



# Drivers' Behavior Effects in the Occurrence of Dangerous Situations Which May Lead to Accidents

I. M. Almeida<sup>1</sup>, R. C. P. Leal-Toledo<sup>1</sup>(✉), E. M. Toledo<sup>2</sup>, D. C. Cacau<sup>1</sup>,  
and G. V. P. Magalhães<sup>1</sup>

<sup>1</sup> Federal Fluminense University, Niteroi, Brazil  
leal@ic.uff.br

<sup>2</sup> LNCC, UFJF, Petropolis, Brazil

**Abstract.** This paper presents an analysis how different acceleration policies to reach the maximum speed of the road, considered as a heterogeneity unobserved in usual measurements, influence the probability of occurrence of Dangerous Situations (DS) that can lead to accidents between vehicles. For this, a modified version of the NaSch model is proposed. The probability Density Function (PDF) Beta is used to describe these distinct behaviors. The effect of these policies on the traffic dynamics was also analyzed. A new metric is presented so that we can analyze results where real deceleration rates data are used to evaluate accident probability.

**Keywords:** Accidents · Traffic · Cellular automata  
Dangerous Situations · Computer simulation

## 1 Introduction

In densely populated areas the frequent traffic jams cause significant economic and social damages. In order to make effective planning, traffic flow simulations can be of fundamental importance to better understand traffic flow behavior, in different situations, helping to improve traffic networks design and of the definition of more efficient transportation systems. For this purpose, microscopic numerical models, as those based on Cellular Automata (CA), have emerged as an alternative to model traffic flow helping to understand its behavior. Microscopic models typically focus their attention on the behavior of individual vehicles, the road topology and on the influence coming from neighborhood vehicles.

The fundamental traffic model proposed by Nagel and Schreckenberg [1], is a stochastic Cellular Automata model of vehicular traffic, known as the NaSch model. It reproduces the basic features of traffic flow. Many others CA models were proposed trying more realistic traffic representation. Among these we find the so called “slow to start” rules [2–4] to model the meta-stable traffic flows. Some others models embody anticipation rules in order to take into account

drivers' movement at next time step. By including anticipation and the brake lights concept [5–9] in the modeling, the vehicles do not solely determine their velocities based on the distance to the next vehicle in front of it, but they also consider the speed and the deceleration of the ahead vehicle. Others models try to include characteristics of driver's behavior at the moment of the definition of its new speed [6, 10, 11].

The dense road traffic has increased the number of accidents. The absence of observations of behaviors that can potentially cause modifications in traditional analyses, may lead to erroneous inferences or erroneous accident predictions. To carry out simulations that can bring information about the effects of distinct acceleration policies behavior is fundamental to understand occurrence of roads Dangerous Situations (DS) that can lead accidents.

Recently, cellular automata models have been extended to investigate car accidents probabilities. Boccara et al. [12] were the first authors to propose conditions for car accidents occurrences in the deterministic NaSch model. Huang et al. [13] presented analytical expressions for car accidents in this model with lower maximum velocity,  $V_{max} = 1$ , and Fukui et al. [14] for high velocities. In Jiang et al. [15], car accidents probabilities are obtained for the so called velocity effect (VE) model. Moussa [16] analyzes car accidents occurrence based on delayed reaction time of the successor car. More recently, Bentaleb et al. [17] presents car accidents occurrence probability in the extended Nagel-Schreckenberg (NaSch) model considering fast and slow vehicles. It also analyzes the effect of damaged vehicles evacuation from the road. Results of car accidents probabilities for the non deterministic NaSch model were obtained [18–20] and also for two-lane CA model [21]. The influence of speed limit zone in roads [22] and intersections [23] were also analyzed for open boundary conditions and Speed Limit Zone. Madani and Moussa [24] present results for NaSch Model and NaSch model with the “slow-to-stop” rule.

In this work we present a modified version of the NaSch model that proposes to evaluate numerically how distinct acceleration policies, to reach the same maximum speed of the road, can influence the traffic dynamics and the Dangerous Situations (DS) evaluation that can result in traffic accidents. Simulating these behaviors, unobserved in usual measurements, can contribute to improve the procedures that evaluate the probability of accidents on roads, actual deceleration data were used to evaluate the accidents probability and the results were compared to those obtained when the road maximum velocity changes. The paper is structured as follows: Sect. 2 presents the NaSch modified model, with heterogeneity in acceleration and deceleration policies and its influence in the traffic flow. Section 3 describes conditions for the occurrence of Dangerous Situations and analyzes results for distinct acceleration policies. In Sect. 4 we show results when actual deceleration data were used to evaluate the probability of accidents. Discussions and conclusions are presented in Sect. 5.

