Kriging-Based Surrogate Modeling for Rotordynamics Prediction in Rotor-Bearing System

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Abstract. In this work, it was proposed to use Kriging surrogate models for rotordynamics prediction in rotor-bearing systems. The motivation is to significantly reduce computation effort when evaluating the design space. First, fundamentals of rotordynamics are reviewed and the rotor-bearing system is modeled using the Finite Element (FE) method. Modal analysis is used to determine whirl frequencies and critical speeds while system dynamic behavior is evaluated in terms of the unbalance response. Subsequently, approximations of the input/output relationships created by the FE simulations are obtained by applying the Kriging interpolating method. The derived models work as fast-running surrogates for the full model. Comparison of the results from Kriging surrogates obtained using different training samples shows that the proposed methodology provides a computationally efficient and low-dimension mathematical relationship that can accurately predict rotor-bearing system outputs with considerably low training effort.

Keywords: Kriging surrogate modeling · Rotor-bearing systems
Rotordynamics prediction

1 Introduction

In engineering design, surrogate modeling techniques are of particular interest when high-fidelity, thus computationally expensive analysis are required, such as rotordynamics analysis. The dynamic behavior of complex rotor systems is usually solved by means of computational models such as the Finite Element (FE), where the complexity of the model increases with the wealth of information it contains.

Also, computational cost increases when the values of system parameters are indeterminate, i.e. they may vary within particular ranges, what is called uncertainty.

Although the recurrent call of the deterministic computational model for processing uncertain quantities through Monte Carlo sampling is robust and independent of the model dimension, it has remarkably slow convergence rate and requires large number of time-consuming simulations to guarantee an accurate and efficient coverage of the design space, which is sometimes impractical for rotordynamics analysis in most rotating machinery design.

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In this context, the use of an adequate approximation of the system response appears as an effective way to accomplish reasonable reduction of numerical effort. Surrogate models, or metamodels, are analytic relationships that approximate the multivariable input/output dynamic behavior of higher order models, based on a limited set of computationally expensive simulations. A surrogate is built from sampled data obtained by intelligently exploring the design space. It only requires tens to a few hundreds of full computational model runs for the training. Once validated, the surrogate model becomes a very effective low cost substitute of the original model for a wide variety of purposes, such as robust optimization, design automation, parametric studies and uncertainty analysis [1].

There are several surrogate models techniques available in the literature, such as polynomial response surface models, radial basis functions, Kriging, support vector regression and artificial neural networks [2].

Here, the method known as Kriging surrogate modeling is approached. Kriging is a statistics-based interpolating technique capable of handling deterministic noise-free data, which drew a lot of attention during the past decade.

It was first used in mining and geostatistical applications and has been increasingly used, especially in structural and aerodynamic optimization. Kriging-based surrogate provides an explicit function to represent the relationship between the inputs and outputs with a small initial training sample set in linear or nonlinear system.

The purpose of this work is to evaluate the use Kriging-based surrogate modeling for rotordynamics prediction (natural frequencies and unbalance response) of rotor-bearing systems. Results of Kriging surrogates are compared for different training samples and the efficiency of the Kriging model for rotordynamics prediction is further analyzed.

Section 2 brings the fundamentals of rotor-bearing system modeling and an overview of the Kriging surrogate modeling. The application of Kriging interpolation to the rotordynamics system is described in Sect. 3, followed by the assessment of the accuracy of the Kriging surrogate. The main conclusions are finally presented in Sect. 4.

2 Mathematical Modeling and Kriging Surrogates: A Brief Description

2.1 Rotor-Bearing Model

The dynamic behavior of rotor-bearing systems depends considerably on the geometry and properties of the rotor and bearing parameters, which in the sense of dynamics have corresponding inertial, elastic, gyroscopic and damping forces [3].

A rotor-bearing system model is typically composed of three essential components: the shaft, the disks and the bearings. In most cases, a common source of rotor excitation resultant from a mass unbalance is also present on the rotor, which must also be considered [4].

In current industry practice, each component of the rotating systems is discretized using the Finite Element method in order to model and predict its dynamic behavior and bearing performance. A common approach for discretization is to use a shaft-line model, in which the mesh is created by simply choosing nodes at key locations along the shaft line [5].

The shaft with distributed mass and elasticity is represented as a set of two-node circular cross-section Timoshenko beam elements, each with eight degrees of freedom, and characterized by both strain and kinetic energies.

