# Asynchronous Control of Switched Nonlinear Systems

**Jiaojiao Ren, Xinzhi Liu, Hong Zhu and Shouming Zhong**

**Abstract** This paper studies the problem of asynchronous control of switched nonlinear systems. The asynchronous control means that the switchings between the candidate controllers and system models are asynchronous. By using the piecewise Lyapunov function and average dwell time approach, the asynchronously switched stabilizing control problem for nonlinear systems is solved under the proposed switching law, which allows us to have a stable or unstable subnonlinear system. Illustrative examples are provided to show the effectiveness of the results.

**Keywords** Asynchronous control · Switched nonlinear system
Exponential stability · Average dwell time

## 1 Introduction

Switched systems [1, 2], consisting of a family of subsystems and a switching rule that orchestrates the switching between them, have been used to model many physical

J. Ren (✉)
School of Information Science and Engineering,
Chengdu University, Chengdu 610106, People's Republic of China
e-mail: jiaojiaoren06@163.com

J. Ren · X. Liu
Department of Applied Mathematics, University of Waterloo,
Waterloo, ON N2L 3G1, Canada
e-mail: xzliu@uwaterloo.ca

H. Zhu
School of Automation Engineering, University of Electronic Science
and Technology of China, Sichuan 611731, People's Republic of China
e-mail: zhuhong@uestc.edu.cn

S. Zhong
School of Mathematical Sciences, University of Electronic Science
and Technology of China, Sichuan 611731, People's Republic of China
e-mail: zhongsm@uestc.edu.cn



D. M. Kilgour et al. (eds.), *Recent Advances in Mathematical and Statistical Methods*, Springer Proceedings in Mathematics & Statistics 259,
https://doi.org/10.1007/978-3-319-99719-3_55

or man-made systems displaying switching features. The diverse switching signals differentiate switched systems from general time-varying systems, since the solutions of the switched systems are dependent on not only the system's initial conditions but also the switching signals. This class of systems have numerous applications in the control of mechanical systems, the automotive industry, air traffic control, switching power converters and many other fields [2].

In switched systems, each subsystem is called a mode, and control problems are said to design a set of mode-dependent controllers or mode-independent controllers for the unforced system and find admissible switching signals such that the resulting systems is stable or satisfies certain performance criteria [2–4]. As we know, mode-dependent controller design is less conservative. However, for the control problem, it inevitably takes some time to identify the system modes and apply the matched controller. So, a very common assumption in the mode-dependent context, the controllers are switched synchronously with the switching of system modes, is quite unpractical. Therefore, the asynchronous phenomena between the system modes and the controller modes always exists. Recently, some efforts have been made to study asynchronous control problems [5–9]. In [5–8], each subsystem is stable. In [5], desirable controller is designed such that the energy function is decreasing in each switching interval (both mismatched period and matched period). This requirement is weakened in [6–8]. The energy function is not required decreasing in mismatched period any more. Most recently, in [9], the authors deal with asynchronous stabilization problem of switched system, which contains stable and unstable subsystems. However, the condition $\inf_{t\geq t_0}[\frac{T^-(t)}{T^+(t)}] \geq -\frac{\beta}{\alpha}$ can not guarantee the condition $-\gamma t = T^-(t)\alpha + T^+(t)\beta$ holds, which only can guarantee $T^-(t)\alpha + T^+(t)\beta < 0$ holds, where, $T^-(t)$ and $T^+(t)$ represent, respectively, the total active time of subsystems that are stable, not stable subsystems over $(0, t)$; $\alpha$, $\beta$ and $\gamma$ are constants. Therefore, a switching law is need.

In this paper, the problem of asynchronous control of switched nonlinear systems is studied. By using the piecewise Lyapunov function and average dwell time approach, the asynchronously switched stabilizing control problem for nonlinear systems is solved under the proposed switching law , which allows us to have stable and unstable subnonlinear system. Some examples are provided to show the effectiveness of the results.

