Problems of Rotordynamic Modeling for Built-Up Gas Turbine Rotors with Central Tie Rod Shaft

Konstantin Shaposhnikov[1(✉)] and Chuang Gao[2]

[1] Helan Turbines Co., Ltd., No. 1180 Xingxian Road, Shanghai 201815, People's Republic of China
kvshaposhnikov@helanturbines.com

[2] Shanghai Advanced Research Institute, Chinese Academy of Sciences, No. 99 Haike Road, Pudong New Area, Shanghai 201210, People's Republic of China

Abstract. Tie rod built-up rotor structures are widely used in power machinery for different types of gas turbine engines. Typical tie rod rotor structure consists from several disks and intermediate parts that are tightened together by central tie rod shaft. This type of construction allows assembling together compressor or turbine disks made from high strength materials whose welding is impossible or hard. Another benefit over solid cast rotor of the same size is lighter weight and possibility to replace damaged parts/disks during repair or retrofit. However modeling of this type of rotors is more complicated, time consuming and different from modeling of solid cast rotors or rotors with shrink fit disks/parts, since multiple interfaces between the built-up rotor components can reduce the shaft stiffness significantly. Fine meshed solid models are known to get a very accurate and close value with natural frequencies of real structures, however significant amount of time usually is required to get solution for them and further application of these models for rotordynamic simulations is not convenient. Thus beam models are still widely used, but cautions must be taken when preparing them, since obtained beam rotor model might be much more rigid than the real structure. Current paper is focused on rotordynamic modeling of typical built-up gas turbine rotor with central tie rod shaft. Paper describes a method how to correct beam model in order to achieve a better matching with fine meshed solid model. Described method was further used for rotor modeling of real 2 MW gas turbine rotor. Obtained simulation results were compared with experimental results from modal testing and good agreement was achieved.

Keywords: Built-up rotor · Tie-rod shaft · Rotordynamics · Modal testing
Natural frequencies · Free-free modes

1 Introduction

Tie rod built-up rotors are widely used by engineers for design of rotors working in different industries. Example of built-up rotors with central tie rod shafts can be found in stationary gas turbines used for power generation [1], in aviation and aircraft engines [2, 3], among turbocharger rotors [4]. Application of built-up rotors usually helps

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designers to increase speed of machine manufacturing due to parallel processing of all assembled parts. Work pieces for single disks usually have a quality advantage over the work pieces for solid-forged rotors. Built-up rotors have possibility to have a lighter weight over the same size solid rotors and allow assembling the rotor from components manufactured from materials which are hard to weld or impossible. Another benefit of their application is that built-up rotors are easier for integration of cooling system in structure design. Moor and Lerche pointed out in [5] that multiple interfaces between the built-up rotor components can reduce the shaft stiffness, depending on the interface diameter and design used. API standard recognizes built-up rotors [6] and recommends to approximate joints being an integral piece of metal when creating a rotor model for rotor dynamic simulation, however there is no detail description for rotordynamic modeling of built-up rotors with central tie rod. This brings to difficulties in modeling of such rotors especially on design stage when exact values of connection stiffness between rotor components are not known. Despite of tendency to more and more use of solid finite element models for rotordynamic analysis [7–10], beam models are still widely used [5, 11, 12] due to significant gain in time used for simulation. However cautions must be taken during preparation of beam model since obtain beam rotor model might be much more rigid than the real structure. Books on rotordynamics [13–16] usually give a general guide for rotor modeling: nodal points must be placed at each location along the rotor with the step change in the diameter and each location with inertia disk, bearing, seal and any other source of external disturbance force. These guidelines are well working for heavy industrial cast rotors or rotors with shrunk disks, but bring to difficulties when implementing for modeling of disks in gas turbine engines which are usually integrated in rotor structure and significantly influence on its bending stiffness. In addition complex geometries such as disks used in aircraft engines are hard to model with single layer beam elements. To increase accuracy of modeling Lalanne [17] recommended to divide the disk on more lumped elements with addition of inertia properties for each element. Vance et al. [18] showed another approach of modeling disks which eliminates the need to add concentrated masses: to model the core structure of each disk with beam elements. This method is efficient for modeling of turbomachinery rotors with huge impeller disks. Hence stiffness and inertia properties of impeller disks are provided by element definition. However the bending stiffness of the structure is not able to change as sharp as its diameter, hence choice of inner and outer diameters of rotor elements depends a lot from experience of the user and may influence the obtained result significantly. In such a way application of solid models built from CAD becomes more and more popular, since meshing features in commercial codes make obtained mesh very accurate and accomplish in very little time compared to procedures in beam modeling [19]. Though, significant size of the solid model meshed with lots of elements, and huge memory slot required to store it are the other side of medal. Therefore beam models are still widely used due to their fast speed solution (what is very important on the design stage, when final design is not chosen yet and engineer should perform multiple analysis to develop it), small size and possibility to cooperate with different bearing codes. Current paper is focused on rotordynamic modeling of typical built-up gas turbine rotor with central tie rod shaft. Paper also describes a method how to correct beam model in order to achieve a better matching with fine meshed solid model.

