John Nash, His Life

Sylvia Nasar

Abstract The life and work of John Forbes Nash, Jr.

A few years ago another journalist and I went to St. Petersburg to track down the Russian mathematician who had solved the Poincare Conjecture. Described in the media as a hermit with wild hair and long nails, Grigori Perelman had dropped out of the mathematics community, and given every indication of intending to turn down a Fields medal. His extraordinary decision to refuse the ne plus ultra of honors for a young mathematician—and a Chinese-American rival's attempt to claim credit for solving the 200-year-old problem—was a terrific story... but only if we could find Perelman and convince him to talk to us.

After four frustrating days of searching St. Petersburg we had found no one who had seen Perelman in years or had any clue to his whereabouts. The notes we left outside what we thought might be his apartment remained untouched. A neighbor told us that she had never seen the flat's occupant. But then, by chance, after we had given up, we stumbled onto his mother's apartment... A moment or two later, I was introducing myself to the alleged "hermit," a scholarly looking, youngish man neatly dressed in a sports jacket and Italian loafers. We had apparently interrupted him while he was watching a soccer match on big TV.

I started to say that we were doing a piece for the *New Yorker* magazine when Perelman interrupted: "You're a writer?" he asked in flawless English. "I didn't read the book, but I saw the movie with Russell Crowe."

> I shall not look upon his like again.
> Hamlet, Act 1, Scene 2

A father once asked me after a talk if John Nash's life was more important than that of his son who also suffered from schizophrenia. Of course not, I answered. But

S. Nasar (✉)
Graduate School of Journalism, Columbia University, New York, NY, USA
e-mail: szn1@columbia.edu

H. Holden, R. Piene (eds.), *The Abel Prize 2013–2017*,
https://doi.org/10.1007/978-3-319-99028-6_17

some lives resonate more, touch more of us. John Nash's life was one of these partly because it was so many things: a drama about the mystery of the human mind, an epic of a creative genius, a tale of triumph over incredible adversity, and, not least, a love story.

At one point in the movie, when it looked as if things were all over for Nash, his wife Alicia took his hand, placed it over her heart, and said, "I have to believe that something extraordinary is possible."

Something extraordinary was possible.

Those of you who are mathematicians have probably studied or used one of Nash's stunning contributions to mathematics. I'm going to tell you about the man. Not, almost certainly, what he would have said about himself had he lived to write an autobiographical essay, but some of the things I learned, first, as a New York Times reporter, then, his un-authorized biographer, and, later, simply as a friend.

Before I studied economics, I majored in literature. Starting with the myths of Icarus and Faust, there are many, many stories about the meteoric rise and equally meteoric fall of a remarkable individual. There are very few stories—much less true ones—with a genuine third act. But Nash's life had such a third act.

That third act drew me to his story in the first place. In the early 1990s at the Times, I heard a rumor that a mad mathematician at Princeton University was probably on a short list for a Nobel prize in economics. Nash was hardly a household name, but everyone who had studied economics, as I did in graduate school, was familiar with game theory and the so-called "Nash equilibrium."

Two or three phone calls later, I had learned that by the time he was 30 years old, Nash was a celebrity in the rarified world of mathematics. As a brilliant student at Princeton in the late 1940s, and a rising star on the MIT faculty in the 1950s, before he had succumbed to the most devastating of mental illnesses, he made major contributions not only in game theory for which he would one day win a Nobel, but to several branches of pure mathematics.

Over the next three decades, the ideas Nash had when he was in his twenties had become influential in disciplines as disparate as economics and biology, algebraic geometry and partial differential equations. But, Nash, the man, was all but forgotten.

Generations of students at Princeton University knew him only as the Phantom of Fine Hall, a silent, ghost-like figure who left mysterious messages on the blackboards of Fine Hall. A lot of people like me who knew of Nash's work simply assumed that he had died long ago.

I was naturally intrigued to learn that Nash was alive, apparently recovered from a disease widely considered incurable, and possibly soon to be the recipient of the ultimate intellectual honor. That someone who had been lost for so long could be found again—that someone who had fallen so far could come back—struck me as incredible, something plucked from a fairy tale, a Greek myth, or a Shakespeare tragedy.