## 2 Modified Nagel-Schreckenberg Model

The Nagel-Schreckenberg (NaSch) model is a one-dimensional probabilistic cellular automata traffic model, that represents the lane as a lattice of cells, where a vehicle occupies one cell and each cell is either empty or occupied by one vehicle. At any instant of time  $t$ , a vehicle occupies the cell  $x(i, t)$  and has the velocity  $v(i, t)$ , which tells how many cells it will move at that instant of time. The number of unoccupied cells in front of each vehicle, generally called as gap, is denoted by  $d(i, t) = x(i + 1, t) - x(i, t) - L$ , where  $L = 1$  is the vehicles' length, and the vehicle  $i+1$  is considered to be in front of the vehicle  $i$ . A periodic boundary condition is considered. The four distinct rules applied in parallel for all vehicles are given by *Algorithm 1* (Table 1):

**Table 1.** Algorithm 1.

(1) Acceleration	$v(i, t + 1) = \min[v(i, t) + A, V_{max}]$
(2) Deceleration	$v(i, t + 1) = \min[v(i, t + 1), d(i, t)]$
(3) Random deceleration	$v(i, t + 1) = \max[v(i, t + 1) - A, 0]$ , with a probability $p$
(4) Movement	$x(i, t + 1) = x(i, t) + v(i, t + 1)$

The model uses parameters such as:  $V_{max}$ , the maximum velocity that a vehicle can reach;  $A$ , the acceleration rate of the vehicles;  $p$ , the stochastic parameter that represents the probability through which a vehicle randomly slows down, aiming to model the uncertainty about the drivers' behavior.

The traditional NaSch model sets  $A = 1 \text{ cell/s}^2$ . The typical length of a cell is 7.5 m. Each time step corresponds to one second, resulting vehicles' speed multiples of 1 cell/s, which is equivalent to 27 km/h. Also,  $V_{max}$  is typically set as 5 cell/s, corresponding to 135 km/h.

Although being a simple model, the NaSch model is able to represent traffic's main characteristics such as the spontaneous occurrence of traffic jams and to show the relation between traffic flow and density, representing two different phases (free and congested flow) and a transition stage between them [25].

### 2.1 The Proposed Modification in the NaSch Model

Despite the random deceleration rule in the NaSch model, the parameter  $A$  is a constant. We investigate whether different acceleration policies influence in traffic dynamics or not. A more refined lattice discretization is proposed to allow the representation of these different policies and each driver's profile tends to accelerate in a characteristic way: abruptly (aggressive profile) or more smoothly (non aggressive profile). A non-uniform Probability Density Function (PDF) is used to describe trends in the drivers' acceleration policy. The new acceleration parameter is stochastic and is calculated as  $A = \text{int}[(1 - \alpha)A_{max}]$ , where  $\alpha$  is a random value between 0 and 1 and  $\text{int}$  is the function that returns the nearest integer of its argument. Therefore, the probability  $p$  models the

drivers' intention to accelerate while  $\alpha$  models how they will accelerate. In this work,  $\alpha$  is modeled by a continuous Beta Function (PDF), defined as  $B(a, b) = \Gamma(a + b) / [\Gamma(a)\Gamma(b)x^{a-1}(1-x)^{b-1}]$ , where  $0 \leq x \leq 1$  and  $\Gamma(n+1) = n!$ ,  $n$  is a positive integer. Depending on the values of the parameters  $a$  and  $b$ , majority of  $\alpha$  values will tend to different values between 0 and 1 and those closer to 0 will produce accelerations  $A$  closer to  $A_{max}$ , while those closer to 1 will produce accelerations  $A$  closer to 0. In fact, the  $\alpha$  values float around the Beta mean value, which are given by  $\mu = \frac{a}{a+b}$ . Thus, it is possible to predict each profile acceleration trend based on the average of the Beta function used to model it. Therefore, each profile is defined by a different pair  $(a, b)$  of parameters, that defines a Beta function, and the different mean values of these distributions model the desired acceleration tendencies.