## 2 Problem Descriptions and Preliminaries

Consider the following switched nonlinear system:

$$\dot{x}(t) = f_{\sigma(t)}(x(t), u(t)), \tag{1}$$

where $x(t) \in R^n$ is a state vector and $u(t) \in R^m$ is a control input vector. $f_{\sigma(t)}$ are a set of regularly nonlinear functions. $\sigma(t) : [0, \infty) \to \mathbb{S}$ is the switching signal, i.e., $\sigma(t) = i_k \in \mathbb{S}$ for $t \in [t_k, t_{k+1})$, where $t_k$ is the $k$th switching time instant, $\mathbb{S} = \{1, 2, \ldots, s\}$, $s, k \in \mathbb{N}$. $0 = t_0 < t_1 < \ldots < t_k < \ldots$, $\lim_{k\to\infty} t_k = \infty$, which can rule out Zeno behavior automatically.

In fact, for the control problem, it inevitably takes some time to identify the system modes and apply the matched controller. Therefore, the asynchronous phenomena between the system modes and the controller modes always exists. In this paper, we assume that the time lag of controllers modes to system modes is $t_d > 0$ ($t_d < t_{k+1} - t_k, k \in N$). The state feedback control input can be written as $u(t) = K_{\sigma(t-t_d)} x(t)$.

Before proceeding further, the following definitions are introduced.

**Definition 2.1** [10] For a switching signal $\sigma(t)$ and any $t^{''} > t^{'} > t_0$, let $N_\sigma(t^{'}, t^{''})$ be the switching numbers of $\sigma(t)$ over the interval $[t^{'}, t^{''})$. If $N_\sigma(t^{'}, t^{''}) \le N_0 + \frac{t^{''} - t^{'}}{\tau_a}$ holds for $N_0 \ge 1$, $\tau_a > 0$, then $N_0$ and $\tau_a$ are called the chatter bound and the average dwell time, respectively.

**Note that**: When the active subsystems are changed at some time instant, a switching happens. Therefore, switching numbers mean the total numbers of switching.

**Definition 2.2** [6] The equilibrium point of system (1) is globally uniformly exponentially stable under certain switching signals $\sigma(t)$ if, for $u(t)$, there exist constants $K > 0$ and $\delta > 0$ such that the solution of the system satisfies $\|x(t)\| \le K e^{-\delta(t-t_0)} \|x(t_0)\|$, $\forall t \ge t_0$.

# 3 Main Results

In this section, we first proposed a switching law for the system (1). Under this switching law, the sufficient condition is given to guarantee the system (1) without control input is exponentially stable by using average dwell time. Second, the obtained result is extended to the system with control input.

## 3.1 Exponential Stability for the System (1) Without Control Input

**Switching law 3.1** *[11] Let* $0 = t^0 < t^1 < t^2 < \ldots$ $(\lim_{j\to\infty} t^j = \infty)$ *be a specified sequence of time instants satisfying* $\sup_j \{t^{j+1} - t^j\} = T < \infty$. *Determine the switching signal* $\sigma(t)$ *such that the inequality* $T^-(t^j, t^{j+1}) / T^+(t^j, t^{j+1}) \ge -(\beta + \alpha^*)/(\alpha + \alpha^*)$ *holds on every time interval* $[t^j, t^{j+1})(j = 0, 1, \ldots)$, *where* $0 < \alpha^* < -\alpha$, $\alpha$ *and* $\beta$ *are given constants,* $T^-(t^j, t^{j+1})$ *and* $T^+(t^j, t^{j+1})$ *denote the total active time of stable and unstable subsystems respectively over* $(t^j, t^{j+1})$.

Based on the given switching law 3.1, the following theorem is presented to guarantee the system is exponentially stable.

