Splitting Permutation Representations of Finite Groups by Polynomial Algebra Methods

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Abstract. An algorithm for splitting permutation representations of a finite group over fields of characteristic zero into irreducible components is described. The algorithm is based on the fact that the components of the invariant inner product in invariant subspaces are operators of projection into these subspaces. An important part of the algorithm is the solution of systems of quadratic equations. A preliminary implementation of the algorithm splits representations up to dimensions of hundreds of thousands. Examples of computations are given in the appendix.

1 Introduction

One of the central problems of group theory and its applications in physics is the decomposition of linear representations of groups into irreducible components. In general, the problem of splitting a module over an associative algebra into irreducible submodules is quite nontrivial. An overview of the algorithmic aspects of this problem can be found in Chap. 7 of [1]. For vector spaces over finite fields, the most efficient is the Las Vegas algorithm [1] called *MeatAxe* [2]. This algorithm played an important role in solving the problem of classifying finite simple groups. However, the approach used in the *MeatAxe* is ineffective in characteristic zero, whereas quantum-mechanical problems are formulated just in Hilbert spaces over zero characteristic fields. Our algorithm deals with representations over such fields, and its implementation copes with dimensions up to hundreds of thousands, which is not less than the dimensions achievable for the *MeatAxe*. The algorithm requires knowledge of the centralizer ring of the considered group representation. In the general case, the calculation of the centralizer ring is a problem of linear algebra, namely, solving matrix equations of the form $AX = XA$. For permutation representations, there is an efficient way to compute the centralizer ring, which reduces to constructing the set of orbitals. In addition, permutation representations are fundamental in the sense that any linear representation of a finite group is a subrepresentation of some permutation

[1] A *Las Vegas algorithm* is a randomized algorithm, each iteration of which either produces the correct result, or reports a failure. An algorithm of this type always gives the correct answer, but the run time is indeterminate.

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representation, and we use this fact in some quantum mechanical considerations [3,4]. Thus, we consider only permutation representations here.

2 Mathematical Preliminaries

Let G (or $\mathsf{G}(\Omega)$) be a *transitive* permutation group on a set $\Omega \cong \{1, \ldots, \mathsf{N}\}$. We will denote the action of $g \in \mathsf{G}$ on $i \in \Omega$ by i^g. A representation of G in an N-dimensional vector space over a field $\mathcal{F}$ by the matrices $\mathrm{P}(g)$ with the entries $\mathrm{P}(g)_{ij} = \delta_{i^g j}$, where δ_{ij} is the Kronecker delta, is called a *permutation representation*. We assume that the permutation representation space is a Hilbert space $\mathcal{H}_\mathsf{N}$. We will assume that the base field $\mathcal{F}$ is a constructive *splitting field* of the group G. In particular, such a field can be a subfield of the mth cyclotomic field, where m is a suitable divisor of the exponent of G. Such a constructive field $\mathcal{F}$, being an abelian extension of the field $\mathbb{Q}$, is a dense subfield of $\mathbb{R}$ or $\mathbb{C}$.

An orbit of G on the Cartesian square $\Omega \times \Omega$ is called an *orbital* [5]. The number of orbitals, called the *rank* of $\mathsf{G}(\Omega)$, will be denoted by R. An orbital Δ is called *self-paired*, if $(i,j) \in \Delta \Rightarrow (j,i) \in \Delta$, i.e., $\Delta = \Delta^{\mathrm{T}}$. Among the orbitals of a transitive group, there is one *diagonal* orbital, $\Delta_1 = \{(i,i) \mid i \in \Omega\}$, which will always be fixed as the first element in the list of orbitals: $\{\Delta_1, \ldots, \Delta_\mathrm{R}\}$. For transitive action of G, there is a natural one-to-one correspondence between the orbitals of G and the orbits of a point stabilizer G_i:

$$\Delta \longleftrightarrow \Sigma_i = \{j \in \Omega \mid (i,j) \in \Delta\}\,.$$

The G_i-orbits are called *suborbits* and their cardinalities will be called the *suborbit lengths*. Note that $|\Delta| = \mathsf{N}\,|\Sigma_i|$.

The invariance condition for a bilinear form A in the Hilbert space $\mathcal{H}_\mathsf{N}$ can be written as the system of equations $A = \mathrm{P}(g)\,A\mathrm{P}\left(g^{-1}\right),\ g \in \mathsf{G}$. It is easy to verify that in terms of the entries these equations have the form $(A)_{ij} = (A)_{i^g j^g}$. Thus, the basis of all invariant bilinear forms is in one-to-one correspondence with the set of orbitals: with each orbital $\Delta_r \in \{\Delta_1, \ldots, \Delta_\mathrm{R}\}$ is associated a *basis matrix* $\mathcal{A}_r$ of the size $\mathsf{N} \times \mathsf{N}$ with the entries

$$(\mathcal{A}_r)_{ij} = \begin{cases} 1, & \text{if } (i,j) \in \Delta_r, \\ 0, & \text{if } (i,j) \notin \Delta_r. \end{cases}$$

It is clear that the matrix of a self-paired orbital is symmetric. For the diagonal orbital, we have $\mathcal{A}_1 = \mathbb{1}_\mathsf{N}$. The matrices

$$\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_\mathrm{R} \tag{1}$$

form a basis of the *centralizer ring* (or *centralizer algebra*) of the representation P. The multiplication table for this basis has the form

$$\mathcal{A}_p\mathcal{A}_q = \sum_{r=1}^{\mathrm{R}} C_{pq}^r \mathcal{A}_r, \tag{2}$$

where C_{pq}^r are non-negative integers. The commutativity of the centralizer ring indicates that the permutation representation P is *multiplicity-free*.

3 Algorithm

Let T be a transformation (we can assume that T is a unitary matrix) that splits the permutation representation P into M irreducible components:

$$T^{-1}\mathrm{P}(g)\,T = 1 \oplus \mathsf{U}_{d_2}(g) \oplus \cdots \oplus \mathsf{U}_{d_m}(g) \oplus \cdots \oplus \mathsf{U}_{d_M}(g)\,,$$

where U_{d_m} is a d_m-dimensional irreducible subrepresentation, $\oplus$ denotes the direct sum of matrices, i.e., $A \oplus B = \mathrm{diag}(A, B)$.

The identity matrix $\mathbb{1}_{\mathsf{N}}$ is the *standard inner product* in any orthonormal basis. In the splitting basis, we have the following decomposition of the standard inner product

$$\mathbb{1}_{\mathsf{N}} = \mathbb{1}_{d_1=1} \oplus \cdots \oplus \mathbb{1}_{d_m} \oplus \cdots \oplus \mathbb{1}_{d_M}\,.$$

The *inverse image* of this decomposition in the original permutation basis has the form

$$\mathbb{1}_{\mathsf{N}} = \mathcal{B}_1 + \cdots + \mathcal{B}_m + \cdots + \mathcal{B}_M, \tag{3}$$

where $\mathcal{B}_m$ is defined by the relation

$$T^{-1}\mathcal{B}_m T = \mathbb{0}_{1+d_2+\cdots+d_{m-1}} \oplus \mathbb{1}_{d_m} \oplus \mathbb{0}_{d_{m+1}+\cdots+d_M} \equiv \mathcal{D}_m. \tag{4}$$

The set $\mathcal{B}_1, \ldots, \mathcal{B}_M$ contains complete information about irreducible decomposition of the representation P. In particular, the transformation matrix can be obtained from the linear system $\mathcal{B}_1 T - T\mathcal{D}_1 = \cdots = \mathcal{B}_M T - T\mathcal{D}_M = \mathbb{0}_{\mathsf{N}}$.