> He was a man. Take him as all in all

John with sister Martha circa 1939. (Courtesy of Martha Nash Legg and John D. Stier)

Act One of Nash's life is the story of creative genius. John Forbes Nash Jr. was born in Bluefield in West Virginia coal country on the eve of the Great Depression. He was a peculiar, solitary, precocious child. Other children called him Bug Brains. He amused himself in un-childlike ways. At 10, he was doing sophisticated chemical experiments and tricking other children with electrical shocks. At 15, he was building pipe bombs. . . and simultaneously re-proving classical theorems by great mathematicians of the past such as Fermat and Gauss.

The summer that World War II ended, the 16 year old Nash went off to Carnegie Tech in Pittsburgh, Pennsylvania to become an engineer like his father. Within months, his professors spotted him as "a young Gauss"—a mathematical prodigy of extraordinary promise.

Three years later they sent him off to Princeton with what was likely the shortest letter of recommendation in the university's history. It consisted of a single line: "This man is a genius."

By the late 1940s, Princeton had become home to the popes of Twentieth-century science: Albert Einstein, Kurt Goedel, Robert Oppenheimer, John von Neumann. A classmate of Nash's, the mathematician John Milnor, recalled, "The notion was that the human mind could accomplish anything with mathematical ideas."

Nash attracted attention as soon as he landed at the center of the mathematical universe. "Genius" was not then the overused term that it has since become. The old Webster's Dictionary defined genius as "transcendent mental superiority," but added that such superiority had to be of a "peculiar, distinctive or identifying character."

At 19, Nash was conspicuous for his movie star looks and his Olympian manner. Over 6 ft tall and heavily muscled, he spoke in a soft southern drawl.

His manners and dress were also southern, slightly formal. But his classmates considered him “weird” “haughty” “spooky.” He wore his fingernails unusually long. His conversation had a stilted, ornamental quality. He avoided classes as a matter of principle. He rarely opened a book, telling classmates that he did not wish to endanger his originality. On the few occasions when he was spotted in the Fine Hall library, he would be lying on one of the tables, his arms folded behind his head, staring up at the ceiling.

Like the Cambridge mathematician GH Hardy, Nash thought of mathematics as a ferociously competitive sport. “I imagine that by now you are indeed used to miscalculation,” sneers the Russell Crowe’s character to a rival. “What if you never come up with your original idea? What if you lose?,” says the other man as he beats Nash at Go. For Nash, who craved recognition, mathematics was about winning. He wasn’t alone either. “Competitiveness, It was sort of like breathing,” another graduate student told me. “We thrived on it.” Nash may have skipped lectures, but he never missed afternoon tea. That’s where the graduate students and professors played Kriegspiel and Go and traded put downs and mathematical gossip. “Trivial” was Nash’s pet putdown. “Hacker” was another. Ranking students and professors—with himself in the Number One spot—was a favorite pastime. He was by no means a brilliant chess player, only an unusually aggressive one. “He managed not just to overwhelm me but to destroy me by pretending to have made a mistake,” recalled a man who had made the mistake of challenging Nash to a game.

Outside of the common room, Nash was always pacing. Always whistling Bach. Or riding a bicycle peremptorily commandeered from one of the racks outside the graduate students’ residence in tight, concentric circles. Always, it seemed, he was working inside his own head. Lloyd Shapley, a game theoriest and friendly rival of Nash’s at Princeton who won a Nobel in 2012, admitted, “He was obnoxious, immature, a brat. What redeemed him was a keen, logical, beautiful mind.”

His ambition was awesome. Milnor, a freshman the year that Nash entered the Ph.D. program, ‘It was as if he wanted to rediscover, for himself, 300 years of mathematics.’ Always on the lookout for a straight line to fame, Nash would corner visiting lecturers, clipboard and writing pad in hand. “He was very much aware of unsolved problems,” said Milnor. “He really cross-examined people.”

But he was also bursting with his own ideas. Norman Steenrod, Nash’s faculty adviser, recalled:

“During his first year of graduate work, he presented me with a characterization of a simple closed curve in the plane. This was essentially the same one given by Wilder in 1932. Some time later he devised a system of axioms for topology based on the primitive concept of connectedness. I was able to refer him to papers by Wallace. During his second year, he showed me a definition of a new kind of homology group which proved to be the same as the Reidemeister group based on homotopy chains.”

