The Mereologies of Upper Ontologies

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Abstract. Mereology, the formal theory of parts and wholes, has a played a prominent role within applied ontology. As a fundamental set of concepts for commonsense reasoning, it also appears in a number of upper level ontologies. Furthermore, such upper-level ontologies provide an account of the most basic, domain-independent, existing entities, such as time, space, objects, and processes. In this paper, we verify the core characterization of mereologies of the Suggested Upper Merged Ontology (SUMO), and the mereology of the Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE), while relating their axiomatizations via ontology mapping. We show that the existing axiomatization of SUMO omits some of the intended models of classical mereology, and we propose the correction and addition of axioms to address this issue. In addition, we show the formal relationship between the axiomatization of mereology in both upper-level ontologies.

Keywords: DOLCE · DOLCE-CORE · SUMO · Ontology mapping
Ontology verification · Upper-level ontology · Mereology · Topology
Mereotopology

1 Introduction

Automatic applications appealing to ontologies for interoperation are unambiguously integrated only when the models of their shared features are equivalent. However, ontologies admitting unintended models ambiguously characterize their vocabularies, which can generate misunderstandings that hinder interoperability.

Upper-level ontologies, also called *foundational* ontologies, provide an account of the most basic, domain independent, existing entities, such as time, space, objects and processes. As ontologies are crucial for the Semantic Web, upper level ontologies are essential for the ontology engineering cycle in activities such as ontology building and integration. Upper level ontologies can be used as the foundational substratum on which new ontologies are developed, because they provide some fundamental ontological distinctions, which can help

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the designer in her task of conceptual analysis, [11]. They can be used as a backbone on top of which more specific concepts can be characterized while reusing their root vocabulary and their general knowledge. In ontology integration, they can be used as oracles for meaning clarification [6].

Upper-level ontologies are expected to be mostly consulted for meaning negotiation and not for terminological reasoning, therefore they have to be represented in expressive languages that can convey every feature of the characterized entities. However, less accurate representations in lightweight languages should be available too, which can be extended with domain specific axiomatizations when the use of lightweight languages is necessary.

Since foundational ontologies are expected to be broadly reused and extended, they are not expected to admit unintended models which the expressivity of their representation language can rule out, and it is also expected that they do not miss intended models. If a upper-level ontology does not admit those models which it is expected to admit, misunderstandings among applications who subscribe to their account of the world can occur.

Various upper level ontologies have been developed in languages with higher or equivalent expressivity to first-order logic, such as SUMO [18] and DOLCE [2,8], and translations of them, with loss, to lightweight language OWL[1], made available. Therefore, semantic mappings connecting their axiomatizations are necessary to facilitate interoperability among applications that commit to the characterizations provided by different upper level ontologies. Those mappings need to be formal, which guarantees their interpretability by automatic agents, and also need to be represented in an expressive language such as standard first-order logic.[2]

Ontology verification [10] is the process by which a theory is checked to rule out unintended models, and possibly characterize missing intended ones. Therefore, ontology verification reduces semantic ambiguity. Since foundational ontologies are expected to be broadly reused, their verification results necessary.

In this paper[3], we verify the subtheory of core mereotopological concepts of the SUMO foundational ontology and the mereology of the DOLCE-CORE, the fragment of DOLCE focused on entities that exist on time. In addition, we formally relate their respective axiomatizations via first-order logic mappings. As a result, we propose the correction, and addition, of some axioms which rule out unintended models or characterize missing ones. As an additional outcome of our work, we have produced a modular representation stated in standard

[1] https://www.w3.org/2001/sw/wiki/OWL.

[2] The expressive power of first-order logic makes its use necessary for the representation of mappings that characterize features that are not representable in lightweight languages, such as Description Logics. In addition, checking the correctness of those mappings results facilitated by the fact that first-order theorem proving in standard first-order logic is a mature field, and, although semi-decidable, first-order reasoning on small modules results in an acceptable trade-off among expressivity and efficiency.

[3] This paper is an extended and expanded version of the paper "Verifying and Mapping the Mereotopology of Upper-Level Ontologies" that originally appeared in the Proceedings of Knowledge Engineering and Ontology Design (KEOD) 2016 [16].

first-order logic of the complete SUMO subtheory of mereotopology. We have used automatic theorem prover Prover9 and model finder Mace4 [15] for the automatic tasks involved in the work described in this paper.

2 Ontology Mapping and Verification

Ontology mapping, also called *ontology matching*, and *ontology alignment*, is concerned with the explicit representation of the existing semantic correspondences among the axiomatizations of different ontologies[4] via *bridge axioms* [6], which are called *translations definitions* in the context of first-order logic.

Building a map between two first-order logic ontologies T_1 and T_2 that interprets the first into the second involves translating every symbol of theory T_1 into the language of T_2, translating every sentence of T_1 into the language of T_2, and checking the ability of T_2 to entail every axiom of T_1. The following definition formalizes the notion of relative interpretation between first-order logic theories.

Definition 1. *A map π interprets a theory T_1 into a theory T_2 iff for every sentence α in the language of T_1, $T_1 \models \alpha \Rightarrow T_2 \models \alpha^\pi$; being α^π the syntactic translation of α into the language of T_2.*

The following theorem that follows fom [5], introduces a fundamental relation between the models of a theory and the models of the theories that it interprets. Given such a relation, in order to demonstrate that a given theory T_2 can represent every feature that another theory T_1 represents, it suffices to demonstrate that theory T_2 is able to interpret theory T_1.

Theorem 1. *If a theory T_1 is interpreted by a theory T_2 by means of a given map π, there is another map δ that sends every model of T_2 into a model of T_1.*

An ontology admits unintended models when it is possible to find features of its underlying conceptualization which are not characterized by its axiomatization. *Ontology Verification* in first-order logic [10] is based on the fact that theories with different vocabularies unambiguously characterize the same concepts only if their sets of models are equivalent. Verifying an ontology T ideally consists of classifying the actual models $\mathfrak{M}$ of T by means of a *representation theorem*,[5] which relates the models of T with the models $\mathfrak{M}^{intended}$ of an alternative axiomatization of T built with well understood theories. Such a representation theorem must be either proved or disproved. The following definition from [19] relates the notion of *ontology mapping* with the fundamentals of *ontology verification*:

4 We assume that an ontology is a set of sentences called *axioms* closed under logical entailment that state the properties that characterize the behaviour of a set of symbols representing constants, relations and functions, called the *signature* of the ontology.

5 A *representation theorem* is a theorem that formally classifies a given class of structures as equivalent to another class of structures whose properties are better understood. The stated equivalence makes possible the extrapolation of those properties to the classified structures, facilitating their understanding.