**Theorem 3.1** *For the given scalars $\alpha_{\sigma(t)}$ and $\mu \geq 1$, the system (1) with $u(t) = 0$, under the switching law 3.1, is exponentially stable if there exist Lyapunov functions $V_{\sigma(t)}(t) : R^n \to R$, and two positive constants $K_1$ and $K_2$ such that $\forall \sigma(t) = i \in \mathbb{S}$ the following inequalities hold*

$$K_1 \|x(t)\|^2 \leq V_i(t) \leq K_2 \|x(t)\|^2, \tag{2}$$

$$\dot{V}_i(t) \leq \alpha_i V_i(t), t \in [t_k, t_{k+1}) \tag{3}$$

$$V_{\sigma(t_k)}(t_k) \leq \mu V_{\sigma(t_k^-)}(t_k^-), \tag{4}$$

$$\tau_a > \frac{\ln \mu}{\alpha^*}. \tag{5}$$

*Proof* When $\forall t \in [t_k, t_{k+1})$, for $\sigma(t) = i \in \mathbb{S}$, $k \in N$, it means the switched system is active within the $i$th subsystem. From (3) and (4), we have

$$\begin{aligned} V_{\sigma(t)}(t) &\leq e^{\alpha_i (t-t_k)} V_{\sigma(t_k)}(t_k) \\ &\leq \mu e^{\alpha_i (t-t_k)} V_{\sigma(t_k^-)}(t_k^-) \\ &\leq \mu e^{\alpha_i (t-t_k) + \alpha_{\sigma(t_{k-1})}(t_k - t_{k-1})} V_{\sigma(t_{k-1})}(t_{k-1}) \\ &\leq \mu^2 e^{\alpha_i (t-t_k) + \alpha_{\sigma(t_{k-1})}(t_k - t_{k-1})} V_{\sigma(t_{k-1}^-)}(t_{k-1}^-) \\ &\leq \ldots \\ &\leq \mu^{N_\sigma(t_0,t)} V_{\sigma(t_0)}(t_0) e^{\alpha T^-(t_0,t) + \beta T^+(t_0,t)}, \end{aligned} \tag{6}$$

where $\alpha = \sup_{i \in \mathbb{S}}\{\alpha_i : \alpha_i < 0\}$, $\beta = \sup_{i \in \mathbb{S}}\{\alpha_i : \alpha_i \geq 0\}$, $T^-(t_0, t)$ and $T^+(t_0, t)$ denote the total active time of those subsystems that are stable, not stable subsystems over $(t_0, t)$, respectively.

Suppose $0 = t^0 < t^1 < t^2 < \ldots (\lim_{j\to\infty} t^j = \infty)$ be a specified sequence of time instants satisfying Switching law 3.1. For any $t$, we have two cases:

(1) For $t_0$ and $t$ satisfying $t^{j-1} < t_0 \leq t^j < t^{j+1} < \ldots < t^k \leq t$, one has

$$\begin{aligned} &\frac{T^-(t^j, t^{j+1})}{T^+(t^j, t^{j+1})} \geq -\frac{\beta + \alpha^*}{\alpha + \alpha^*} \\ &\Rightarrow T^-(t^j, t^{j+1})(-\alpha - \alpha^*) \geq T^+(t^j, t^{j+1})(\beta + \alpha^*) \\ &\Rightarrow -\alpha^*(T^-(t^j, t^{j+1}) + T^+(t^j, t^{j+1})) \geq \alpha T^-(t^j, t^{j+1}) + \beta T^+(t^j, t^{j+1}) \\ &\Rightarrow -\alpha^*(t^{j+1} - t^j) \geq \alpha T^-(t^j, t^{j+1}) + \beta T^+(t^j, t^{j+1}), \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{T^-(t^j, t^{j+1})}{T^+(t^j, t^{j+1})} \geq -\frac{\beta + \alpha^*}{\alpha + \alpha^*} \\ &\Rightarrow T^-(t^j, t^{j+1})(-\alpha - \alpha^*) \geq T^+(t^j, t^{j+1})(\beta + \alpha^*) \\ &\Rightarrow T^-(t^j, t^{j+1})(-\alpha - \alpha^*) + T^+(t^j, t^{j+1})(-\alpha - \alpha^*) \end{aligned}$$