The main idea of the algorithm is based on the fact that $\mathcal{B}_m$'s form a complete set of *orthogonal projectors*, i.e., in addition to the *completeness* (3), we have the *idempotency*

$$\mathcal{B}_m^2 = \mathcal{B}_m \tag{5}$$

and the mutual *orthogonality*

$$\mathcal{B}_m\mathcal{B}_{m'} = \mathbb{0}_{\mathsf{N}} \text{ if } m \neq m'. \tag{6}$$

It follows from (4) that

$$\mathrm{tr}\,\mathcal{B}_m = d_m. \tag{7}$$

We see that all $\mathcal{B}_m$'s can be obtained as solutions of the equation

$$X^2 - X = \mathbb{0}_{\mathsf{N}} \tag{8}$$

for the generic invariant form

$$X = x_1\mathcal{A}_1 + \cdots + x_{\mathrm{R}}\mathcal{A}_{\mathrm{R}}.$$

Using the multiplication table (2), we can write the left-hand side of (8) as a set of R polynomials (we will call them *idempotency polynomials*)

$$E(x_1, \ldots, x_{\mathrm{R}}) = \{E_1(x_1, \ldots, x_{\mathrm{R}})\,, \ldots, E_{\mathrm{R}}(x_1, \ldots, x_{\mathrm{R}})\} \tag{9}$$

and Eq. (8) can be written symbolically as

$$E(x_1, \ldots, x_{\mathrm{R}}) = 0. \tag{10}$$

Each polynomial in (9) has the structure $E_r(x_1, \ldots, x_{\mathrm{R}}) = Q_r(x_1, \ldots, x_{\mathrm{R}}) - x_r$, where $Q_r(x_1, \ldots, x_{\mathrm{R}})$ is a homogeneous quadratic polynomial in the indeterminates $x_1, \ldots, x_{\mathrm{R}}$.

In the basis (1), the projector $\mathcal{B}_m$ can be represented as

$$\mathcal{B}_m = b_{m,1}\mathcal{A}_1 + b_{m,2}\mathcal{A}_2 + \cdots + b_{m,\mathrm{R}}\mathcal{A}_{\mathrm{R}},$$

where the vector $B_m = [b_{m,1}, \ldots, b_{m,\mathrm{R}}]$ is a solution of Eq. (10).

Since only $\mathcal{A}_1$ has nonzero diagonal elements, we have

$$\mathrm{tr}\,\mathcal{B}_m = b_{m,1}\mathsf{N}.$$

Combining this with (7) we can fix the coefficient $b_{m,1}$:

$$b_{m,1} = d_m/\mathsf{N}.$$

Thus, the only relevant values of x_1 in (10) are d/N for some d's from the interval $[1, \ldots, \mathsf{N} - 1]$. Any relevant natural number d is either an irreducible dimension or a sum of such dimensions. Using the orthogonality condition (6) for the irreducible projectors, we can exclude the consideration of dimensions that are sums of irreducible ones. The generic orthogonality condition can be written as

$$BX = 0, \tag{11}$$

where $B = b_1\mathcal{A}_1 + \cdots + b_{\mathrm{R}}\mathcal{A}_{\mathrm{R}}$. Equation (11) is a system of linear equations for the indeterminates $x_1, \ldots, x_{\mathrm{R}}$ with the parameters $b_1, \ldots, b_{\mathrm{R}}$. Again, using the multiplication table (2), we can write the left-hand side of (11) as a system of R bilinear forms, which we denote by

$$O(b_1, \ldots, b_{\mathrm{R}}; x_1, \ldots, x_{\mathrm{R}}) \tag{12}$$

and call *orthogonality polynomials*.

The core part of the algorithm is a loop on dimensions that starts with $d = 1$ and ends when the sum of irreducible dimensions becomes equal to N.

The current d is processed as follows.

- We solve [2] the system of equations $E(d/\mathsf{N}, x_2, \ldots, x_{\mathrm{R}}) = 0$.
- If the system is incompatible, then go to the next d.
- If $E(d/\mathsf{N}, x_2, \ldots, x_{\mathrm{R}})$ describes a zero-dimensional ideal, then we have k (including $k = 1$) different d-dimensional irreducible subrepresentations.

[2] The solution is always algorithmically realizable, since the problem involves only polynomial equations with abelian Galois groups.

– If the polynomial ideal has dimension $h > 0$, then we encounter an irreducible component with a multiplicity $k > 1$. The corresponding component of the centralizer algebra has the form $A \otimes \mathbb{1}_d$, where A is an arbitrary $k \times k$ matrix, and $\otimes$ denotes the Kronecker product. The idempotency condition $(A \otimes \mathbb{1}_d)^2 = A \otimes \mathbb{1}_d$ implies $A^2 - A = 0$. The complete family of solutions of this equation [3] is a manifold of dimension $\lfloor k^2/2 \rfloor = h$. In this case, we select, by a somewhat arbitrary procedure, k convenient mutually orthogonal representatives in the family of equivalent subrepresentations.
– In any case, if at the moment we have a solution $\mathcal{B}_m$, we append $\mathcal{B}_m$ to the list of irreducible projectors, and exclude from the further consideration the corresponding invariant subspace by adding the linear orthogonality polynomials $\mathcal{B}_m X$ to the polynomial system:

$$E(x_1, x_2, \ldots, x_{\mathrm{R}}) \leftarrow E(x_1, x_2, \ldots, x_{\mathrm{R}}) \cup \{\mathcal{B}_m X\}\,.$$

– After processing all $\mathcal{B}_m$'s of dimension d, go to the next d.

4 Implementation

Our approach involves some widely used methods of polynomial computer algebra. Therefore, it is reasonable, at least for the preliminary experience, to take advantage of computer algebra systems with developed tools for working with polynomials.

The complete algorithm is implemented by two procedures, the pseudocodes of which are given below.

1. The procedure ***PreparePolynomialData*** is a program written in **C**. The input data for this program is a set of permutations of Ω that generates the group $\mathsf{G}(\Omega)$. The program computes the basis of the centralizer ring and its multiplication table, constructs the idempotency and orthogonality polynomials, and generates the code of the procedure ***SplitRepresentation*** that processes the polynomial data. The main parameter that determines the run time for ***PreparePolynomialData*** is the dimension of the representation. The example in Sect. A.3 shows that the PC implementation copes with a dimension of about one hundred thousand in a time of about one hour.
2. The procedure ***SplitRepresentation*** implements the above described loop on dimensions that splits the representation of the group into irreducible components. It is generated by the **C** program ***PreparePolynomialData***. Currently, the code is generated in the **Maple 2017.3** language, and the polynomial equations are processed by the **Maple** implementation of the Gröbner bases algorithms. The run time for ***SplitRepresentation*** depends mainly on the rank of the representation. Problems of rank $\mathrm{R} = 17$ take about 8 hours on a PC.

[3] It is well known that any solution of the matrix equation $A^2 = A$ can be represented as $A = Q^{-1}(\mathbb{1}_r \oplus \mathbb{0}_{k-r})\,Q$, where Q is an arbitrary invertible $k \times k$ matrix and $r \in [0, k]$.

