



Numerical Modeling of Crack Propagation in Masonry Structures

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Abstract. Before implementing permanent monitoring systems or reinforcements on a historic structure with identified cracks, it is imperative to understand how cracking patterns may have originated and how they affect the current state of the structure. This work outlines a mixed numerical approach for understanding the causes of masonry cracking and the resulting effects on structural performance. By combining Finite Element Modeling and Distinct Element Modeling, the structural response of an undamaged version of a building can be calculated for a combination of loading scenarios (including dead load, lateral loads, and differential settlement). The results of the loading scenarios can be compared to the current state of the building for a probabilistic understanding of potential causes. This combined approach was used to examine the large cracks running longitudinally on the subterranean walls of the Florence Baptistery. Various combinations of loading scenarios were run for an undamaged model of the wall. Additionally, various geometries of the joints were simulated to understand how the accuracy of input geometries can affect simulation results.

Keywords: Crack propagation · Masonry · Distinct element modeling
Finite element modeling · Florence Baptistery · Foundations

1 Introduction

Masonry, as one of the oldest building materials, is still one of the more complex to analyze. Composed of several discrete units of stones, bricks, and sometimes mortar, it is a heterogeneous, anisotropic, and jointed material. The joints themselves act as planes of weakness in the structure and introduce nonlinearities into the behavior. For this reason, it is imperative that the methods used for analysis are rigorous enough to capture the material complexities of masonry so that the findings and the ensuing preservation plans can be as accurate as possible.

A thorough analysis of masonry construction must consider both the behavior of the masonry blocks (bricks, stones, etc.) and the behavior of the joints (lime mortar, dry joints, etc.). In this paper, detailed micro-modeling is explored. Detailed micro-modeling

models not only the blocks or bricks explicitly, but also the mortar joints themselves. For this approach, the mortar in the joints is modeled as a continuum whereas the interactions between the blocks and the mortar are modeled discontinuously.

This objective of this work is two-fold. The first aim is to apply a mixed numerical method for detailed micro-modeling to the foundations of the Baptistery of San Giovanni to understand the evolution of existing cracks. The second aim is to understand how alterations in geometries input to detailed micro-modeling approaches can alter the results and ensuing preservation plans. The Baptistery di San Giovanni in Florence, Italy was chosen as a case study since (1) the authors had access to accurate 3D models of the existing geometry and (2) there are existing cracks within the foundation wall. This work is a preliminary study where only dead load and settlement have been simulated. In future iterations, additional loading conditions should be examined to get a probabilistic understanding of how the cracks in the foundation wall occurred.

2 Literature Review

2.1 Baptistery di San Giovanni

Dating back to the 3rd century AD, the foundations of the Baptistery di San Giovanni include instances of Roman construction. Building upon this foundation, additions were made throughout the centuries and the structure was officially named the Florence's Baptistery in 1128 [1, 2]. For in-depth discussion about the history and contextualization of the Baptistery see [3–5]. Previous study of the Baptistery includes dynamic response based on ambient vibration tests [6] and scientific dating of the timber structures inside the dome [7]. Additionally, the Baptistery was used as a recent case study for developing an integrative methodology for repeatable data acquisition [2].

The plan of the Baptistery is an octagon topped with an 8-sided dome. Without any central supporting structures, the flow of forces is through the dome, down the side walls of the structure, terminating in the masonry foundations. Figure 1 is a plan view of the building showing the wall and column measurement that was extracted from the terrestrial LiDAR data [2]. For this case study, the portion of the foundation wall which is highlighted in Fig. 1 was modeled and analyzed.

2.2 Brief Background on Numerical Methods

The need for a micro-modeling approach for masonry that could account for the cracking of a structure at its joints was first discussed in [8] where a possible method which could fill this gap was outlined. A multi-surface interface model was developed which could simulate cracking failure in masonry construction such as joint tension and crushing [8]. This approach was later validated for tracking the development of cracks by comparing the results to experimental testing [9]. Further work has been done in the field by studying the homogenized limit analysis of masonry walls [10].