Definition 2. *Two theories T_1 and T_2 are* ***synonymous*** *iff there exist two sets of translation definitions Δ and Π, respectively from T_1 to T_2 and from T_2 to T_1, such that $T_1 \cup \Pi$ is logically equivalent to $T_2 \cup \Delta$.*

Given Definition 2, from Theorem 1 follows that the models of synonymous theories are equivalent, and therefore *ontology mapping* can be used for classifying the sets of models of two ontologies as equivalent.

3 SUMO

SUMO [18] is a freely available upper level ontology intended to describe the world as perceived by humans, based on human knowledge and culture, in opposition to *ontological realism* [9], which is meant to present the world as it is, independently of the bias of human perception. In addition to the main ontology, which contains about 4000 axioms, SUMO has been extended with a mid-level ontology and a number of domain specific ontologies, all of which account for 20,000 terms and 70,000 axioms. SUMO has been translated into OWL and WordNet [17]. The representation language of SUMO is SUO-KIF[6], a very expressive dialect of KIF[7] with many-sorted features, whose syntax permits higher-order constructions such as predicates that have other predicates, or formulas, as their arguments, and the existence of predicates and functions of variable arity [1].

We have translated (with loss) into standard first-order logic, and modularized, the subset of SUMO that characterizes the notion of mereotopology, which resulted in the hierarchy of subtheories shown in Fig. 1, where each theory conservatively extends[8] its related theories below. Due to space limitations, we only address in this work the study of modules T_{part}, T_{sum}, $T_{product}$, $T_{decomposition}$, $T_{topology}$, and $T_{mereotopology}$. The first-order logic axiomatization of all the modules shown in Fig. 1 can be found at colore.oor.net/ontologies/sumo/modules.

SUMO adopts various partial orderings to address the part-whole relationship in different categories. Regarding entities that are in space and time, classified as *Physical* in SUMO, relations *part* and *subProcess* respectively characterize part-whole relations for members of *Object* and *Process*, while relation *temporalPart* represents part-whole for members of *TimePosition*, which extends to points and intervals of time.

3.1 The Subtheory T_{part}

The subtheory T_{part} represents the relation among a whole and its parts by axiomatizing the primitive relation *part* and using conservative definitions for the *overlapsSpatially*, *overlapsPartially*, and *properPart* relations. Extracting

6 http://suo.ieee.org/SUO/KIF/suo-kif.html.

7 http://logic.stanford.edu/kif/kif.html.

8 A theory T' is a *conservative extension* of a theory T if every theorem of T is a theorem of T', and every theorem of T' in the signature of T is also a theorem of T.

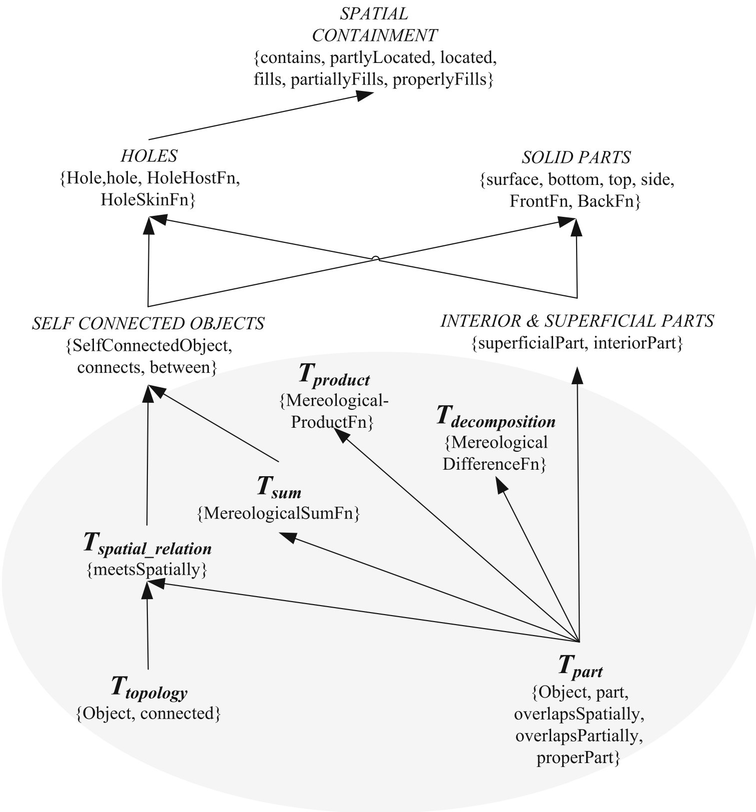

Fig. 1. Modular decomposition of the SUMO axiomatization of concepts related to mereotopology. Theories in the shaded region are discussed in this paper; the remaining subtheories of SUMO are left for future work. Arrows point to conservative extensions among modules. Signature members are shown in the module that first introduces them. (Original figure from [16].)

the sentences from SUMO that use the primitive signature $\{Object, part\}$, we obtain the following subtheory:

Definition 3. *T_{part} is the subtheory composed by axioms (1) to (7).*

$$(\forall x, y) part(x, y) \rightarrow Object(x) \wedge Object(y) \tag{1}$$

$$(\forall x) Object(x) \rightarrow part(x, x) \tag{2}$$

$$(\forall x, y) part(x, y) \wedge part(y, x) \rightarrow (x = y) \tag{3}$$

$$(\forall x, y, z) part(x, y) \wedge part(y, z) \rightarrow part(x, z) \tag{4}$$

$$(\forall x, y) overlapsSpatially(x, y) \leftrightarrow (\exists z (part(z, x) \wedge part(z, y))) \tag{5}$$

$$(\forall x, y) overlapsPartially(x, y) \leftrightarrow \neg part(x, y) \wedge \neg part(y, x) \wedge (\exists z) part(z, x) \wedge part(z, y) \quad (6)$$

$$(\forall x, y) properPart(x, y) \leftrightarrow part(x, y) \wedge \neg part(y, x) \quad (7)$$

The first question we need to address is whether or not this subtheory is a module of SUMO, or whether there are additional sentences in the signature $\{Object, part\}$ that are entailed by the remaining axioms of SUMO. Before we can fully answer this question, we need to consider the other subtheories of SUMO that are related to mereology.

3.2 Subtheory T_{sum}

The mereological sum of two parts into a whole is represented in the subtheory T_{sum} by the function symbol $MereologicalSumFn$.

Definition 4. *The subtheory T_{sum} is the subtheory that extends T_{part} in the expanded signature $\{Object, part, MereologicalSumFn\}$ by means of axioms (8) and (9).*

$$(\forall x, y, z) Object(x) \wedge Object(y) \rightarrow ((z = MereologicalSumFn(x, y)) \rightarrow (\forall p)(part(p, z) \leftrightarrow (part(p, x) \vee part(p, y))) \quad (8)$$

$$(\forall x, y) Object(x) \wedge Object(y) \rightarrow Object(MereologicalSumFn(x, y)) \quad (9)$$

Given two objects, the existence of their mereological sum is guaranteed in this theory due to the use of a function to represent such an operation (since functions in first-order logic are total).

