A Conflict Analysis Model Based on Three-Way Decisions

Yan Fan[1], Jianjun Qi[2], and Ling Wei[1](✉)

[1] School of Mathematics, Northwest University, Xi'an 710127, People's Republic of China
fyan0411@163.com, wl@nwu.edu.cn

[2] School of Computer Science and Technology, Xidian University, Xi'an 710071, People's Republic of China
qijj@mail.xidian.edu.cn

Abstract. In decision-making, three-way decisions play an essential role and have been widely used in many fields and disciplines. In this paper, we propose a conflict analysis model based on three-way decisions, so as to explore the inter structure of conflict situation. Firstly, by adopting including degree, two pairs of evaluation functions are defined specifically based on the conflict situation. After that, with restricting the evaluations, three regions of agent set and issue set can be obtained. Comparing with existing conflict analysis models, this trisection model is more efficient, practical and pragmatical. Finally, the trisection of agent set and issue set could be used to ascertain sub-optimal feasible consensus strategies, and determine the scope of the kernel issues in conflict situation, respectively.

Keywords: Three-way decisions · Conflict analysis · Including degree

1 Introduction

Conflict, as an essential characteristic of human life, exists in a wide variety of social problems. To make proper decisions in conflict situations, conflict study is of significance both in theory and practice. Conflict analysis, purposed to explore the structure of conflict, has attracted enormous attention [1–13]. For example, Pawlak initially proposed discernibility matrix and distance functions based on rough set [2,3], then presented an approach dividing the agent set into several coalitions. Deja [4,5] subsequently extended Pawlak conflict analysis model through adding three basic questions:

(1) What are the intrinsic reasons for the conflict?
(2) How can a feasible consensus strategy be found?
(3) Is it possible to satisfy all the agents?

To tackle the problems mentioned by Deja, Sun et al. [6,7] developed a rough set-based conflict analysis model. However, there are still many problems should

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be studied further, such as the more feasible strategy. Ali et al. [8] provided a new conflict analysis model based on soft preference relation and soft dominance relation, disclosing the information more efficiently. Nevertheless, this model paid more attention to domination relations between agents, so that the relations between issues and agents were ignored. That would end up with missing more benefit strategies.

In conflict situations, the main problem is how find an efficient way to model uncertainty in conflict situations [4,5]. For a feasible consensus strategy, the way of model uncertainty is to ascertain the agents' attitudes towards any strategy: agreed, opposed or neutral.

The notion of three-way decisions was proposed and used to interpret three regions in rough set. More specifically, positive, negative and boundary region are viewed respectively as acceptance, rejection, and non-commitment in a ternary classification [14–17]. The intrinsic ideas of three-way decisions has been widely applied to many fields, for instance, medical decision-making [18], management sciences [19], and peering review process [20].

The essential ideas of three-way decisions are described in terms of a ternary classification according to the evaluations of a set of criteria [17]. This kind of classification is, to some extent, consensus with the trisection of agent set based on every agent's attitude to a specific strategy, and the trisection of issue set based on agent group's whole attitude to every single issue. Therefore, our main research are as follows. On the one hand, we define a pair of evaluation functions to estimate the extent to which agent u accepts or opposes a strategy Y. Then, the three regions of agents could be determined through restricting the value of the evaluation function subsequently; On the other hand, another pair of evaluation functions is also defined to estimate the extent to which an issue a is accepted or opposed by the whole agent group X. Then, three regions of issues could be determined as well. Finally, we can find that this model is more appropriate than existing conflict analysis models.

Basic notions of Sun's conflict analysis model and three-way decisions are recalled in Sect. 2. Then, the conflict analysis model based on three-way decisions is proposed in Sect. 3. Finally, we conclude our researches and give further research directions in Sect. 4.