2 Numerical Simulation

2.1 Modeling of Built-Up Rotors with Central Tie Rod Shaft

Before performing rotordynamic analysis of the engine its rotor model should be created and verified. As mentioned by Vance and Murphy [20] identification of rotor free-free modes using modal analysis is an excellent way of checking the accuracy of the mass-elastic model without involving uncertainties in the bearing parameters. For prediction of rotor free-free modes the problem is reduced to eigenvalue problem:

$$[\mathrm{K} - \lambda \mathrm{M}]\{\mathrm{u}\} = 0, \tag{1}$$

where $\lambda = \omega_i^2$ – are eigenvalues (i = 1, 2…, n); ω_i – are natural frequencies; $\{\mathrm{u}\}$ – eigenvector; $[\mathrm{K}] = [\mathrm{K_s}] + [\mathrm{K_{bear}}]$ – rotor stiffness matrix, which consists from shaft and bearing stiffness matrix; $[\mathrm{M}] = \left[\mathrm{M}_T^s + \mathrm{M}_R^s\right] + \left[\mathrm{M}_T^d + \mathrm{M}_R^d\right]$ – rotor mass matrix which includes translational mass and rotational inertia matrices for shaft and disk components. Since the structure is assumed to be freely supported stiffness matrix is positive singular matrix where some eigenvalues become zero and are associated with rigid body modes, while the others are positive and associated with bending modes.

Simplified model of overhang gas turbine rotor with central tie rod shaft was used for study in the current paper, Fig. 1. For simplicity blades were not taken into account. In the same manner, for simplification all components for model were set to be made of steel (ρ = 7800 kg/m^3, E = 2.0E+11 N/m^2, G = 7.7E+10 N/m^2). Spline joint was assumed to be used for coupling shaft connection and Curvic coupling joints for shaft-turbine disk connection and disks connection. Assembled condition is achieved with help of central tie rod shaft which has 3 steps and goes through the main shaft. Front nut is tightened from the coupling side to implement tie rod shaft pretension. Model was prepared from 3D geometry in ANSYS using solid elements (Solid 186), Fig. 2.

The main difficulty for creation of this type of model consists from correct understanding of applied boundary conditions between rotor components and parts. Some of these values are hard to obtain experimentally, but they have significant

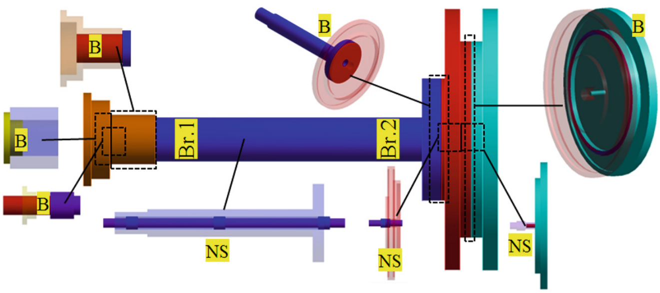

Fig. 1. Model of overhang built-up gas turbine rotor with central tie rod shaft with highlighted boundary conditions used for simulation in ANSYS: B-bounded; NS-no separation

influence on rotor free-free bending mode natural frequencies and mode shapes. On the design stage for the rotor which is assumed to be well assembled two types of contact connections can be used in ANSYS: bounded contact and no separation contact. Bounded contact is a contact when two contacting surfaces are not allowed to separate (assumed to be glued) or slide. No separation is similar to bounded type of connection, but connected parts are allowed to slide slightly.