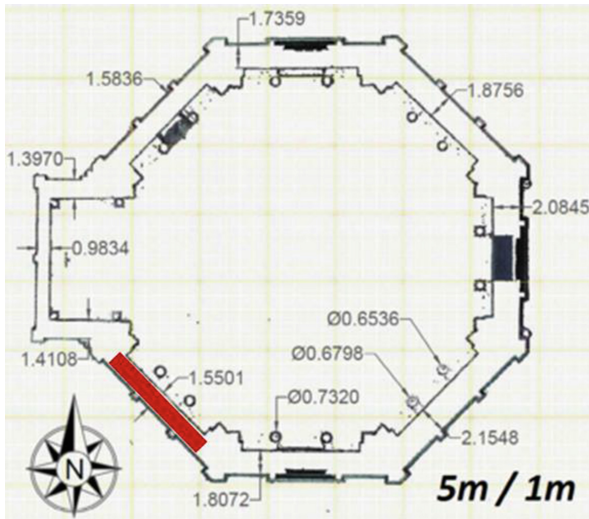


Fig. 1. Plan view of the Baptistery extracted from terrestrial LiDAR data [2]

Around the same time that work was being done on understanding the homogenized limit analysis of masonry walls, a different method for the use of rigid particle contact for analyzing crack propagation was delineated [11]. This work was expanded upon in 2006 when a comprehensive method described how a combination of the finite element method and distinct element modeling could accurately model the evolution of cracks [12]. (This work built upon earlier studies [13] where a combination of the numerical methods was used to describe the fracturing of rocks.) The distinct element method (DEM) is a discontinuous numerical method which uses an explicit solution to Newton's laws of motion [14]. It allows for finite displacements and rotations of discrete blocks (including complete detachment) and recognizes new contacts as the calculation progresses [14]. The highly nonlinear behavior which characterizes masonry joints and crack propagation [14] is more efficiently solved using an explicit solution such as UDEC or 3DEC instead of an implicit method such as DDA which is more efficient for linear or well-behaved systems [15].

Generally, for numerical analysis of masonry, a simplified micro-modeling approach using rigid blocks is sufficient [16, 17]. However, for the purposes of this paper, a detailed micro-modeling approach, which models the joints as a continuous body (soft-contact or deformable finite element zones) and the stones as distinct, rigid elements, was used. Using this combined method, stresses can be calculated in the continuum elements as they occur, however, a finite element mesh between the stones and the mortar is not required—full detachment and cracking can occur at this interface. This method has been applied and validated in previous case studies for historic masonry construction. [18–21]. For further and more in-depth discussion of numerical modeling of masonry construction, see [12, 14, 16].

While a great deal of work has been carried out at the intersection of these two numerical methods, an area which has yet to be explored is the accuracy of an input

model. With the increase in accuracy of photogrammetry and laser scanning data [2], there is a decrease in the difficulty of generating a detailed micro-model for analysis. Additionally, with the current advances in automating the point cloud to polygon or mesh process [2, 22], understanding how accurate the input models need to be is crucial.

3 Methodology

The geometry of the Baptistery di San Giovanni was captured with sub-millimeter accuracy using terrestrial LiDAR. The geometry was then converted into NURBS surfaces (a type of surface in Computer Aided Design) which were separated into mortar and stone. The mortar was imported into 3DEC, a numerical modeling software, as a continuous, finite element mesh while the stones were imported as rigid, distinct blocks. There was no physical testing done of the materials since the stones and mortar were in-situ and destructive testing was not possible. Therefore, the properties of the joints and the stones for the wall were obtained from similar construction typologies in the literature; these can be seen in Table 1.

Table 1. Material properties for Baptistery foundation simulation

Material property	Value
Stone density (kg/m ³)	2713 [23]
Mortar density (kg/m ³)	1540 [24]
Mortar Young’s modulus (N/m ²)	18 × 10 ⁹
Joint normal stiffness (N/m ³)	1 × 10 ⁹ [25]
Joint shear stiffness (N/m ³)	1 × 10 ⁹ [25]
Joint friction angle (degrees)	37 [25]

In the foundation wall of the Baptistery, there is a nearly vertical construction joint (Fig. 2A). It is unknown if that existed in the original Roman wall or if this was created when the site was being converted for later use. Presently, at the site of the persistent joint, the wall is cracked. By modeling the existing geometry of the foundation wall, the results of finite-distinct element method can be used to understand how cracks might have formed. In addition to modeling the wall as it currently stands, the wall was also modeled two other ways. In the first additional model, changes were made to only small a region of the wall which was susceptible to cracking (Fig. 2B). In the second additional model, the wall was idealized to an isodomic pattern (Fig. 2C). The three configurations can be seen in Fig. 2. The red regions in Fig. 2 indicate the major joint which persists throughout the wall and was subtly changed in the second configuration.