One afternoon during Nash’s first term at Princeton, John von Neumann, the great, the Hungarian polymath best known as a father of the atomic bomb and the digital computer, was in the common room when he noticed two students hunched over a rhombus covered in hexagons and black and white go stones . “What they were playing, he asked a colleague?" "Nash," came the answer, "Nash." Parker Bros. later called Nash's nifty game, which was invented independently by the Danish mathematician and poet Piet Hein, "Hex."

Nash proved a beautiful and surprising theorem showing that the player who makes the first move can always win. But his own story proves that in real life—as opposed to the game—outcomes aren't necessarily determined by the first move, or the second, or even the 50th.

Rebecca West, the English novelist and lover of H. G. Wells, once described genius as "the abnormal justifying itself." Excluded and isolated the genius tries to win acceptance, she speculated, by "some magnificent act of creation." For John Nash several such magnificent acts were to follow before the curtain fell.

Nash's playful foray into mathematical games foreshadowed a far more serious involvement in a novel branch of mathematics. Today, the language of game theory permeates the social sciences. In 1948, game theory was brand-new and very much in the air at Princeton's Fine Hall.

The notion that games could be used to analyze strategic thinking has a long history. Such games as Kriegspiel, a form of blind chess, were used to train Prussian officers. And renowned mathematicians like Emile Borel, Ernst Zermelo, and Hugo Steinhaus studied parlor games to derive novel mathematical insights. The first formal attempt to create a theory of games was von Neumann's 1928 article, "Zur Theorie der Gesellschaftsspiele," in which he developed the concept of strategic interdependence.

But game theory as a basic paradigm for studying decision making in situations where one actor's best options depend on what others do did not come into its own until World War II when the British navy used it to improve its hit rate in the campaign against German submarines. Social scientists discovered it in 1944 when von Neumann and the Princeton economist Oskar Morgenstern published their masterpiece, *Theory of Games and Economic Behavior*, the first attempt to derive logical and mathematical rules about social dynamics, strategies involving conflict and cooperation. The authors predicted that game theory would eventually do for the study of market what calculus had done for physics in Newton's day. Von Neumann's interest in the field lent it irresistible cache for Nash and his fellow graduate students in mathematics.

Nash wrote his first major paper—his now-classic article on bargaining—while attending Albert Tucker's weekly game theory seminar during his first year at Princeton. That is also where he met von Neumann and Morgenstern for the first time. But he had come up with the basic idea as an undergraduate at Carnegie Tech in the only economics course—international trade—he ever took.

Bargaining had long posed a conundrum for economists. Despite the rise of the marketplace with millions of buyers and sellers who never interact directly, one-on-one deals—between individuals, corporations, governments, or unions—have always been a ubiquitous feature economic life. Yet, before Nash, economists assumed that the outcome of a two-way bargaining was determined by psychology and was therefore outside the realm of economics. (Think of Donald Trump's *The*

Art of the Deal.) They had no formal framework for thinking about how parties to a bargain would interact or how they would split the pie.

Obviously, each participant in a negotiation expects to benefit more by cooperating than by acting alone. Equally obviously, the terms of the deal depend on the bargaining power of each. Beyond this, economists had little to add. No one had discovered principles by which to winnow unique predictions from a large number of potential outcomes. Little if any progress had been made since Edgeworth conceded, in 1881, "The general answer is ... contract without competition is indeterminate."

In their game theory opus, von Neumann and Morgenstern suggested that "a real understanding" of bargaining lay in defining bilateral exchange as a "game of strategy." But they, too, came up empty. It is easy to see why: real-life negotiators have an overwhelming number of potential strategies to choose from—what offers to make, when to make them, what information, threats, or promises to communicate, and so on.