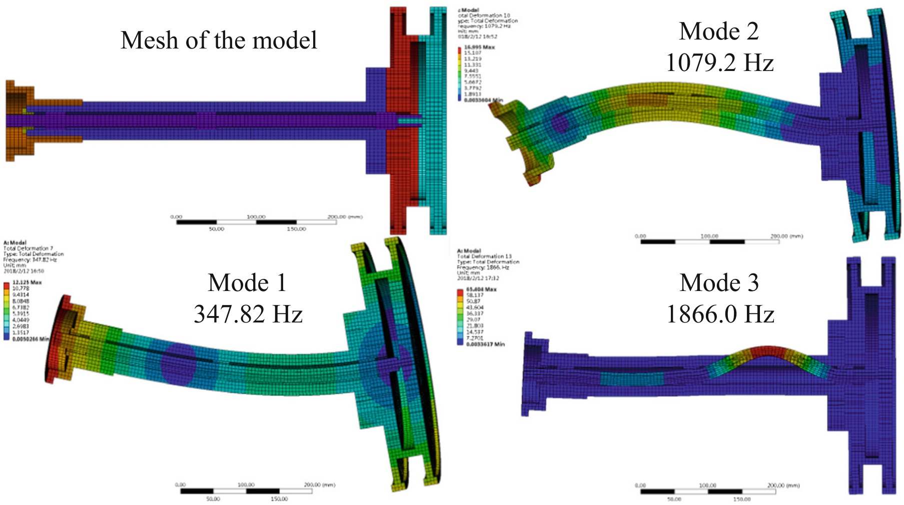

Fig. 2. Simulation results for solid rotor model – free-free bending modes

In addition, several beam models for the same type of rotor were prepared in XLRotor rotordynamic code for comparison with solid model. Consistent mass option was used for formulation of mass matrix of each model and solver based on finite element method was used for eigenanalysis. Model No. 1 is a multi-level model where each component (coupling, shaft, tie rod shaft, turbine disks) were modeled as separate parts connected by user-defined spring elements with 16 input columns: 4 translational (K_T) and 4 rotational stiffness (K_R). Damping was set to zero. Connection elements were highlighted in Fig. 3. For initial model high translational (1E+12 N/mm) and rotational (1E+12 N-mm/rad) stiffness were set. Inner and outer diameters for the beams were set to follow component geometry obtained from CAD. Model No. 2 is a 2 level model where coupling, shaft and turbine disks were modeled as one structure, assuming that joints between parts are stiff and approximated as integral piece of metal as recommended in [6]. Tie rod shaft was modeled as separate rotor connected with the main shaft using the same user-defined spring elements. Model 3 is a single level model built using beam layers. During eigenvalue and response calculations XLRotor automatically merges layers for beams which are at the same station [21]. Element number used for this model is twice less than for Model No. 1. Comparison of mass properties for all obtained models is shown in Table 1. All models had mass properties close to exact 3D model. Obtained simulation results for first 3 ordinal free-free bending modes for all beam models were summarized in Fig. 4.

Table 1. Mass properties for model

3D model	Solid model (51294 elements), kg	Beam models, kg		
		Model No. 1 (101 elements)	Model No. 2 (92 elements)	Model No. 3 (53 elements)
44.193	44.193	44.172	44.172	44.172

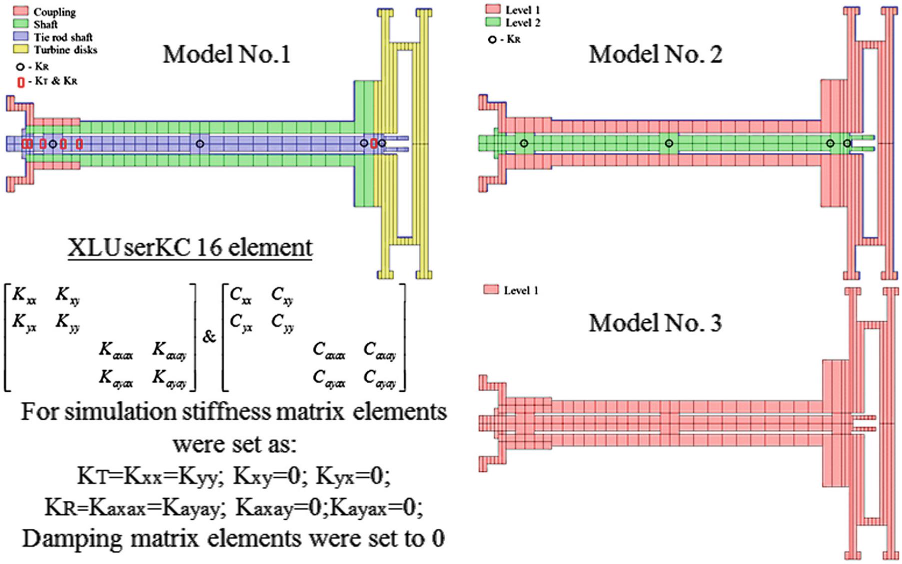

Fig. 3. Beam models for overhang built-up gas turbine rotor with central tie rod shaft

