







# Application of Belief Functions to Levee Assessment

Théo Dezert<sup>1,2</sup> , Yannick Fargier<sup>2,3</sup> , Sérgio Palma Lopes<sup>1</sup> ,  
and Philippe Côte<sup>1</sup> 

<sup>1</sup> IFSTTAR, GERS, GeoEND, 44344 Bouguenais, France  
{theo.dezert,sergio.lopes,philippe.cote}@ifsttar.fr

<sup>2</sup> Cerema Direction territoriale Normandie-Centre, 41000 Blois, France

<sup>3</sup> Univ Lyon, IFSTTAR, GERS, RRO, 69675 Bron, France  
yannick.fargier@ifsttar.fr

**Abstract.** We propose the use of Smets and PCR5 rules to merge artificial geophysical and geotechnical data, as part of fluvial levee assessment. It highlights the ability to characterize the presence of interfaces and a geological anomaly.

**Keywords:** Levee assessment · Geophysics · Geotechnical testing  
Belief functions · Data fusion

## 1 Introduction

Fluvial levees are manmade structures built for flood protection. They are considered as hazardous structures that can fail and lead to disastrous consequences such as human or material loss and economic disasters. There are globally acknowledged methodologies for levee assessment that include geophysical and geotechnical investigation methods [1]. Geophysical methods are non-intrusive and provide physical information on large volumes of subsoil with high output and potentially significant related uncertainties. These associated uncertainties are notably due to the indirect and integrating aspects of the methods and to the resolution of inverse problems. Geotechnical investigation methods are intrusive and provide more punctual and more accurate information. An important issue of assessment of levees is to be able to combine geophysical and geotechnical data taking into consideration their respective associated uncertainties, imprecisions and spatial distributions. In this work, we suggest the use of Belief Functions (BFs) and combination rules to merge artificial geophysical (electrical resistivities) and geotechnical (cone bearing) data to display their ability to discriminate three sets of soils. We assume that the reader is familiar with the BFs introduced by Shafer in [2]. The use of BFs requires: (1) to select a common frame of discernment (FoD) of the considered problem, (2) to determine the masses of belief or Basic Belief Assignments (BBAs) from available data (geophysical and geotechnical) and (3) to choose a rule of combination.

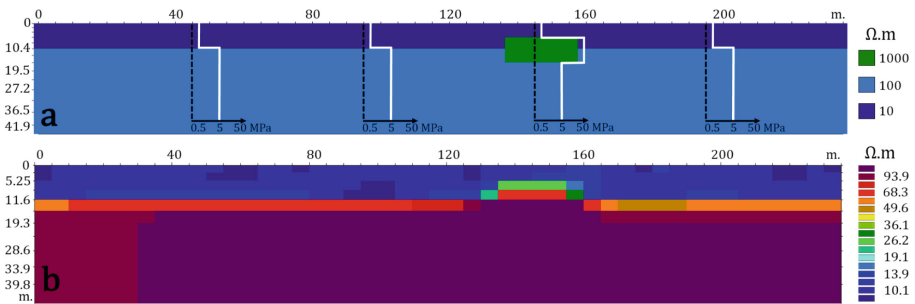
---

Supported by the Pays de la Loire Region.

## 2 FoD and BBAs Construction

For the addressed levee problematic, we consider three classes of distinct soils  $\theta_1$ ,  $\theta_2$  and  $\theta_3$ . Because the FoD,  $\Theta$ , must consist of a set of exhaustive and exclusive hypotheses, we will be using a fourth class  $\theta_4$  to cover the physical characteristics not included in the three first sets. The FoD is common to both information sources. We use  $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$ . The construction of the BBAs for each data source consists in assigning each data type to  $\Theta$ .