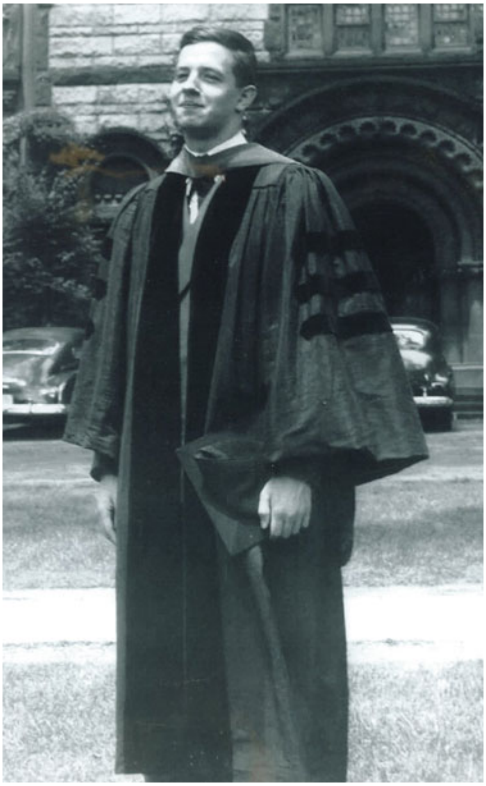
Nash took a novel tack: he simply finessed the process. He visualized a deal as the outcome of either a process of negotiation or else independent strategizing by individuals each pursuing his own interest. Instead of defining a solution directly, he asked what reasonable conditions any division of gains from a bargain would have to satisfy. He then posited four conditions and, using an ingenious mathematical argument, showed that, if the axioms held, a unique solution existed that maximized the product of the participants' utilities.

Essentially, he reasoned, how gains are divided reflects how much the deal is worth to each party and what other alternatives each has. By formulating the bargaining problem simply and precisely, Nash showed that a unique solution exists for a large class of such problems. His approach has become the standard way of modeling the outcomes of negotiations in a huge theoretical literature spanning many fields, including labor-management negotiations and international trade agreements.

Nash was naturally irreverent and iconoclastic. When Princeton asked him, on his graduate school application, for his religion, he wrote "Shinto." When he cast about for a thesis topic, he zeroed in on a problem that he knew had eluded the great von Neumann.

A mere 14 months after he enrolled at Princeton, Nash discovered the original idea that got him a Princeton doctorate in 1950 a few days short of his 21st birthday and would ultimately lead to a Nobel. Ironically, it failed to impress Princeton's pure mathematicians. Most considered game theory slightly déclassé because it was actually... useful.

Since 1950, the Nash equilibrium has become "the analytical structure for studying all situations of conflict and cooperation." Nash made his breakthrough at the beginning of his second year at Princeton. As soon as he described his idea David Gale, a fellow graduate student, the latter insisted Nash "plant a flag" by submitting the result as a note to the Proceedings of the National Academy of Sciences. In the note, "Equilibrium Points in n-Person Games," Nash gives the general definition of equilibrium for a large class of games and provides a proof using the Kakutani fixed



Graduation from Princeton 1950. (Courtesy of Martha Nash Legg and John D. Stier)

point theorem to establish that equilibria in randomized strategies must exist for any finite normal form game.

After wrangling for months with Al Tucker, his thesis adviser, Nash provided an elegantly concise doctoral dissertation which contained a second, alternative proof, using the Brouwer fixed point theorem. In his thesis, titled "Non-Cooperative Games," Nash drew the all-important distinction between games where players act on their own "without collaboration or communication with any of the others," and ones where players have opportunities to share information, make deals, and join coalitions. Nash's theory of games—especially his notion of equilibrium for such games—significantly extended the boundaries of economics as a discipline.

All social, political, and economic theory is about interaction among individuals, each of whom pursues his own objectives (whether altruistic or selfish). Before Nash, economics had only one way of formally describing how economic agents interact, namely, the impersonal market. Classical economists like Adam Smith assumed that each participant regarded the market price beyond his control and simply decided how much to buy or sell. By some means—i.e., Smith's famous Invisible Hand—a price emerged that brought overall supply and demand into balance.

Even in economics, the market paradigm sheds little light on less impersonal forms of interaction between individuals with greater ability to influence outcomes. For example, even in markets with vast numbers of buyers and sellers, individuals have information that others do not, and decide how much to reveal or conceal and how to interpret information revealed by others. And in sociology, anthropology, and political science, the market as explanatory mechanism was even more undeveloped. A new paradigm was needed to analyze a wide array of strategic interactions and to predict their results.