### 2.2 Numerical Results

For all results presented in this paper, the parameters of the model are set as: size of the cell equal to 1.5 m;  $V_{max} = 25 \text{ cell/s} = 135 \text{ km/h}$ ;  $A_{max} = 5 \text{ cell/s}^2 = 7.5 \text{ m/s}^2$ ;  $p = 0.30$ . To maintain analogy with the traditional NaSch model, a vehicle in our model occupies 5 cells = 7.5 m. Beta functions were chosen to represent the different acceleration policies, with distinct averages and similar variance. Besides the results from traditional NaSch model, four different profiles were considered in this work, the Beta functions that describe their acceleration are:  $B(10, 30)$ , Aggressive profile, with an average acceleration of  $\mu = 4 \text{ cells/s}^2$ ;  $B(20, 28)$ , called Intermediary I, with  $\mu = 3 \text{ cells/s}^2$ ;  $B(28, 20)$ , Intermediary II, with  $\mu = 2 \text{ cells/s}^2$  and  $B(30, 10)$ , Non-Aggressive profile, with  $\mu = 1 \text{ cells/s}^2$ .

All the simulations were performed with a lane composed of 10,000 cells, with density varying from 1 to 100 (given in percentage of occupied cells). An usual simulation varies the density  $\rho$ , while keeping constant the parameters  $V_{max}$ ,  $A$ , and  $p$ . A total of 15,000 time units were simulated, but only the data from the last 5,000 units were taken into consideration since transient effects were not the target. The modified model was configured, to every profile, with  $A_{max} = 5 \text{ cell/s}^2$ ,  $V_{max} = 25 \text{ cell/s}$ ,  $p = 0.30$ . The simulation starts with vehicles at random positions and  $V = 0$ .

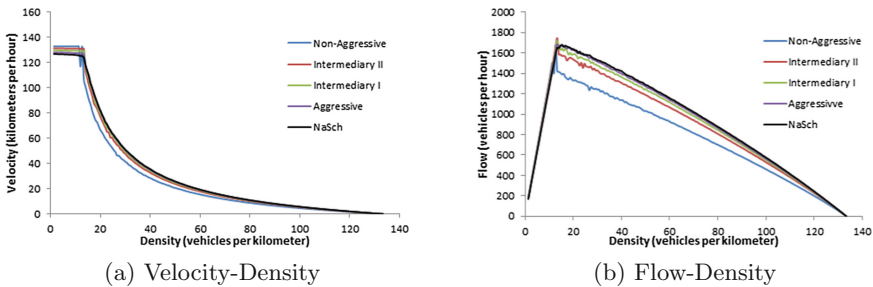


Fig. 1. Diagrams

**Traffic Flow,  $V_{max} = 25$  cell/s.** In this section, we present in Fig. 1, fundamental (flow-density) and velocity-density diagrams for traditional and modified NaSch model. Since  $V_{max} = 135$  km/h in all simulations, Fig. 1(a) and (b) show the impact of the different acceleration policies on traffic dynamics. In the free flow region of the diagram presented in Fig. 1(a), the mean velocity of the *Aggressive* driver is lower than the *Non-Aggressive* one, under the same speed limit. However, the inverse happens when the interaction between vehicles begins. Even though the modified model takes the NaSch as base, the *Non-Aggressive* profile starts to represent the meta-stability region in Fig. 1(b).

### 3 Conditions for the Occurrence of Dangerous Situations

In this work we analyzed the impact the consideration of different drivers profiles has in the occurrence of situations that can lead to traffic accidents, which is a heterogeneity unobserved in usual measurements in a usual scenario. It should be noted, in rule (2) of *Algorithm1*, that the models prevent collisions between vehicles. Thus, we analyze the occurrence of dangerous situations (DS) which could lead to collisions between vehicles in a real scenario. As usual, we consider the DS caused by sudden deceleration and sudden stop and adapt the conditions utilized by Moussa [12, 16, 24].

