Applying Compressed Sensing to Blade Tip Timing Data: A Parametric Analysis

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Abstract. Monitoring the behavior of rotating blades is a critical procedure to ensure safety and proper performance of turbomachinery. For a long time strain gages have been the only solution to measure blade vibrations in a rotating scenario, but with the advances in software and hardware over the last decades, the research on blade tip timing (BTT) data analysis, a non-intrusive technique has gained momentum. A major drawback comes with this approach: undersampling. Several methodologies can be found in the literature do deal with this undersampled signal and this work presents a parametric study of the most recent type of approach that has gained momentum in the BTT research: compressed sensing (CS). The results show what are the best conditions to apply CS on BTT data, in terms of probe placement, amount of sensors and number of rotations.

Keywords: Blade tip timing · Blade vibrations
Compressed sensing · Signal reconstruction · Spectral analysis

1 Introduction

Blades are a fundamental component in turbomachinery and they are often subjected to induced vibrations. For a typical gas turbine, these vibrations are originated through mainly four types of stresses [1], but the alternating stresses, that are originated by forced response to excitations at multiples of the rotating speed, are the most common cause of high cycle fatigue. These vibrations will diminish the fatigue life and the performance of the blades, which can ultimately lead to catastrophic results. Therefore, carrying on-line vibration monitoring is an intrinsic duty when dealing with turbomachinery.

For a long time strain gages have been the only solution to measuring flexible blade vibrations in rotating machinery. However, the installation of such sensors consists in a very tedious, laborious, costly and time consuming task. This is due specially to the number of blades per stage of the turbine and the requirement of having telemetry or slip rings to acquire the signals. On top of these disadvantages, the sensors can alter the blade dynamics and, more importantly,

K. L. Cavalca and H. I. Weber (Eds.): IFToMM 2018, MMS 62, pp. 121–134, 2019.
https://doi.org/10.1007/978-3-319-99270-9_9

this monitoring system tends to not be very durable due to the harsh working conditions [2]. In light of these issues, the development of non-intrusive vibration measuring system for rotating blades was boosted.

Describing it in a simple manner, the Blade Tip-Timing (BTT) methodology consists in monitoring blade vibrations by measuring passing times of the blades' tip with static sensors, usually optic, located at the turbine casing. Since the blades are vibrating, the deformation induced will alter these passing times, known as arrival times, when compared with the arrival times of the blades in a non-vibrating referential. This referential can be obtained by placing a sensor on the shaft to measure the actual rotating speed, and by knowing the angular position of the blades, the non-vibrating signal can be generated. This sensor is known as the Once Per Revolution (OPR) sensor.

Despite being a simplistic and straightforward methodology with several advantages over the strain gage measuring procedure, the technique suffers from bad sampling, since the sampling frequency is directly related to the rotating speed and the number of sensors used. To ensure a sampling frequency that attends to the Nyquist criterion, the quantity of sensors involved can be quite substantial. As a result, a significant effort in the BTT scope was devoted to the development and application of techniques capable of identifying vibration properties from a under sampled signal.

Recently, Compressed Sensing (CS), a digital signal reconstruction technique, has gained momentum in the field of BTT data analysis. The idea behind the theory of Compressed Sensing consists in a search for the reconstruction of sparse signals with a number of samples that is way lower than the number of samples required by the classic methodologies that need to attend to the Nyquist criterion. This methodology is being explored for some time, especially in the field of image compression, in which the image is recovered directly by a compressed representation, instead of capturing the whole image to then compress it and decompress it in a computer, eliminating an entire step and reducing the amount of data that was needed to be stored. The pioneer work on this technique can be found in [3–5]. The motivation for utilizing this methodology on the BTT scenario comes from the fact that if some criteria is met, that will be shown in the next section, it is possible to perfectly reconstruct a signal with a sampling frequency lower than the Nyquist rate.

The goal of this present work is to produce a parametric analysis of a CS implementation on BTT data. To achieve this goal, the frequency spectrum of a simulated vibration signal, obtained by BTT, is reconstructed in the frequency domain by the CS methodology, and several parameters of the BTT system are altered so that it is possible to identify its effects on the frequency spectrum obtained.

2 BTT Data

A BTT measuring system scheme is presented in Fig. 1 and to extract the information of blade tip deflection, it is necessary to first generate a virtual signal, based on the OPR sensor, that corresponds to the expected passing times of the blades in a non-vibrating referential. The non-vibrating time sequence, $\hat{t}(i,n,k)$, can be described as

$$\hat{t}(i,n,k) = \frac{\alpha_i + 2\pi n - \theta_k}{2\pi \Omega}, \tag{1}$$

where α_i is the angular position of the ith probe, θ_k is the initial angular position of the kth blade, n is the rotation number and Ω is the constant rotating speed of the assembly, in Hz.