Nash's solution concept for games with many players provided that alternative. Economists usually assume that each individual will act to maximize his or her own objective. The concept of the Nash equilibrium, as Roger Myerson has pointed out, is essentially the most general formulation of that assumption. Nash formally defined equilibrium of a non-cooperative game to be "a configuration of strategies, such that no player acting on his own can change his strategy to achieve a better outcome for himself." The outcome of such a game must be a Nash equilibrium if it is to conform to the assumption of rational individual behavior. That is, if the predicted behavior doesn't satisfy the condition for Nash equilibrium, then there must be at least one individual who could achieve a better outcome if she were simply made aware of her own best interests.

In one sense, Nash made game theory relevant to economics by freeing it from the constraints of von Neumann and Morgenstern's two-person, zero-sum theory. By the time he was writing his thesis, even the strategists at RAND had come to doubt that nuclear warfare, much less post-war reconstruction, could usefully be modeled as a game in which the enemy's loss was a pure gain for the other side.

Nash had the critical insight that most social interactions involve neither pure competition nor pure cooperation but rather a mix of both. From a perspective of half a century later, Nash did much more than that. After Nash, the calculus of rational choice could be applied to situations beyond the market itself to analyze the system of incentives created by any social institution. Myerson's eloquent assessment of Nash's influence on economics is worth quoting at length:

Before Nash, price theory was the one general methodology available to economics. The power of price theory enabled economists to serve as highly valued guides in practical policy making to a degree that was not approached by scholars in any other social science. But even within the traditional scope of economics, price theory has serious limits. Bargaining situations where individuals have different information ... the internal organization of a firm ... the defects of a command economy ... crime and corruption that undermine property rights. ... and so on.

The broader analytical perspective of non-cooperative game theory has liberated practical economic analysis from these methodological restrictions. Methodological limitations no longer deter us from considering market and non-market systems on an equal footing, and from recognizing the essential interconnections between economic, social, and political institutions in economic development. By accepting non-cooperative game theory as a core analytical methodology alongside price theory, economic analysis has returned to the breadth of vision that characterized the ancient Greek social philosophers who gave economics its name.

Von Neumann, the dominant figure in mathematics at the time, didn't think much of the Nash equilibrium. When Nash met with him, the Hungarian polymath dismissed the younger man's result as "trivial." The second edition of *The Theory of Games and Economic Behavior* included only a perfunctory mention of "non-cooperative games" in the Preface. Nash didn't care: "If you're going to develop exceptional ideas, it requires a type of thinking that is not simply practical thinking."

His doctorate in his pocket, Nash headed off to RAND, the ultra-secret cold war think tank, in the summer of 1950. He would be part of "the Air Force's big-brain-buying venture"—whose stars would eventually serve as models for Dr. Strangelove—for the next 4 years, spending every other summer in Santa Monica. With the Cold War and the nuclear arms race in full swing, game theory was considered RAND's secret weapon in a war of wits against the Soviet Union. "We hope [the theory of games] will work, just as we hoped in 1942 that the atomic bomb would work," a Pentagon official told Fortune magazine.

At Rand, Nash got an excited reception. Researchers like Kenneth Arrow, who later won a Nobel for his social choice theory, were already chafing at RAND's "preoccupation with the two-person zero-sum game." As weapons became ever more destructive, all-out war could not be seen as a situation of pure conflict in which opponents shared no common interests. Nash's model thus seemed more promising than von Neumann's.

Probably the single most important work Nash did at RAND involved an experiment. Designed with a team that included Milnor and published as "Some Experimental n-Person Games," it anticipated by several decades the now-thriving field of experimental economics. At the time the experiment was regarded as a failure, Alvin Roth has pointed out, casting doubt on the predictive power of game theory. But it later became a model because it drew attention to two aspects of interaction.

First, it highlighted the importance of information possessed by participants. Second, it revealed that players' decisions were, more often than not, motivated by concerns about fairness. Despite the experiment's simplicity, it showed that watching how people actually play a game drew researchers' attention to elements of interaction—such as signaling and implied threats—that weren't part of the original model. Nash, whose own interests were rapidly shifting away from game theory to pure mathematics, became fascinated with computers at RAND. Of the dozen or so working papers he wrote during his summers in Santa Monica, none is more visionary than one, written in his last summer at the think tank, called "Parallel Control."