Disks represent the rotating components, either attached to the shaft or an integral part of the shaft, with relatively short axial length and large diameter (e.g., compressor impellers, turbine wheels and balancing rings). They are characterized by the kinetic energy, and modeled as rigid elements when studying its effects on rotordynamics [6]. Usually present on disks, mass unbalance is also defined in terms of the kinetic energy.

The elements that support the shaft are the bearings, which may be classified into rigid or elastic. In practice, a rigid bearing is equivalent to a high stiffness bearing whereas an elastic bearing is characterized by finite stiffness properties and by viscous damping properties [7]. In this work, we use hydrodynamic journal bearings as the shaft elastic support elements. These fluid-film bearings have noticeable speed-dependent properties, which has to be incorporated in the rotor-bearing model by changing the linear stiffness and damping elementary matrices as the shaft speed varies [5].

The speed-dependent linearized stiffness and damping coefficients for the journal bearing can be calculated analytically as a function of the journal eccentricity and the modified Sommerfeld number, assuming Ocvirk's short-bearing approximation [5, 8–10]. As expected, the stiffness matrix is not symmetric, introducing anisotropy into the model.

Rotor-bearing system time-domain equations of motion, including the effects of rotatory inertia, gyroscopic moments and damping, are obtained by assembling element matrices derived from Lagrange's equations and is written in matrix form as:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = F(t) \tag{1}$$

where q are the generalized coordinate displacement vector, M is the inertia matrix, C contains the linearized bearing damping matrix and the gyroscopic matrix, K is the stiffness matrix and F is the force vector.

This matrix equation represents a set of n second-order ordinary differential equations. This system is solved for two different cases: first for eigenvalues and eigenvectors (i.e., natural frequencies and mode shapes) using modal analysis and lately for frequency response to harmonic excitation forces (i.e., unbalance forces) [11].

Regarding the modal analysis, due to the nonproportional damping the standard eigenvalue problem cannot be used, since the normal modes do not decouple the damping matrix [12]. The solution of the free vibration system leads to a quadratic eigenvalue problem. To solve it, it is convenient to reformulate the second-order equation of motion into a set of $2n$ first-order differential equations:

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2}$$

Thus, writing in the state space form, the equation of motion becomes:

$$Ax + \dot{x}x = 0 \tag{3}$$

where the state vector with $2n$ elements is:

$$x = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} \tag{4}$$

Solutions are sought of the form:

$$x(t) = \; ve^{st} \tag{5}$$

Thus, the eigenvalue problem is defined as:

$$[sA + B]v = 0 \quad \text{or} \quad \left[-A^{-1}B - sI\right]v = 0 \tag{6}$$

where the dynamic matrix is $-A^{-1}B$, v are the eigenvectors and the eigenvalues are:

$$s_i, s_{n+i} = \omega_i\left(-\zeta_i \pm j\sqrt{1-\zeta_i^2}\right) = -\zeta_i\omega_i \pm j\omega_{d_i} \tag{7}$$

The response of the system to a generic force varying harmonically in time such as synchronous mass unbalance can be determined by assuming a harmonic solution for the equation of motion and obtaining a solution in the frequency domain:

$$Y(\omega) = \left[-\omega^2 M + j\omega C + K\right]^{-1} F(\omega) \tag{8}$$

2.2 Overview of Kriging Method

The interpolation method known as Kriging is popular in approximating computation-intensive generated data which are deterministic in nature [13]. It was conceived by the mining engineer Krige [14] in geoestatistics and later developed by Matheron [15]. Kriging was definitely introduced into engineering design following the work of Sacks *et al.* [16], who applied the method to construct an approximation model based on data from computer experiments [17].

The Kriging approach treats the function of interest as a realization of a stochastic process [18, 19]. It is a statistical-based approximation method for design and analysis of computer experiments [20]. Prediction with a Kriging model requires the inversion and multiplication of several matrices, thus the Kriging model does not exists as a "closed-form" polynomial equation.

In order to train the Kriging model it is necessary to start with a set of sample data and observed responses. After a first identification of k input variables that have a significant impact on system output, the design variable vector $x = \{x_1, x_2, \ldots, x_k\}^T$ is determined as well as the ranges of each variable. The next step is the definition of n of these k-vectors, $X = \left\{x^{(1)}, x^{(2)}, \ldots, x^{(n)}\right\}^T$, that will represent the design space. The notion of design space covers the set of all possible experiments or simulations that may interest the analyst. In this regard, the design of experiments is used to intelligently determine a few points out of the full factorial set that provide sufficient information about the full response space instead of covering the whole design space. The Latin hypercube sampling (LHS) is a commonly used technique to select the values of input variables [21]. The observed responses are stored in a vector $y = \left\{y^{(1)}, y^{(2)}, \ldots, y^{(n)}\right\}^T$.