We can immediately find some straightforward consequences[9] of the axioms of T_{sum}:

Proposition 1.

$$T_{sum} \models (\forall x, y, z) Object(x) \wedge Object(y) \wedge (z = MereologicalSumFn(x, y)) \rightarrow part(z, x) \vee part(z, y) \quad (10)$$

$$T_{sum} \models (\forall x, y, z) Object(x) \wedge Object(y) \wedge (z = MereologicalSumFn(x, y)) \rightarrow part(x, z) \wedge part(y, z) \quad (11)$$

Given theorems (10) and (11), and due to the antisymmetry of relation *part*, it holds that z must be x or y, this fact entails that every pair of objects in the universe of every interpretation of SUMO must be in relation *part*, which shows that SUMO omits models where there exist objects that are disjoint, or that overlap without being one part of the other, as depicted in parts *(b)* and *(c)* of Fig. 2. The following proposition formalizes this claim:

[9] The proofs for all Propositions have been found using the Prover9 automated theorem prover, and models were constructed using Mace4. Results are available at: colore.oor.net/ontologies/sumo/mereotopology/proofs.

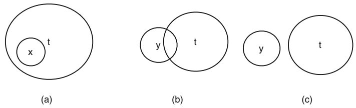

Fig. 2. With the original characterization of mereological sum, every two objects in every model of SUMO must be in relation *part*, such as objects x and y in *(a)*. Models corresponding to *(b)* and *(c)* with overlapping objects without being one part of the other, or with disjoint objects, are not admitted by SUMO submodule T_{sum}. (Original figure from [16].)

Proposition 2. $T_{sum} \models (\forall x, y) Object(x) \wedge Object(y) \rightarrow part(x, y) \vee part(y, x)$.

In other words, SUMO entails that the *part* relation is synonymous with a linear ordering. Although there is much discussion in the philosophical and applied ontology literature [20] over which axioms should constitute a mereology, there is nobody who proposes that all models of an axiomatization of mereology be synonymous with linear orderings. In order to allow those omitted models that Proposition 2 identifies, we propose a modification of T_{sum}:

Definition 5. *$T_{extended\_sum}$ is the theory which extends T_{part} with the sentences*

$$(\forall x, y, z) Object(x) \wedge Object(y) \rightarrow$$
$$((z = MereologicalSumFn(x, y) \rightarrow (\forall p)(part(z, p) \leftrightarrow part(x, p) \wedge part(y, p))) \tag{12}$$

$$(\forall x, y) Object(x) \wedge Object(y) \rightarrow Object(MereologicalSumFn(x, y)) \tag{13}$$

The following proposition shows that $T_{extended\_sum}$ does not rule out models in which overlapping or disjoint objects exist.

Proposition 3.

$$T_{extended\_sum} \not\models (\forall x, y) Object(x) \wedge Object(y) \rightarrow (part(x, y) \vee part(y, x)$$

If we look more closely at the proposed axiomatization, we can see that the $MereologicalSumFn$ function is commutative and idempotent:

Proposition 4.

$$T_{extended\_sum} \models (\forall x, y, z) Object(x) \wedge Object(y) \wedge Object(z) \wedge$$

$$(MereologicalSumFn(x, y) = z) \rightarrow (MereologicalSumFn(y, x) = z)$$

$$T_{extended\_sum} \models (\forall x, y, z) part(x, y) \rightarrow (MereologicalSumFn(x, y) = y)$$

This leads us to consider how $T_{extended\_sum}$ is related to lattice theory [4].

Theorem 2. *$T_{extended\_sum}$ is synonymous with $T_{join\_semilattice}$*[10].

[10] colore.oor.net/ontologies/lattices/join_semilattice.clif.

Proof. Let Δ be the sentence

$$(\forall x, y, z)\,(MereologicalSumFn(y, x) = z) \leftrightarrow (join(x, y) = z)$$

Using Prover9, we can show that $T_{extended\_sum} \cup \Delta \models T_{join\_semilattice}$, and $T_{join\_semilattice} \cup \Delta \models T_{extended\_sum}$ □

We can also specify an extension of T_{part} in the same signature.

Definition 6. *T_{part_sum} is the extension of T_{part} with the sentence*

$$(\forall x, y)(\exists j)\,(part(x, j) \wedge part(y, j) \wedge ((\forall z)\,(part(x, z) \wedge part(y, z) \supset part(j, z))))$$

Theorem 3. *T_{part_sum} is synonymous with $T_{strong\_lub\_mereology}$*[11]

Proof. To disambiguate the signatures of SUMO and other existing mereologies, let $part^s umo(x, y)$ be the relation in the signature of SUMO.

Let Δ be the sentence

$$(\forall x, y)\, part^{sumo}(x, y) \leftrightarrow part(x, y))$$

Using Prover9, we can show that $T_{part\_sum} \models T_{strong\_lub\_mereology}$, and $T_{strong\_lub\_mereology} \models T_{part\_sum}$ □

It should be noted that the axiomatization of $T_{strong\_lub\_mereology}$ corresponds to the sentence SA13 in [20].

Proposition 5.

$$SUMO \models T_{part\_sum}$$

3.3 Subtheory $T_{product}$

Given two objects, their mereological product intuitively corresponds to their intersection. SUMO represents the notion of mereological product by means of the function $MereologicalProductFn$.

Definition 7. *$T_{product}$ is the subtheory of SUMO that extends theory T_{part} in the expanded signature $\{Object, part, MereologicalProductFn\}$ by the sentences:*

$$(\forall x, y, z)Object(x) \wedge Object(y) \rightarrow ((z = MereologicalProductFn(x, y)) \rightarrow (\forall p)(part(p, z) \leftrightarrow part(p, x) \wedge part(p, y))) \quad (14)$$

$$(\forall x, y)Object(x) \wedge Object(y) \rightarrow Object(MereologicalProductFn(x, y)) \quad (15)$$

[11] colore.oor.net/ontologies/mereology/strong_lub_mereology.clif.

Given two objects, the existence of their mereological product is guaranteed due to the use of a function to represent such an operation.

The characterization of mereological product in SUMO corresponds to the $infimum$ or *meet* of the corresponding arguments on the lattice that relation *part* defines. We have found that from the characterization of mereological product of SUMO follows that every pair of objects must overlap, which indicates that SUMO omits those models where there exist objects that do not overlap (in other words, all elements overlap each other in all models of SUMO):

Proposition 6. $T_{product} \models (\forall x, y)Object(x) \wedge Object(y) \rightarrow (overlapsSpatially\ (x, y))$.