2 Preliminaries

Conflict situation consists of agents and their attitudes to some issues. In Pawlak's model, conflict situation can be presented as a pair (U, V), where $U = \{ u_1, ..., u_m \}$ is the universe of agents, and $V = \{ a_1, ..., a_n \}$ is the universe of issues. The attitude of agent u to an issue a can be interpreted as a function $a : U \rightarrow V_a$, where $V_a = \{+, -, 0\}$. $a(u) = +$ represents agent u agrees with issue a, $a(u) = -$ means agent u objects to issue a, and $a(u) = 0$ means agent u is neutral towards issue a. An example of conflict situation is presented in Table 1. The relationship of each agent u_i to a specific issue a_j could be clearly shown in this table.

Table 1. The conflict situation of the Middle East conflict.

	a_1	a_2	a_3	a_4	a_5
u_1	−	+	+	+	+
u_2	+	0	−	−	−
u_3	+	−	−	−	0
u_4	0	−	−	0	−
u_5	+	−	−	−	−
u_6	0	+	−	0	+

Sun et al. [7] focused on the first two questions proposed by Deja [4,5], "What are the intrinsic conflict reasons" and "How can a feasible consensus strategy be found". Inspired by Pawlak's model, they tried to introduce a new analysing method of conflict situation based on rough set theory over two universes.

According to [7], for any subset $Y \subseteq V$, Y is called a strategy. Subsequently, Y is called a feasible consensus strategy if it satisfies all agents. A sub-optimal feasible consensus strategy Y satisfies the agents as many as possible. The feasible consensus strategy does not exist usually since there are different opinions for every issue. Thus, it is more meaningful to determine sub-optimal feasible consensus strategies. In order to find a sub-optimal feasible consensus strategy, the most important thing is to determine the attitudes of all agents to every strategy. On the basis of Pawlak rough set, Sun et al. [7] described an agent's attitude in the conflict situation as follows:

Let $f = \{f^+, f^-\}$ be the set valued mappings from U to $P(V)$, where

$$f^+ : U \to P(V),\ f^+(u) = \{a \in V | a(u) = +\},\ \forall u \in U,$$
$$f^- : U \to P(V),\ f^-(u) = \{a \in V | a(u) = -\},\ \forall u \in U.$$

The image of f^+ represents the subset of issue universe V which satisfy agent u. The image of f^- represents the subset of issue universe V which are opposed by agent u.

For any strategy $Y \subseteq V$, the lower and upper approximations are:

$$\underline{apr}^+_f(Y) = \{u \in U | f^+(u) \subseteq Y\},\ \overline{apr}^+_f(Y) = \{u \in U | f^+(u) \cap Y \neq \varnothing\};$$
$$\underline{apr}^-_f(Y) = \{u \in U | f^-(u) \subseteq Y\},\ \overline{apr}^-_f(Y) = \{u \in U | f^-(u) \cap Y \neq \varnothing\}.$$

Then the agreement subset, disagreement subset, neutral subset for the strategy Y are denoted as follows:

Agreement subset: $R^+_f(Y) = \underline{apr}^+_f(Y) - \underline{apr}^-_f(Y)$;
Disagreement subset: $R^-_f(Y) = \underline{apr}^-_f(Y) - \underline{apr}^+_f(Y)$;
Neutral subset: $R^0_f(Y) = U - R^+_f(Y) \cup R^-_f(Y)$.

Thus, a sub-optimal feasible consensus strategy Y can be found through selecting the maximum cardinality of the agreement subset $R^+_f(Y)$.

Example 1. We consider the Middle East conflict in Table 1. Given strategy $Y = \{a_2, a_3, a_5\} \subseteq V$, and then $\underline{apr}^+_f(Y) = \{u_6\}$, $\overline{apr}^+_f(Y) = \{u_1, u_6\}$, $\underline{apr}^-_f(Y) = \{u_4, u_6\}$, $\overline{apr}^-_f(Y) = \{u_2, u_3, u_4, u_5, u_6\}$. According to Sun et al. [7], there is no agent agrees with the strategy Y, since $R^+_f(Y) = \varnothing$. Additionally, the agents in $R^-_f(Y) = \{u_4\}$ oppose the strategy Y, and all the agents in $R^0_f(Y) = \{u_1, u_2, u_3, u_5, u_6\}$ hold neutral attitude.