As stated before, the observed responses are considered as if they were from the realization of a random process. The vector $Y = \left\{Y\left(x^{(1)}\right), Y\left(x^{(2)}\right), \ldots, Y\left(x^{(n)}\right)\right\}^T$ denotes the random field and the random variables are correlated using the Kriging basis function with Gaussian form:

$$cor\left[Y\left(x^{(i)}\right), Y\left(x^{(l)}\right)\right] = exp\left(-\sum_{j=1}^{k} \theta_j \left|x_j^{(i)} - x_j^{(l)}\right|^{p_j}\right) \tag{9}$$

This correlation function shows that if $x_j^{(i)} = x_j^{(l)}$ the correlation is one. Likewise, if the distance between the two points grows, the correlation tends to zero. The parameter θ_j allows the width of the basis function to change from variable to variable. By looking at the elements of the vector θ, the most important variables can be determined. The smoothness parameter p_j is typically fixed at two (Gaussian basis exponent) for smooth correlations. From this expression, we obtain the correlation matrix:

$$\Psi = \begin{bmatrix} cor\left[Y\left(x^{(1)}\right), Y\left(x^{1}\right)\right] & \cdots & cor\left[Y\left(x^{(1)}\right), Y\left(x^{(n)}\right)\right] \\ \vdots & \ddots & \vdots \\ cor\left[Y\left(x^{(n)}\right), Y\left(x^{(1)}\right)\right] & \cdots & cor\left[Y\left(x^{(n)}\right), Y\left(x^{(n)}\right)\right] \end{bmatrix} \tag{10}$$

One advantage of using a basis function with Gaussian form is that it always lead to a symmetric positive definite correlation matrix, thereby guaranteeing the computation of its inverse via Cholesky factorization [17].

Once we have a set of correlated random variables Y, where the correlations depend on the absolute distance between the sample data and the parameters θ and p, the next step is to tune the Kriging model by choosing θ and p to maximize the likelihood of the observed data y. For this, we use a metaheuristic global search method such as a genetic algorithm or simulated annealing, which has proved to produce the best results.

After the search, we can finally use the maximum likelihood estimation (MLE) values for the model parameters θ and p to calculate the correlation matrix and make predictions of the response at new points using the Kriging model.

According to Jones [22], the standard formula for the Kriging prediction at a new point x^*, can be written as:

$$\widehat{y}(x^*) = \widehat{\mu} + \psi^T \Psi^{-1}(y - 1\widehat{\mu}) \tag{11}$$

where $\hat{\mu}$ is the MLE for the mean μ:

$$\hat{\mu} = (1^T\Psi^{-1}y)/\left(1^T\Psi^{-1}1\right) \tag{12}$$

and ψ is the vector of correlations between the observed data and our new prediction:

$$\psi = \begin{Bmatrix} cor\left[Y\left(x^{(1)}\right), Y(x^*)\right] \\ \vdots \\ cor\left[Y\left(x^{(n)}\right), Y(x^*)\right] \end{Bmatrix} = \begin{Bmatrix} \psi^{(1)} \\ \vdots \\ \psi^{(n)} \end{Bmatrix} \tag{13}$$

3 Numerical Results and Discussion

In order to demonstrate the applicability and accuracy of the Kriging-based surrogate in rotordynamics prediction, a numerical study of a rotor-bearing system was carried out using the MATLAB® software package. Figure 1 illustrates the FE model to be evaluated [5].

The rotor-bearing system is composed of a steel shaft (E = 211 GPa, ν = 0.3 and ρ = 7,810 kg/m^3) with 6 beam elements, two rigid steel disk elements, and it is supported by oil-film journal bearings located in both ends. The nominal bore diameter of the journal bearing is the same as the shaft.

A synchronous excitation is also added by an unbalance moment of 5×10^{-4} kg-m positioned at the right disk element. Details of the rotor-bearing baseline parameters are presented in Table 1.

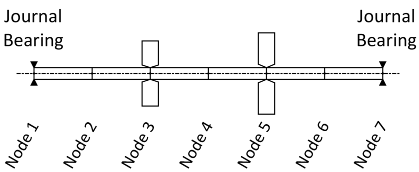

Fig. 1. Rotor-bearing FE model.