$$\geq T^{+}(t^{j}, t^{j+1})(\beta + \alpha^{*}) + T^{+}(t^{j}, t^{j+1})(-\alpha - \alpha^{*})$$
$$\Rightarrow T^{+}(t^{j}, t^{j+1}) \leq \frac{-\alpha - \alpha^{*}}{\beta - \alpha} T, \tag{8}$$

and whether or not the activated subsystems over the interval $[t_0, t^j]$ and $[t^k, t]$ are stable subsystems, we consider that the activated subsystems over the interval $[t_0, t^j]$ and $[t^k, t]$ are unstable subsystems. Then, one obtain

$$e^{\alpha T^{-}(t_0,t)+\beta T^{+}(t_0,t)} \leq e^{\beta(t^j - t_0)+\sum_{q=j}^{k-1}[\beta T^{+}(t^q,t^{q+1})+\alpha T^{-}(t^q,t^{q+1})]+\beta(t-t^k)}$$

According to (7) and (8), one obtains

$$\begin{aligned} e^{\alpha T^{-}(t_0,t)+\beta T^{+}(t_0,t)} &\leq e^{\beta(t-t^k)-\alpha^{*}\sum_{q=j}^{k-1}(t^{q+1}-t^q)+\beta(t^j-t_0)} \\ &= e^{\beta(t-t^k)-\alpha^{*}(t^k-t^j)+\beta(t^j-t_0)} \\ &= e^{(\beta+\alpha^{*})(t-t^k+t^j-t_0)-\alpha^{*}(t-t_0)} \\ &\leq e^{(\beta+\alpha^{*})(T^{+}(t^k,t^{k+1})+T^{+}(t^{j-1},t^j))-\alpha^{*}(t-t_0)} \\ &\leq e^{\gamma-\alpha^{*}(t-t_0)}, \end{aligned} \tag{9}$$

where $\gamma = \frac{-2(\beta+\alpha^{*})(\alpha+\alpha^{*})}{\beta-\alpha} T$.

(2) For $t_0$ and $t$ satisfying $t^q \leq t_0 < t \leq t^{q+1}$, one has

$$\begin{aligned} e^{\alpha T^{-}(t_0,t)+\beta T^{+}(t_0,t)} &\leq e^{\beta(t-t_0)} \\ &= e^{(\beta+\alpha^{*})(t-t_0)-\alpha^{*}(t-t_0)} \\ &\leq e^{\gamma-\alpha^{*}(t-t_0)}, \end{aligned} \tag{10}$$

where $\gamma$ has the same value as the one above.

Based on (6), (9) and (10) and average dwell time, for any $t$, the following inequality holds

$$\begin{aligned} V_{\sigma(t)}(t) &\leq \mu^{N_\sigma(t_0,t)} V_{\sigma(t_0)}(t_0) e^{\alpha T^{-}(t_0,t)+\beta T^{+}(t_0,t)} \\ &\leq e^{N_0 \ln\mu+\gamma} e^{(\frac{\ln\mu}{\tau_a}-\alpha^{*})(t-t_0)} V_{\sigma(t_0)}(t_0). \end{aligned} \tag{11}$$

From (2) and (5), we have

$$\|x(t)\| \leq K e^{-\kappa(t-t_0)} \|x(t_0)\|, \tag{12}$$

where $K = (\frac{K_2}{K_1} e^{N_0 \ln\mu+\gamma})^{1/2}$ and $\kappa = \frac{1}{2}(\alpha^{*} - \frac{\ln\mu}{\tau_a})$.

Therefore, the system (1) without control input is exponentially stable.