The following facts can be observed: (1) For agents u_4 and u_5, they agree on strategy Y, but they are grouped into different coalitions. (2) In Table 1, the issues in Y are all agreed by agent u_1, but according to the above method, agent u_1 is considered neutral about strategy Y. Both of the two aspects are not very suitable for assuring the agents' attitude to a specific strategy in practice. Moreover, more feasible strategy may be missed. Actually, the reason for these confusions is the inconformity between the approximation in rough set based on two universes and semantics of the three subsets of agents for a strategy. Therefore, we need more efficient conflict analysis model to determine the structure in conflict situation.

The theory of three-way decisions can be used to interpret the regions of acceptance, rejection, and non-commitment in a ternary classification. This theory is applicable to divide agent universe U into three subsets according to their attitude to a strategy. Three kinds of evaluation-based three-way decisions are proposed in [17], and then the corresponding three-way decision models are introduced and studied. Among these three kinds of models, the first one as follows is more consensus to the semantics of determining the three subsets in conflict analysis.

Definition 1 [17]**.** *Suppose U is a finite nonempty set and $(L_a, \preceq_a)$, $(L_r, \preceq_r)$ are two posets. A pair of functions $v_a : U \to L_a$ and $v_r : U \to L_r$ is called an acceptance evaluation and a rejection evaluation, respectively. For $u \in U$, $v_a(u)$ and $v_r(u)$ are called the acceptance and rejection values of u, respectively.*

In conflict situation (U, V), the acceptance value $v_a(u)$ and rejection value $v_r(u)$ can be constructed by evaluating the extent to which agent u agrees with or disagrees with strategy Y, respectively. What's more, if the agent u_1 accepts strategy Y, $v_a(u_1)$ must be in a certain subset of L_a representing the acceptance region of L_a. Similarly, $v_a(u_2)$ included in the rejection region of L_r means agent u_2 reject strategy Y to a large extent. Therefore, L_a and L_r should be defined. These values are called designated values for acceptance and designated values for rejection, respectively. Based on the two sets of designated values, one can easily obtain three regions for three-way decisions.

Definition 2 [17]**.** *Let $\varnothing \neq L^+_a \subseteq L_a$ be a subset of L_a called the designated values for acceptance, and $\varnothing \neq L^-_r \subseteq L_r$ be a subset of L_r called the designated* *values for rejection. The positive, negative, and boundary regions of three-way decisions induced by* (v_a, v_r) *are defined by:*

$$POS_{(L_a^+, L_r^-)}(v_a; v_r) = \{u \in U | v_a(u) \in L_a^+ \wedge v_r(u) \notin L_r^-\},$$

$$NEG_{(L_a^+, L_r^-)}(v_a; v_r) = \{u \in U | v_a(u) \notin L_a^+ \wedge v_r(u) \in L_r^-\},$$

$$\begin{aligned} BND_{(L_a^+, L_r^-)}(v_a; v_r) &= (POS_{(L_a^+, L_r^-)}(v_a, v_r) \cup NEG_{(L_a^+, L_r^-)}(v_a, v_r))^c \\ &= \{u \in U | (v_a(u) \notin L_a^+ \wedge v_r(u) \notin L_r^-) \vee (v_a(u) \in L_a^+ \wedge v_r(u) \in L_r^-)\}. \end{aligned}$$

From the above analysis, we know that there are two essential problems. One is how to evaluate the extent to which agent u agrees and disagrees with a certain strategy Y, and the other is how to define the designated values for acceptance and rejection.