In order to allow models in which nonoverlapping elements exist, we propose a modification of SUMO:

Definition 8. *$T_{extended\_product}$ is the theory which extends T_{part} by the sentences*

$$\begin{gathered}(\forall x, y, z) overlapsSpatially(x, y) \rightarrow \\ ((z = MereologicalProductFn(x, y)) \rightarrow (\forall p)(part(p, z) \leftrightarrow part(p, x) \wedge part(p, y)))\end{gathered} \tag{16}$$

$$(\forall x, y)Object(x) \wedge Object(y) \rightarrow Object(MereologicalProductFn(x, y)) \tag{17}$$

The following propositions show that $T_{extended\_product}$ does not rule out models in which there exist nonoverlapping objects, and that $MereologicalProductFn$ is commutative and idempotent.

Proposition 7.

$$T_{extended\_product} \not\models (\forall x, y)Object(x) \wedge Object(y) \rightarrow overlapsSpatially(x, y)$$

Proposition 8.

$$T_{extended\_product} \models (\forall x, y, z)Object(x) \wedge Object(y) \wedge Object(z) \wedge$$

$$(MereologicalProdFn(x, y) = z) \rightarrow (MereologicalProdFn(y, x) = z)$$

$$T_{extended\_product} \models (\forall x, y, z)part(x, y) \rightarrow (MereologicalProdFn(x, y) = x)$$

We can also specify an extension of T_{part} in the same signature.

Definition 9. *T_{part_prod} is the extension of T_{part} with the sentence*

$$(\forall x, y)\ overlapsSpatially(x, y) \supset (\exists z)\ ((\forall u)\ (part(u, z) \leftrightarrow (part(u, x) \wedge part(u, y))))$$

Calling $part^{sumo}$ to the relation *part* of T_{part}, the following property holds:

Theorem 4. *T_{part_prod} is synonymous with $T_{prod\_mereology}$*[12].

Proof. Let Δ be the sentence

$$(\forall x, y)\ part^{sumo}(x, y) \leftrightarrow part(x, y))$$

Using Prover9, we can show that $T_{part\_prod} \models T_{prod\_mereology}$, and $T_{prod\_mereology} \models T_{part\_prod}$ $\square$

[12] colore.oor.net/ontologies/mereology/prod_mereology.clif.

3.4 Subtheory $T_{decomposition}$

The remainder between a whole and its proper parts is represented by the function *MereologicalDifferenceFn*.

Definition 10. *$T_{decomposition}$ is the subtheory of SUMO that extends T_{part} by the sentences*

$$(\forall x, y, z) Object(x) \wedge Object(y) \rightarrow ((z = MereologicalDifferenceFn(x, y)) \rightarrow (\forall p) properPart(p, z) \leftrightarrow properPart(p, x) \wedge \neg properPart(p, y)) \quad (18)$$

$$(\forall x, y) Object(x) \wedge Object(y) \rightarrow Object(MereologicalDifferenceFn(x, y)) \quad (19)$$

Because the mereological difference, or remainder, between a whole and one of its parts is represented in SUMO by a function, its existence is guaranteed in every case at the expenses of having arbitrary values of the function $MereologicalDifferenceFn$.

We have found that the axiomatization of *MereologicalDifferenceFn* given by (18) and (19) entails an unusual result in which the remainder overlaps with the subtrahend:

Proposition 9.

$$T_{decomposition} \models (\forall x, y, z) Object(x) \wedge Object(y)$$
$$\wedge (z = MereologicalDifferenceFn(x, y)) \wedge properPart(y, x) \rightarrow properPart(y, z))$$

In order to eliminate such a class of unintended models, we propose the following modification, and prove by means of Proposition 10 that the new theory does not admit these models.

Definition 11. *$T_{extended\_decomp}$ is the theory that extends T_{part} with the following sentences*

$$(\forall x, y, z) Object(x) \wedge Object(y) \rightarrow ((MereologicalDifferenceFn(x, y) = z) \rightarrow (\forall p)(part(p, z) \leftrightarrow part(p, x) \wedge \neg overlapsSpatially(p, y))) \quad (20)$$

$$(\forall x, y) Object(x) \wedge Object(y) \rightarrow Object(MereologicalDifferenceFn(x, y)) \quad (21)$$

Proposition 10.

$$T_{extended\_decomp} \not\models (\forall x, y, z) Object(x) \wedge Object(y)$$
$$\wedge (z = MereologicalDifferenceFn(x, y)) \wedge properPart(y, x) \rightarrow properPart(y, z))$$

We can also specify an extension of T_{part} in the same signature.

Definition 12. *T_{part_decomp} is the extension of T_{part} with the sentence*

$$(\forall x, w)\, \neg part(w, x) \supset (\exists z)\, ((\forall y)\, (part(y, z) \leftrightarrow \neg overlapsSpatially(y, x)))$$

Theorem 5. *T_{part_decomp} is synonymous with $T_{comp\_mereology}$*[13].

Proof. Let Δ be the sentence

$$(\forall x, y)\, part^{sumo}(x, y) \leftrightarrow part(x, y))$$

Using Prover9, we can show that $T_{part\_decomp} \models T_{comp\_mereology}$, and $T_{comp\_mereology} \models T_{part\_decomp}$ □

[13] colore.oor.net/ontologies/mereology/comp_mereology.clif.

3.5 Relationship to Classical Mereologies

We have so far evaluated four subtheories of SUMO and proposed revisions to their axiomatizations to address the problem of classes of omitted and unintended models regarding those normally associated with mereology. We now take a closer look at these revised theories.

Proposition 11. *The theory*

$$T_{part} \cup T_{extended\_sum} \cup T_{extended\_product} \cup T_{extended\_decomp}$$

is consistent.

Given that the revised theories are consistent, can we characterize their models? In particular, how is the mereology within SUMO related to the classical mereologies that have been explored by the philosophical and applied ontology communities? Regarding the supplementation principles (22) to (25), respectively named in [21] as *weak company, strong company, supplementation*, and *strong supplementation*, Proposition 12 shows that those principles are not theorems of SUMO.