### 3.2 Exponential Stability for the System (1) With Control Input

**Switching law 3.2** *Let* $0 = t^0 < t^1 < t^2 < \ldots (\lim_{j\to\infty} t^j = \infty)$ *be a specified sequence of time instants satisfying* $\sup_j\{t^{j+1} - t^j\} = T < \infty$. *Determine the switching signal* $\sigma(t)$ *such that the inequality* $(T^-(t^j, t^{j+1}) - N_\sigma^-(t^j, t^{j+1})t_d)/(T^+(t^j, t^{j+1}) + N_\sigma^-(t^j, t^{j+1})t_d) \geq -(\beta + \alpha^*)/(\alpha + \alpha^*)$ *holds on every time interval* $[t^j, t^j + 1)(j = 0, 1, \ldots)$, *where* $0 < \alpha^* < -\alpha$, $\alpha$ *and* $\beta$ *are given constants.* $T^-(t^j, t^{j+1})$, $T^+(t^j, t^{j+1})$ *and* $N_\sigma^-(t^j, t^{j+1})$ *denote the total active time of those subsystems that are stable, not stable subsystems and the total switching numbers of stable subsystems over* $(t^j, t^{j+1})$, *respectively.*

Based on the given switching law 3.2, the following theorem is presented to guarantee the system is exponentially stable.

**Theorem 3.2** *For the given scalars* $\alpha_{\sigma(t),\sigma(t-t_d)}$ *and* $\mu \geq 1$, *the system (1) with* $u(t) = K_{\sigma(t-t_d)}x(t)$, *under the switching law 3.2, is exponentially stable if there exist Lyapunov functions* $V_{\sigma(t),\sigma(t-t_d)}(t) : R^n \to R$, *and two positive constants* $\hat{K}_1$ *and* $\hat{K}_2$ *such that* $\forall \sigma(t) = i$, $\sigma(t - t_d) = p$, $\forall i, p \in \mathbb{S}$ *the following inequalities hold*

$$\hat{K}_1\|x(t)\|^2 \leq V_{i,p}(t) \leq \hat{K}_2\|x(t)\|^2, \tag{13}$$

$$\dot{V}_{i,p}(t) \leq \begin{cases} \alpha_{i,p}V_{i,p}(t), & t \in [t_k, t_k + t_d), i \neq p, \\ \alpha_{i,i}V_{i,i}(t), & t \in [t_k + t_d, t_{k+1}), i = p, \end{cases} \tag{14}$$

$$V_{\sigma(t_k),\sigma(t_k-t_d)}(t_k) \leq \hat{\mu}V_{\sigma(t_k^-),\sigma(t_k^- - t_d)}(t_k^-), \tag{15}$$

$$V_{\sigma(t_k+t_d),\sigma(t_k)}(t_k + t_d) \leq \hat{\mu}V_{\sigma(t_k^-+t_d),\sigma(t_k^-)}(t_k^- + t_d), \tag{16}$$

$$\tau_a > \frac{2\ln\hat{\mu}}{\alpha^*}. \tag{17}$$

*Proof* When $\forall t \in [t_k + t_d, t_{k+1})$, $\sigma(t) = i \in \mathbb{S}$; $\forall t \in [t_k, t_k + t_d)$, $\sigma(t - t_s) = p \in \mathbb{S}$, $k \in N$. From (14), (15) and (16), we have

$$\begin{aligned} V_{\sigma(t),\sigma(t-t_d)}(t) &\leq e^{\alpha_{i,i}(t-t_k-t_d)}V_{\sigma(t_k+t_d),\sigma(t_k)}(t_k + t_d) \\ &\leq \hat{\mu}e^{\alpha_{i,i}(t-t_k-t_d)}V_{\sigma(t_k^-+t_d),\sigma(t_k^-)}(t_k^- + t_d) \\ &\leq \hat{\mu}e^{\alpha_{i,i}(t-t_k-t_d)+\alpha_{i,p}t_d}V_{\sigma(t_k),\sigma(t_k-t_d)}(t_k) \\ &\leq \hat{\mu}^2e^{\alpha_{i,i}(t-t_k-t_d)+\alpha_{i,p}t_d}V_{\sigma(t_k^-),\sigma(t_k^--t_d)}(t_k^-) \\ &\leq \ldots \\ &\leq \hat{\mu}^{2N_\sigma(t_0,t)}e^{\alpha(T^-(t_0,t)-N_\sigma^-(t_0,t)t_d)+\beta(T^+(t_0,t)+N_\sigma^-(t_0,t)t_d)}\times \\ & V_{\sigma(t_0),\sigma(t_0-t_d)}V(t_0), \end{aligned} \tag{18}$$