**BBA Construction From Geophysical Data:** since the electrical resistivity (ER) tomography method is one of the most employed, we propose the use of ER as geophysical data. We consider two soil layers: an upper conductive layer ( $10 \Omega.m$ ) standing for clays [3] and a subjacent and more resistive one ( $10^2 \Omega.m$ ) standing for silts starting at 10.4 m depth. A very resistive anomaly ( $10^3 \Omega.m$ ) standing for a sandy lens of about 10.5 m high and 21.25 m wide, is finally positioned between these two first media. We then associate ER classes to specific soils (split into ranges of ER) to  $\Theta$ :  $\theta_1 = [5, 20]$ ,  $\theta_2 = [50, 2 \cdot 10^2]$ ,  $\theta_3 = [5 \cdot 10^2, 2 \cdot 10^3]$  and  $\theta_4 = [1, 5 \cup ]20, 50 \cup ]2 \cdot 10^2, 5 \cdot 10^2 \cup ]2 \cdot 10^3, 10^4]$ . We use Res2Dmod free software [4] to simulate noised data acquisition from a chosen resistivity model (Fig. 1a) and then use the Res2Dinv software [5] to obtain the inverted ER section as one would get from the processing of survey data (Fig. 1b). The distinction between clays and silts can easily be made while the discrimination of the anomaly is not obvious. We finally use the Res2dinv discretization grid for the BBA  $m_1(\cdot)$  corresponding to each event of  $2^\Theta$ . The values of the masses are set using the Wasserstein distances between an inverted ER value  $\pm$  its uncertainty issued from Res2dinv and the interval corresponding to each event, so as each cell of the grid gets a normalized BBA.



**Fig. 1.** 2D section of subsoil displaying (a) true ER with boreholes position in dashed line and associated cone bearing values in white and (b) inverted ER.

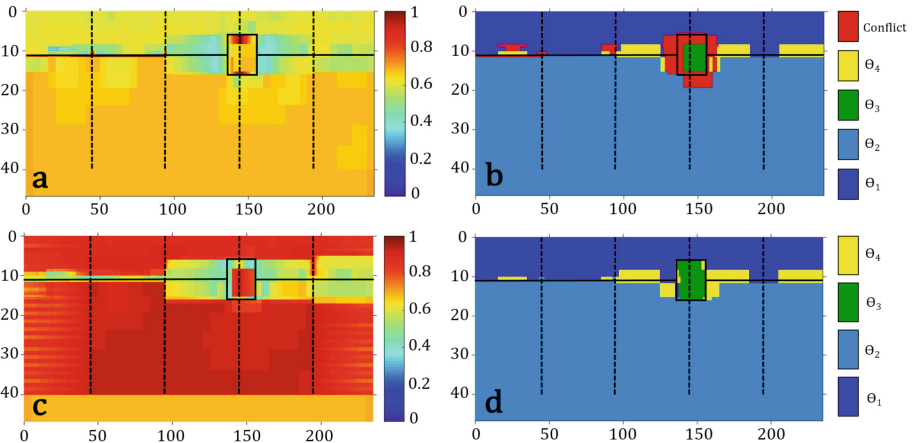
**BBA Construction From Geotechnical Data:** as geotechnical data, we use artificial cone bearing values (expressed in  $MPa$ ). These information could be obtained from a cone penetrometer test investigation campaign. We simulate a

data acquisition from 4 boreholes with an interspacing of 50 m (as recommended in [6]), drilled to 40 m depth with an acquisition every meter (Fig. 1a). One of the boreholes is positioned so that it goes through the resistive anomaly. We consider the following assignment of intervals of cone bearing values to  $\Theta$ :  $\theta_1 = [0.3, 0.7]$ ,  $\theta_2 = [3, 7]$ ,  $\theta_3 = [30, 70]$  and  $\theta_4 = [10^{-2}, 0.3[\cup]0.7, 3[\cup]7, 30[\cup]70, 10^2]$  that can be associated to specific soils [7], such as clays for low values, silty soils for intermediate values and sands for higher ones. We assume a belief mass equal to 1 in the borehole and impose a lateral decrease of the trust in the data. The geotechnical grid depends on the boreholes distance and on the acquisition rate. Thus, for each cell, a second BBA  $m_2(\cdot)$  is fixed, entering in the fusion process.