Proposition 12. *The following sentences are not entailed by* $T_{part} \cup T_{extended\_sum} \cup T_{extended\_product} \cup T_{extended\_decomp}$*:*

$$(\forall x, y) properPart(x, y) \rightarrow \exists z(properPart(z, y) \wedge -(z = x)) \tag{22}$$

$$(\forall x, y) properPart(x, y) \rightarrow (\exists z)(properPart(z, y) \wedge \neg part(z, x)) \tag{23}$$

$$(\forall x, y) properPart(x, y) \rightarrow (\exists z)(Part(z, y) \wedge \neg overlapsSpatially(z, x)) \tag{24}$$

$$(\forall x, y) \neg part(y, x) \rightarrow (\exists z)(Part(z, y) \wedge \neg overlapsSpatially(z, x)) \tag{25}$$

This is closely related to the question of whether or not T_{part} is a module of SUMO. We have already seen that the subtheories of SUMO entail additional mereological theories that are not entailed by T_{part} alone. However, we can see that the addition of these new sentences does form a module of the revised axioms by combining the earlier theorems:

Theorem 6. $T_{part\_sum} \cup T_{part\_prod} \cup T_{part\_decomp}$ *is a module of* $T_{part} \cup T_{extended\_sum} \cup T_{extended\_product} \cup T_{extended\_decomp}$ *that is synonymous with* $T_{strong\_lub\_mereology} \cup T_{prod\_mereology} \cup T_{comp\_mereology}$

3.6 Mereotopology in SUMO

Since mereology can only represent the relation of parts with their respective wholes, predicate *connected* is characterized in SUMO to represent a more general symmetric and reflexive spatial relationship among objects which are not necessarily in a part-whole relation.

Definition 13. *$T_{topology}$ is the subtheory of SUMO consisting of the following axioms:*

$$(\forall x) Object(x) \rightarrow connected(x, x) \tag{26}$$

$$(\forall x, y) connected(x, y) \rightarrow Object(x) \wedge Object(y) \tag{27}$$

$$(\forall x, y) connected(x, y) \rightarrow connected(y, x) \tag{28}$$

The subtheory of SUMO that axiomatizes mereotopology, which we have called $T_{spatial\_relation}$ is intended to characterize the relationship between the notions of mereology and topology. In it, both predicates, *meetsSpatially*, which represents external connection among objects, and *overlapsSpatially*, are declared disjoint specializations of predicate *connected*.

Definition 14. *$T_{spatial\_relation}$ is the subtheory of SUMO which is an extension of $T_{part} \cup T_{topology}$ consisting of the sentences*

$$(\forall x, y)\, meetsSpatially(x, y) \rightarrow connected(x, y) \tag{29}$$

$$(\forall x)\, \neg meetsSpatially(x, x) \tag{30}$$

$$(\forall x, y)\, meetsSpatially(x, y) \rightarrow meetsSpatially(y, x) \tag{31}$$

$$(\forall x, y)\, overlapsSpatially(x, y) \rightarrow connected(x, y) \tag{32}$$

$$(\forall x)\, overlapsSpatially(x, x) \tag{33}$$

$$(\forall x, y)\, overlapsSpatially(x, y) \rightarrow overlapsSpatially(y, x) \tag{34}$$

$$(\forall x, y)\, meetsSpatially(x, y) \rightarrow \neg overlapsSpatially(x, y) \tag{35}$$

$$(\forall x, y) connected(x, y) \rightarrow (meetsSpatially(x, y) \vee overlapsSpatially(x, y)) \tag{36}$$

However, the axiomatization of this theory is already entailed by the following definitional extension of $T_{part} \cup T_{topology}$:

Definition 15. *$T_{mereotop\_def}$ is the definitional extension of $T_{part} \cup T_{topology}$ consisting of the sentences*

$$(\forall x, y) overlapsSpatially(x, y) \leftrightarrow connected(x, y) \wedge (\exists z)\, part(z, x) \wedge part(z, y) \tag{37}$$

$$(\forall x, y) meetsSpatially(x, y) \leftrightarrow connected(x, y) \wedge \neg(\exists z)\, part(z, x) \wedge part(z, y) \tag{38}$$

Proposition 13.

$$T_{mereotop\_def} \models T_{spatial\_relation}$$

We have found that the monotony of relation *connected* with respect to parthood was not characterized in SUMO, which introduces unintended models as the one represented in Fig. 3, where all parts share one point, but only shaded ones result to be connected.

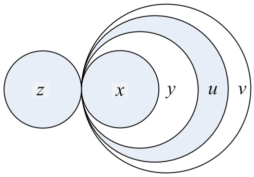

Fig. 3. Model of SUMO where the monotony of relation *connected* with respect to parthood was not characterized. Even though $connected(z, x)$, $part(x, y)$, $part(y, u)$, and$part(u, v)$ hold, $connected(z, y)$ and $connected(z, v)$ do not hold, while $connected(z, u)$ does hold. (Original figure from [16].)

Proposition 14.

$$T_{mereotop\_def} \not\models (\forall x, y) part(x, y) \rightarrow \forall z(connected(z, x) \rightarrow connected(z, y))$$

In order to rule out those unintended models that proposition 14 identifies, we propose the following extension:

Definition 16. *$T_{extend\_mereotop}$ is the theory which extends $T_{part} \cup T_{topology}$ with sentence (39).*

$$(\forall x, y) part(x, y) \rightarrow \forall z(connected(z, x) \rightarrow connected(z, y)) \tag{39}$$

4 DOLCE

The Descriptive Ontology for Linguistic and Cognitive Engineering DOLCE [8, 14] is a freely available upper ontology that is part of the WonderWeb project[14], which is aimed to provide the infrastructure required for a large-scale deployment of ontologies intended to be the foundation for the Semantic Web. DOLCE has a cognitive approach, i.e., it presents the world as it is grasped by humans, based on human knowledge and culture, in opposition to *ontological realism* [9], which intends to present the world as it is, independently of the bias of human perception. The development of DOLCE has followed the principles of the OntoClean methodology [12]. The first version of DOLCE had a representation in Modal Logic, a translation with loss into standard first-order logic, a translation with further loss into OWL, and also an alignment with WordNet [7]. A new version of the fragment of the original ontology that focuses on entities that exist on time, called *temporal particulars*, was presented in [2], called DOLCE-CORE; we will circumscribe our work to the axiomatization of DOLCE-CORE.

At the top of DOLCE-CORE the category of temporal-particulars PT is partitioned into six basic categories: objects O, events E, individual qualities Q, regions R, concepts C, and arbitrary sums AS. Categories ED (*endurant*) and PD (*perdurant*) of DOLCE were, respectively, renamed O (*object*) and

[14] http://wonderweb.semanticweb.org.