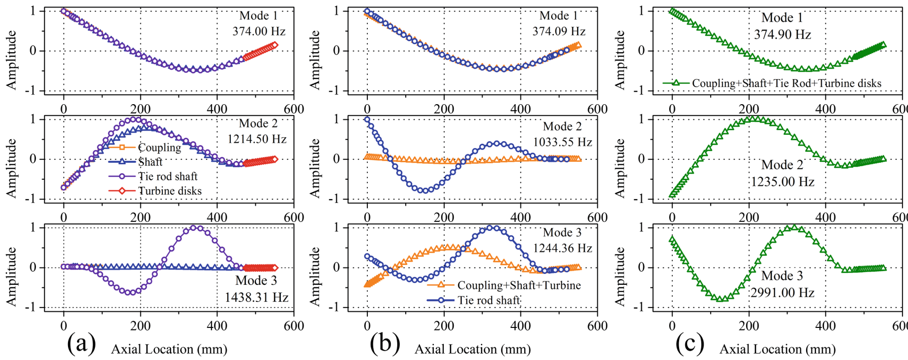

Fig. 4. Simulation results for beam models - free-free bending modes: (a) Model No. 1; (b) Model No. 2; (c) Model No. 3

Simulation results revealed that only for Model No. 1 all 3 ordinal bending modes, Fig. 4(a), were close with solid model results, Fig. 2. Model No. 2 and No. 3 were able to repeat first and second shaft bending modes but failed to present tie rod shaft bending mode. In addition they had shown much higher natural frequency in comparison with Model No. 1. Hence modeling of the structure using beam layers should be performed with caution and requires experience. When components are modeled as integral piece, beam element properties (OD and ID) should be set reasonably in order not to obtain much rigid structure. Model No. 1 in this case has an advantage over the other beam models described in this paper. Although natural frequencies for it also didn't match well with solid model (mode 1 difference - $\Delta = 7.52\%$, mode 2 - $\Delta = 12.54\%$, mode 3 - $\Delta = 22.92\%$), a good matching in mode shapes was reached. For natural frequencies a better matching can be achieved by setting the connection stiffness between model components and method for its implementation will be described in next section of this paper.

2.2 Method for Correction of Beam Model for Built-Up Rotor with Central Tie Rod Shaft

During free-free modal testing structure is freely supported and its bearing stiffness are set to zero, but internal connections between rotor components exist and influence on $[K_{bear}]$ component of matrix [K] in Eq. (1). Rotor structure in Model No. 1 has only 10 connection elements but more complicated models may have much more internal connections, hence the question is how to understand which connection should be chosen and set to get a better matching for exact rotor mode shape.

The answer could be found with help of vibration energy analysis. Industrial application of energy distribution method in lateral analysis for rotor modeling was described by Gunter and Gaston [22]. Various forms of energy, work and their contribution to the dynamics of the system were also described by Chen [23]. Vibration energy analysis is currently integrated in all advanced rotordynamic commercial codes [15]. For this purpose the system damping is neglected and supports are considered as isotropic. Obtained shaft whirl modes are circular and energy distribution remains constant through the orbit. Since the amplitude of a free whirl mode is arbitrary, energy distribution is usually displayed as percentage of total energy of the system for the mode of interest. Potential energy of rotating shaft element in matrix form could be written as:

$$\mathrm{V} = \frac{1}{2} q^T \left(K_b + K_\beta + K_a\right) q, \tag{2}$$

where q – is the displacement vector for rotor elements; K_b – bending stiffness; K_β – shear stiffness; K_a – geometric stiffness due to axial force. Ehrich pointed out in [13] that energy maps are very useful on the design stage, especially in preliminary design, but they also can be used for model refining to increase its accuracy and get better matching with experiment when rotor model is going to be prepared for further rotordynamic analysis. Evaluation of rotor potential energy distribution for Model No. 1 revealed that modes 1–2 were pure shaft bending modes since more than 90% of energy was concentrated on the shaft, Fig. 5(a). However small concentration of potential energy for turbine disks - shaft connection element brought to conclusion that natural frequency of these modes can be changed by decreasing the stiffness for identified dominate element. Reduction of rotational stiffness for turbine disks - shaft connection element can help to achieve better matching between beam and solid model. When rotational stiffness was reduced to 5E+10 N-mm/rad simulation results can match closer with experiment: Mode 1 - Δ = 0.08%; Mode 2 - Δ = 0.09%, Fig. 5(c).