### 3 BBAs Combination and Preliminary Results

We propose a fusion mesh containing all the meshes from the geotechnical and geophysical grids in order to avoid the unnecessary data alteration due to interpolations. The merging process is carried out on two meshes of same dimension. The data fusion consists in combining  $m_1(\cdot)$  and  $m_2(\cdot)$  assigned to each cell of the grid. Many rules of BBA combination have been proposed. Here we present only two of them: Smets' rule [8] and the Proportional Conflict Redistribution rule no. 5 (PCR5) [9] allowing the redistribution of all partial conflicts proportionately to the masses involved in them. We use PCR5 since we combine only two sources of evidence thus PCR6 is equivalent to PCR5 rule [9] in this case. Smets' rule (conjunctive rule under an open-world assumption) allows the quantification of the classical conflict level represented by (Eq. 1):

$$m_{12}(\emptyset) = \sum_{X_1, X_2 \subseteq \Theta | X_1 \cap X_2 = \emptyset} m_1(X_1)m_2(X_2) \quad (1)$$



**Fig. 2.** Data fusion with Smets' combination rule (a, b) and with PCR5 (c, d). (a) and (c) represent the BBAs associated to the events with the highest mass, presented in (b) and (d) respectively. The black lines stand for the interfaces fixed in the ER model (Fig. 1a) while the dashed lines stand for the boreholes position.

Thanks to it, we are able to point out the conflictual zones around the horizontal interfaces and the resistive anomaly (Fig. 2b). The fusion, following PCR5 (closed world assumption)[9] (Fig. 2d) is very close to the true model we imposed (Fig. 1a), giving a clearer view of the interface and of the vertical and horizontal extension of the resistive anomaly compared to the image given by the inverted ER (Fig. 1b). As a decision-making support, we choose to represent the events having the highest belief masses (Fig. 2b and d) and their related degrees of belief (Fig. 2a and c).

## 4 Conclusion

The use of BFs for investigation of levees is promising. It is able to highlight the presence of an interface between two media much more precisely than the geophysical method alone. Furthermore, it enables the reliable estimation of the complete extension of an anomaly with high ER and cone bearing values. Without normalization, Smets' combination rule easily spotlights the conflicting zones. Such information could be precious during an investigation campaign, indicating areas where survey has to be reinforced. In future work, we will focus on parametric studies to choose the best decreasing functions for the lateral propagation of the geotechnical information. Finally, we will test our algorithm using real data acquired on a scale model and on a levee.

## References

1. Fauchard, C., Mériaux, P.: Geophysical and geotechnical methods for diagnosing flood protection dikes: guide for implementation and interpretation. Quae (2007)
2. Shafer, G.: A Mathematical Theory of Evidence, Princeton University Press, Princeton (1976)
3. Palacky, G., West, G.F.: Electromagnetic methods in applied geophysics. Resistivity Characteristics of Geologic Targets, pp. 52–129 (1987)
4. Loke, M.H.: RES2DMOD ver. 3.01: Rapid 2D resistivity forward modelling using the finite difference and finite-element methods. Software manual (2002)
5. Loke, M.H., Barker, R.D.: Rapid least-squares inversion of apparent resistivity pseudosections by a quasi-Newton method. Geophys. Prospect. **44**(1), 131–152 (1996)
6. Phoon, K.K., Kulhawy, F.H.: Characterization of geotechnical variability. Can. Geotech. J. **36**(4), 612–624 (1999)
7. Robertson, P.K.: Soil classification using the cone penetration test. Can. Geotech. J. **27**(1), 151–158 (1990)
8. Smets, P.: The combination of evidence in the transferable belief model. IEEE Trans. Pattern Anal. Mach. Intell. **12**(5), 447–458 (1990)
9. Smarandache, F., Dezert, J.: Advances and applications of DSMT for information fusion- Collected works - ARP, volume 3 (2009)