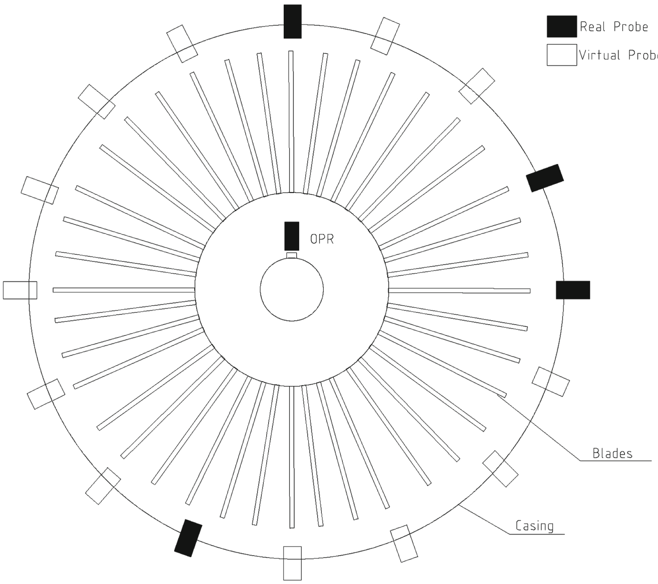

Fig. 1. Representation of the BTT measuring system.

Due to the fact that the blades are vibrating, the real measured time sequence will be distorted by the angular deflection of the blade's tip, $\delta(i,n,k)$, altering the real timing sequence $t(i,n,k)$ to

$$t(i,n,k) = \frac{\alpha_i + 2\pi n - \theta_k - \delta(i,n,k)}{2\pi \Omega}. \tag{2}$$

Subtracting Eqs. (2) from (1) and knowing the distance between the rotating center and the tip of the blade, R, the linear displacement can be obtained by

$$\mathbf{d}(i,n,k) = \delta(i,n,k)R = 2\pi\Omega R\big[t(i,n,k) - \hat{t}(i,n,k)\big] = 2\pi\Omega R\Delta t. \tag{3}$$

With the obtained $\mathbf{d}$ signal, the vibration amplitude of each blade, measured at each sensor, in each rotation is extracted, but the sampling rate of this signal is directly related to the amount of probes used, and the rotating speed of the assembly. To exemplify the typical sampling obtained with a BTT system, Fig. 2 presents a 30 Hz vibration signal on a 10 Hz rotating speed assembly and it shows the collection of samples obtained by four probes. This case shows an Engine Order (EO) excitation, since the vibrating frequency is an integer multiple of the rotating frequency. The bad sampling can clearly be seen due to the fact that each probe needs to wait three entire periods (3 EO) of the vibrating signal to collect new information, for each blade.

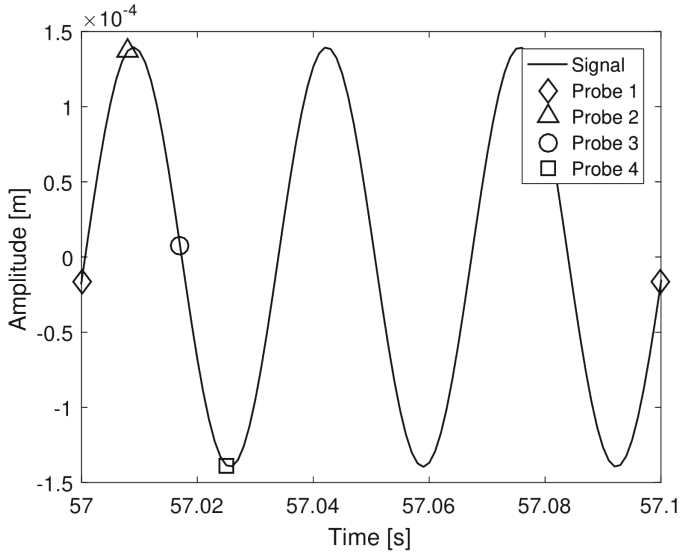

Fig. 2. Example of a BTT signal sampled with 4 probes.

Several techniques are being utilized and explored in the context of BTT undersampled data. A new approach that has gained track in recent years is the Compressed Sensing methodology.

3 CS Theory

The CS methodology consists in recovering a signal, sparse in some domain, with fewer samples than the classic methodologies that are based on the Shanon-Nyquist criterion. The motivation for this approach is to recover the signal directly from compressed samples (lower sampling rate) instead of firstly sampling the signal at a high rate to then compress it, eliminating a stage of compression and decompression [6]. This problem can be formulated in mathematical form through an optimization task, searching for the sparsest solution to the problem:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_0} \quad \text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{b}, \tag{4}$$

where $\mathbf{x}$ is the sparse signal with N samples, sampled above the Nyquist rate; ℓ_0 is the "0-norm", a quasi-norm that represents the amount of non zero elements in a vector, indicating the quantity k of sparse elements. The vector $\mathbf{b}$ is the M observations vector, where M << N, while $\mathbf{A}$ is the MxN sensing matrix.