3 Conflict Analysis Model Based on Three-Way Decisions

This section mainly introduces an conflict analysis model on the basis of three-way decisions, which is considered from two perspectives. Based on three-way decisions, Sect. 3.1 shows how to obtain three subsets of agents subjecting to each agent's attitude to a specific strategy, which helps to determine the sub-optimal feasible consensus strategy. Similarly, Sect. 3.2 proposes an approach to get trisection of the issue set related to the unitary attitude of an agent group to a specific strategy. The outcome helps to determine the scope of the core issues causing conflict. Furthermore, compared with Sun's conflict analysis model, the superiorities of this model are showed as well.

3.1 Trisection of Agent Set Based on Each Agent's Attitude to a Specific Strategy

To trisect the agent set, we just have to tackle the problems in the last paragraph of Sect. 2. That is how to evaluate the extent to which agent u agrees and disagrees with strategy Y, and how to define the designated values for acceptance and rejection. Including degree can be adopted to estimate the extent to which agent u accepts or opposes strategy Y. Then the designated values can be determined through restricting the including degree.

Definition 3 [21]**.** *Let* $(L, \leq)$ *be a partially ordered set. If for any* $X, Y \subseteq L$*, there is a real number* $D(Y/X)$ *with the following properties:*

(1) $0 \leq D(Y/X) \leq 1$
(2) $X \subseteq Y$ *implies* $D(Y/X) = 1$
(3) $X \subseteq Y \subseteq Z$ *implies* $D(X/Z) \leq D(X/Y)$

then D *is called an including degree on* L*.*

The including degree $D(Y/X)$ represents the extent to which set Y contains the set X. It is obvious that $D(Y/X) = \frac{|X \cap Y|}{|X|}$ is an including degree.

Definition 4. *Let (U, V) be a conflict situation. $([0,1], \leq)$ a totally ordered set. $Y \subseteq V$, Y is a strategy. A pair of evaluation functions v_a and v_r are defined as:*

$$v_a : U \times P(V) \to [0,1], v_a(u, Y) = D(f^+(u)|Y),$$

$$v_r : U \times P(V) \to [0,1], v_r(u, Y) = D(f^-(u)|Y).$$

v_a is called agent acceptance evaluation function, and $v_a(u, Y)$ evaluates the extent to which agent u accepts strategy Y; v_r is called agent rejection evaluation function, and $v_r(u, Y)$ evaluates the extent to which agent u rejects strategy Y, where, $D(f^+(u)|Y)$ and $D(f^-(u)|Y)$ are defined as

$$D(f^+(u)|Y) = \frac{|f^+(u) \cap Y|}{|Y|}, \ D(f^-(u)|Y) = \frac{|f^-(u) \cap Y|}{|Y|}.$$

Property 1. Let (U, V) be a conflict situation. $\forall u \in U, Y \subseteq V$, we have $v_a(u, Y) + v_r(u, Y) \leq 1$.

Proof. It is obvious that $f^+(u) \cap f^-(u) = \varnothing$. Then $(f^+(u) \cap Y) \cap (f^-(u) \cap Y) = \varnothing$, so $|f^+(u) \cap Y| + |f^-(u) \cap Y| \leq |Y|$. Therefore, $\frac{|f^+(u) \cap Y|}{|Y|} + \frac{|f^-(u) \cap Y|}{|Y|} \leq 1$. That is, $v_a(u, Y) + v_r(u, Y) \leq 1$.

Example 2. Consider the Middle East conflict presented in Table 1. For strategy $Y = \{a_2, a_3, a_5\} \subseteq V$, we obtain the following results:

Table 2. Evaluations for the Middle East conflict.

U	u_1	u_2	u_3	u_4	u_5	u_6
$v_a(u_i, Y)$	1	0	0	0	0	$\frac{2}{3}$
$v_r(u_i, Y)$	0	$\frac{2}{3}$	$\frac{2}{3}$	1	1	$\frac{1}{3}$

From Table 2, we know that the extent to which agent u_6 accepts strategy Y is $\frac{2}{3}$, and the extent to which agent u_6 opposes strategy Y is $\frac{1}{3}$ and so on.