Input: $S = \{s_1, \ldots, s_K\}$ // set of permutations of Ω that generates group G
Output: $E(x_1, \ldots, x_{\mathrm{R}})$, $O(b_1, \ldots, b_{\mathrm{R}}; x_1, \ldots, x_{\mathrm{R}})$, ***SplitRepresentation***
compute basis of centralizer ring $\mathcal{A}_1, \ldots, \mathcal{A}_{\mathrm{R}}$
compute multiplication table $\mathcal{A}_p\mathcal{A}_q = \sum_{r=1}^{\mathrm{R}} C_{pq}^r \mathcal{A}_r$
construct idempotency polynomials $E(x_1, \ldots, x_{\mathrm{R}})$
construct orthogonality polynomials $O(b_1, \ldots, b_{\mathrm{R}}; x_1, \ldots, x_{\mathrm{R}})$
construct code ***SplitRepresentation*** for processing polynomial data
return ***SplitRepresentation*** $(E(x_1, \ldots, x_{\mathrm{R}}), O(b_1, \ldots, b_{\mathrm{R}}; x_1, \ldots, x_{\mathrm{R}}))$

Algorithm 1: *PreparePolynomialData*

Input: $E(x_1, \ldots, x_{\mathrm{R}})$, $O(b_1, \ldots, b_{\mathrm{R}}; x_1, \ldots, x_{\mathrm{R}})$
Output: $IrreducibleProjectors = [(1, \mathcal{B}_1), \ldots, (d_m, \mathcal{B}_m) \ldots, (d_M, \mathcal{B}_M)]$
$IrreducibleProjectors \leftarrow \left[\left(1, \frac{1}{\mathsf{N}}[1, \ldots, 1]\right)\right]$ // trivial subrepresentation
$E(x_1, \ldots, x_{\mathrm{R}}) \leftarrow E(x_1, \ldots, x_{\mathrm{R}}) \cup O(1, \ldots, 1; x_1, \ldots, x_{\mathrm{R}})$
$Sdim \leftarrow 1$ // sum of dimensions, global variable
$D \leftarrow 0$ // current dimension, global variable
while $Sdim < \mathsf{N}$ **do**
  $D \leftarrow$ ***NextRelevantDimension*(D)**
  $all\_solutions \leftarrow$ ***SolveAlgebraicSystem***$(E(D/\mathsf{N}, x_2, \ldots, x_{\mathrm{R}}))$
  if $all\_solutions \neq \varnothing$ **then**
    $h \leftarrow$ ***NumberOfFreeParameters***$(all\_solutions)$
    if $h = 0$ **then**
      for $solution \in all\_solutions$ **do**
        UseSingleSolution$(solution)$
    else
      repeat
        $solution \leftarrow$ ***PickBestSolution***$(all\_solutions)$
        UseSingleSolution$(solution)$
        $all\_solutions \leftarrow$ ***SolveAlgebraicSystem***$(E(D/\mathsf{N}, x_2, \ldots, x_{\mathrm{R}}))$
      until $all\_solutions = \varnothing$
return $IrreducibleProjectors$

Algorithm 2: *SplitRepresentation*

Input: $solution = [\beta_1, \ldots, \beta_{\mathrm{R}}]$
$E(x_1, \ldots, x_{\mathrm{R}}) \leftarrow E(x_1, \ldots, x_{\mathrm{R}}) \cup O(\beta_1, \ldots, \beta_{\mathrm{R}}; x_1, \ldots, x_{\mathrm{R}})$
$IrreducibleProjectors \leftarrow [IrreducibleProjectors, (D, solution)]$
$Sdim \leftarrow Sdim + D$

Algorithm 3: *UseSingleSolution*

Comments on the procedure ***SplitRepresentation***:

- The procedure ***NextRelevantDimension*** can be implemented in different ways, depending on the available information about the group and the representation:
  - The simplest implementation is "$D \leftarrow D + 1$".
  - The implementation "**repeat** $D \leftarrow D + 1$ **until** $D \mid \mathrm{Ord}(\mathsf{G})$" is about 25% faster than the simplest one. In fact, the size of the group is always known.
  - Knowledge of the character decomposition provides the most efficient loop on dimensions. Sometimes this information is available. Actually, computing the character decomposition is much easier than computing the decomposition of the representation.
- The procedures ***SolveAlgebraicSystem*** and ***NumberOfFreeParameters*** involve the polynomial algebra functions available in the computer algebra system used. At present, we use the **Maple** implementation of Gröbner basis techniques.
- The ***PickBestSolution*** procedure is applied in the case of nontrivial multiplicity of the irreducible component. It selects a particular solution in the parametric set of solutions. Currently, the choice of solutions with zero values of parameters is used. Such an oversimplified approach sometimes leads to "ugly roots" that go beyond the "natural" splitting field. This can be illustrated by the example of a 29155-dimensional representation of the Held group whose decomposition into irreducible components is given in Sect. A.2. The decomposition contains a 1275-dimensional irreducible component of multiplicity two. Representatives of this component obtained by the simple version of ***PickBestSolution*** contain irrationality $\mathbf{i}\sqrt{231}$ (see $\mathcal{B}^{(1)}_{\mathbf{1275}}$ and $\mathcal{B}^{(2)}_{\mathbf{1275}}$ expressions), which belongs to the quadratic field $\mathbb{Q}\left(\sqrt{-231}\right)$, while the representation in question splits over the "much smaller" field $\mathbb{Q}\left(\sqrt{-7}\right)$. Therefore, the ***PickBestSolution*** procedure requires improvement using strategies that lead to minimal extensions of the field $\mathbb{Q}$.

4.1 Comparison with the Magma Implementation of the *MeatAxe*

The **Magma** database contains a 3906-dimensional permutation representation of the exceptional group of Lie type $G_2(5)$. The decomposition into irreducible components of this representation over the field GF(2) is given in [6] as an illustration of the possibilities of the *MeatAxe*.

The application of our algorithm to this problem shows that in the characteristic zero, the considered representation is split over the field $\mathbb{Q}$. The calculation produces the following data:

Rank: 4. Suborbit lengths: $1, 30, 750, 3125$.

$$\underline{\mathbf{3906}} \cong \mathbf{1} \oplus \mathbf{930} \oplus \mathbf{1085} \oplus \mathbf{1890}$$

$$\mathcal{B}_{\mathbf{1}} = \frac{1}{3906}\sum_{k=1}^{4}\mathcal{A}_k$$

$$\mathcal{B}_{\mathbf{930}} = \frac{5}{21}\left(\mathcal{A}_1 + \frac{3}{10}\mathcal{A}_2 + \frac{1}{50}\mathcal{A}_3 - \frac{1}{125}\mathcal{A}_4\right)$$

$$\mathcal{B}_{\mathbf{1085}} = \frac{5}{18}\left(\mathcal{A}_1 - \frac{1}{5}\mathcal{A}_2 + \frac{1}{25}\mathcal{A}_3 - \frac{1}{125}\mathcal{A}_4\right)$$

$$\mathcal{B}_{\mathbf{1890}} = \frac{15}{31}\left(\mathcal{A}_1 - \frac{1}{30}\mathcal{A}_2 - \frac{1}{30}\mathcal{A}_3 + \frac{1}{125}\mathcal{A}_4\right)$$

Time **C**: 0.5 s. Time **Maple**: 0.8 s.