The sensing matrix $\mathbf{A}$ is usually formed by the multiplication of two matrices, $\mathbf{\Phi}$ and $\mathbf{\Psi^{-1}}$. The matrix $\mathbf{\Phi}$ consists in the sensing part of the matrix, i.e., the relationship between the selected samples and the reconstructed signal, while $\mathbf{\Psi^{-1}}$ consists in the sparse basis of the signal. In most cases the signal in time is not sparse, so the matrix $\mathbf{\Psi^{-1}}$ is usually the inverse Fourier basis, so that the signal in time can be represented in the sparse frequency domain.

Despite being a straightforward and well defined approach, finding the sparsest solution is a non trivial task, in fact it is known as a NP-hard problem, being computationally unsolvable even for modest values of k, M and N. Fortunately, [3] proposed a substitution of ℓ_0 by the norm ℓ_1, defined as

$$\|\mathbf{x}\|_1 = \sum_{n=0}^{N-1} |x(n)|, \tag{5}$$

making the optimization problem convex and linear. Despite not being the same exact solution to the ℓ_0 problem, a graphic representation of different norms in $\mathbb{R}^2$ shows that the closest solution to the sparsest search, is the ℓ_1 norm, as seen in Fig. 3. The optimization task of Eq. (4) with the norm ℓ_1 instead of ℓ_0 is called Basis Pursuit (BP).

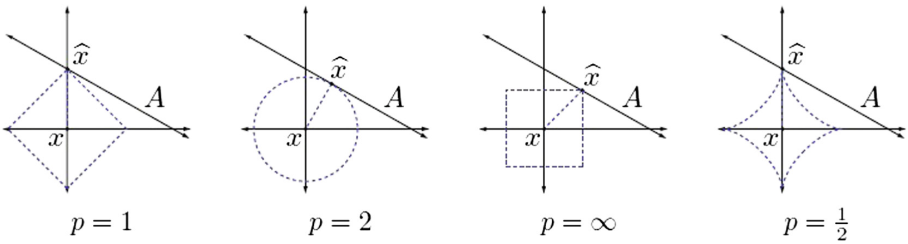

Fig. 3. Approximation of a point in $\mathbb{R}^2$ by a one-dimensional subspace for ℓ_p norms, with p =1, 2, ∞ and $\frac{1}{2}$. Source:[6].

To ensure unique solution to the BP problem, the matrix $\mathbf{A}$ needs to respect the Restricted Isometry Property (RIP) described in [7]. This property basically requires that the columns of $\mathbf{A}$ to be quasi-orthonormal. Another way of ensuring the unique solution is to have minimal coherence between the matrices $\mathbf{\Phi}$ and $\mathbf{\Psi^{-1}}$. Since $\mathbf{A} = \mathbf{\Phi\Psi^{-1}}$ the coherence is defined as

$$\mu(A) = \max_{1 \le i < j \le N} \frac{|\langle a_i, a_j \rangle|}{\|a_i\|_2 \|a_j\|_2}, \tag{6}$$

where $a_{i,j}$ are the column vectors of $\mathbf{A}$.

Usually random $\mathbf{\Phi}$ matrices with a fixed $\mathbf{\Psi}^{-1}$ basis attend to this property with a high probability, and to exemplify the potential of this approach, a signal composed by three sinusoids of frequencies 50, 75 and 100 Hz, with unitary amplitudes, is reconstructed with this technique. The original time signal to be reconstructed, $x(t)$, is 512 samples long with a sampling frequency higher than the Nyquist rate, while the observed signal, $y(t)$, is made of 128 random samples. The results are shown in Fig. 4 and it can be seen perfect reconstruction of the signal.