Table 1. Rotor-bearing model baseline

Shaft length, m	1.500
Shaft diameter, m	0.050
Left disk diameter, m	0.280
Right disk diameter, m	0.350
Disk thickness, m	0.070
Bearing diameter, m	0.050
Bearing length, m	0.030
Bearing radial clearance, μm	100
Bearing oil film viscosity, Pa-s	0.1
Bearing static load, N	525

The rotor-bearing system dynamic behavior was assessed with the shaft spinning at 4,000 rev/min. The modal analysis for this condition indicates a first undamped natural frequency of 17.08 Hz and the first six modes shapes are illustrated in Fig. 2. Observe that when the displacement at the bearings location is small, fluid-film damping is expected to be less effective for the rotor-bearing system damping as a whole.

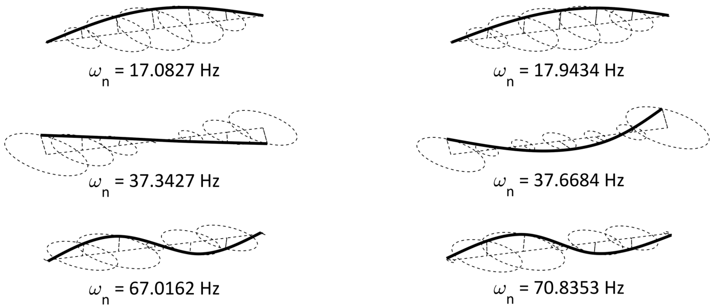

Fig. 2. Rotor-bearing mode shapes at 4,000 rev/min.

The frequency response to the mass unbalance in Fig. 3 indicates that the system passes through a first critical speed near 1,000 rev/min, which corresponds exactly to the first mode excitation. In the proximities of 4,000 rev/min, a second critical speed is experienced, however with a higher damping. There is at least an order of magnitude between the amplitudes at the right disk element (node 5) and the amplitudes at the bearings location (nodes 1 and 7).

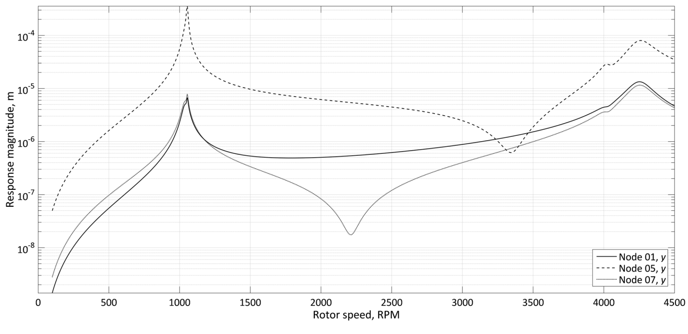

Fig. 3. Rotor-bearing unbalance response magnitude at different locations.

Since the system dynamic behavior is extremely dependent on bearing stiffness and damping, an evaluation of bearing parameters influence is desirable to rotordynamics predictions.

Here, both hydrodynamic journal bearings' length, radial clearance and oil film viscosity were chosen as the critical design variables, which are allowed to vary at specific ranges. Table 2 indicates the baseline, minimum, and maximum values for these parameters.

Table 2. Critical design variables baseline parameters.

Parameter	Baseline	Minimum	Maximum
Left bearing length, m	0.030	0.020	0.038
Left bearing radial clearance, μm	100	50	125
Left bearing oil film viscosity, Pa-s	0.100	0.010	0.200
Right bearing length, m	0.030	0.020	0.038
Right bearing radial clearance, μm	100	50	125
Right bearing oil film viscosity, Pa-s	0.100	0.010	0.200

Kriging methodology was then applied to construct a surrogate for the FE model based on initial samples of design points. The objective is to assess Kriging-based surrogate accuracy and efficiency in predicting system dynamic behavior.

The proposed application of Kriging interpolation to the rotor-bearing system for rotordynamics predictions is illustrated in Fig. 4 flowchart.