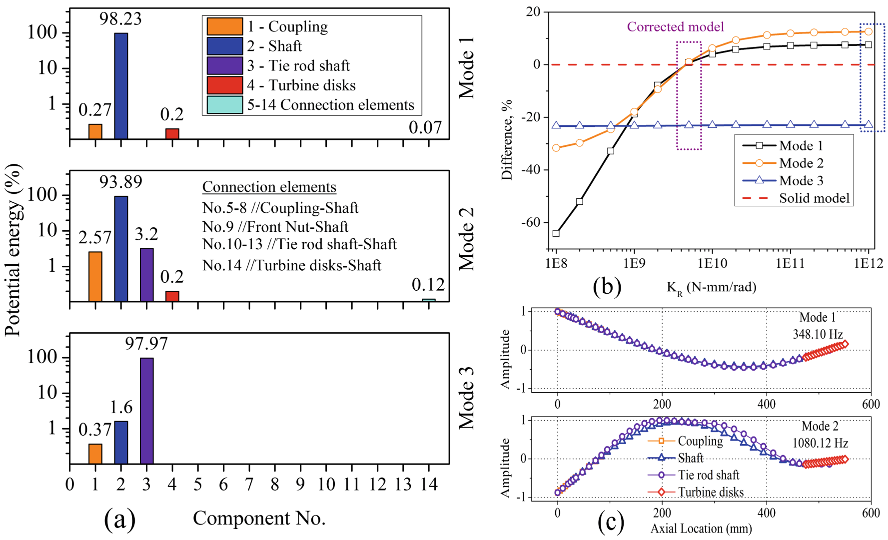

Fig. 5. (a) Potential energy map for initial Model No. 1 (b) Influence of rotational stiffness used for shaft-turbine connection on natural frequencies of beam model (c) Modes 1–2 mode shapes and natural frequencies for corrected beam model

Meanwhile for mode 3 most of potential energy was concentrated on the tie rod shaft and inspection of the plot in Fig. 5(a) from the first glance didn't bring to conclusion which connection was dominant for this mode. However comparison of the mode shapes obtained for solid and beam models revealed that tie rod central step in solid model had much larger area of contact surface with the rotor shaft. As a results additional connection element with only translational stiffness K_T was added to the beam model in area of tie rod central step. Influence of translational stiffness for added connection element on natural frequencies of mode 3 is shown in Fig. 6. A better matching with solid model was achieved when translational stiffness for additional connection element at the central step of tie rod shaft was increased to K_T = 1.7E+07 N/mm. Influence of this connection element on mode 3 natural frequency is clear from potential energy map obtained for rotor model when all bending modes were corrected to be close to solid model result, Fig. 6(c). Comparison of the obtained simulation results for initial (Model No. 1), corrected beam model and solid model was summarized in form of Table 2.

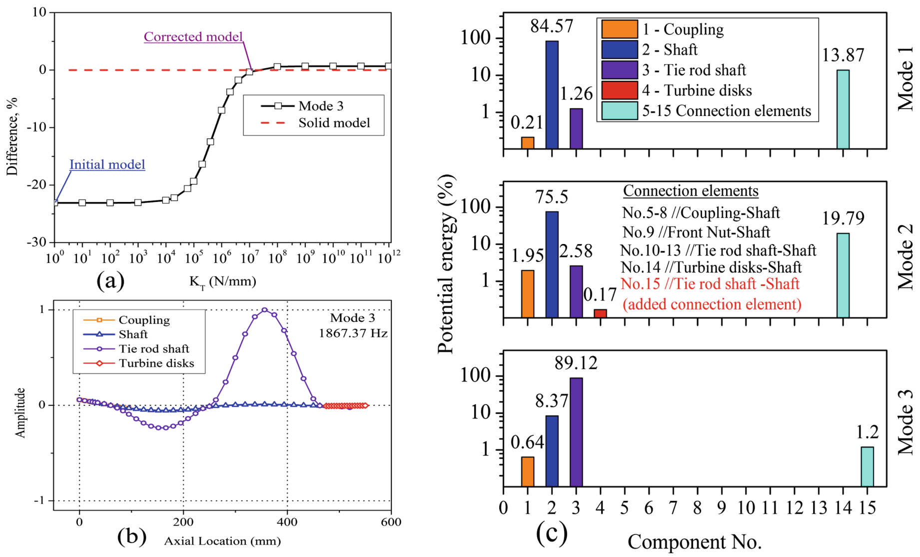

Fig. 6. (a) Influence of translational stiffness for added connection element on mode 3 for beam model; (b) Mode 3 mode shape and natural frequency for corrected beam model; (c) Potential energy map for corrected beam model

Table 2. Comparison of natural frequencies for solid and beam models

Mode No.	Natural frequency, Hz		
	Solid model	Model No. 1 (initial)	Model No. 1 (corrected)
1	347.82	374.00 (Δ = 7.53%)	348.10 (Δ = 0.08%)
2	1079.20	1214.50 (Δ = 12.54%)	1080.12 (Δ = 0.09%)
3	1866.00	1438.31 (Δ = 22.92%)	1867.37 (Δ = 0.07%)
Average difference, %		Δ = 14.33%	Δ = 0.08%

2.3 Influence of Axial Load on Natural Frequencies of Built-Up Rotor with Central Tie Rod

Tightening of the front nut for tie rod shaft helps to achieve assembled condition for the built-up rotor described in Sect. 2.1, Fig. 1. At the same time it brings to appearance of tension load in the tie rod shaft. Importance of bolt pretension incorporation in simulations for assembled built-up rotors was highlighted in [5, 11, 15, 24]. Influence of pre-tightening force on modal parameters for simplified rod-fastened rotors was also confirmed by experimental modal testing in [25].