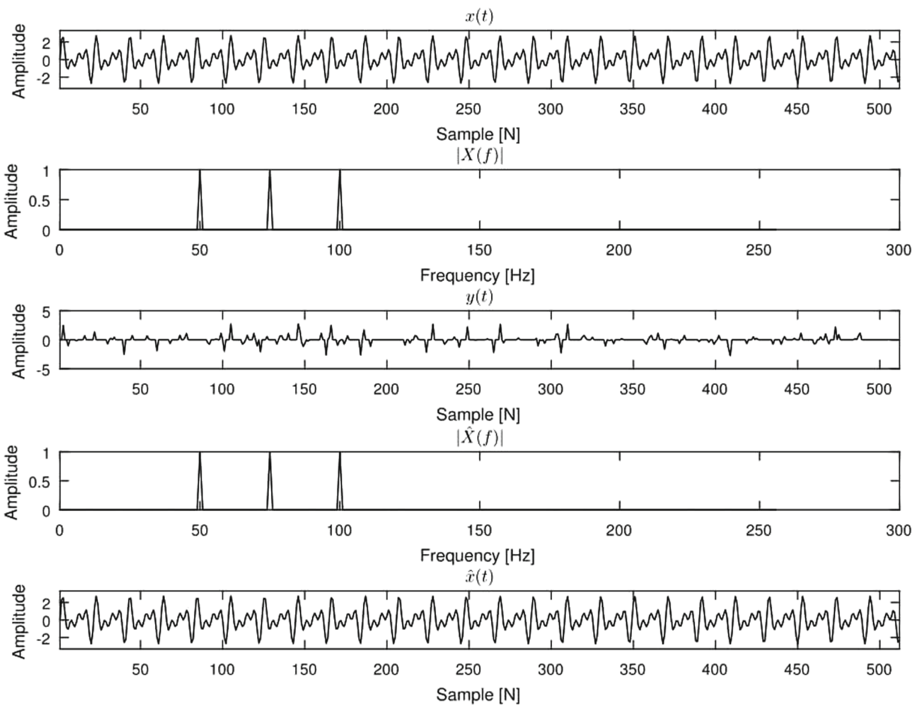

Fig. 4. Example of CS reconstruction with random sensing matrix.

4 Applying CS on BTT Data

As presented before, the CS consists in retrieving a sparse signal with N samples trough an optimization task based only on M observations, usually M<<N. To reproduce this conditions in the BTT context, the number N of samples of the sparse vector, in this case the frequency spectrum sampled over the Nyquist rate, is produced trough placing a number L of imaginary probes equispaced around the casing and rotating the blade assembly for N_r revolutions, resulting in $N = LN_r$. The real measurements made are obtained trough the real probes, positioned in any of the possible virtual positions L. This results in a real amount

of data $M = lN_r$, where l is the number of real probes placed around the casing. From this, observations it is easy to see that the ratio $\frac{M}{N}$ is always defined by the ratio of the number of real sensors in respect with the number of virtual sensors $(\frac{l}{L})$. The idea behind this formulation is to retrieve a frequency spectrum of a scenario of L probes, that attends to the Nyquist sampling rate, with only l real probes being utilized.

In this context, the matrix $\mathbf{\Psi^{-1}}$ will be the inverse Fourier basis NxN, since the signal is not sparse in the time domain, but it is in the frequency domain. With respect to the sensing matrix $\mathbf{\Phi}$, it is constructed as

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 0 & \dots\dots\dots & 0 \\ 0 \dots & 1 & 0 & \dots 0 \\ \vdots & & & \vdots \\ \vdots & \dots\dots\dots\dots & & \vdots \\ 0 \dots\dots & & 1 & \dots 0 \end{bmatrix}_{M \times N}, \tag{7}$$

that is determined by the sampling sequence of the probes throughout the N_r revolutions. Each row contains a unitary value on a single column $j = \frac{\alpha_i L}{2\pi} + nL$, where α_i is the angular position of the ith probe and n is the current rotation number.

With these matrices assembled, it is seen that the observations made by l probes are:

$$\mathbf{y} = \mathbf{\Phi \Psi^{-1} X}, \tag{8}$$

where $\mathbf{X}$ is the FFT of length N of the signal sampled by the L imaginary probes, which will be recovered through the BP algorithm.

As described in the last section, the matrix $\mathbf{A} = \mathbf{\Phi \Psi^{-1}}$ needs to comply with the RIP condition, and, as noted before, it complies, with high probability, if the matrix is random. From the direct formulation of the CS on BTT data, it is seen that this matrix is not random, since the real probes are fixed throughout the revolutions. To deal with this problem [8] proposes the realization of a orthogonalization preprocessing procedure, in which the matrix $\mathbf{A}$ is reformulated as a orthogonal basis for the range of $\mathbf{A^T}$, as in $\mathbf{Q} = \left[\mathrm{orth}(\mathbf{A^T})\right]^{\mathbf{T}}$. The procedure results in

$$\mathbf{QA^\dagger y} = \mathbf{QA^\dagger AX}, \tag{9}$$

where † denotes the pseudoinverse. From [8] it is also described that $\mathbf{QA^\dagger A} = \mathbf{Q}$ and renaming $\mathbf{z} = \mathbf{QA^\dagger y}$ results in the formulated BP problem:

$$\min_{\mathbf{X}} \|\mathbf{X}\|_{\ell_1} \quad \text{s.t.} \quad \mathbf{QX} = \mathbf{z}. \tag{10}$$

It is worth noticing that the optimization task can be quite demanding, computationally, since the amount of variables N are directly related to the amount of virtual probes L and the number of revolutions N_r. In some cases it can easily be in the range of 10^3–10^4 variables.