Magma failed to split the 3906-dimensional representation over the field $\mathbb{Q}$ due to memory exhaustion after long computation, but we can simulate to some extent the case of characteristic zero, using a field of a characteristic that does not divide $\mathrm{Ord}(G_2(5))$. The smallest such field is GF(11).

Below is the session of the corresponding **Magma V2.21-1** computation on a computer with two Intel Xeon E5410 2.33 GHz CPUs (time is given in seconds).

```
> load "g25";
Loading "/opt/magma.21-1/libs/pergps/g25"
The Lie group G( 2, 5 ) represented as a permutation
group of degree 3906.
Order: 5 859 000 000 = 2^6 * 3^3 * 5^6 * 7 * 31.
Group: G
> time Constituents(PermutationModule(G,GF(11)));
[
    GModule of dimension 1 over GF(11),
    GModule of dimension 930 over GF(11),
    GModule of dimension 1085 over GF(11),
    GModule of dimension 1890 over GF(11)
]
Time: 282.060
```

5 Conclusion

The algorithm described here is based on the use of methods of polynomial algebra, which are considered algorithmically difficult. However, our approach leads to a small number (in typical cases) of low-degree polynomials. Recall that the idempotency system (9) is a set of R square polynomials. Calculations of Gröbner bases in **Maple** on PC are limited in practice to $\mathrm{R} = 17$. Among the 886 permutation representations available in the ATLAS [7], 761 (i.e., 86%) have ranks $\mathrm{R} \leq 17$. As can be seen in Appendix A, even a straightforward implementation of the approach can cope with rather large tasks. The data presented in the appendix shows that the most restrictive parameter for the **Maple** part of the implementation is the rank of representations, i.e., the number of polynomial indeterminates. A possible way to improve performance is to try to develop specialized algorithms that take into account the very special type of polynomial equations that arise in the problem instead of the universal Gröbner basis methods.

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A Examples of Computations

- Generators of representations are taken from the section "Sporadic groups" of the ATLAS [7].
- For a group G
  - $\mathrm{M}(\mathsf{G})$ denotes the *Schur multiplier*, the 2nd homology group $H_2(\mathsf{G},\mathbb{Z})$,
  - $\mathrm{Out}(\mathsf{G})$ denotes the *outer automorphism group* of G,
  - $n.\mathsf{G}$ denotes a *covering group* of G, a *central extension* of G by C_n.
- The results presented below assume the following ordering for the centralizer ring basis matrices

$$\mathcal{A}_1=\mathbb{1}_{\mathsf{N}},\quad \underbrace{\mathcal{A}_2,\ldots,\mathcal{A}_k,}_{\text{symmetric matrices}}\quad \underbrace{\mathcal{A}_{k+1},\mathcal{A}_{k+2}=\mathcal{A}_{k+1}^{\mathrm{T}},\ldots,\mathcal{A}_{\mathrm{R}-1},\mathcal{A}_{\mathrm{R}}=\mathcal{A}_{\mathrm{R}-1}^{\mathrm{T}}}_{\text{asymmetric matrices}}.$$

  The matrices within the first sublist are ordered by the rule: $A<B$ if $i_A<i_B$, where $i_X=\min\left(i\mid (X)_{i1}=1\right)$. The same rule is applied to the first elements of the pairs of asymmetric matrices.
- Representations are denoted by their dimensions in bold (possibly with some signs added to distinguish different representations of the same dimension). Permutation representations are underlined. Multiple subrepresentations are underbraced in the decompositions.
- We omit the irreducible projectors related to the trivial subrepresentation: these projectors have the standard form $\mathcal{B}_{\mathbf{1}}=\frac{1}{\mathsf{N}}\sum_{k=1}^{\mathrm{R}}\mathcal{A}_k$.
- All timing data refer to a PC with 3.30 GHz Intel Core i3 2120 CPU.

A.1 Higman–Sims Group *HS*

Main properties: $\mathrm{Ord}(HS)=44352000=2^9\cdot3^2\cdot5^3\cdot7\cdot11$.
$\mathrm{M}(HS)=\mathsf{C}_2$. $\mathrm{Out}(HS)=\mathsf{C}_2$.

11200-dimensional Representation of 2.*HS*

Rank: 16. Suborbit lengths: $1^2,110,132^2,165^2,660^2,792^2,990,1320^2,1980^2$.

$$\underline{\mathbf{11200}}\cong\mathbf{1}\oplus\mathbf{22}\oplus\mathbf{56}\oplus\mathbf{77}\oplus\mathbf{154}\oplus\mathbf{175}\oplus\mathbf{176}\oplus\overline{\mathbf{176}}\oplus\mathbf{616}\oplus\overline{\mathbf{616}}$$
$$\oplus\,\mathbf{770}\oplus\mathbf{825}\oplus\mathbf{1056}\oplus\mathbf{1980}\oplus\overline{\mathbf{1980}}\oplus\mathbf{2520}$$

$$\mathcal{B}_{\mathbf{22}} = \frac{11}{5600}\Big(\mathcal{A}_1 + \frac{13}{33}\mathcal{A}_2 - \frac{7}{33}\mathcal{A}_3 + \frac{1}{11}\mathcal{A}_4 + \frac{1}{11}\mathcal{A}_5 + \frac{13}{33}\mathcal{A}_6 + \frac{1}{11}\mathcal{A}_7 - \frac{7}{33}\mathcal{A}_8 + \frac{13}{33}\mathcal{A}_9 + \mathcal{A}_{10} - \frac{7}{33}\mathcal{A}_{11} - \frac{7}{33}\mathcal{A}_{12} + \frac{1}{11}\mathcal{A}_{13} + \frac{1}{11}\mathcal{A}_{14} - \frac{17}{33}\mathcal{A}_{15} - \frac{17}{33}\mathcal{A}_{16}\Big)$$

$$\mathcal{B}_{\mathbf{56}} = \frac{1}{200}\Big(\mathcal{A}_1 + \frac{1}{4}\mathcal{A}_3 + \frac{1}{4}\mathcal{A}_4 - \frac{1}{4}\mathcal{A}_5 + \frac{1}{4}\mathcal{A}_6 - \frac{1}{4}\mathcal{A}_8 - \frac{1}{4}\mathcal{A}_9 - \mathcal{A}_{10}\Big)$$

$$\mathcal{B}_{\mathbf{77}} = \frac{11}{1600}\Big(\mathcal{A}_1 + \frac{1}{11}\mathcal{A}_2 + \frac{17}{132}\mathcal{A}_3 - \frac{23}{132}\mathcal{A}_4 - \frac{23}{132}\mathcal{A}_5 + \frac{37}{132}\mathcal{A}_6 - \frac{4}{11}\mathcal{A}_7 + \frac{17}{132}\mathcal{A}_8 + \frac{37}{132}\mathcal{A}_9 + \mathcal{A}_{10} - \frac{2}{33}\mathcal{A}_{11} - \frac{2}{33}\mathcal{A}_{12} + \frac{1}{66}\mathcal{A}_{13} + \frac{1}{66}\mathcal{A}_{14} + \frac{8}{33}\mathcal{A}_{15} + \frac{8}{33}\mathcal{A}_{16}\Big)$$