E (*event*) in DOLCE-CORE. The axiomatization of mereology in DOLCE-CORE is as follows,[15] where predicate P represents parthood, and (40)–(42) respectively stand for the reflexivity, transitivity, and antisymmetry of relation P. Overlap of parts and mereological sum representing binary fusion of parts are respectively defined in (43) and (44), while (46)–(50) characterize the dissectivity of P across categories, and (51)–(56) close the sum of parts inside each category.

$$(\forall x)P(x,x) \tag{40}$$

$$(\forall x,y)P(x,y) \wedge P(y,z) \rightarrow P(x,z) \tag{41}$$

$$(\forall x,y)P(x,y) \wedge P(y,x) \rightarrow (x=y) \tag{42}$$

$$(\forall x,y)Ov(x,y) \equiv (\exists z)(P(z,x) \wedge P(z,y)) \tag{43}$$

$$(\forall x,y,z)SUM(z,x,y) \equiv (\forall v)Ov(v,z) \leftrightarrow Ov(v,x) \vee Ov(v,y) \tag{44}$$

$$(\forall x,y)\neg P(x,y) \rightarrow (\exists z)P(z,x) \wedge \neg Ov(z,y) \tag{45}$$

$$(\forall x,y)O(y) \wedge P(x,y) \rightarrow O(x) \tag{46}$$

$$(\forall x,y)E(y) \wedge P(x,y) \rightarrow E(x) \tag{47}$$

$$(\forall x,y)T(y) \wedge P(x,y) \rightarrow T(x) \tag{48}$$

$$(\forall x,y)TQ(y) \wedge P(x,y) \rightarrow TQ(x) \tag{49}$$

$$(\forall x,y)C(y) \wedge P(x,y) \rightarrow C(x) \tag{50}$$

$$(\forall x,y,z)O(x) \wedge O(y) \wedge SUM(z,x,y) \rightarrow O(z) \tag{51}$$

$$(\forall x,y,z)E(x) \wedge E(y) \wedge SUM(z,x,y) \rightarrow E(z) \tag{52}$$

$$(\forall x,y,z)T(x) \wedge T(y) \wedge SUM(z,x,y) \rightarrow T(z) \tag{53}$$

$$(\forall x,y,z)TQ(x) \wedge TQ(y) \wedge SUM(z,x,y) \rightarrow TQ(z) \tag{54}$$

$$(\forall x,y,z)C(x) \wedge C(y) \wedge SUM(z,x,y) \rightarrow C(z) \tag{55}$$

$$(\forall x,y,z)AS(x) \wedge AS(y) \wedge SUM(z,x,y) \rightarrow AS(z) \tag{56}$$

Due to the ontological commitment represented by axiom (45), the mereology characterized in DOLCE-CORE is an *extensional mereology*[16] according to [3, 21].

[15] Axioms (48), (49), (53), and (54) are the instantiation of DOLCE higher-order axiom schemas for the subcategories of main categories Q and R which are relevant for our work. A complete version of DOLCE-CORE mereology represented in first-order logic is available at colore.oor.net/ontologies/dolce-core/mereology.in.

[16] It can be proved that in an *extensional mereology* non-atomic entities whose proper parts are the same, are identical, i.e., every entity is exhaustively defined by its parts.

5 Mapping the Mereologies of SUMO and DOLCE

In order to relate SUMO and DOLCE we assume that the changes that we have proposed in Sect. 3 for eliminating unintended models and characterizing missing intended ones have been performed in SUMO. There is no axiomatization in DOLCE-CORE, neither in DOLCE, that corresponds to the notion of topology, therefore our mappings are circumscribed to the axiomatization of mereology in both theories.

By examining the predicates that characterize the participation of objects in events in both ontologies, and also by the type of relation that the main categories of SUMO and DOLCE-CORE have with time and space, we have built the translation definitions of Table 1 for the main these categories.

Table 1. Mapping of SUMO and DOLCE main categories.

$(\forall x) Object(x) \leftrightarrow O(x)$	(57)
$(\forall x) Process(x) \leftrightarrow E(x)$	(58)
$(\forall x) TimeInterval(x) \leftrightarrow T(x)$	(59)
$(\forall x) Region(x) \leftrightarrow S(x)$	(60)

Table 2. Translation definitions for $T_{dolce\_part\_t}$ into $T_{time\_mereology}$.

$(\forall x) T(x) \leftrightarrow TimeInterval(x)$	(61)
$(\forall x, y) P(x, y) \leftrightarrow temporalPart(x, y)$	(62)
$(\forall x, y) Ov(x, y) \leftrightarrow overlapsTemporally(x, y)))$	(63)

Table 3. Translation definitions for $T_{time_mereology}$ into $T_{dolce\_part\_t}$.

$(\forall x) TimeInterval(x) \leftrightarrow T(x)$	(64)
$(\forall x, y) temporalPart(x, y) \leftrightarrow P(x, y) \wedge T(x) \wedge T(y)$	(65)
$(\forall x, y) overlapsTemporally(x, y) \leftrightarrow Ov(x, y) \wedge T(x) \wedge T(y)$	(66)

5.1 Mapping Time

The subtheory T_{sumo_time}, whose modular structure is shown in Fig. 5, characterizes the axiomatization of time in SUMO. This theory, which was verified in [20], includes 3 submodules[17] $T_{sumo\_ordered\_timepoints}$, $T_{sumo\_timeintervals}$, and $T_{time\_mereology}$, such that each module is a conservative extension of each connected subtheory below it in Fig. 5. These 3 subtheories respectively characterize a linear ordering between instants of time, a part-whole relation among intervals of time, and an account of Allen's interval relations *starts, finishes, during, earlier*, and *meetsTemporally* [13]. Finally, T_{sumo_time} characterizes a part-whole relationship that includes intervals and instants of time.

On the other hand, T_{dolce_part} characterizes parthood by unique predicate P across every category, including T. By means of the following theorems we can characterize the relationship that exists among $T_{dolce\_part\_t}$ and T_{sumo_time}.

[17] Available at colore.oor.net/ontologies/sumo/modules.

Theorem 7. *Let $T_{dolce\_part\_t}$ be the subtheory of T_{dolce_part} with signature $\{T, part\}$, and Let $T_{time\_mereology}$ be the theory given by axioms (67)–(72). Then, $T_{time\_mereology}$ is synonymous with $T_{dolce\_part\_T}$.*

$$(\forall x)TimeInterval(x) \rightarrow temporalPart(x, x). \tag{67}$$

$$(\forall x, y)temporalPart(x, y) \wedge temporalPart(y, x) \rightarrow (x = y). \tag{68}$$

$$(\forall x, y, z)temporalPart(x, y) \wedge temporalPart(y, z) \rightarrow temporalPart(x, z). \tag{69}$$

$$(\forall x, y)overlapsTemporally(x, y) \rightarrow TimeInterval(x) \wedge TimeInterval(y) \tag{70}$$

$$(\forall x)TimeInterval(x) \rightarrow overlapsTemporally(x, x)). \tag{71}$$

$$\begin{aligned}&(\forall x, y)TimeInterval(x) \wedge TimeInterval(y) \rightarrow (overlapsTemporally(x, y) \leftrightarrow \\ &\quad ((\exists z)(TimeInterval(z) \wedge temporalPart(z, x) \wedge temporalPart(z, y))))\end{aligned} \tag{72}$$

Proof. Let Δ be the set of translations shown in Table 2, and Υ the set of translations shown in Table 3. Using Prover9 we have shown that $T_{time\_mereology} \cup \Delta \models T_{dolce\_part\_T}$, and $T_{dolce\_part\_T} \cup \Upsilon \models T_{time\_mereology}$. □

5.2 Mapping Events

Regarding the representation of events in SUMO and DOLCE, by means of the following definition and theorem we classify the relationship that their respective part-whole axiomatizations have as *synonymy*.