#### 3.1 Dangerous Situations Caused by Sudden Deceleration

Real accidents frequently happen when vehicles are at high speeds and a sudden deceleration occurs. If the vehicle  $i$ , that is behind the  $i + 1$ , is near enough, this situation may lead to an accident. Hence, we consider a Dangerous Situation (DS) due to sudden deceleration when the following conditions are satisfied:

$$\text{Condition 1: } \tau \cdot v(i, t) > d(i, t) + v(i + 1, t + 1)$$

$$\text{Condition 2: } v(i + 1, t) - v(i + 1, t + 1) \geq V_d$$

$\tau$  is a reaction time and the parameter  $V_d$  is the deceleration limit, beyond which the risk of an accident exists. In Condition 1, the vehicle  $i$  has a velocity  $v$  greater than the space  $d$  it has to move at the current time. The Condition 2 indicates when the front car has decelerated more than a limit  $V_d$ , previously defined.

#### 3.2 Dangerous Situations Caused by Sudden Stop

In this definition of DS, the vehicle  $i + 1$  will stop at the next instant of time and, since the vehicle  $i$ , that is behind it, is close enough, this situation might lead to an accident. In this context, the following conditions are satisfied:

$$\text{Condition 1: } \tau \cdot v(i, t) > d(i, t)$$

$$\text{Condition 2: } v(i + 1, t) \geq V_{min}$$

$$\text{Condition 3: } v(i + 1, t + 1) = 0$$

where  $V_{min}$  is a velocity limit, beyond which the risk of an accident exists. In Condition 1, the vehicle  $i$  is close enough to the vehicle ahead, i.e. it is at a speed  $v$  greater than the space it has to move at the current time. In Condition 2, the vehicle ahead  $i + 1$  is moving with a velocity higher than or equal to  $V_{min}$  at the current instant of time. The Condition 3 indicates that the vehicle ahead  $i + 1$  will stop at the next instant of time  $t + 1$ . Moussa [16] and Madani and Moussa [24], in their work with the NaSch model, consider  $V_{min} = 1 \text{ cell/s}$ , what corresponds to  $27 \text{ km/h}$  in their discretization. In this work we can represent velocities smaller than  $27 \text{ km/h}$ .

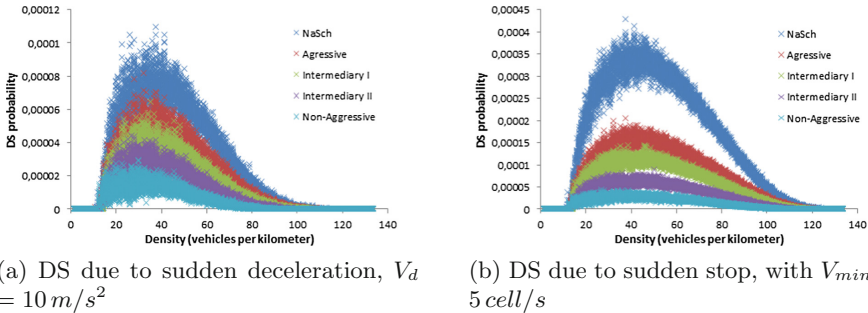


Fig. 2. Analysis of Dangerous Situations (DS)

### 3.3 Numerical Results

The probability per vehicle and per time step for a DS to occur is denoted by Pds. Figure 2(a) presents the results for the probability of Dangerous Situations (Pds) due to sudden deceleration, using  $V_d = 10 \text{ cell/s} = 15 \text{ m/s}^2$ , while Fig. 2(b) presents results for the Pds due to sudden stop, using  $V_{min} = 5 \text{ cell/s} = 27 \text{ km/h}$ , to compare with results presented in Madani and Moussa [24] for the NaSch model. In all simulations we consider  $\tau = 1 \text{ s}$ . We can observe that in the free flow region, since vehicles do not stop, there are no vehicle accidents. The value of the critical density where DS is maximum appears to remain unchanged with respect to the four different Beta functions.

Note that in the NaSch model, drivers have a constant acceleration rate  $A$  and, for the discretization used in this work,  $A = 5 \text{ cell/s}^2$ . The most aggressive driver considered in the modified NaSch model here proposed, accelerates  $A = 4 \text{ cell/s}^2 = 6.0 \text{ m/s}^2$  in average. For comparison reasons, the results presented for the NaSch model were obtained following the propositions of Madani and Moussa [24]. It is noticeable in Fig. 2(b) that, even under the same speed limit, the more aggressive the profile is, the higher is the probability of occurrence of DS.