$$\mathcal{B}_{\mathbf{154}} = \frac{11}{800}\Big(\mathcal{A}_1 + \frac{3}{55}\mathcal{A}_2 + \frac{7}{55}\mathcal{A}_3 + \frac{1}{11}\mathcal{A}_4 + \frac{1}{11}\mathcal{A}_5 - \frac{1}{11}\mathcal{A}_6 - \frac{19}{55}\mathcal{A}_7 + \frac{7}{55}\mathcal{A}_8 - \frac{1}{11}\mathcal{A}_9 + \mathcal{A}_{10} - \frac{1}{55}\mathcal{A}_{11} - \frac{1}{55}\mathcal{A}_{12} - \frac{3}{55}\mathcal{A}_{13} - \frac{3}{55}\mathcal{A}_{14} - \frac{7}{55}\mathcal{A}_{15} - \frac{7}{55}\mathcal{A}_{16}\Big)$$

$$\mathcal{B}_{\mathbf{175}} = \frac{1}{64}\Big(\mathcal{A}_1 + \frac{7}{55}\mathcal{A}_2 - \frac{1}{15}\mathcal{A}_3 + \frac{1}{33}\mathcal{A}_4 + \frac{1}{33}\mathcal{A}_5 + \frac{1}{33}\mathcal{A}_6 + \frac{7}{55}\mathcal{A}_7 - \frac{1}{15}\mathcal{A}_8 + \frac{1}{33}\mathcal{A}_9 + \mathcal{A}_{10} + \frac{1}{33}\mathcal{A}_{11} + \frac{1}{33}\mathcal{A}_{12} - \frac{1}{15}\mathcal{A}_{13} - \frac{1}{15}\mathcal{A}_{14} + \frac{37}{165}\mathcal{A}_{15} + \frac{37}{165}\mathcal{A}_{16}\Big)$$

$$\mathcal{B}_{\mathbf{176}} = \frac{11}{700}\Big(\mathcal{A}_1 + \frac{2}{33}\mathcal{A}_3 - \frac{1}{11}\mathcal{A}_4 + \frac{1}{11}\mathcal{A}_5 + \frac{7}{33}\mathcal{A}_6 - \frac{2}{33}\mathcal{A}_8 - \frac{7}{33}\mathcal{A}_9 - \mathcal{A}_{10} + \mathbf{i}\frac{1}{33}\mathcal{A}_{11} - \mathbf{i}\frac{1}{33}\mathcal{A}_{12} + \mathbf{i}\frac{2}{33}\mathcal{A}_{13} - \mathbf{i}\frac{2}{33}\mathcal{A}_{14} + \mathbf{i}\frac{7}{33}\mathcal{A}_{15} - \mathbf{i}\frac{7}{33}\mathcal{A}_{16}\Big)$$

$$\mathcal{B}_{\mathbf{616}} = \frac{11}{200}\Big(\mathcal{A}_1 - \frac{7}{132}\mathcal{A}_3 + \frac{1}{44}\mathcal{A}_4 - \frac{1}{44}\mathcal{A}_5 + \frac{13}{132}\mathcal{A}_6 + \frac{7}{132}\mathcal{A}_8 - \frac{13}{132}\mathcal{A}_9 - \mathcal{A}_{10} - \mathbf{i}\frac{1}{66}\mathcal{A}_{11} + \mathbf{i}\frac{1}{66}\mathcal{A}_{12} - \mathbf{i}\frac{1}{33}\mathcal{A}_{13} + \mathbf{i}\frac{1}{33}\mathcal{A}_{14} + \mathbf{i}\frac{4}{33}\mathcal{A}_{15} - \mathbf{i}\frac{4}{33}\mathcal{A}_{16}\Big)$$

$$\mathcal{B}_{\mathbf{770}} = \frac{11}{160}\left(\mathcal{A}_1 - \frac{1}{165}\mathcal{A}_2 - \frac{1}{60}\mathcal{A}_3 - \frac{1}{44}\mathcal{A}_4 - \frac{1}{44}\mathcal{A}_5 + \frac{13}{132}\mathcal{A}_6 - \frac{4}{55}\mathcal{A}_7 - \frac{1}{60}\mathcal{A}_8 + \frac{13}{132}\mathcal{A}_9 + \mathcal{A}_{10} + \frac{7}{165}\mathcal{A}_{11} + \frac{7}{165}\mathcal{A}_{12} - \frac{1}{110}\mathcal{A}_{13} - \frac{1}{110}\mathcal{A}_{14} - \frac{16}{165}\mathcal{A}_{15} - \frac{16}{165}\mathcal{A}_{16}\right)$$

$$\mathcal{B}_{\mathbf{825}} = \frac{33}{448}\left(\mathcal{A}_1 + \frac{13}{495}\mathcal{A}_2 + \frac{7}{220}\mathcal{A}_3 - \frac{13}{396}\mathcal{A}_4 - \frac{13}{396}\mathcal{A}_5 - \frac{1}{12}\mathcal{A}_6 + \frac{12}{55}\mathcal{A}_7 + \frac{7}{220}\mathcal{A}_8 - \frac{1}{12}\mathcal{A}_9 + \mathcal{A}_{10} - \frac{1}{990}\mathcal{A}_{13} - \frac{1}{990}\mathcal{A}_{14} - \frac{8}{165}\mathcal{A}_{15} - \frac{8}{165}\mathcal{A}_{16}\right)$$

$$\mathcal{B}_{\mathbf{1056}} = \frac{33}{350}\left(\mathcal{A}_1 - \frac{23}{495}\mathcal{A}_2 + \frac{3}{220}\mathcal{A}_3 + \frac{1}{36}\mathcal{A}_4 + \frac{1}{36}\mathcal{A}_5 + \frac{13}{132}\mathcal{A}_6 + \frac{6}{55}\mathcal{A}_7 + \frac{3}{220}\mathcal{A}_8 + \frac{13}{132}\mathcal{A}_9 + \mathcal{A}_{10} - \frac{1}{55}\mathcal{A}_{11} - \frac{1}{55}\mathcal{A}_{12} - \frac{2}{495}\mathcal{A}_{13} - \frac{2}{495}\mathcal{A}_{14} + \frac{4}{165}\mathcal{A}_{15} + \frac{4}{165}\mathcal{A}_{16}\right)$$

$$\mathcal{B}_{\mathbf{1980}} = \frac{99}{560}\left(\mathcal{A}_1 + \frac{1}{132}\mathcal{A}_3 - \frac{1}{396}\mathcal{A}_4 + \frac{1}{396}\mathcal{A}_5 - \frac{7}{132}\mathcal{A}_6 - \frac{1}{132}\mathcal{A}_8 + \frac{7}{132}\mathcal{A}_9 - \mathcal{A}_{10} - \mathbf{i}\frac{1}{33}\mathcal{A}_{11} + \mathbf{i}\frac{1}{33}\mathcal{A}_{12} + \mathbf{i}\frac{1}{99}\mathcal{A}_{13} - \mathbf{i}\frac{1}{99}\mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{2520}} = \frac{9}{40}\left(\mathcal{A}_1 - \frac{1}{165}\mathcal{A}_2 - \frac{1}{60}\mathcal{A}_3 + \frac{1}{396}\mathcal{A}_4 + \frac{1}{396}\mathcal{A}_5 - \frac{7}{132}\mathcal{A}_6 - \frac{4}{55}\mathcal{A}_7 - \frac{1}{60}\mathcal{A}_8 - \frac{7}{132}\mathcal{A}_9 + \mathcal{A}_{10} - \frac{1}{330}\mathcal{A}_{11} - \frac{1}{330}\mathcal{A}_{12} + \frac{1}{90}\mathcal{A}_{13} + \frac{1}{90}\mathcal{A}_{14} + \frac{4}{165}\mathcal{A}_{15} + \frac{4}{165}\mathcal{A}_{16}\right)$$

Time **C**: 8 s. Time **Maple**: 1 h 39 min 6 s.