Table 4. Translation definitions for $T_{dolce\_part\_E}$ into $T_{sumo\_subprocess}$.

$(\forall x)E(x) \leftrightarrow Process(x)$	(73)
$(\forall x, y)P(x, y) \leftrightarrow subProcess(x, y)$	(74)
$(\forall x, y)Ov(x, y) \leftrightarrow (\exists z)(subProcess(z, x) \wedge subProcess(z, y))$	(75)

Table 5. Translation definitions for $T_{sumo\_subprocess}$ into $T_{dolce\_part\_E}$.

$(\forall x)Process(x) \leftrightarrow E(x)$	(76)
$(\forall x, y)subProcess(x, y) \leftrightarrow E(x) \wedge E(y) \wedge P(x, y)$	(77)

Definition 17. *$T_{sumo\_subprocess}$ is the theory given by the axioms:*

$$(\forall x, y) subProcess(x, y) \rightarrow Process(x) \wedge Process(y) \tag{78}$$

$$(\forall x) Process(x) \rightarrow subProcess(x, x) \tag{79}$$

$$(\forall x, y) subProcess(x, y) \wedge subProcess(y, z) \rightarrow subProcess(x, z) \tag{80}$$

$$(\forall x, y) subProcess(x, y) \wedge subProcess(y, x) \rightarrow (x = y) \tag{81}$$

Theorem 8. *Let $T_{dolce\_part\_E}$ be the theory given by axioms (40)–(42) and (47). $T_{sumo\_subprocess}$ is synonymous with $T_{dolce\_part\_E}$.*

Proof. Let Δ be the set of translations shown in Table 4 and Γ the set of translations shown in Table 5. Using Prover9 we have demonstrated that $T_{sumo\_subprocess} \cup \Delta \models T_{dolce\_part\_E}$ and $T_{dolce\_part\_E} \cup \Gamma \models T_{sumo\_subprocess}$. □

5.3 Mapping Objects

Regarding the representation of objects in SUMO and DOLCE-CORE, by means of the following theorem we classify the relationship among their respective part-whole axiomatizations as *synonymy*.

Definition 18. SUMO PART *is the theory given by axioms (1)–(7), and* DOLCE PART-T *is the theory given by axioms (40)–(43) and (46).*

Theorem 9. *Let T_{sumo_part} be the theory given by axioms (1)–(7), and $T_{dolce\_part\_O}$ the theory given by axioms (40)–(43) and (46). Then, T_{sumo_part} is synonymous with $T_{dolce\_part\_O}$.*

Proof. Let us call Δ to the set of translations shown in Table 6, and Π the set of translations shown in Table 7. Using Prover9 we have shown that $T_{sumo\_part} \cup \Delta \models T_{dolce\_part\_O}$ and $T_{dolce\_part\_O} \cup \Pi \models T_{sumo\_part}$. □

Table 6. Translations DOLCE PART-O into SUMO PART.

$(\forall x, y) P(x, y) \leftrightarrow part(x, y))$	(82)
$(\forall x, y) Ov(x, y) \leftrightarrow overlapsSpatially(x, y))$	(83)

Table 7. Translations SUMO PART into DOLCE PART-O.

$(\forall x, y) part(x, y) \leftrightarrow O(x) \wedge O(y) \wedge P(x, y)$	(84)
$(\forall x, y) properPart(x, y) \leftrightarrow O(x) \wedge O(y) \wedge P(x, y) \wedge \neg P(y, x)$	(85)
$(\forall x, y) overlapsSpatially(x, y) \leftrightarrow O(x) \wedge O(y) \wedge Ov(x, y))$	(86)
$(\forall x, y) overlapsPartially(x, y) \leftrightarrow Ov(x, y) \wedge \neg P(x, y))) \wedge \neg P(y, x))))$	(87)

5.4 Mapping Mereologies with Sums

The theories T_{dolce_sum} in DOLCE and $T_{extended\_sum}$ for SUMO are both intended to axiomatize the intuitions regarding the fusion of parts. The key question is now whether or not they actually axiomatize the same class of intended models.

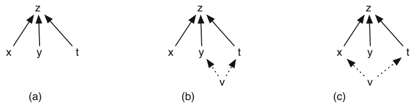

Fig. 4. Objects x, y, z, t, which do not hold $SUM(z,x,y)$ but hold $MereologicalSumFn(x,y) = z$. Arrows represent relation *part* of theory $T_{extended\_sum}$, and relation P of theory T_{dolce_sum}. (Original figure from [16]).

Table 8. Translations DOLCE SUM into SUMO SUM.

$$(\forall x, y, z)SUM(z, x, y) \leftrightarrow Object(x) \wedge Object(y) \wedge (MereologicalSumFn(x, y) = z) \quad (88)$$

The axiomatization of $T_{extended\_sum}$ from SUMO is weaker than the axiomatization of T_{dolce_sum} in DOLCE. In fact, let us consider objects x, y, z, t of Fig. 4, such that $properPart(x, z)$, $properPart(y, z)$, and $properPart(t, z)$ hold, while none of $overlapsSpatially(x, y)$, $overlapsSpatially(t, y)$, or $overlapsSpatially(x, t)$ hold. In parts (a), (b), and (c) of the bottom of Fig. 4 parthood is indicated with arrows from the part to the whole. According to the characterization of mereological sum in $T_{extended\_sum}$, we must have $(MereologicalSumFn(x, y) = z)$. However, parts (b) and (c) of Fig. 4 depict alternative additional conditions that the characterization of mereological sum in T_{dolce_sum} must satisfy. In DOLCE, any other object t which overlaps with the sum z must also overlap with at least one of the addends x or y, which is a condition indicated by dotted lines. Because neither $overlapsSpatially(x, t)$ nor $overlapsSpatially(y, t)$ hold, then $SUM(z, x, y)$ does not hold in DOLCE. The following theorem formalizes our claim.