where $\alpha = \sup_{i \in \mathbb{S}}\{\alpha_{i,i}, \alpha_{i,i} < 0\}, \beta = \sup_{i,p \in \mathbb{S}, i \neq p}\{\alpha_{i,p}, \alpha_{i,i} > 0\}, T^-(t_0, t), T^+(t_0, t)$ and $N_\sigma^-(t_0, t)$ denote the total active time of those subsystems that are stable, not stable subsystems and the total switching numbers of stable subsystems over $(t_0, t)$, respectively.

Combining Switching law 3.2 and following the similar proof procedure, we can conclude that the system (1) with $u(t) = K_{\sigma(t-t_d)}x(t)$, under the switching law 3.2, is exponentially stable.

## 4 Numerical Examples

*Example 1* Consider the following switched nonlinear system without control input

**Switching Region 1**: $\sigma(t) = 1$

$$\begin{aligned} \dot{x}_1(t) &= 0.2x_1(t) + 0.1x_2(t) - 0.15|sin(10x_2(t))|e^{-sin(10x_2(t))}x_2(t) \\ \dot{x}_2(t) &= 0.7x_1(t) + 0.02x_2(t) \end{aligned} \tag{19}$$

**Switching Region 2**: $\sigma(t) = 2$

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) \\ \dot{x}_2(t) &= -0.2|cos(10x_1(t))|x_1(t) - x_2(t) \end{aligned} \tag{20}$$

Here, let $\alpha_1 = 1.5$, $\alpha_2 = -0.6$, $\alpha^* = 0.3$ and $\mu = 1.2$. According to Switching law 3.1, for $t \in [0, 12]$, the switching signal $\sigma(t)$ is given as follows:

$$\begin{aligned} \sigma(t) = 1: \quad & t \in [0, 0.3), \quad [2.9, 3.4), \quad [6.8, 7.1), \quad [8.9, 9.4), \\ \sigma(t) = 2: \quad & t \in [0.3, 2.9), \quad [3.4, 6.8), \quad [7.1, 8.9), \quad [9.4, 12], \end{aligned}$$

where the specified sequence of time instants $\{t_n\}_{n=0}^4$ is given as $\{0, 3, 6, 9, 12\}$. Note that the condition $1.5 = \tau_a \geq \frac{\ln \mu}{\alpha^*} = 0.6077$ also holds. The simulation results are shown in Figs. 1 and 2, which well illustrate Theorem 3.1.

*Example 2* Consider the following switched nonlinear system

**Switching Region 1**: $\sigma(t) = 1$

$$\begin{aligned} \dot{x}_1(t) &= x_1(t) + 0.1x_2(t) + 0.15|sin(6x_1(t))|e^{-sin(6x_1(t))}x_2(t) \\ &\quad + (0.1 + \frac{0.4}{e}|sin(6x_1(t))|e^{-sin(6x_1(t))})u_1(t) \\ \dot{x}_2(t) &= 0.7x_1(t) + 0.2x_2(t), \end{aligned} \tag{21}$$