A.2 Held Group *He*

Main properties: $\mathrm{Ord}(He) = 4030387200 = 2^{10} \cdot 3^3 \cdot 5^2 \cdot 7^3 \cdot 17$.
$\mathrm{M}(He) = 1$. $\mathrm{Out}(He) = \mathsf{C}_2$.

29155-dimensional Representation of *He*

Rank: 12. Suborbit lengths: $1, 90, 120, 384, 960^2, 1440, 2160, 2880^2, 5760, 11520$.

$$\underline{\mathbf{29155}} \cong \mathbf{1} \oplus \mathbf{51} \oplus \overline{\mathbf{51}} \oplus \mathbf{680} \oplus \underbrace{(\mathbf{1275} \oplus \mathbf{1275})} \oplus \mathbf{1920} \oplus \mathbf{4352} \oplus \mathbf{7650} \oplus \mathbf{11900}$$

$$\mathcal{B}_{\mathbf{51}} = \frac{3}{1715}\left\{\mathcal{A}_1 + \frac{5}{12}\mathcal{A}_2 - \frac{1}{48}\mathcal{A}_3 + \frac{1}{8}\mathcal{A}_4 + \frac{1}{8}\mathcal{A}_5 + \frac{13}{48}\mathcal{A}_6 - \frac{1}{6}\mathcal{A}_7 - \frac{1}{6}\mathcal{A}_8 \right.$$
$$- \frac{1}{32}\left(3 - \mathbf{i}\frac{7\sqrt{7}}{3}\right)\mathcal{A}_9 - \frac{1}{32}\left(3 + \mathbf{i}\frac{7\sqrt{7}}{3}\right)\mathcal{A}_{10}$$
$$\left. + \frac{1}{96}\left(5 + 7\mathbf{i}\sqrt{7}\right)\mathcal{A}_{11} + \frac{1}{96}\left(5 - 7\mathbf{i}\sqrt{7}\right)\mathcal{A}_{12}\right\}$$

$$\mathcal{B}_{\mathbf{680}} = \frac{8}{343}\left(\mathcal{A}_1 + \frac{3}{10}\mathcal{A}_2 - \frac{1}{48}\mathcal{A}_3 - \frac{23}{1440}\mathcal{A}_4 - \frac{1}{20}\mathcal{A}_5 + \frac{1}{8}\mathcal{A}_6 + \frac{1}{120}\mathcal{A}_7 \right.$$
$$\left. + \frac{13}{90}\mathcal{A}_8 + \frac{1}{36}\mathcal{A}_9 + \frac{1}{36}\mathcal{A}_{10} + \frac{1}{15}\mathcal{A}_{11} + \frac{1}{15}\mathcal{A}_{12}\right)$$

$$\mathcal{B}^{(1)}_{\mathbf{1275}} = \frac{15}{343}\left\{\mathcal{A}_1 + \frac{1}{4280}\left(\frac{331}{3} - 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_2 - \frac{1}{25680}\left(13 - \mathbf{i}\frac{7\sqrt{231}}{3}\right)\mathcal{A}_3 \right.$$
$$- \frac{1}{25680}\left(\frac{1381}{3} + 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_4 + \frac{1}{25680}\left(2101 + 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_5$$
$$- \frac{1}{1712}\left(13 - \mathbf{i}\frac{7\sqrt{231}}{3}\right)\mathcal{A}_6 + \frac{1}{2568}\left(\frac{109}{3} - \mathbf{i}\frac{7\sqrt{231}}{5}\right)\mathcal{A}_7$$
$$+ \frac{1}{4815}\left(1571 - \mathbf{i}\frac{7\sqrt{231}}{2}\right)\mathcal{A}_8 - \frac{1}{38520}\left(467 - 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_9$$
$$\left. - \frac{1}{38520}\left(467 - 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_{10}\right\}$$

$$\mathcal{B}^{(2)}_{\mathbf{1275}} = \frac{15}{343}\left\{\mathcal{A}_1 + \frac{1}{4280}\left(\frac{1381}{3} + 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_2 + \frac{1}{25680}\left(227 - \mathbf{i}\frac{7\sqrt{231}}{3}\right)\mathcal{A}_3 \right.$$
$$- \frac{1}{25680}\left(\frac{331}{3} - 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_4 - \frac{1}{25680}\left(389 + 7\mathbf{i}\sqrt{231}\right)\mathcal{A}_5$$
$$+ \frac{1}{1712}\left(227 - \mathbf{i}\frac{7\sqrt{231}}{3}\right)\mathcal{A}_6 + \frac{1}{2568}\left(\frac{319}{3} + \mathbf{i}\frac{7\sqrt{231}}{5}\right)\mathcal{A}_7$$
$$- \frac{1}{4815}\left(394 - \mathbf{i}\frac{7\sqrt{231}}{2}\right)\mathcal{A}_8 - \frac{7}{38520}\left(\frac{157}{2} + \mathbf{i}\sqrt{231}\right)\mathcal{A}_9$$
$$\left. - \frac{7}{38520}\left(\frac{157}{2} + \mathbf{i}\sqrt{231}\right)\mathcal{A}_{10} - \frac{1}{16}\mathcal{A}_{11} - \frac{1}{16}\mathcal{A}_{12}\right\}$$

$$\mathcal{B}_{\mathbf{1920}} = \frac{384}{5831}\left(\mathcal{A}_1 + \frac{1}{120}\mathcal{A}_2 - \frac{7}{384}\mathcal{A}_3 + \frac{1}{120}\mathcal{A}_4 + \frac{7}{160}\mathcal{A}_5 - \frac{7}{384}\mathcal{A}_6 + \frac{1}{120}\mathcal{A}_7 \right.$$
$$\left. - \frac{2}{15}\mathcal{A}_8 + \frac{5}{192}\mathcal{A}_9 + \frac{5}{192}\mathcal{A}_{10} - \frac{13}{480}\mathcal{A}_{11} - \frac{13}{480}\mathcal{A}_{12}\right)$$

$$\mathcal{B}_{\mathbf{4352}} = \frac{256}{1715}\left(\mathcal{A}_1 + \frac{1}{8}\mathcal{A}_2 + \frac{7}{768}\mathcal{A}_3 - \frac{5}{576}\mathcal{A}_4 - \frac{7}{128}\mathcal{A}_6 - \frac{1}{48}\mathcal{A}_7 - \frac{1}{18}\mathcal{A}_8 + \frac{1}{576}\mathcal{A}_9 + \frac{1}{576}\mathcal{A}_{10} - \frac{1}{192}\mathcal{A}_{11} - \frac{1}{192}\mathcal{A}_{12}\right)$$

$$\mathcal{B}_{\mathbf{7650}} = \frac{90}{343}\left(\mathcal{A}_1 - \frac{1}{20}\mathcal{A}_2 + \frac{1}{120}\mathcal{A}_4 - \frac{7}{360}\mathcal{A}_5 - \frac{1}{90}\mathcal{A}_7 + \frac{1}{10}\mathcal{A}_8 + \frac{1}{240}\mathcal{A}_9 + \frac{1}{240}\mathcal{A}_{10} - \frac{1}{80}\mathcal{A}_{11} - \frac{1}{80}\mathcal{A}_{12}\right)$$

$$\mathcal{B}_{\mathbf{11900}} = \frac{20}{49}\left(\mathcal{A}_1 - \frac{1}{20}\mathcal{A}_2 - \frac{1}{720}\mathcal{A}_4 + \frac{1}{120}\mathcal{A}_7 - \frac{1}{18}\mathcal{A}_8 - \frac{1}{180}\mathcal{A}_9 - \frac{1}{180}\mathcal{A}_{10} + \frac{1}{60}\mathcal{A}_{11} + \frac{1}{60}\mathcal{A}_{12}\right)$$