Let $\alpha \geq 0.5$, $\beta \geq 0.5$, and then $(\alpha, 1]$ represent the designated values for acceptance, which are used to restrict the extent to which an agent accepts strategy Y in the agreement subset. $(\beta, 1]$ represent the designated values for rejection, which are used to restrict the extent to which an agent rejects the strategy Y in the disagreement subset. On the basis of two sets of designated values, we can easily obtain three regions of agents based on their attitudes to strategy Y.

Definition 5. *Let* (U, A) *be a conflict situation,* $(\alpha, 1]$ *the designated values for acceptance,* $(\beta, 1]$ *the designated values for rejection,* $Y \subseteq V$ *a strategy,* $v_a(u, Y) = D(f^+(u)|Y)$ *and* $v_r(u, Y) = D(f^-(u)|Y)$*. Then, we denote:*

$$AS_{\alpha,\beta}(Y) = \{u \in U | v_a(u, Y) \in (\alpha, 1] \wedge v_r(u, Y) \notin (\beta, 1]\},$$

$$DS_{\alpha,\beta}(Y) = \{u \in U | v_a(u, Y) \notin (\alpha, 1] \wedge v_r(u, Y) \in (\beta, 1]\},$$

$$NS_{\alpha,\beta}(Y) = U - AS_{\alpha,\beta}(Y) \cup DS_{\alpha,\beta}(Y).$$

We call $AS_{\alpha,\beta}(Y)$ *the* $(\alpha, \beta)-$*agreement subset of strategy* Y*,* $DS_{\alpha,\beta}(Y)$ *the* $(\alpha, \beta)-$*disagreement subset of strategy* Y*, and* $NS_{\alpha,\beta}(Y)$ *the* $(\alpha, \beta)-$*neutral subset of strategy* Y*.*

Remark. It should be noted that when $\alpha \geq 0.5$ and $\beta \geq 0.5$, we have $v_a(u, Y) \in (\alpha, 1] \Longleftrightarrow v_r(u, Y) \notin (\beta, 1]$, and $v_a(u, Y) \notin (\alpha, 1] \Longleftrightarrow v_r(u, Y) \in (\beta, 1]$. It can be proved easily through Property 1, $v_a(u, Y) + v_r(u, Y) \leq 1$. Therefore, the definition of $AS_{\alpha,\beta}(Y)$ and $DS_{\alpha,\beta}(Y)$ can be simplified as

$$AS_\alpha(Y) = \{u \in U | v_a(u) \in (\alpha, 1]\},$$

$$DS_\beta(Y) = \{u \in U | v_r(u) \in (\beta, 1]\}.$$

Similarly, $AS_\alpha(Y)$ is named the $\alpha-$agreement subset of strategy Y, and $DS_\beta(Y)$ is called the $\beta-$disagreement subset of strategy Y. Therefore, the agents in $AS_\alpha(Y)$ agree with strategy Y to designated value α, the agents in $DS_\beta(Y)$ object to strategy Y to designated value β, and the agents in $NS_{\alpha,\beta}(Y)$ have neutral attitude for strategy Y to designated values (α, β).

Proposition 1. *Let* (U, A) *be a conflict situation,* $Y \subseteq V$ *a strategy.* $\alpha \geq 0.5$*, and* $\beta \geq 0.5$*. The following relations hold:* $AS_\alpha(Y) \cap DS_\beta(Y) = \varnothing$*,* $AS_\alpha(Y) \cap NS_{\alpha,\beta}(Y) = \varnothing$*, and* $DS_\beta(Y) \cap NS_{\alpha,\beta}(Y) = \varnothing$*.*