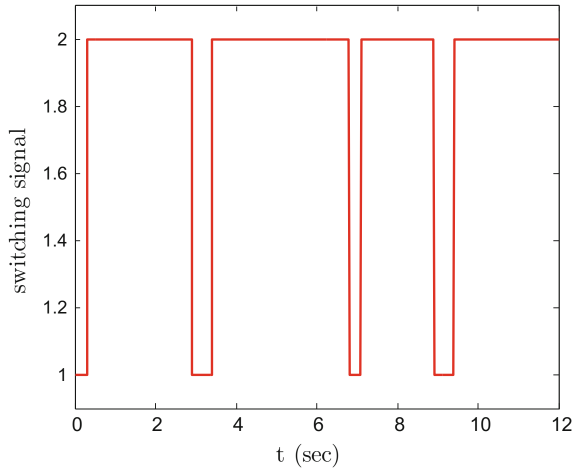


**Fig. 1** The switching signal $\sigma(t)$

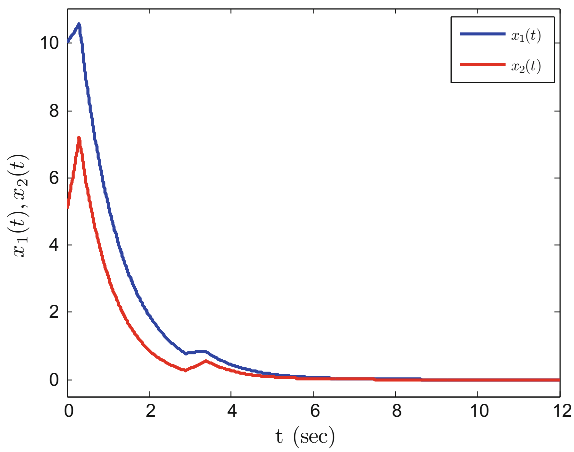


**Fig. 2** The state trajectories of the system (3)

**Switching Region 2**: $\sigma(t) = 2$

$$\begin{aligned}\dot{x}_1(t) &= 0.6x_1(t) + 0.3u_1(t) + 0.7|cos(6x_2(t))|u_1(t) \\ \dot{x}_2(t) &= 0.2|cos(10x_2(t))|x_1(t) - 0.7x_2(t),\end{aligned} \tag{22}$$

Here, let $\alpha_{11} = 0.2$, $\alpha_{12} = 0.5$, $\alpha_{21} = 0.1$, $\alpha_{22} = -0.3$, $\alpha^* = 0.2$, $\mu = 1.1$ and $t_d = 0.2$. According to Switching law 3.2, for $t \in [0, 12]$, the switching signal $\sigma(t)$ and $\sigma(t - t_d)$ are given as follows:

$$\begin{aligned}\sigma(t) = 1: &\quad t \in [0, 0.3), \quad [2.9, 3.4), \quad [6.8, 7.1), \quad [9.0, 9.4), \\ \sigma(t) = 2: &\quad t \in [0.3, 2.9), \quad [3.4, 6.8), \quad [7.1, 9.0), \quad [9.4, 12],\end{aligned}$$

$$\sigma(t-t_d) = 1: \quad t \in [0, 0.5), \quad [3.1, 3.6), \quad [7.0, 7.3), \quad [9.2, 9.6),$$
$$\sigma(t-t_d) = 2: \quad t \in [0.5, 3.1), \quad [3.6, 7.0), \quad [7.3, 9.2), \quad [9.6, 12],$$

where the specified sequence of time instants $\{t_n\}_{n=0}^{4}$ is given as {0, 3, 6, 9, 12}. Note that the condition $1.5 = \tau_a \geq \frac{2\ln\mu}{\alpha^*} = 0.9532$ also holds. The simulation results are shown in Figs. 3 and 4, which well illustrate Theorem 3.2.