Time **C**: 47 s. Time **Maple**: 15 s.

A.3 Suzuki Group *Suz*

Main properties: $\mathrm{Ord}(Suz) = 448345497600 = 2^{13} \cdot 3^7 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$. $\mathrm{M}(Suz) = \mathsf{C}_6$. $\mathrm{Out}(Suz) = \mathsf{C}_2$.

65520-dimensional Representation of 2.*Suz*

Rank: 10. Suborbit lengths: $1^2, 891^2, 2816^2, 3960, 12672, 20736^2$.

$$\underline{\mathbf{65520}} \cong \mathbf{1} \oplus \mathbf{143} \oplus \mathbf{364}_\alpha \oplus \mathbf{364}_\beta \oplus \overline{\mathbf{364}_\beta} \oplus \mathbf{5940} \oplus \mathbf{12012} \oplus \mathbf{14300} \oplus \mathbf{16016} \oplus \overline{\mathbf{16016}}$$

$$\mathcal{B}_{\mathbf{143}} = \frac{11}{5040}\left(\mathcal{A}_1 + \mathcal{A}_2 + \frac{2}{11}\mathcal{A}_3 - \frac{1}{11}\mathcal{A}_4 + \frac{2}{11}\mathcal{A}_5 - \frac{1}{11}\mathcal{A}_6 + \frac{3}{11}\mathcal{A}_9 + \frac{3}{11}\mathcal{A}_{10}\right)$$

$$\mathcal{B}_{\mathbf{364}_\alpha} = \frac{1}{180}\left(\mathcal{A}_1 + \mathcal{A}_2 + \frac{1}{16}\mathcal{A}_3 + \frac{1}{6}\mathcal{A}_4 + \frac{1}{16}\mathcal{A}_5 - \frac{1}{24}\mathcal{A}_6 - \frac{1}{144}\mathcal{A}_7 - \frac{1}{144}\mathcal{A}_8 - \frac{1}{9}\mathcal{A}_9 - \frac{1}{9}\mathcal{A}_{10}\right)$$

$$\mathcal{B}_{\mathbf{364}_\beta} = \frac{1}{180}\left(\mathcal{A}_1 - \mathcal{A}_2 - \frac{1}{8}\mathcal{A}_3 + \frac{1}{8}\mathcal{A}_5 + \mathbf{i}\frac{\sqrt{3}}{72}\mathcal{A}_7 - \mathbf{i}\frac{\sqrt{3}}{72}\mathcal{A}_8 + \mathbf{i}\frac{\sqrt{3}}{9}\mathcal{A}_9 - \mathbf{i}\frac{\sqrt{3}}{9}\mathcal{A}_{10}\right)$$

$$\mathcal{B}_{\mathbf{5940}} = \frac{33}{364}\left(\mathcal{A}_1 + \mathcal{A}_2 + \frac{1}{352}\mathcal{A}_3 + \frac{1}{66}\mathcal{A}_4 + \frac{1}{352}\mathcal{A}_5 + \frac{1}{66}\mathcal{A}_6 - \frac{7}{864}\mathcal{A}_7 - \frac{7}{864}\mathcal{A}_8 + \frac{1}{27}\mathcal{A}_9 + \frac{1}{27}\mathcal{A}_{10}\right)$$

$$\mathcal{B}_{\mathbf{12012}} = \frac{11}{60}\left(\mathcal{A}_1 + \mathcal{A}_2 + \frac{1}{88}\mathcal{A}_3 - \frac{1}{66}\mathcal{A}_4 + \frac{1}{88}\mathcal{A}_5 + \frac{1}{264}\mathcal{A}_6 - \frac{1}{33}\mathcal{A}_9 - \frac{1}{33}\mathcal{A}_{10}\right)$$

$$\mathcal{B}_{\mathbf{14300}} = \frac{55}{252}\left(\mathcal{A}_1 + \mathcal{A}_2 - \frac{5}{352}\mathcal{A}_3 + \frac{1}{330}\mathcal{A}_4 - \frac{5}{352}\mathcal{A}_5 - \frac{1}{132}\mathcal{A}_6 + \frac{1}{288}\mathcal{A}_7 + \frac{1}{288}\mathcal{A}_8 + \frac{1}{99}\mathcal{A}_9 + \frac{1}{99}\mathcal{A}_{10}\right)$$

$$\mathcal{B}_{\mathbf{16016}} = \frac{11}{45}\left(\mathcal{A}_1 - \mathcal{A}_2 + \frac{1}{352}\mathcal{A}_3 - \frac{1}{352}\mathcal{A}_5 - \mathbf{i}\frac{\sqrt{3}}{288}\mathcal{A}_7 + \mathbf{i}\frac{\sqrt{3}}{288}\mathcal{A}_8 + \mathbf{i}\frac{\sqrt{3}}{99}\mathcal{A}_9 - \mathbf{i}\frac{\sqrt{3}}{99}\mathcal{A}_{10}\right)$$

Time **C**: 6 min 3 s. Time **Maple**: 10 s.

98280-dimensional Representation of 3.*Suz*

Rank: 14. Suborbit lengths: $1^3, 891^3, 2816^3, 5940, 19008, 20736^3$.

$$\underline{\mathbf{98280}} \cong \mathbf{1} \oplus \mathbf{78} \oplus \overline{\mathbf{78}} \oplus \mathbf{143} \oplus \mathbf{364} \oplus \mathbf{1365} \oplus \overline{\mathbf{1365}} \oplus \mathbf{4290} \oplus \overline{\mathbf{4290}} \oplus \mathbf{5940} \oplus \mathbf{12012} \oplus \mathbf{14300} \oplus \mathbf{27027} \oplus \overline{\mathbf{27027}}$$