Each of the configurations was simulated under the loading of the superstructure. A column rests above this foundation wall and transmits 1/16 of the weight of the dome and the roof to these foundations. In addition to this load, the dead load of the Baptistery’s first level was transmitted directly to the foundation walls. The four upper blocks which can be seen in Fig. 2 were loaded proportionally to their length.

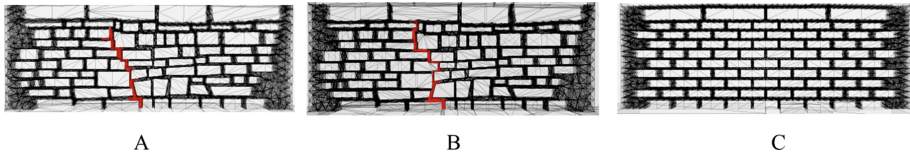


Fig. 2. Varied input geometries for the foundation wall of the Baptistery: (A) Existing pattern of stones and mortar, (B) Wall with small alterations in the pattern of the stones and mortar along the red joint, (C) Isodomic patterning of stones and mortar

4 Results and Discussion

A preliminary investigation into the cause of the foundation crack was carried out by simulating the existing geometry under various loading conditions: (1) dead load of the superstructure, (2) dead load and settlement of the left side of the foundation, and (3) dead load and settlement of the right side of the foundation. These loading conditions were selected for preliminary investigation due to variable settlement across the foundations reported in Hess et al. 2017 [2]. Furthermore, understanding (1) that the foundations of the baptistery are composite and not originally designed for this superstructure and (2) that the foundations are on silty-gravel [2], the settlement is further supported as a viable preliminary study. Figure 3 is a preliminary result which compares the displacement magnitude of the stones for each of the loading cases examined for the existing geometry.

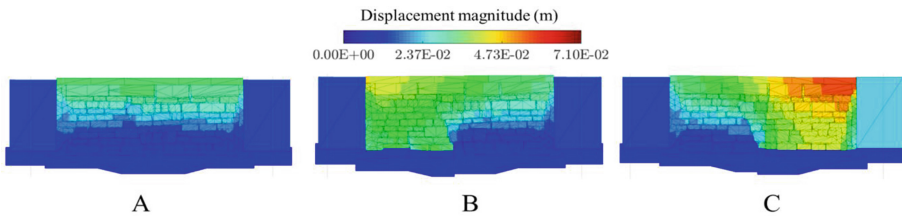


Fig. 3. Simulated displacement magnitude for the existing geometry for each of the three settlement cases: (A) No settlement, (B) Left side settled 0.05 m, (C) Right side settled 0.05 m

As can be seen in Fig. 3A, when there is no settlement, the displacement that occurs is almost zero throughout the wall and it is nearly symmetric. When the wall is settled on the left (Fig. 3B) and on the right (Fig. 3C) there are higher levels of displacement which correlate with the side that was simulated to settle 0.05 m. By just using the plots of displacement magnitude however, there is not much information about how the joints are transmitting the loads throughout the wall. Plots of maximum principal strain did not show indications of cracking, even under circumstances where cracking should have occurred. Therefore, the maximum principal stresses were considered instead to garner understanding about crack growth (Figs. 4A–C).

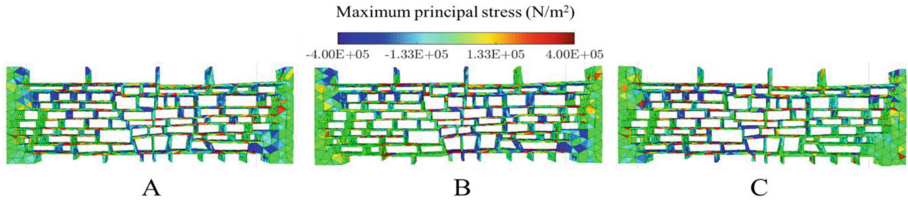


Fig. 4. Simulated maximum principal stress for the existing geometry for each of the three settlement cases: (A) No settlement, (B) Left side settled 0.05 m, (C) Right side settled 0.05 m