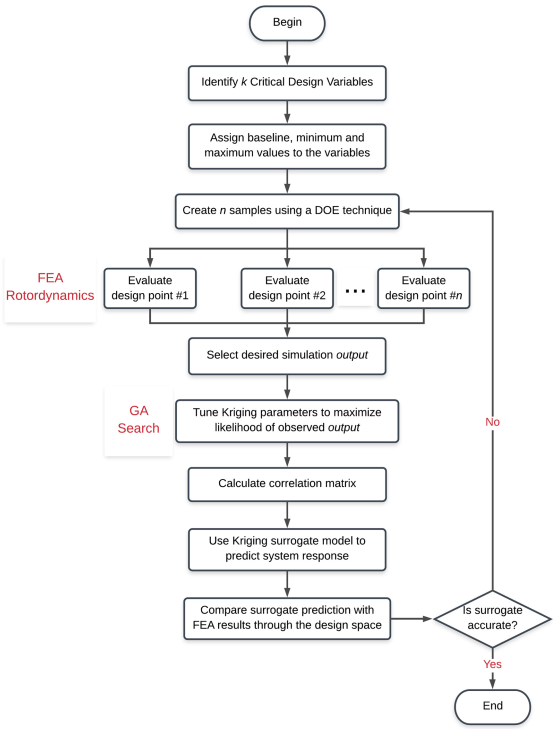

Fig. 4. Kriging method flowchart.

The LHS technique was used to generate the values of input variables, which are evaluated by the FE model. To investigate the effect of sampling size, different training samples, containing 40, 70,100 or 200 design points, were used to construct the surrogates, as suggested by Han *et al.* [23] in his work with bearing parameter identification.

The 1st mode undamped natural frequency and the unbalance response amplitude at the right disk element, both calculated with the rotor spinning at 4,000 rev/min, were used as the observed outputs for the sample data.

3.1 Kriging-Based Surrogate for Eigenfrequency Prediction

Figure 5 shows the tile plot obtained from the Kriging-based surrogate prediction for the system 1st mode undamped natural frequency. Each tile presents a filled contour of the eigenfrequency, in hertz, versus two of the six design variables, keeping the remaining variables at the baseline value. This plot is very useful to understanding how the variables involved impact the evaluated function, in this case the natural frequency.

Clearly, the fluid-film viscosity plays a very important role in changing system natural frequency. In general, high lengths, high viscosities and low radial clearances reduce journal-bearing eccentricity, leading to higher bearing stiffness and damping. The consequence to the rotor-bearing system is increased natural frequencies.

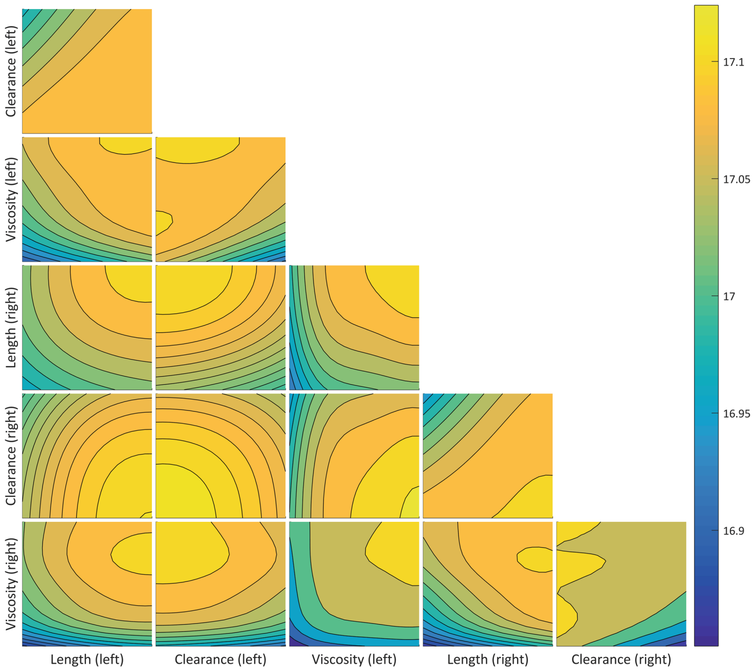

Fig. 5. 1st mode undamped natural frequency landscape, in Hz.

Figures 6 and 7 show the good agreement between Kriging-based surrogate prediction and the result from the FE model. The normalized root mean square errors

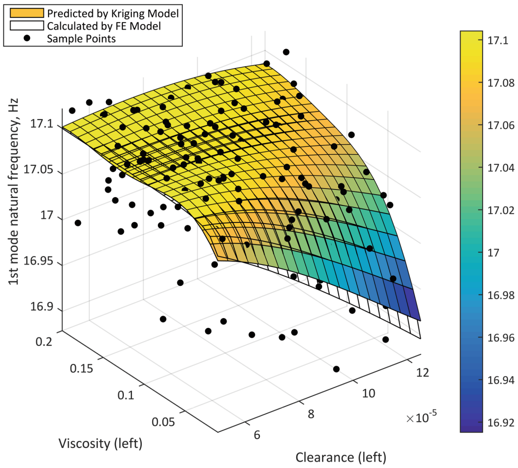

Fig. 6. Left bearing viscosity vs. clearance influence on 1st mode undamped natural frequency. NRMSE = 1.76%.