$$\mathcal{B}_{\mathbf{78}} = \frac{1}{1260}\left(\mathcal{A}_1 - \frac{1}{12}\mathcal{A}_2 - \frac{1}{3}\mathcal{A}_4 + \frac{1}{4}\mathcal{A}_6 - \frac{\mathsf{r}}{12}\mathcal{A}_7 - \frac{\mathsf{r}^2}{12}\mathcal{A}_8 + \frac{\mathsf{r}}{4}\mathcal{A}_9 + \frac{\mathsf{r}^2}{4}\mathcal{A}_{10} - \frac{\mathsf{r}^2}{3}\mathcal{A}_{11} - \frac{\mathsf{r}}{3}\mathcal{A}_{12} + \mathsf{r}\mathcal{A}_{13} + \mathsf{r}^2\mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{143}} = \frac{11}{7560}\left(\mathcal{A}_1 - \frac{1}{11}\mathcal{A}_3 + \frac{3}{11}\mathcal{A}_4 - \frac{1}{11}\mathcal{A}_5 + \frac{2}{11}\mathcal{A}_6 + \frac{2}{11}\mathcal{A}_9 + \frac{2}{11}\mathcal{A}_{10} + \frac{3}{11}\mathcal{A}_{11} + \frac{3}{11}\mathcal{A}_{12} + \mathcal{A}_{13} + \mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{364}} = \frac{1}{270}\left(\mathcal{A}_1 - \frac{1}{144}\mathcal{A}_2 + \frac{1}{6}\mathcal{A}_3 - \frac{1}{9}\mathcal{A}_4 - \frac{1}{24}\mathcal{A}_5 + \frac{1}{16}\mathcal{A}_6 - \frac{1}{144}\mathcal{A}_7 - \frac{1}{144}\mathcal{A}_8 + \frac{1}{16}\mathcal{A}_9 + \frac{1}{16}\mathcal{A}_{10} - \frac{1}{9}\mathcal{A}_{11} - \frac{1}{9}\mathcal{A}_{12} + \mathcal{A}_{13} + \mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{1365}} = \frac{1}{72}\left(\mathcal{A}_1 + \frac{1}{144}\mathcal{A}_2 + \frac{1}{9}\mathcal{A}_4 + \frac{1}{16}\mathcal{A}_6 + \frac{\mathsf{r}}{144}\mathcal{A}_7 + \frac{\mathsf{r}^2}{144}\mathcal{A}_8 + \frac{\mathsf{r}}{16}\mathcal{A}_9 + \frac{\mathsf{r}^2}{16}\mathcal{A}_{10} + \frac{\mathsf{r}^2}{9}\mathcal{A}_{11} + \frac{\mathsf{r}}{9}\mathcal{A}_{12} + \mathsf{r}\mathcal{A}_{13} + \mathsf{r}^2\mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{4290}} = \frac{11}{252}\left(\mathcal{A}_1 + \frac{1}{72}\mathcal{A}_2 - \frac{5}{99}\mathcal{A}_4 + \frac{1}{88}\mathcal{A}_6 + \frac{\mathsf{r}}{72}\mathcal{A}_7 + \frac{\mathsf{r}^2}{72}\mathcal{A}_8 + \frac{\mathsf{r}}{88}\mathcal{A}_9 + \frac{\mathsf{r}^2}{88}\mathcal{A}_{10} - \frac{5\mathsf{r}^2}{99}\mathcal{A}_{11} - \frac{5\mathsf{r}}{99}\mathcal{A}_{12} + \mathsf{r}\mathcal{A}_{13} + \mathsf{r}^2\mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{5940}} = \frac{11}{182}\left(\mathcal{A}_1 - \frac{7}{864}\mathcal{A}_2 + \frac{1}{66}\mathcal{A}_3 + \frac{1}{27}\mathcal{A}_4 + \frac{1}{66}\mathcal{A}_5 + \frac{1}{352}\mathcal{A}_6 - \frac{7}{864}\mathcal{A}_7 \right.$$
$$\left. - \frac{7}{864}\mathcal{A}_8 + \frac{1}{352}\mathcal{A}_9 + \frac{1}{352}\mathcal{A}_{10} + \frac{1}{27}\mathcal{A}_{11} + \frac{1}{27}\mathcal{A}_{12} + \mathcal{A}_{13} + \mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{12012}} = \frac{11}{90}\left(\mathcal{A}_1 - \frac{1}{66}\mathcal{A}_3 - \frac{1}{33}\mathcal{A}_4 + \frac{1}{264}\mathcal{A}_5 + \frac{1}{88}\mathcal{A}_6 + \frac{1}{88}\mathcal{A}_9 \right.$$
$$\left. + \frac{1}{88}\mathcal{A}_{10} - \frac{1}{33}\mathcal{A}_{11} - \frac{1}{33}\mathcal{A}_{12} + \mathcal{A}_{13} + \mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{14300}} = \frac{55}{378}\left(\mathcal{A}_1 + \frac{1}{288}\mathcal{A}_2 + \frac{1}{330}\mathcal{A}_3 + \frac{1}{99}\mathcal{A}_4 - \frac{1}{132}\mathcal{A}_5 - \frac{5}{352}\mathcal{A}_6 + \frac{1}{288}\mathcal{A}_7 \right.$$
$$\left. + \frac{1}{288}\mathcal{A}_8 - \frac{5}{352}\mathcal{A}_9 - \frac{5}{352}\mathcal{A}_{10} + \frac{1}{99}\mathcal{A}_{11} + \frac{1}{99}\mathcal{A}_{12} + \mathcal{A}_{13} + \mathcal{A}_{14}\right)$$

$$\mathcal{B}_{\mathbf{27027}} = \frac{11}{40}\left(\mathcal{A}_1 - \frac{1}{432}\mathcal{A}_2 + \frac{1}{297}\mathcal{A}_4 - \frac{1}{176}\mathcal{A}_6 - \frac{\mathsf{r}}{432}\mathcal{A}_7 - \frac{\mathsf{r}^2}{432}\mathcal{A}_8 \right.$$
$$\left. - \frac{\mathsf{r}}{176}\mathcal{A}_9 - \frac{\mathsf{r}^2}{176}\mathcal{A}_{10} + \frac{\mathsf{r}^2}{297}\mathcal{A}_{11} + \frac{\mathsf{r}}{297}\mathcal{A}_{12} + \mathsf{r}\mathcal{A}_{13} + \mathsf{r}^2\mathcal{A}_{14}\right)$$

$\mathsf{r} = \exp(2\pi\mathbf{i}/3) = -\frac{1}{2} + \mathbf{i}\frac{\sqrt{3}}{2}$ is the basic primitive 3rd root of unity.
Time **C**: 57 min 58 s. Time **Maple**: 7 min 41 s.

References

1. Holt, D.F., Eick, B., O'Brien, E.A.: Handbook of Computational Group Theory. Chapman & Hall/CRC, Boca Raton (2005)
2. Parker, R.: The computer calculation of modular characters (the Meat-Axe). In: Atkinson, M.D. (ed.) Computational Group Theory, pp. 267–274. Academic Press, London (1984)
3. Kornyak, V.V.: Quantum models based on finite groups. J. Phys.: Conf. Ser. **965**, 012023 (2018). http://stacks.iop.org/1742-6596/965/i=1/a=012023
4. Kornyak, V.V.: Modeling quantum behavior in the framework of permutation groups. EPJ Web Conf. **173**, 01007 (2018). https://doi.org/10.1051/epjconf/201817301007
5. Cameron, P.J.: Permutation Groups. Cambridge University Press, Cambridge (1999)
6. Bosma, W., Cannon, J., Playoust, C., Steel, A.: Solving Problems with Magma. University of Sydney. http://magma.maths.usyd.edu.au/magma/pdf/examples.pdf
7. Wilson, R., et al.: Atlas of finite group representations. http://brauer.maths.qmul.ac.uk/Atlas/v3