In the current paper effect of axial load on natural frequencies for free-free bending modes was also studied for built-up rotor with central tie rod shaft on the base of beam corrected model described in Sect. 2.2. Based on ISO 898 [26] the minimum ultimate tensile load for M20 thread diameter of the tie rod shaft was identified to be 299 kN (for 12.8 property class). Pretension force was applied on beam model of tie rod shaft in form of axial load. At the same time equal value of compression load was applied on rotor components which were held together by tie rod shaft. Simulation results revealed that increase of pretension force applied on tie rod shaft in comparison with non-loaded condition had a minor effect on modes 1–2, but on mode 3 natural frequencies influence was considerable, Fig. 7. With maximum applied pretension force natural frequencies: for mode 1 – decreased on 0.07% (due to compression load on the shaft), for mode 2 – increased on 0.05%, for mode 3 – increased on 10.2%. Influence of increase of pretension force on natural frequencies for modes 1–2 in comparison with mode 3 was smaller, since modes 1–2 were identified as full structure bending modes, Fig. 4(a), while mode 3 was a tie rod shaft bending mode. Thus, received simulation results confirmed necessity to include axial load from bolt pretension for assembled structures when preparing rotor model for rotordynamic simulations, since neglect of it may lead to additional decrease of accuracy of obtained model when it is going to be compared with experimental results from modal testing, especially for bending modes associated with tie rod shaft.

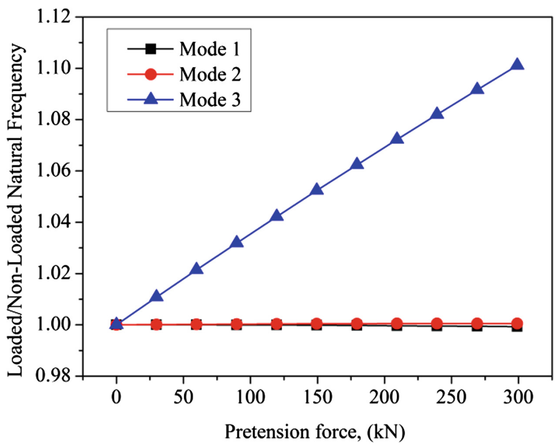

Fig. 7. Influence of applied pretension force on natural frequencies for first 3 ordinal free-free bending modes of built-up rotor with central tie rod shaft

3 Experimental Results

3.1 Modal Testing for Components of Built-Up Rotor with Central Tie Rod Shaft

Experimental modal testing was performed for components (tie rod shaft and rotor shaft) of real 2 MW built-up gas turbine rotor of ZK2000 engine in order to identify structure free-free bending modes and to compare them with simulation results. Both shafts were hanged using knitted ropes. Schematic view of used data acquisition system for modal testing is shown in Fig. 8. For every shaft 5 acceleration probes (Type 8774B050A Kistler) were used for measurements. Probes location is shown in Fig. 9. DASP software was used for processing of experiment results. Impact testing helped to get frequency response functions (FRF) for each rotor shaft. Mode shapes were obtained from FRF by measuring the peak amplitudes for its imaginary parts.

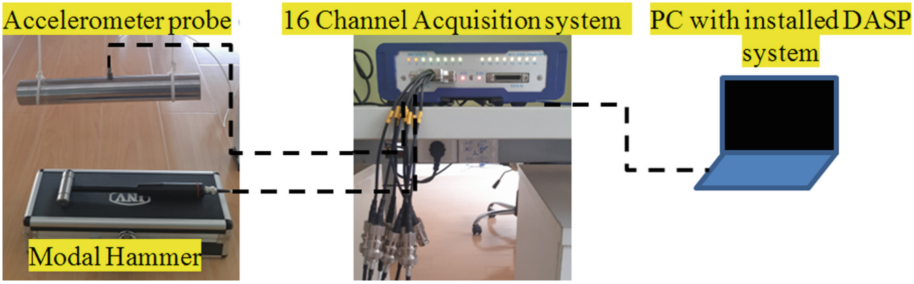

Fig. 8. Scheme of data acquisition system used for modal testing

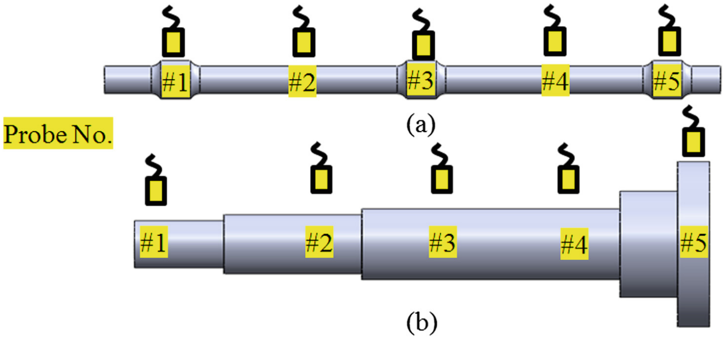

Fig. 9. Schematic view on probes location for components of built-up 2 MW gas turbine rotor: (a) Tie rod shaft; (b) Rotor shaft