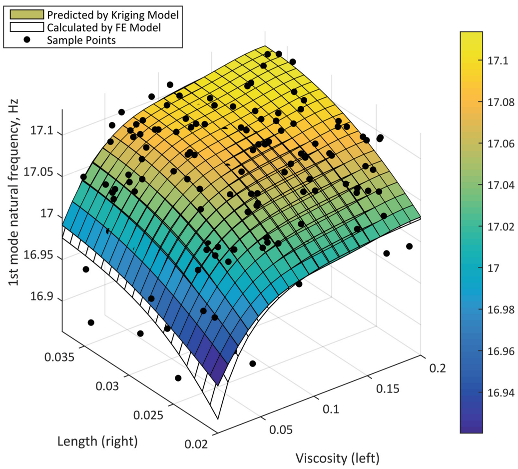

Fig. 7. Right bearing length vs. left bearing viscosity influence on 1st mode undamped natural frequency. NRMSE = 2.39%.

(NRMSE) are 1.76% and 2.39%, respectively. To give an idea of the goodness of fit, a coefficient of determination $R^2 = 0.8$ roughly corresponds to a NRMSE of 10% [17]. Notice the fine resolution of the vertical axis, also indicated in the color bar. The surrogate was trained using 200 design points, which are also displayed.

3.2 Kriging-Based Surrogate for Unbalance Response Prediction

The Kriging-based surrogate prediction for the system unbalance response amplitude at the right disk element is shown in Fig. 8 tile plot. Clearance appears to have great influence in the frequency response.

Figures 9 and 10 show that low radial clearances and high oil-film viscosities increase system response to unbalance, especially due to the stiffening effect observed at the bearings. Although journal-bearing damping is also higher in these conditions, its effectiveness in reducing system unbalance response might be low if the displacement at the bearings location is small. As observed, Kriging model predictions present good correlation with the FE results. The normalized root mean square errors (NRMSE) are 2.36% and 5.81%, respectively. Again, a fine resolution was used to enhance the visualization of the predicted surfaces.

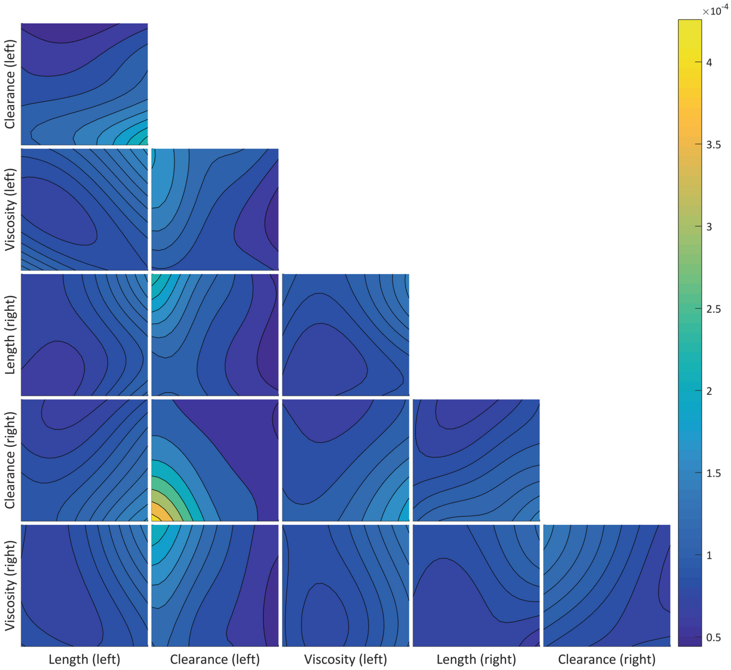

Fig. 8. Unbalance response amplitude at the right disk element landscape, in m.

3.3 Effectiveness of Kriging-Based Surrogate for Rotordynamics

The efficiency and accuracy of Kriging-based surrogate models for rotordynamics were assessed by quantifying the computational effort reduction and the normalized root mean square error (NRMSE) for Kriging predictions when compared with the full FE model predictions.