Under the dead load of the superstructure, there is a consistent line of tension (blue) in the persistent joint of the existing wall. If the left portion of the wall is settled (Fig. 4B), the persistent joint essentially experiences no stress, and if the right portion of the wall is settled (Fig. 4C), the persistent joint experiences similar stress regions as the case of dead loading. However, it should be noted that in Fig. 4C, the tensile stresses are not as consistent or as large throughout the whole vertical section of the wall like they are under dead loading (0.4 MPa vs 0.2 MPa). Considering the tensional strength of Roman mortar, low-strength lime mortar has a tensile capacity below 0.35 MPa, mid-strength hydraulic lime mortar has a tensile capacity of 0.35–0.55 MPa, and high-strength natural pozzolanic mortars have a tensile capacity greater than 0.6 MPa [26]. Since the dark blue regions on the plots of maximum stress correspond to 0.4 MPa or larger, this is a preliminary indicator that cracking due to tensile failure could have occurred in this joint under either the dead load or settlement of the right side, but not necessarily under settlement of the left side. Further study of how material parameters, settlement depths, and other loading conditions affect the stresses in this region are required.

In addition to studying how cracks could have formed in this foundation wall under various loading conditions, additional geometric configurations were considered to understand how accurate an input 3D model must be for reasonable results in micro-modeling. Figure 5 is a comparison of the existing wall, a wall with small alterations as illustrated originally in Fig. 2B, and an “ideal” wall (Fig. 2C) under the three loading conditions.

As expected, the stress distribution within the idealized wall differs greatly from the existing wall. However, what was notable was how different the stress distributions are between the existing wall and the wall with small alterations. Overall, the wall with small alterations experiences a higher distribution of tensile forces and a reduction in compressive forces in regions neighboring the persistent joint/alterations. If the cases where the left side was settled are compared, while the existing wall experiences no stress in the persistent joint, the wall with small alterations does experience tensile stress in some regions. Therefore, in this preliminary study, the importance of accurate input geometry can be highlighted.

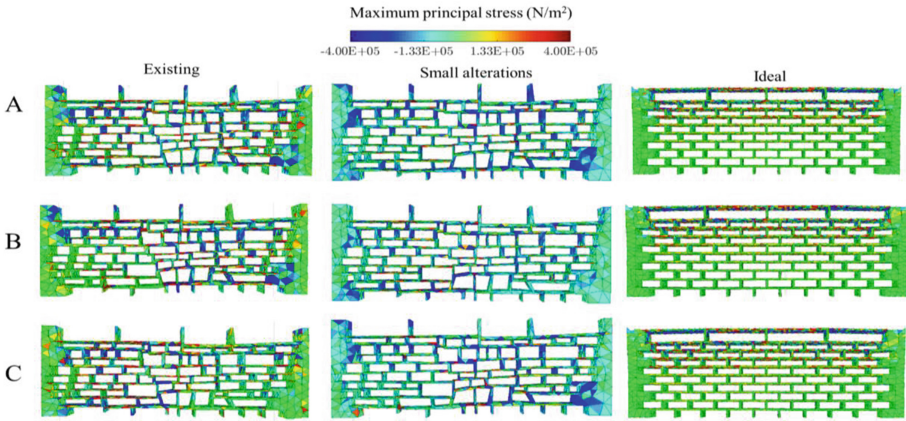


Fig. 5. Simulated maximum principal stress for the each of the geometries for each of the three settlement (A) No settlement, (B) Left side settled 0.05 m, (C) Right side settled 0.05 m

5 Conclusions

This work illustrated how a mixed numerical approach such as Finite-Distinct Element Modeling can be used to capture crack propagation in masonry construction. While the distinct element modeling allowed the stones to fully settle in the simulation since complete detachment could occur, displacement data did not play a crucial role in understanding how the crack in the mortar could have formed. By additionally considering the maximum principal stresses which formed in the mortar joints, initial insights could be gained into the cracking behavior. In future work, the effects of different material parameters, settlement depths, and loading conditions need to be further explored to understand the evolution of cracks within the Baptistery foundation walls. Additionally, this study illustrated the importance of geometry during detailed micro-modeling. By comparing the plots of maximum principal stress across the three different input geometries, it is evident that not only large, but also small alterations can greatly affect the results of a simulation and plans for preservation. In future studies, it would be interesting to understand if the need for high accuracy is limited to detailed micro-modeling and how susceptible other forms such as simplified micro-models can be to small alterations in input geometry.

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