Proof. For any $u \in AS_\alpha(Y)$, we have $v_a(u, Y) > \alpha \geq 0.5$. Since $v_a(u, Y) + v_r(u, Y) \leq 1$, then $v_r(u, Y) \leq 1 - v_a(u, Y) < 1 - \alpha \leq 0.5$, so $v_r(u, Y) \not> 0.5$, which means $u \notin DS_\beta(Y)$. Thus, we obtain $AS_\alpha(Y) \cap DS_\beta(Y) = \varnothing$. According to the definition of neutral subset $NS_{\alpha,\beta}(Y)$, we have $AS_\alpha(Y) \cap NS_{\alpha,\beta}(Y) = \varnothing$ and $DS_\beta(Y) \cap NS_{\alpha,\beta}(Y) = \varnothing$. Therefore, the three regions are pair-wise disjoint.

For simplicity, we denote $I_1 = AS_\alpha(Y)$, $I_2 = DS_\beta(Y)$, and $I_3 = NS_{\alpha,\beta}(Y)$.

Proposition 2. *Let* (U, A) *be a conflict situation,* $Y \subseteq V$ *a strategy.* $\forall u_1, u_2 \in U$*, if* $f^+(u_1) \cap Y = f^+(u_2) \cap Y$ *and* $f^-(u_1) \cap Y = f^-(u_2) \cap Y$*, then* $u_1 \in I_t \Longleftrightarrow u_2 \in I_t$*,* $t = \{1, 2, 3\}$*.*

Proof. If $f^+(u_1) \cap Y = f^+(u_2) \cap Y$, and $f^-(u_1) \cap Y = f^-(u_2) \cap Y$, then $v_a(u_1, Y) = v_a(u_2, Y)$ and $v_r(u_1, Y) = v_r(u_2, Y)$. Furthermore, we have that

$$u_1 \in I_1 \Longleftrightarrow v_a(u_1, Y) > \alpha \Longleftrightarrow v_a(u_2, Y) > \alpha \Longleftrightarrow u_2 \in I_1;$$

$$u_1 \in I_2 \Longleftrightarrow v_r(u_1, Y) > \beta \Longleftrightarrow v_r(u_2, Y) > \beta \Longleftrightarrow u_2 \in I_2;$$

$$u_1 \in I_3 \Longleftrightarrow v_a(u_1, Y) < \alpha \& v_r(u_1, Y) < \beta$$
$$\Longleftrightarrow v_a(u_2, Y) < \alpha \& v_r(u_2, Y) < \beta \Longleftrightarrow u_2 \in I_3.$$

The proposition is proved.

This proposition shows that if two agents of universe U have the same attitude to strategy Y, they will be grouped together. That is to say, in the terms of determining agreement subset, disagreement subset and neutral subset for strategy Y, the model proposed in this paper improves the first inconformity in Sun's model, which is presented in Example 1.

Proposition 3. *Let (U, A) be a conflict situation, $Y \subseteq V$ a strategy. $\forall u_1, u_2 \in U$, if $v_a(u_1, Y) \geq v_a(u_2, Y)$, and $u_2 \in AS_\alpha(Y)$, then we have $u_1 \in AS_\alpha(Y)$; Similarly, if $v_r(u_1, Y) \geq v_r(u_2, Y)$, and $u_2 \in DS_\beta(Y)$, then we have $u_1 \in DS_\beta(Y)$.*

Proof. If $v_a(u_1, Y) \geq v_a(u_2, Y)$ and $u_2 \in AS_\alpha(Y)$, then we have $v_a(u_1, Y) > \alpha$, which means $u_1 \in AS_\alpha(Y)$. Similarly, If $v_r(u_1, Y) \geq v_r(u_2, Y)$ and $u_2 \in DS_\beta(Y)$, then we conclude $v_r(u_1, Y) > \beta$, which means $u_1 \in DS_\beta(Y)$.