5 Results

To make the analysis, two scenarios were created, with signals in the form of $\sum_{i=1}^{p} a(i)\sin(2\pi f(i)\frac{n}{N} + \phi(i))$, whose parameters are described in the Table 1.

Scenario 1 consists of a signal with three close frequencies, all asynchronous (not integer multiples of the rotating speed Ω). The reconstruction of the signal is made from cases of two to six probes. For scenario 2 a synchronous frequency of 75 Hz replaces the 88 Hz frequency, so that it is possible to verify a hybrid vibration case and in this situation only five and six probes were utilized, due to the results seen in scenario 1.

Table 1. Simulated signals.

Scenario	$\mathbf{a(i)}$	$\mathbf{f(i)}$ [Hz]	$\phi(\mathbf{i})$ rad	$\mathbf{\Omega}$ [Hz]	# Probes
1	$\begin{bmatrix}3\ 2\ 1\end{bmatrix}$	$\begin{bmatrix}88\ 89\ 90\end{bmatrix}$	$[0 - 2\pi]$	25	2-6
2	$\begin{bmatrix}3\ 2\ 1\end{bmatrix}$	$\begin{bmatrix}75\ 89\ 90\end{bmatrix}$	$[0 - 2\pi]$	25	5-6

For both cases evaluated, the number of virtual probes was fixed in L = 10. The rotating speed was also fixed at 25 Hz. This conditions implicate that the signal to be reconstructed had a $F_s = L\Omega = 250$ Hz sampling frequency, being over the Nyquist rate for both scenarios. The parametric analysis is based on the amount of sensors and the amount of revolutions utilized in the analysis.

Firstly, as pointed in Eq. (6), the coherence of $\mathbf{Q}$ should be closer to 0 to ensure the RIP condition. With that in mind, the minimal coherence for all the combinations of probes, from two probes up to six are calculated and shown in Table 2, for $L = 10$ to 25 imaginary probes. It can be seen that increasing the number of real probes elevates the incoherence in the matrix $\mathbf{Q}$, but increasing the amount of virtual probes and maintaining the number of real ones results in lower incoherence. The amount of possible probe arrangements also increase with a higher number of virtual probes as it is given by the total $\binom{L}{l}$ combinations of probe placement.

Table 2. Minimal $\mu(Q)$ for different amount of real and virtual probes.

Probes	$\mu(\mathbf{Q})$			
	$\mathbf{L = 10}$	$\mathbf{L = 15}$	$\mathbf{L = 20}$	$\mathbf{L = 25}$
2	0.95	0.98	0.99	0.99
3	0.72	0.76	0.83	0.86
4	0.54	0.50	0.56	0.69
5	0.43	0.48	0.53	0.50
6	0.36	0.40	0.44	0.47

5.1 Scenario 1

Even though the phase of the signals is generated randomly, between 0 and 2π radians, the expected results throughout the course of 50 reconstructions with the BP algorithm are expected to be the same. If they are different it means that the proposed sensing matrix does not attend the RIP condition, resulting in a poor recovery. Using two sensors resulted in extremely poor results, same as the case of three probes, presented in the Fig. 5, that was obtained with $N_r = 300$ revolutions, resulting in 900 observations to reconstruct a 3000 samples long signal. The probe positioning was $p = [1\ \ 2\ \ 4]$, one of the optimal arrangements that give minimal $\mu(\mathbf{Q})$.

In the Fig. 5 and for the others to follow, the criteria utilized to analyze the results were the energy of the signal. E_T stands for the total energy of the signal, E_f is the energy of the correct frequency components and E_R is the real energy of the correctly sampled signal. The wanted results are those in which the ratio $\frac{E_f}{E_T}$ is 100%, meaning that all the energy of the reconstructed signal is located only on the correct frequencies. The ratio $\frac{E_T}{E_R}$ is also wanted to be 100% meaning that the amount of energy in the reconstructed signal is equal to the desired correctly sampled signal. It is obvious from Fig. 5 that three probes are not enough to assure proper reconstruction, there are even cases that the energy of the correct frequency components surpass the total energy. This occurs due to the fact that the E_f calculation is made as twice the energy of the positive frequencies, which means that the situations that $E_f > E_T$ are those that the reconstructed signal is complex, not having the symmetry property of the frequency spectrum.

