style="list-style-type: none"> • Location science • Standards based coverage • Site selection context and history • Surveillance, sensors, and warning systems • Emergency response • Nature reserve protection • Spatial separation • Expanded locational constructs • General forms of coverage
2	Classic beginnings	<ul style="list-style-type: none"> • Location set covering problem • Maximal covering location problem • Theoretical linkages • Fixed charges
3	Extended forms of coverage	<ul style="list-style-type: none"> • Multiple service • Existing service system • Site quality • Multiple objectives • Backup coverage • Coordinated systems • Hierarchical services • Multiple optima
4	Probabilistic coverage	<ul style="list-style-type: none"> • Reliable coverage • Expected coverage • Maximum reliable coverage • Queuing • Facility availability • Extensions
5	Anti-covering	<ul style="list-style-type: none"> • Separation context • Model construct • Mathematical structure • Relaxations and extensions • Inefficiency • Facets and more
6	Weighted benefit, variable radius, and gradual coverage	<ul style="list-style-type: none"> • Equity and implied value of service • Generalized maximal covering location problem • Expanded forms of generalized coverage • Endogenously determined coverage • Continuous endogenous coverage • Gradual coverage
7	Capture, capacities, and thresholds	<ul style="list-style-type: none"> • Maximum capture • Capturing/intercepting flow • Capacities • Thresholds • Franchise territory design
8	Continuous space coverage	<ul style="list-style-type: none"> • Problems • Formulations • Simplification and relaxation • Solution

(continued)

Table 1.2 (continued)

Chapter	Topic	Major themes
9	Disruption, protection, and resilience	<ul style="list-style-type: none"> • r-interdiction • Design sensitive to facility loss • Planning for greater levels of disruption • Issues when providing complete coverage • Coverage loss when supporting infrastructure is lost • Defensive coverage • Facility fortification • Applications
10	Coverage of network-based structures: paths, tours and trees	<ul style="list-style-type: none"> • Shortest covering path • Salesman and tour coverage • Maximal Covering Trees • Maximal coverage variants • Arc improvement • Applications
11	Grand challenges	<ul style="list-style-type: none"> • Big data • Developing better models and service assumptions • Problem transformation • Flexible and accessible location application software systems • GIS developers • Artificial intelligence and machine learning

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