Obtained experimentally mode shapes for the first 3 ordinal free-free bending modes are shown in Fig. 10, and were in agreement with obtained simulation results from beam models. Due to company regulations exact values of natural frequencies could not be presented in the paper, but difference for obtained simulation results with experimental was summarized in Table 3. Results revealed that for the first 3 ordinal free-free bending modes average difference between simulation and experiment for tie rod rotor shaft beam model was about $\Delta = 1\%$. For the rotor shaft beam model $\Delta = 1.3\%$.

Table 3. Comparison of simulation results with experiment for beam models of components of built-up 2 MW gas turbine rotor

Mode No.	Difference with experiment Δ, %	
	Tie rod shaft	Rotor shaft
1	−1.41	−0.60
2	−1.17	0.96
3	−0.53	2.23
Average Δ, %	1.04	1.26

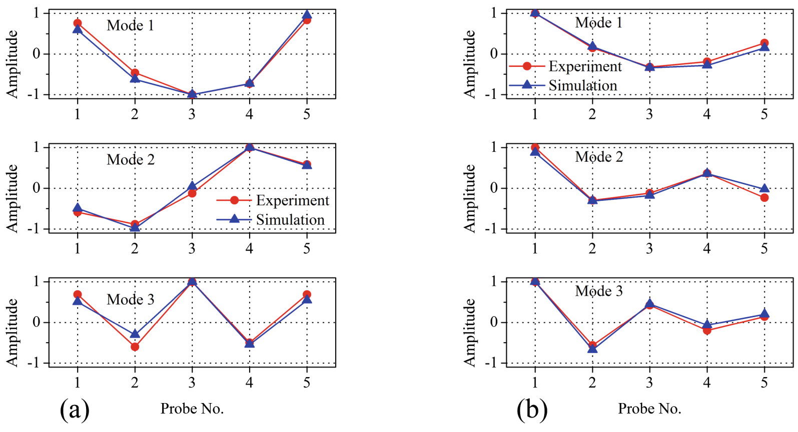

Fig. 10. Mode shapes comparison – experiment vs. simulation: (a) Tie rod shaft; (b) Rotor shaft

3.2 Modal Testing for Assembled Built-Up Rotor with Central Tie Rod Shaft

In the same manner as in previous section experimental modal testing was performed for assembled built-up gas turbine rotor of ZK2000 engine. The rotor was hung using single rope. Rope was girding the rotor in the zone between compressor-impeller and turbine disks, close to its center of gravity, what made the shaft silent and balanced during experimental modal testing. Eleven acceleration probes were used for measurements, Fig. 11. Impact excitation with modal hammer and measurements were performed in horizontal plane in order to reduce influence of the rope on natural frequencies.

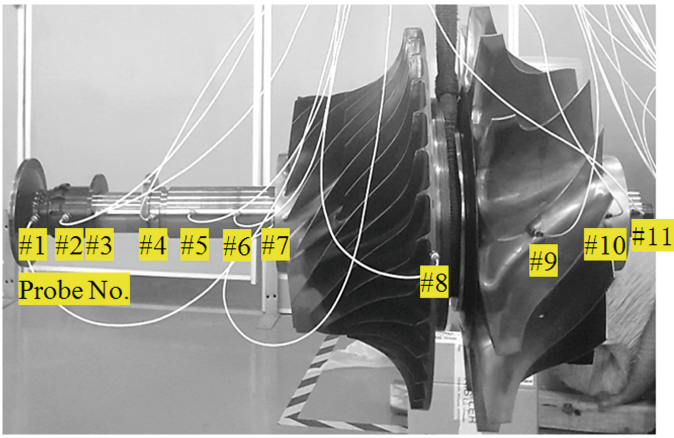

Fig. 11. Probes location for modal testing of built-up gas turbine rotor of ZK2000 engine

Obtained experimentally mode shapes for the first 3 ordinal free-free bending modes are shown in Fig. 12(a). Beam model for assembled built-up rotor of ZK2000 with included pretension load at tie rod shaft was also prepared. In general obtained by simulation mode shapes were in agreement with experimental results, Fig. 12(b). However due to complexity of the assembled rotor consisting from multiple parts and substitution of continuous structure on model with discrete elements with connections between components, matching was worse in comparison with results obtained for modal testing of single components, Fig. 10. In addition, natural frequencies of the initially constructed beam model didn't match easily, with average difference Δ = 12% with experiment for the first 3 ordinal modes. However using the method described in Sect. 2, identifying dominate connection elements for each mode of interest beam model for built-up rotor was corrected and better matching was obtained with average difference Δ = 1.25% for the first 3 ordinal modes, Table 4.