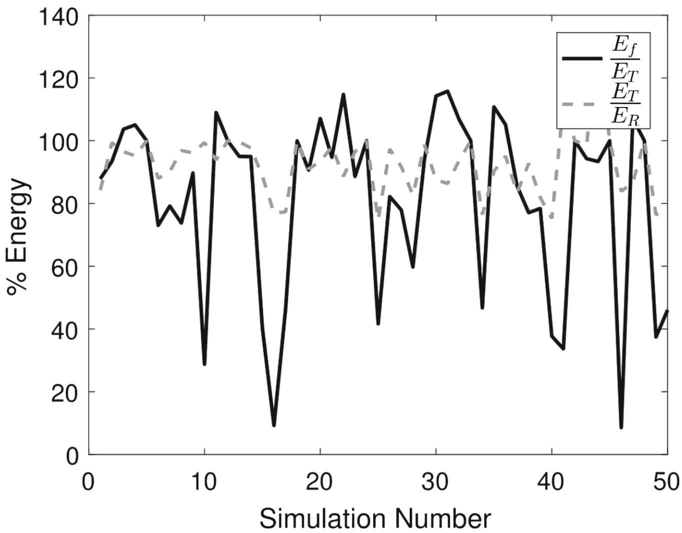

Fig. 5. Ratio of signal energy for the reconstructed signal of scenario 1 using 3 probes and 50 revolutions through 50 simulations.

Seeing that three probes were insufficient for perfect reconstruction, the amount of sensors was increased to four, in the locations $p = [1\ 3\ 4\ 9]$ with a $N_r = 50$ revolutions. It can be seen from Fig. 6 that the results are much improved, but there are still cases of poor reconstruction. In an attempt to improve this results the amount of revolutions was increased to $N_r = 300$ resulting in 1200 observations to reconstruct a 3000 samples long signal. From Fig. 7 it is clear that augmenting the amount of data did not improve the results.

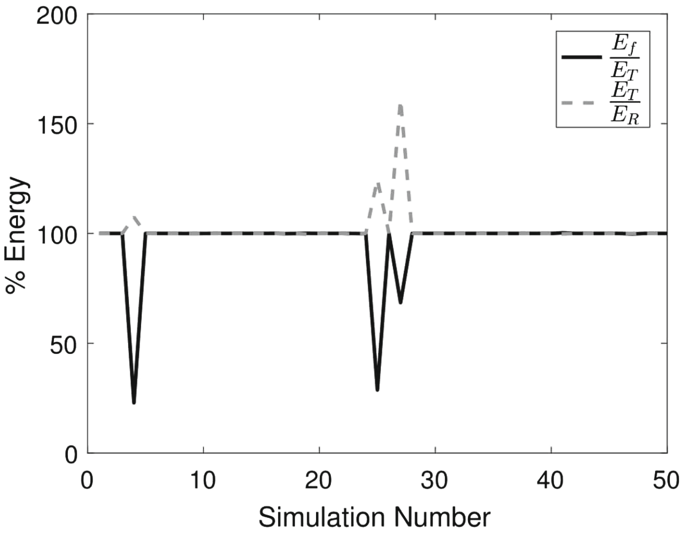

Fig. 6. Ratio of signal energy for the reconstructed signal of scenario 1 using 4 probes and 50 revolutions through 50 simulations.

Now, the number of probes was raised to five and the results from Fig. 8, with only $N_r = 50$ revolutions and with the probes at $p = [1\ 2\ 6\ 9\ 10]$, were extremely satisfactory, showing that perfect reconstruction was achieved, since through the 50 simulations the ratio was around 100%. The same can be said by the results with six probes on Fig. 9 with probes at $p = [1\ 2\ 3\ 4\ 6\ 8]$, for the same number of revolutions.

5.2 Scenario 2

Scenario 2 is expected to be more complicated, because of the presence of a synchronous vibration in the signal. From the results of Scenario 1, utilizing less then five probes is not recommended, since the perfect reconstruction was not ensured.

From Fig. 10 it becomes clear that the introduction of the synchronous frequency has a negative impact on the reconstruction of the frequency spectrum. To elucidate an example of what is happening, Fig. 11 presents the reconstructed