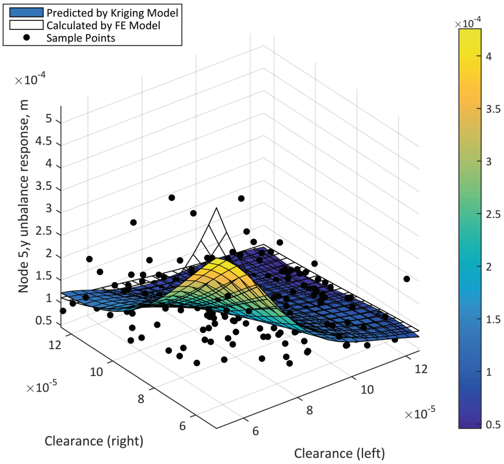

Fig. 9. Right vs. left bearing radial clearance influence on unbalance response amplitude at the right disk element. NRMSE = 2.36%.

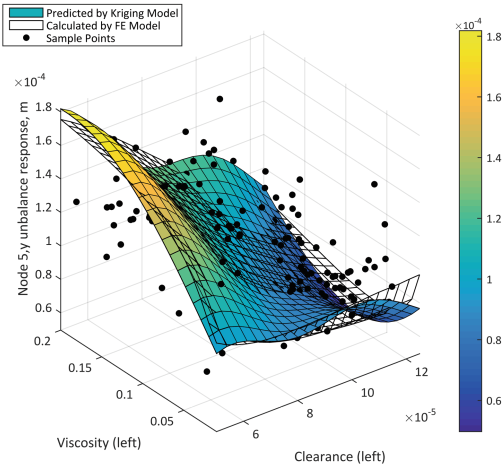

Fig. 10. Left bearing oil-film viscosity vs. left bearing radial clearance influence on unbalance response amplitude at the right disk element. NRMSE = 5.81%.

Figure 11 shows the increasing of time required in the overall training process with the number of training points. It includes sampling, full model simulations and the search for the parameters that maximize the likelihood of the observed data. We observe that time spent on intelligent sampling such as LHS grows exponentially whereas simulation time increases linearly. Thus, depending on the number of design variables and sample size, training the Kriging surrogate model might become costly.

Meanwhile, the great advantage of using a Kriging-based surrogate shows up while using the Kriging correlations to make predictions at new points. Figure 12 brings a

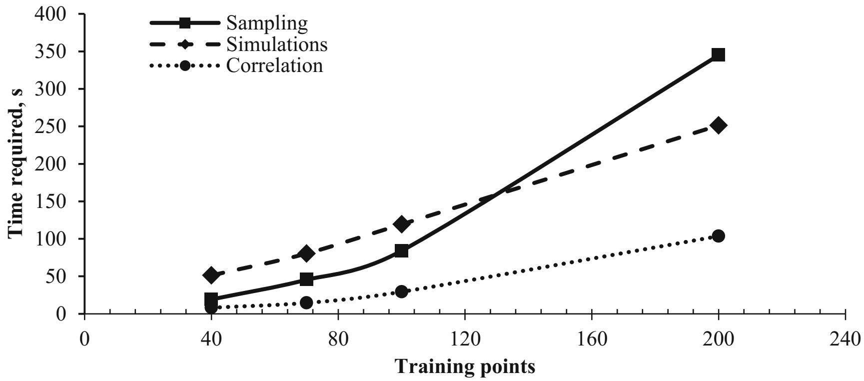

Fig. 11. Time required in the Kriging-based surrogate training.

comparison of the time required by the Kriging model and the FE model to predict the rotor-system response (i.e., eigenfrequency or unbalance response) at a single design point. The result points out that Kriging prediction is three order of magnitude faster than running the full model. Despite the reduction of this advantage with the increase of the training data size (prediction goes through all the training points, i.e., it interpolates the data), the computational effort can be reduced in more than 99% even with large training samples.