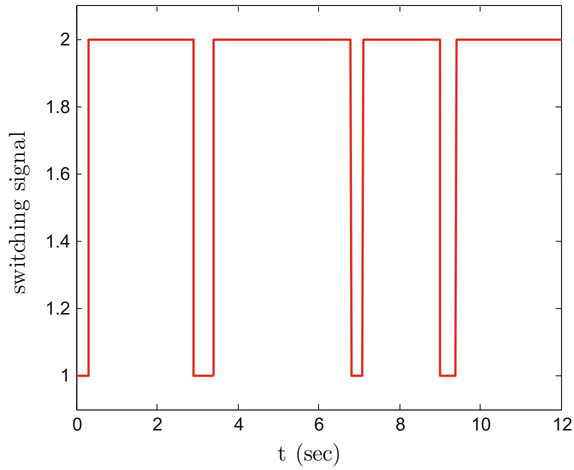

**Fig. 3** The switching signal $\sigma(t)$

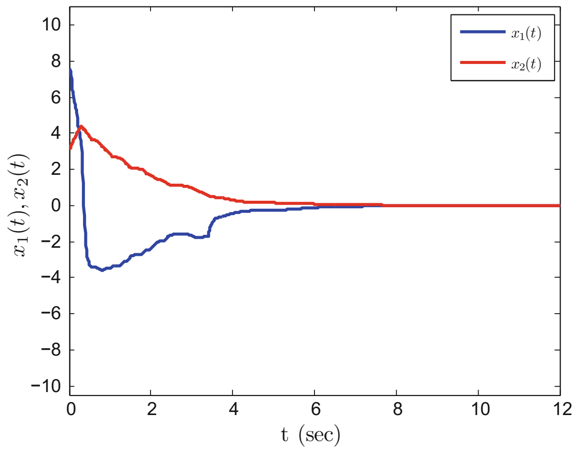

**Fig. 4** The state trajectories of the system (3)

## 5 Conclusion

In this paper, the piecewise Lyapunov function and average dwell time approach have been used to investigate the problem of asynchronous control of switched nonlinear systems. By using the proposed switching law, the asynchronously switched stabilizing control problem for nonlinear systems has been solved, which allows us to have stable and unstable subnonlinear system. Illustrative examples have been provided to show the effectiveness of the results.

## References

1. Hespanha, J.P: Uniform stability of switched linear systems extensions of Lasalle's invariance principle. IEEE Trans. Autom. Control **49**(4), 470–482 (2004)
2. Lu, B., Wu, F., Kim, S.: Switching LPV control of an F-16 aircraft via controller state reset. IEEE Trans. Control Syst. Technol. **14**(2), 267–277 (2003)
3. Rinehart, M., Dahleh, M., Reed, D., Kolmanovsky, I.: Suboptimal control of switched systems with an application to the disc engine. IEEE Trans. Control Syst. Technol. **16**(2), 189–201 (2008)
4. Xu, X., Antsaklis, P.J.: Optimal control of switched systems based on parameterization of the switching instants. IEEE Trans. Autom. Control **49**(1), 2–16 (2004)
5. Zhang, Z.R., Wang, R.H.: Roust control for uncertain switched nonlinear systems with time delay under asynchronous swithching. IET Control Theor. Appl. **3**(8), 1041–1050 (2009)
6. Zhang, L.X., Gao, H.J.: Asynchronously switched control of switched linear systems with average dwell time. Automatic **46**, 953–958 (2010)
7. Xiang, M., Xiang, Z.R., Karimi, H.R.: Asynchronous $L_1$ control of delayed swithched positive systems with mode-dependent average dwell time. Inf. Sci. **278**, 703–714 (2014)
8. Ren, W., Xiong, J.L.: Stability and stabilization of switched stochastic systems under asynchronous switching. Syst. Control Lett. **97**, 184–192 (2016)
9. Mao, Y.B., Zhang, H.B., Xu, S.Y.: The exponential stability and asynchronous stabilization of a class of switched nonlinear system via the T-S fuzzy model. IEEE Trans. Fuzzy Syst. **22**(4), 817–828 (2014)
10. Hespanha, J.P., Morse, A.S.: Stability of switched systems with average dwell time. In: Proceedings of 38th Conference on Decision Control Phoenix, AZ 2655–2660 (1999)
11. Zhai, G.S., Hu, B., Yasuda, K., Michel, A.N.: Disturbance attenuation properties of time-controlled switched systems. J. Frankl. Inst. **338**, 765–779 (2001)