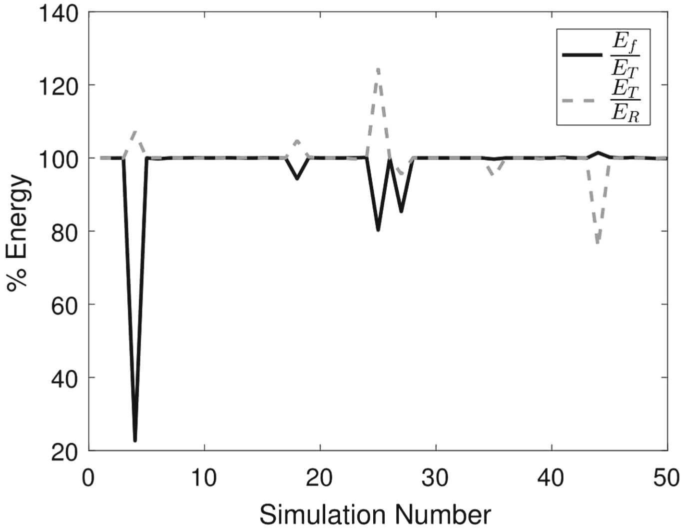

Fig. 7. Ratio of signal energy for the reconstructed signal of scenario 1 using 4 probes and 300 revolutions through 50 simulations.

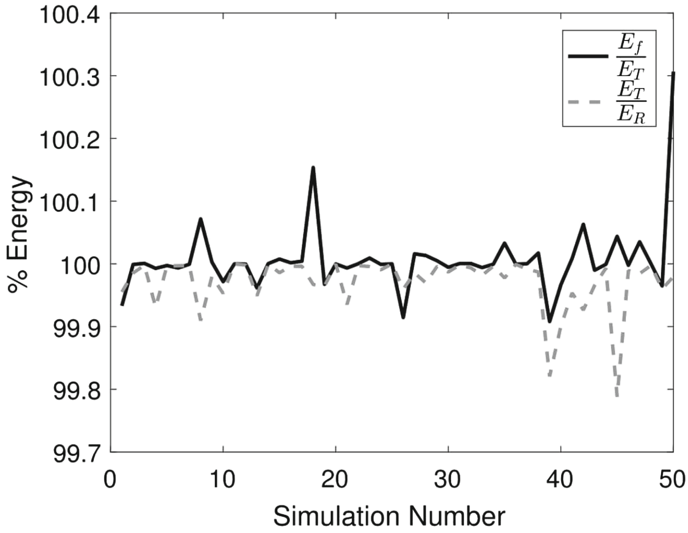

Fig. 8. Ratio of signal energy for the reconstructed signal of scenario 1 using 5 probes and 50 revolutions through 50 simulations.

frequency spectrum of the simulation 25 from Fig. 10. It can be seen that the synchronous frequency started to replicate around multiples of the rotating speed, at 25 and 50 Hz, while the asynchronous frequencies of 89 and 90 Hz were perfectly reconstructed.

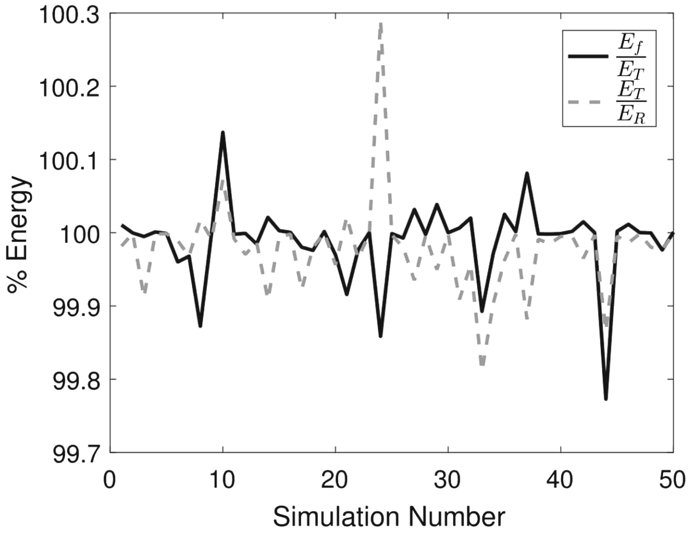

Fig. 9. Ratio of signal energy for the reconstructed signal of scenario 1 using 6 probes and 50 revolutions through 50 simulations.

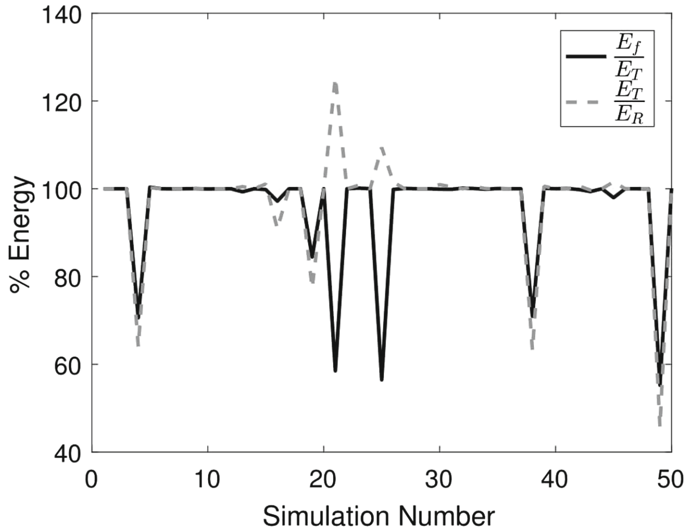

Fig. 10. Ratio of signal energy for the reconstructed signal of scenario 2 using 5 probes and 50 revolutions through 50 simulations.