## 4 Conditions for the Occurrence of Accidents

In the previous section we presented conditions that analyzed the occurrence of dangerous situations (DS) that can cause traffic accidents. However, in some cases an attentive driver would be able to avoid the accident. In this section we propose a new metric to evaluate the existence of DS that are highly probable to lead to accidents in a real scenario.

### 4.1 Accidents Probability

We intended to evaluate if a considered vehicle would be able to brake and avoid collision, given a maximum deceleration rate parameter being counted as an accident which does not occur. The metric is similar to the case of sudden stop, but now it is taken into account the maximum deceleration rate  $MDR$  a real vehicle is capable of performing. Thus, the conditions are defined as:

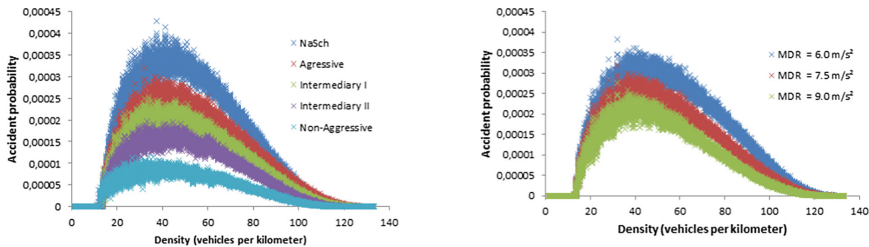
- Condition 1:  $v(i, t) - d(i, t) \geq MDR$
- Condition 2:  $v(i + 1, t) > 0$
- Condition 3:  $v(i + 1, t + 1) = 0$

Condition 1 indicates whether the vehicle  $i$  would have sufficient distance to brake or not. If  $v(i, t) - d(i, t) \geq MDR$ , then the vehicle  $i$  needs to perform a deceleration higher than  $MDR$ , what would be impossible in a real scenario. Conditions 2 and 3 represent the sudden stop.

### 4.2 Numerical Results

**Accident Probability.** Figure 3(a) presents the result obtained for  $MDR = 5 \text{ cell/s}^2 = 7.5 \text{ m/s}^2$  as the maximum deceleration rate that a vehicle could perform in a real scenario. Thus, we consider a real accident when a vehicle needs to decelerate more than  $7.5 \text{ m/s}^2$ .

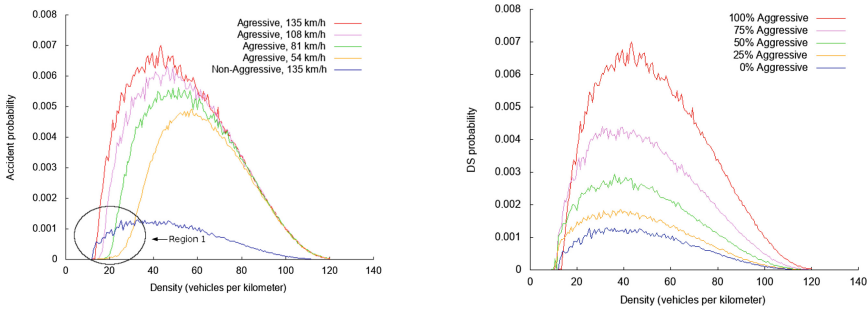
We considered that the Maximum Deceleration Rate ( $MDR$ ) of a normal vehicle is between  $6.0$  and  $9.0 \text{ m/s}^2$ . Figure 3(b) presents the result of accident probability for the *Aggressive* driver, where the parameter  $MDR$  is varied.



(a) Accident probability, for different drivers profile,  $MDR = 5 \text{ cell/s}^2$       (b) Aggressive driver for different  $MDR$