Yet the image that stuck with one of his Rand colleagues for decades afterwards was of Nash running down a street trying to kick some pigeons.

Nash left California determined to prove his prowess as a pure mathematician. Even before completing his doctoral thesis, he turned his attention to the trendy topic of geometric objects called manifolds. Manifolds play a role in many physical problems, including cosmology. Right off the bat, he made what he called "a nice discovery relating to manifolds and real algebraic varieties." Hoping for an appointment at Princeton, he returned there for a post-doctoral year and devoted himself to working out the details of the difficult proof.

Many breakthroughs in mathematics come from seeing unsuspected connections between objects that appear intractable and ones that are already well understood. Dismissing conventional wisdom, Nash argued that manifolds were closely related to a simpler class of objects called algebraic varieties. Loosely speaking, Nash asserted that for any manifold it was possible to find an algebraic variety one of whose parts corresponded in some essential way to the original object. To do this, he showed, one has to go to higher dimensions.

Nash's theorem was initially greeted with skepticism. Experts found the notion that every manifold could be described by a system of polynomial equations simply implausible. "I didn't think he would get anywhere," said his Princeton adviser.

Nash completed "Real Algebraic Manifolds," his favorite paper and the only one he later considered nearly perfect, in the fall of 1951. Its significance was instantly recognized. "Just to conceive the theorem was remarkable," said Michael Artin, an algebraic geometer at MIT. Artin and Barry Mazur, who was a protégé of Nash's as an undergraduate at MIT and later proved the generalized Schoenflies conjecture used Nash's result to resolve a basic problem in dynamics, the estimation of periodic points. Artin and Mazur proved that any smooth map from a compact manifold to itself could be approximated by a smooth map such that the number of periodic points of period p grows at most exponentially with p. The proof relied on Nash's work by translating the dynamic problem into an algebraic one of counting solutions to polynomial equations.

Nash's hoped-for appointment at Princeton did not materialize. Instead, he was forced to accept an offer at MIT, America's leading engineering school but far from the great research university that it was to become. Once there someone dared him to solve a deep problem that had baffled mathematicians since the nineteenth century. So he did.

In 1955, he told a disbelieving audience at the University of Chicago where he had been invited to give a talk, "I did this because of a bet." Two years earlier, a skeptical rival challenged him. "If you're so good, why don't you solve the embedding problem?"

He did. In this instance, he simplified a complex problem that seemed to defy solution by pursuing a strategy that the 'experts' pronounced impossible, if not outlandish. A colleague recalled: 'Everyone else would climb a peak by looking for a path somewhere on the mountain, Nash would climb another mountain altogether and from a distant peak would shine a searchlight back on the first peak.'

When Nash announced that "he had solved it, modulo details," the consensus around Cambridge, Massachusetts was that "he is getting nowhere." The precise question that Nash was posing—"Is it possible to embed any Riemannian manifold in a Euclidian space?"—was a challenge that had frustrated the efforts of eminent mathematicians for three-quarters of a century.

By the early 1950s, interest was shifting to geometric objects in higher dimensions, partly because of the large role played by distorted time and space relationships in Einstein's theory of relativity. Embedding means presenting a given geometric object as a subset of a space of possibly higher dimension, while preserving its essential topological properties. Take, for instance, the surface of a balloon, which is two-dimensional. You cannot put it on a blackboard, which is two-dimensional, but you can make it a subset of a space of three or more dimensions. John Conway, the Princeton mathematician who invented the cellular automaton, the *Game of Life*, called Nash's result "one of the most important pieces of mathematical analysis in this century."

Nash's theorem stated that any surface that embodied a special notion of smoothness could actually be embedded in a Euclidean space. He showed, essentially, that you could fold a manifold like a handkerchief without distorting it. Nobody would have expected Nash's theorem to be true. In fact, most people who heard the result for the first time couldn't believe it. "It took enormous courage to attack these problems," said Paul Cohen, famous for his work on the continuum hypothesis, who knew Nash at MIT.

After the publication of "The Imbedding Problem for Riemannian Manifolds" in the Annals of Mathematics, the earlier perspective on partial differential equations was completely altered. "Many of us have the power to develop existing ideas," said Mikhail Gromov, a geometer and Abel laureate whose work was influenced by Nash. "We follow paths prepared by others. But most of us could never produce anything comparable to what Nash produced. It's like lightening striking . . . there has been some tendency in recent decades to move from harmony to chaos. Nash said that chaos was just around the corner."