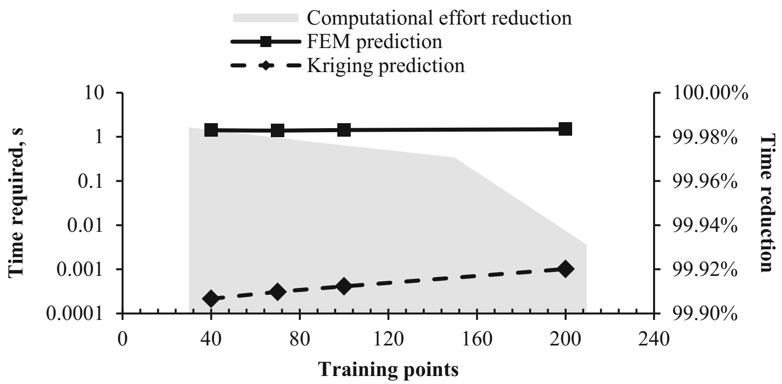

Fig. 12. Comparison of the prediction time using Kriging-based surrogate and the full model.

Regarding the surrogate model accuracy in terms of NRMSE, it is desirable this metric to be as small as possible. Figure 13 shows how Kriging predictions for the 1st undamped natural frequency approximate the FE model results when increasing the initial training data. The NRMSE for all three correlations reached values close to 2% for the training samples with 200 points.

The same is presented in Fig. 14 for the unbalance response at right disk element. The NRMSE was higher but still converged for values below 8% in this case.

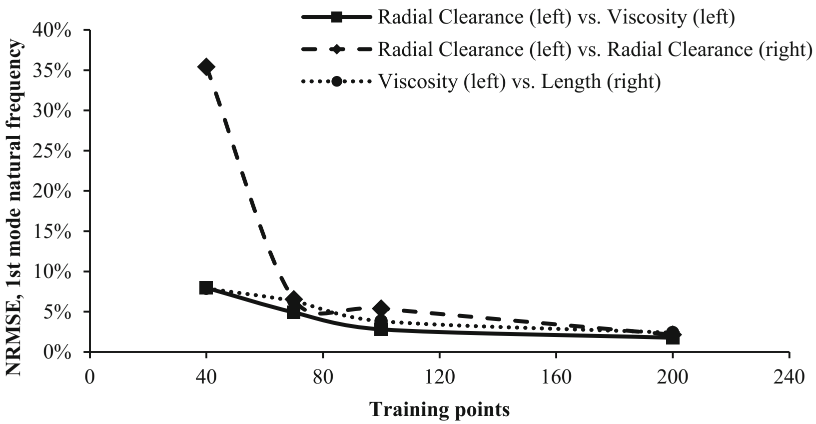

Fig. 13. Normalized root mean square error of the Kriging-based surrogate prediction for the 1st mode undamped natural frequency.

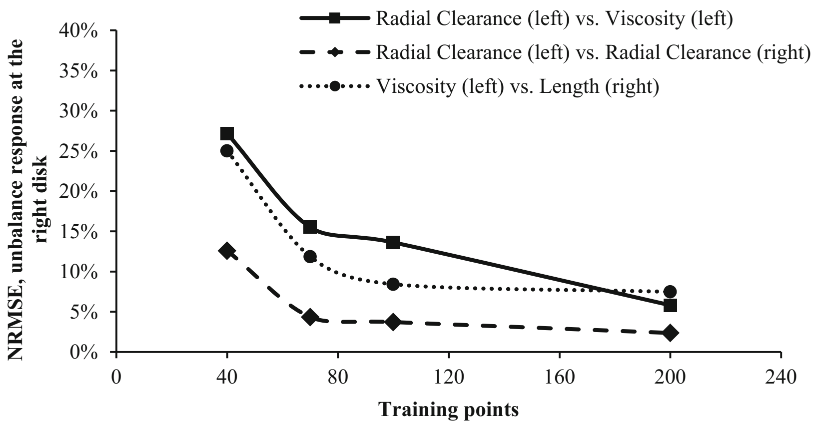

Fig. 14. Normalized root mean square error of the Kriging-based surrogate prediction for the unbalance response amplitude at the right disk element.

4 Conclusion

In this work, the applicability of Kriging-based surrogate modeling for rotordynamics prediction in rotor-bearing systems was assessed. These surrogates are used as substitutes for the rotor-bearing Finite Element model and are capable of quickly predicting responses, thus facilitating the evaluation of different points into the design space.

Kriging surrogate predictions for eigenfrequencies and unbalance response were compared with the FE model using the root mean square error metric for different training samples and the results indicate that the models can accurately predict rotor-bearing system outputs with considerably low computational effort. Ultimately, given Kriging model efficiency in rotordynamics prediction, the results demonstrate the feasibility and effectiveness of the proposed application for purposes such as multi-disciplinary design optimization and uncertainty propagation in rotor-bearing systems.

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