**Fig. 3.** Relations of the accident analysis with maximum deceleration rate ( $MDR$ )

**The Influence of the Speed Limit and the Drivers' Behavior.** Results presented in the previous sections indicate that the acceleration policies impact decisively on the number of accidents in the road. Thus, in order to compare the influence of the speed limit of the road with the impact of the acceleration policy of the drivers in the accident probability, we present a comparative result in Fig. 4(a). Five different driver profiles, with different speed limits, were considered: (1) Aggressive drivers with the speed limit of  $V_{max} = 25$  cell/s; with  $V_{max} = 20$  cell/s; with  $V_{max} = 15$  cell/s; and with  $V_{max} = 10$  cell/s; (2) Non-Aggressive drivers with the speed limit of  $V_{max} = 25$  cell/s. It can be observed in Fig. 4(a) that, for the used metric, even with the decrease of the speed limit for the Aggressive profile, its curve remains well above the curve for the Non-Aggressive profile with a much higher speed limit. This indicates that the vehicle acceleration policy has greater impact on the number of accidents due to collision between vehicles than the speed limit, except in Region 1 (Fig. 4), where in profiles using  $V_{max} = 135$  km/h there is already interaction between vehicles, earlier than the curves using  $V_{max} < 135$  km/h with Aggressive profile.



(a) Influence of  $V_{max}$  against the acceleration profile. (b) Sudden stop, with  $V_{min} = 5$  cell/s, for different profile ratios.

**Fig. 4.** Accident analysis

The situation in which the road is filled with drivers of different acceleration profiles is also analyzed. Aggressive and Non-Aggressive profiles, with the same speed limit of  $V_{max} = 25$  cell/s, are used to simulate that situation. Figure 4(b) presents results obtained due to sudden stop with  $V_{min} = 5$  cell/s, where different ratios for the Aggressive profile were considered. These results suggest that the probability of Dangerous Situations increases as the road's ratio of drivers with aggressive acceleration policies increases.

## 5 Conclusions

In this article we presented a modified version of the NaSch model that proposes to numerically analyze the influence of heterogeneity due to different acceleration



policies for vehicles under the same speed limit, that usually is an unobserved situation in usual traffic flow measuring. To enable this analysis, the lane discretization was refined and heterogeneity was introduced in the drivers' acceleration, using a continuous probability density function, the Beta function, to model it. The usage of functions with different mean values made possible the consideration of drivers with different steering behaviours, given by their acceleration profile. Dangerous Situations on roads, which can cause collisions between vehicles, were also analyzed for this modified NaSch model. Actual deceleration rates data was used to evaluate the probability of accidents and the obtained results were compared to those obtained when varying the road's maximum velocity.

Having distinct drivers' behaviors allowed us to capture its effects on traffic dynamics and evaluate the most important features of the traffic flow phenomena. It was shown, for instance, how the fundamental diagram are affected by these behaviors. We observed that, even under the same maximum velocities, different policies influences the flow, improving the average speed in free flow regimes and altering the region of bottled flow, depending on the considered profile.

Using the Dangerous Situation definition [12,16,24], adapted for our proposed modified NaSch model, we noted that DS decreased with more cautious acceleration policies. We also observed that these behaviors have fundamental importance on avoiding collisions between vehicles and may be more relevant than the maximum speed of the road.

**Acknowledgement.** Authors thank CNPq/PIBIC/PIBIT (UFF, LNCC) scholarship.

## References

1. Nagel, K., Schreckenberg, M.: A cellular automaton model for freeway traffic. *Journal de physique I* **2**(12), 2221–2229 (1992)
2. Takayasu, M., Takayasu, H.:  $1/f$  noise in a traffic model. In: *Fractals in Natural Sciences*, pp. 486–492 (1994)
3. Barlovic, R., Huisinga, T., Schadschneider, A., Schreckenberg, M.: Open boundaries in a cellular automaton model for traffic flow with metastable states. *Phys. Rev. E* **66**(4), 046113 (2002)
4. Kuang, H., Zhang, G.X., Li, X.L., Lo, S.M.: Effect of slow-to-start in the extended BML model with four-directional traffic. *Phys. Lett. A* **378**(21), 1455–1460 (2014)
5. Larraga, M.E., del Río, J.A., Schadschneider, A.: New kind of phase separation in a CA traffic model with anticipation. *J. Phys. A: Math. General* **37**(12), 3769 (2004)
6. Larraga, M.E., Alvarez-Icaza, L.: Cellular automaton model for traffic flow based on safe driving policies and human reactions. *Physica A: Stat. Mech. Appl.* **389**(23), 5425–5438 (2010)
7. Knospe, W., Santen, L., Schadschneider, A., Schreckenberg, M.: Towards a realistic microscopic description of highway traffic. *J. Phys. A: Math. General* **33**(48), L477 (2000)
8. Knospe, W., Santen, L., Schadschneider, A., Schreckenberg, M.: A realistic two-lane traffic model for highway traffic. *J. Phys. A: Math. General* **35**(15), 3369 (2002)