From above we can know that if agent u agrees with all issues of strategy Y, then u would be grouped into the $\alpha-$agreement subset. This conclusion is tenable for any $\alpha \in [0.5, 1]$. Similarly, the model proposed in this paper improves the second inconformity in Sun's model, which is presented in Example 1. Therefore, compared with the outcomes of Sun's conflict analysis model in Sect. 2, the approach to determine the three regions of agent set proposed in this paper is more appropriate.

Example 3 (continued from Example 2). Consider the Middle East conflict presented in Table 1. For strategy $Y = \{a_2, a_3, a_5\}$, let $\alpha = 0.6$, $\beta = 0.6$, and we obtain the following results: $AS_{0.6}(Y) = \{u_1, u_6\}$, $DS_{0.6}(Y) = \{u_2, u_3, u_4, u_5\}$ and $NS_{0.6,0.6}(Y) = \varnothing$.

Therefore, the agents in $AS_{0.6}(Y) = \{u_1, u_6\}$ agree with strategy Y to designated value 0.6, the agents in $DS_{0.6}(Y) = \{u_2, u_3, u_4, u_5\}$ object to strategy Y to designated value 0.6, and no agent has neutral attitude for strategy Y to designated values (0.6,0.6). Furthermore, the agents u_4 and u_5 are grouped together, besides, u_1 is assigned to the 0.6-agreement subset because of its full agreements with the issues in Y.

In this section, we proposed an effective approach to determine three regions of agents for any strategy Y. The result can be used to resolve some problems, such as finding the sub-optimal feasible consensus strategy by selecting the maximum cardinality of the $\alpha-$agreement subset [7].

3.2 Trisection of Issue Set Based on the Whole Attitude of Agent Group to Every Issue

We call $X \subseteq U$ an agent group. This subsection defines two evaluation functions to estimate the extent to which the issue a is accepted or opposed by the whole agent group X. Then three regions of issues: $\alpha-$agreement strategy, $\beta-$disagreement strategy and $(\alpha, \beta)-$noncommittal strategy are determined as well. Since the theories in this section are dual to that in Sect. 3.1. We omit the proofs of theories in this section.

Let $g = \{g^+, g^-\}$ be the set valued mappings from V to $P(U)$, where

$$g^+ : V \to P(U),\ g^+(a) = \{u \in U | a(u) = +\},\ \forall a \in V,$$
$$g^- : V \to P(U),\ g^-(a) = \{u \in U | a(u) = -\},\ \forall a \in V.$$

Definition 6. *Let (U, A) be a conflict situation, $([0, 1], \leq)$ a totally ordered set, $X \subseteq U$ an agent group. A pair of evaluation functions w_a and w_r are defined as:*

$$w_a : V \times P(U) \to [0, 1], w_a(a, X) = D(g^+(a)|X),$$
$$w_r : V \times P(U) \to [0, 1], w_r(a, X) = D(g^-(a)|X).$$

w_a is called issue acceptance evaluation function, and $w_a(a, X)$ evaluates the extent to which agent group X accepts issue a; w_r is called issue rejection evaluation function, and $w_r(a, X)$ evaluates the extent to which agent group X rejects issue a, where $D(g^+(a)|X)$ and $D(g^-(a)|X)$ are defined as

$$D(g^+(a)|X) = \frac{|g^+(a) \cap X|}{|X|},\ D(g^-(a)|X) = \frac{|g^-(a) \cap X|}{|X|}.$$

Property 2. Let (U, A) be a conflict situation. $\forall a \in V$, $X \subseteq U$, we have $w_a(a, X) + w_r(a, X) \leq 1$.

The designated values for acceptance and rejection of issue set are identical to that in Sect. 3.1 numerically. Therefore, the three regions of issues can be determined similarly.