A few years after he published his embedding paper, Nash once again stunned the mathematics profession by solving an equally difficult, contemporary problem.

Nominally attached to the Institute for Advanced Study in Princeton during a leave from MIT in the academic year 1956–1957, Nash gravitated to the grittier Courant Institute at New York University, "the national capital of applied mathematical analysis." At Courant, then housed in a former hat factory off Washington Square in Greenwich Village, a group of young mathematicians, including Louis Nirenberg who later shared the 2015 Abel prize with Nash, was responsible for the rapid progress stimulated by World War II in the field of partial differential equations. Such equations were useful in modeling a wide variety of physical phenomena, from air passing under the wings of a jet to heat passing through metal.

By the mid-1950s, mathematicians knew simple routines for solving ordinary differential equations using computers. But straightforward methods for solving most nonlinear partial differential equations—the kind potentially useful for describing large or abrupt changes—did not exist. Stanislaw Ulam, inventor of the Monte Carlo method and, with Edward Teller, the first hydrogen bomb design, complained that such systems of equations were "baffling analytically," noting that they defied "even qualitative insights by present methods." Nash proved basic local existence, uniqueness, and continuity theorems (and also speculated about relations with statistical mechanics, singularities, and turbulence.) He used novel methods of his own invention.

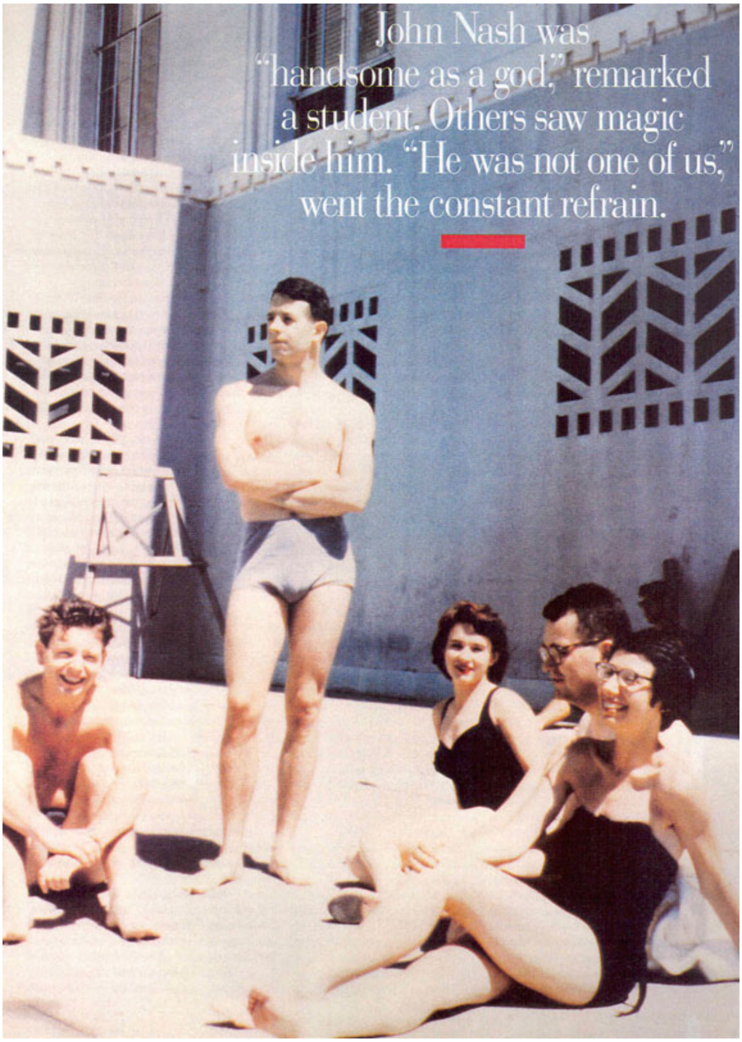
Nash was convinced that deep problems would never yield to a frontal attacks. Taking an ingeniously roundabout approach, he first transformed the non-linear equations into linear ones and then attacked them with non-linear means. Today rocket scientists on Wall Street use Nash inspired methods for solving a particular class of parabolic partial differential equations that arise in finance problems. When he returned to MIT the following fall, there were still gaps in the proof. “It was as if he was a composer and could hear the music, but he didn’t know how to write it down,” a colleague recalled. Instead of struggling on alone, Nash organized a team of mathematicians to help him get the paper ready for publication. “It was like building the atom bomb ... a kind of factory,” said one of them later. The complete proof was published in 1958 in “Continuity of Solutions of Parabolic and Elliptic Equations.”