Meanwhile, the same results were not observed on the six probes case of Fig. 12, that was capable of performing in the same manner as in the asynchronous case, whit perfect reconstruction throughout the test.

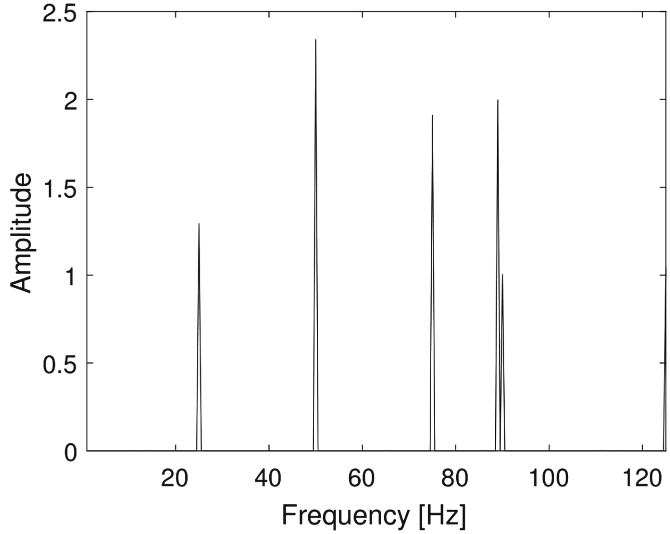

Fig. 11. Reconstructed frequency spectrum for simulation 25 of scenario 2 using 5 probes and 50 revolutions.

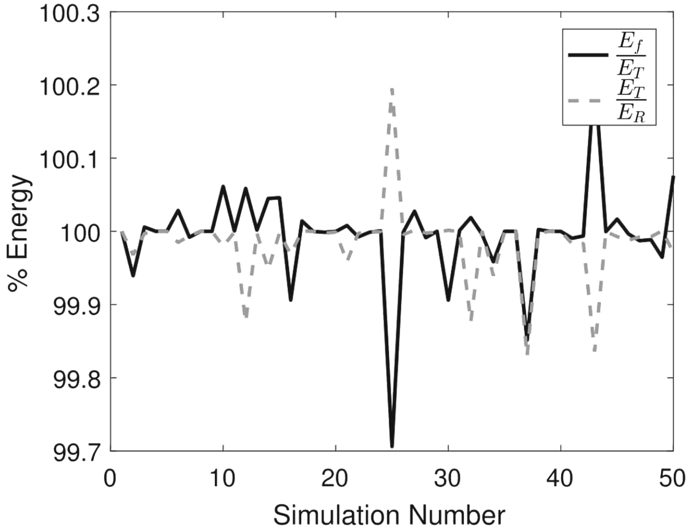

Fig. 12. Ratio of signal energy for the reconstructed signal of scenario 2 using 6 probes and 50 revolutions through 50 simulations.

6 Conclusions

This work proposed the evaluation of several conditions for the application of Compressed Sensing (CS) on Blade Tip Timing (BTT) data. It was described both methodologies and how to produce the reconstruction of signals through undersampled and non uniform data.

From the results of the present research, it was possible to conclude that the CS has potential of performing well on BTT data. It was shown that some special probe positioning impact in the results of the analysis and those are the ones that indicate minimal coherence of the sensing matrix. Furthermore, it was shown that for asynchronous vibrations, five probes are sufficient to ensure perfect reconstruction of the signal generated by BTT sampling. For cases that include synchronous vibrations, six probes were determined as a reliable proposition for perfect recovery.

It is worth noticing that the high number of probes required to retrieve a frequency spectrum of an asynchronous vibration signal is possibly related to the amount of frequencies presented in the simulated signal and due to the fact that increasing the amount of observations did not result in improvements on the reconstruction, surprisingly.

Even though the predominant amplitude of vibration, in real cases, are identified as the result of EO excitations, it is still valid to pursue reconstruction of multi frequency signals, because instabilities, that originates asynchronous excitation, can still occur even when the dominant frequency of vibration has an EO nature.

For future works it is interesting to investigate the ratio of real to virtual probes$\frac{l}{L}$, for increased values L. It was shown that less incoherence comes with higher L which will affect negatively on the reconstruction, but perhaps an optimal ratio can be found while varying L.

Acknowledgments. This project was supported by the Brazilian research foundation CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico).

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