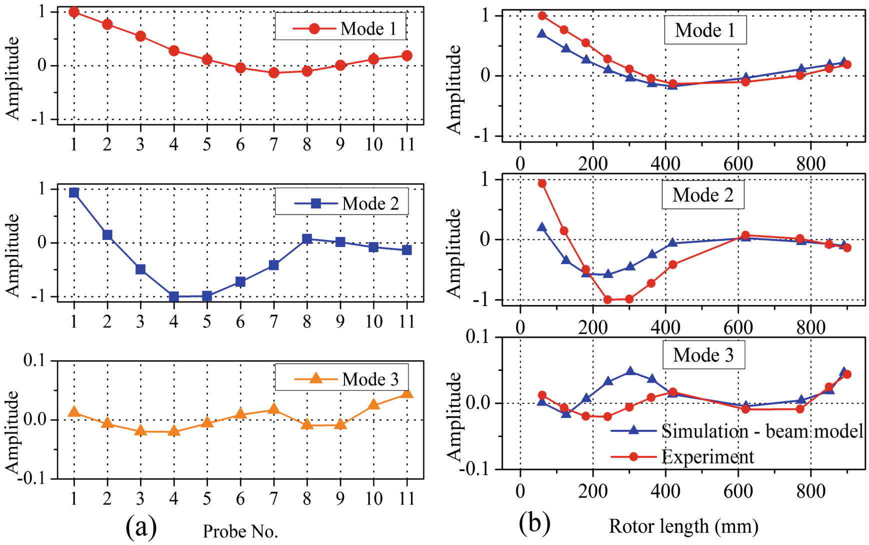

Fig. 12. (a) Experimentally obtained mode shapes for ZK2000 rotor (b) Comparison of simulation results for beam model of ZK2000 rotor with experiment

Table 4. Comparison of simulation results with experiment for different beam models of built-up 2 MW gas turbine rotor

Mode No.	Difference with experiment Δ, %	
	Initial beam model	Corrected beam model
1	12.69	1.36
2	10.56	−1.09
3	−13.15	1.32
Average Δ, %	12.13	1.25

4 Conclusion

It can be summed up in the conclusion:

- Existence of multiple interface surfaces inside built-up rotors can significantly reduce bending stiffness of the assembled rotor structure, what makes rotordynamic modeling of such rotors more complicated and time consuming. Application of modeling method when components are modeled as integral piece using beam elements and method when components are modeled using beam layers requires experience and should be performed with caution. When beam element properties (OD and ID) are not set reasonably, much rigid structure can be obtained. Solid models obtained from CAD and meshed in commercial codes can help to obtain a very accurate and close with experiment result. In this case for creation of beam models, when experimental results are not available, solid models can be used as a reference;
- Rotor model built with multi-level method where components are connected with user-defined elements is usually more convenient for further rotordynamic analysis, since tuning of connection stiffness between components can help to get better matching with experimental results obtained from modal testing. However, when connection stiffness between rotor components are not properly set model can have significant difference both with experiment and solid model;
- Inspection of potential energy maps for beam model of assembled rotor structure for each mode of interest can point out which connection element has a dominate influence on certain mode. Changing connection stiffness in dominate connection element can help to get a better matching with experimental results for certain mode;
- Simulation results revealed that increase of pretension force applied on tie rod shaft in comparison with non-loaded condition brings to increase of natural frequencies for tie rod shaft mode and confirmed necessity to include axial load from bolt pretension for assembled structures when preparing rotor model for rotordynamic simulations. Bending modes of the rotor structure may decrease slightly due to negative axial load from compression when assembled condition is modeled;
- Described in paper method for model tuning using inspection of potential energy maps was successfully implemented for creation of beam model for real gas turbine rotor. Further correction of the beam model helped to get good agreement with experimentally obtained mode shape and reduce difference in natural frequencies with experiment for numerical model to almost 1%;
- Performed experimental modal testing had shown that for single rotor parts like rotor shaft and tie rod shaft very close matching in natural frequencies for free-free bending modes between simulation and experimental results can be achieved for initially created beam models. Once the rotor is assembled, connection stiffness between rotor components and parts may influence the results matching significantly both for mode shapes and natural frequencies. Thus initially created beam model could have difference with experiment and will require further correction.

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