9. Tian, J.F., Jia, N., Zhu, N., Jia, B., Yuan, Z.Z.: Brake light cellular automaton model with advanced randomization for traffic breakdown. *Transp. Res. Part C: Emerg. Technol.* **44**, 282–298 (2014)
10. Zamith, M., Leal-Toledo, R.C.P., Clua, E.: A novel cellular automaton model for traffic freeway simulation. In: Sirakoulis, G.C., Bandini, S. (eds.) *ACRI 2012*. LNCS, vol. 7495, pp. 524–533. Springer, Heidelberg (2012). [https://doi.org/10.1007/978-3-642-33350-7\\_54](https://doi.org/10.1007/978-3-642-33350-7_54)
11. Zamith, M., Leal-Toledo, R.C.P., Clua, E., Toledo, E.M., de Magalhães, G.V.: A new stochastic cellular automata model for traffic flow simulation with drivers' behavior prediction. *J. Comput. Sci.* **9**, 51–56 (2015)
12. Boccara, N., Fuks, H., Zeng, Q.: Car accidents and number of stopped cars due to road blockage on a one-lane highway. *J. Phys. A: Math. General* **30**(10), 3329 (1997)
13. Huang, D.W.: Exact results for car accidents in a traffic model. *J. Phys. A: Math. General* **31**(29), 6167 (1998)
14. Fukui, M., Ishibashi, Y.: Traffic flow in 1D cellular automaton model including cars moving with high speed. *J. Phys. Soc. Jpn.* **65**(6), 1868–1870 (1996)
15. Jiang, R., Jia, B., Wang, X.L., Wu, Q.S.: Dangerous situations in the velocity effect model. *J. Phys. A: Math. General* **37**(22), 5777 (2004)
16. Moussa, N.: Car accidents in cellular automata models for one-lane traffic flow. *Phys. Rev. E* **68**(3), 036127 (2003)
17. Bentaleb, K., Lakouari, N., Marzoug, R., Ez-Zahraouy, H., Benyoussef, A.: Simulation study of traffic car accidents in single-lane highway. *Phys. A: Stat. Mech. Appl.* **413**, 473–480 (2014)
18. Huang, D.W., Wu, Y.P.: Car accidents on a single-lane highway. *Phys. Rev. E* **63**(2), 022301 (2001)
19. Yang, X.Q., Ma, Y.Q.: Car accidents in the deterministic and nondeterministic Nagel-Schreckenberg models. *Modern Phys. Lett. B* **16**(09), 333–344 (2002)
20. Jiang, R., Wang, X.L., Wu, Q.S.: Dangerous situations within the framework of the Nagel-Schreckenberg model. *J. Phys. A: Math. General* **36**(17), 4763 (2003)
21. Moussa, N.: Simulation study of traffic accidents in bidirectional traffic models. *Int. J. Modern Phys. C* **21**(12), 1501–1515 (2010)
22. Zhang, W., Yang, X.Q., Sun, D.P., Qiu, K., Xia, H.: Traffic accidents in a cellular automaton model with a speed limit zone. *J. Phys. A: Math. General* **39**(29), 9127 (2006)
23. Marzoug, R., Echab, H., Lakouari, N., Ez-Zahraouy, H.: Car accidents at the intersection with speed limit zone and open boundary conditions. In: El Yacoubi, S., Waş, J., Bandini, S. (eds.) *ACRI 2016*. LNCS, vol. 9863, pp. 303–311. Springer, Cham (2016). [https://doi.org/10.1007/978-3-319-44365-2\\_30](https://doi.org/10.1007/978-3-319-44365-2_30)
24. Madani, A., Moussa, N.: ICS: an interactive control system for simulating the probability of car accidents with object oriented paradigm and cellular automaton. *Int. J. Comput. Sci. Eng.* **3**(8), 2965 (2011)
25. Gerwinski, M.: Krug: analytic approach to the critical density in cellular automata for traffic flow. *Phys. Rev. E* **60**(1), 188 (1999)