Theorem 10. *$T_{extended\_sum}$ cannot interpret T_{dolce_sum}.*

Proof. Let us call Δ to the translations shown in Table 6, and Π to the translation shown in Table 8, and let T_1 be the theory that results from adding sentence (89) to theory $T_{extended\_sum}$. Using Mace4, we have built a model of $T_1 \cup \Delta \cup \Pi$ (see footnote 17).

$$(\exists x, y, z)SUM(z, x, y) \wedge \neg(\forall w)(Ov(w, z) \leftrightarrow Ov(w, x) \vee Ov(w, y)) \quad (89)$$

$\square$

In order to translate the symbol $MereologicalSumFn$ of theory $T_{extended\_sum}$ into the language of DOLCE-CORE, we have represented the graph[18] of function *MereologicalSumFn* by means of predicate *MSum*, as shown in Table 9.

Theorem 11. *T_{dolce_sum} cannot interpret $T_{extended\_sum}$.*

Proof. Let us call Δ to the translations shown in Table 1, Π to the translations in Table 7, and Υ to the translation in Table 10, and let T_1 be the theory that results from adding sentence (90) to T_{dolce_sum}. Using Mace4, we have built a model of $T_1 \cup \Delta \cup \Pi \cup \Upsilon$ (see footnote 18).

$$(\exists x, y) Object(x) \wedge Object(y) \wedge (\forall z)(\neg Object(z) \vee \neg MSum(z, x, y)) \tag{90}$$

□

Table 9. Characterization of predicate MSum in SUMO.

$$(\forall x, y, z) MSum(z, x, y) \rightarrow (\forall w)(part(z, w) \leftrightarrow (part(x, w) \wedge part(y, w)))) \tag{91}$$

$$(\forall x, y, z) MSum(z, x, y) \rightarrow Object(x) \wedge Object(y) \wedge Object(z) \tag{92}$$

$$(\forall x, y) Object(x) \wedge Object(y) \rightarrow \exists z(Object(z) \wedge MSum(z, x, y)) \tag{93}$$

$$(\forall x, y, z, t) MSum(z, x, y) \wedge MSum(t, x, y) \rightarrow (z = t) \tag{94}$$

Table 10. Translation of $T_{extended\_sum}$ into T_{dolce_sum}.

$$(\forall x, y, z) MSum(z, x, y) \leftrightarrow Object(x) \wedge Object(y) \wedge SUM(z, x, y) \tag{95}$$

Figure 5 shows conservative extensions by means of thin black arrows and relative interpretations (mappings), by thick grey arrows from interpreted to interpreting theories. Because every theorem of a theory is also a theorem of its conservative extensions, each conservative extension is capable of interpreting every theory that the modules that it extends interpret. In particular, the subtheory T_{dolce_part}, shown in Fig. 5, is the theory resulting from the union of $T_{dolce\_part\_T}$, $T_{dolce\_part\_E}$, and $T_{dolce\_part_O}$, plus axioms (49), (50), while the subtheory *DOLCE EXTENSIONAL MEREOLOGY* is the union of T_{dolce_part}, T_{dolce_sum}, and axioms

[18] A n-ary function f from A^n to B is representable by a relation ϱ with arity (n+1), called *the graph of f*, such that:
(a) Every tuple of ϱ is a tuple $\langle \bar{x}, f(\bar{x}) \rangle$ with $\bar{x} \in A^n$ and $f(\bar{x}) \in range(f)$.
(b) If $f(\bar{x}) = b$ and $f(\bar{z}) = c$, then $b = c$.

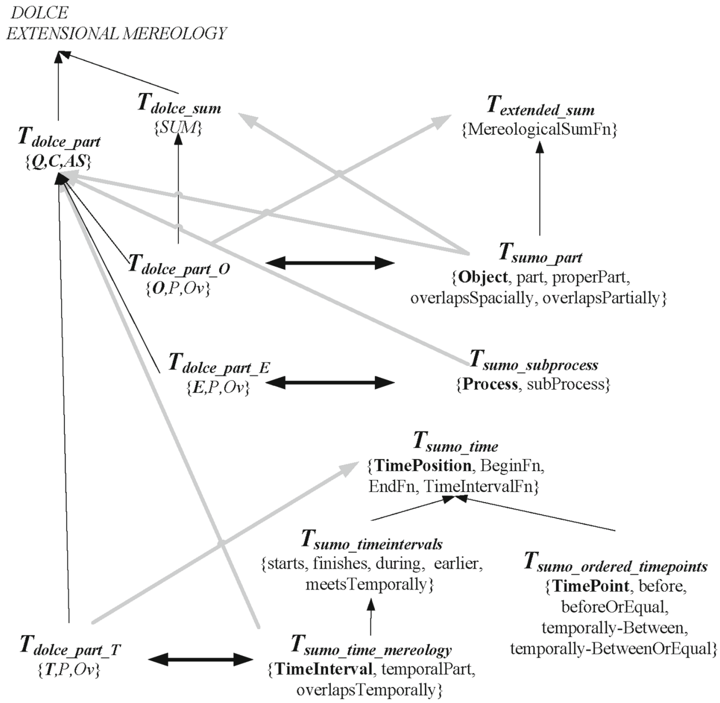

Fig. 5. Mappings between modules of DOLCE-CORE and SUMO (extracted from [16]). Black thin arrows point to conservative extensions, thick grey arrows are directed from interpreted theories to interpreting theories, and thick black arrows connect synonymous theories.

(45), (52), (53), (54), (55), and (56). As indicated by oriented grey arrows, the axiomatization of part-whole relations in categories *Object*, *Process*, and *TimeInterval* of SUMO are mappable to DOLCE minimal axiomatization of mereology represented by the subtheory T_{dolce_part}. Although not represented in Fig. 5, it holds that because *DOLCE EXTENSIONAL MEREOLOGY* extends T_{dolce_part}, it also interprets T_{sumo_part}, $T_{sumo\_subprocess}$, and $T_{time\_mereology}$. In turn, T_{sumo_sum} interprets $T_{dolce\_part\_O}$. The strongest subtheories of SUMO and DOLCE-CORE that are synonymous, and therefore have equivalent models, are the pairs indicated by double black arrows, i.e, $T_{dolce\_part\_O}$ with T_{sumo_part}, $T_{dolce\_part\_E}$ with $T_{sumo\_subprocess}$, and $T_{dolce\_part\_T}$ with $T_{time\_mereology}$.

6 Conclusions

Since the conceptual coverage of upper ontologies needs to be broad enough to cover the underpinnings of domain ontologies, we find a modules of upper ontologies for notions about time, space, objects, and processes. In this paper, we have focussed on how two upper ontologies (SUMO and DOLCE) axiomatize mereotopologies, that is, the notions of parthood (mereology) and connection (topology). By showing how different subtheories of SUMO and DOLCE are logically synonymous with different theories of lattices, we have identified unintended and omitted models of the original axiomatization of SUMO. There are additional subtheories of SUMO that capture additional spatial concepts such as containment, holes, orientation, and betweenness. Future work will provide a verification of these modules, thereby provided a firmer foundation for spatial reasoning.

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