Definition 7. *Let (U, A) be a conflict situation, $(\alpha, 1]$ the designated values for acceptance, $(\beta, 1]$ the designated values for rejection, $X \subseteq U$ an agent group. $w_a(a, X) = D(g^+(a)|X)$ and $w_r(a, X) = D(g^-(a)|X)$, then we denote:*

$$AT_\alpha(X) = \{a \in V | w_a(a, X) \in (\alpha, 1]\},$$
$$DT_\beta(X) = \{a \in V | w_r(a, X) \in (\beta, 1]\},$$
$$NT_{\alpha,\beta}(X) = U - AT_\alpha(X) \cup DT_\beta(X).$$

We name $AT_\alpha(X)$ the $\alpha-$agreement strategy of agent group X, which represents the issues agreed by agent group X to designated value α; $DT_\beta(X)$ is called the $\beta-$disagreement strategy of agent group X, which represents the issues disagreed by agent group X to designated value β; $NT_{\alpha,\beta}(X)$ is called the $(\alpha, \beta)-$noncommittal strategy of agent group X, which represents the noncommittal issues to designated values (α, β).

From Definition 7, the $(\alpha, \beta)-$noncommittal strategy contains issues with $w_a(a, X) \leq \alpha$ and $w_r(a, X) \leq \beta$. Thus, the attitude of the whole agent group X to issue a would be not inclined to agree or disagree greatly. Consequently, and the issues in $NT_{\alpha,\beta}(X)$ could be essential points causing the conflict.

Proposition 4. *Let (U, A) be a conflict situation, $X \subseteq U$ an agent group. $\alpha > 0.5, \beta > 0.5$. The following relations hold: $AT_{\alpha}(X) \cap DT_{\beta}(X) = \varnothing$, $AT_{\alpha}(X) \cap NT_{\alpha,\beta}(X) = \varnothing$, $DT_{\beta}(X) \cap NT_{\alpha,\beta}(X) = \varnothing$.*

For simplicity, we denote $F_1 = AT_{\alpha}(X)$, $F_2 = DT_{\beta}(X)$, and $F_2 = NT_{\alpha,\beta}(X)$.

Proposition 5. *Let (U, A) be a conflict situation, $X \subseteq U$ an agent group. $\forall a_1, a_2 \in V$, if $g^{+}(a_1) \cap X = g^{+}(a_2) \cap X$ and $g^{-}(a_1) \cap X = g^{-}(a_2) \cap X$, then $a_1 \in F_t \Longleftrightarrow a_2 \in F_t$, $t = \{1, 2, 3\}$.*

This proposition shows that if the agents in group X have the same attitude to issues a_1 and a_2, then the two issues will be assigned to identical strategy.

Proposition 6. *Let (U, A) be a conflict situation, $X \subseteq U$ an agent group. $\forall a_1, a_2 \in V$, if $w_a(a_1, X) \geq w_a(a_2, X)$, and $a_2 \in AT_{\alpha}(X)$, then we have $a_1 \in AT_{\alpha}(X)$; Similarly, if $w_r(a_1, X) \geq w_r(a_2, X)$, and $a_2 \in DT_{\beta}(X)$, then we have $a_1 \in DT_{\beta}(X)$.*

4 Conclusion

A new conflict analysis model based on three-way decisions is proposed in this paper. This model analyzes the structure of conflict situation from two aspects.

On the one hand, we define a pair of evaluation functions, through including degree, to estimate the extent to which agent u accepts or opposes a strategy Y, and then trisect the agent set into three regions. Those ideas are all based on the theory of three-way decisions. Subsequently, the better strategy can be acquired.

On the other hand, another pair of evaluation functions are defined to estimate the extent to which issue a is accepted or opposed by an agent group X, and trisection of issue set is confirmed as well. Then the core conflict issues of agent group would be contained in $(\alpha, \beta)-$noncommittal strategy. Moreover, we conclude that this model is more suitable to our cognizance than the existing models.

Open problems remaining for future research include: the algorithm of finding the sub-optimal feasible consensus strategy should be acquired; the determination of core conflict issues need to be studied explicitly further.

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