To his peers, Nash’s was a “bad boy, but a great one.” As his 30th birthday approached, he was about to become a full professor. He was singled out by Fortune magazine as the most brilliant of the younger generation of American mathematicians. He seemed poised to make more groundbreaking contributions. He told colleagues of “an idea of an idea” about a possible solution to the Riemann hypothesis, the deepest puzzle in all of mathematics. He set out “to revise quantum theory,” along lines he had once, as a first-year graduate student, described to Einstein. Writing to Robert Oppenheimer, the physicist who directed the Manhattan Project and subsequently ran the Institute for Advanced Study, in 1957, Nash had proclaimed, “To me one of the best things about the Heisenberg paper is its restriction to observable quantities ... I want to find a different and more satisfying under-picture of a non-observable reality.”

To most observers, Nash’s private life seemed as enviable as his professional accomplishments. He had succeeded in getting a stunningly beautiful, intelligent glamorous woman to fall madly in love with him. “An El Salvadoran princess with a sense of noblesse oblige,” Alicia Larde was one of just 16 women in a class of 800 at MIT. She was a physics major and, a trifle incongruously, a cheerleader. They married in 1958 and within a few months they were expecting a baby. Despite her delicate build, high heels and Elizabeth-Taylor-Butterfield-8 looks, Alicia possessed “a certain steely resolve.” She would need all of the metal she had.

Beneath the shiny facade of John Nash’s successes lurked chaos and confusion. A neglected illegitimate son. A secret former lover. Ambivalence toward his new marriage and his wife’s pregnancy. An undercurrent of anxiety about his abilities as a mathematician.

The first signs of Nash’s slide from eccentricity to psychosis were so ambiguous that most of his colleagues assumed he was making one of his weird private jokes. On New Year’s Eve, 1958, Nash showed up at a costume party wearing a diaper and spent the night sitting in Alicia’s lap, alternately sucking on a pacifier and taking swigs from a baby’s bottle filled with bourbon and milk. One morning, he walked into the math common room carrying a copy of the New York Times and announced that a story on the front page contained encrypted messages from inhabitants of another galaxy that only he could decipher. Another time, he pulled one of his doctoral students aside to hand him an intergalactic driver’s license and offer him a seat on Nash’s newly organized world government...



Left to right: Unidentified person, John, Alicia, Felix and Eva Browder. (From Vanity Fair. Courtesy of John D. Stier)

Initially Alicia tried to cover up or explain away her husband's increasingly bizarre behavior. But soon things spun out of control. In February Nash gave a highly anticipated lecture at Columbia University, claiming that he'd solved the Riemann Hypothesis, the third of the trio of "greatest" then-unsolved mathematics problems. The lecture began normally enough, but soon degenerated into a disjointed series of non-sequiturs.

Something was clearly horribly wrong. Alicia had little choice but to turn to psychiatrists at MIT who urged her to commit her husband to a hospital for

John and Alicia. (Courtesy of John D. Stier)

John with John David. (Courtesy of John D. Stier) observation… against his will if necessary. Nash insisted that he was persecuted not ill. It was a tough call.

In May, 1959, a few weeks before his 31st birthday, two Cambridge police officers took Nash to McLean Hospital, the asylum outside Boston that became the setting for *Girl, Interrupted*. The doctors there diagnosed him with the most devastating and intractable of mental illnesses, paranoid schizophrenia.

A Harvard mathematician who visited Nash at Maclean asked him, "How could you, a mathematician committed to rationality, how could you believe that aliens from outer space were recruiting you to save the world?" Nash replied, "These ideas came to me the same way my mathematical ideas did, so I took them seriously."

The inability to distinguish between delusion and reality, between voices and ones own thoughts, is the tragedy of schizophrenia. We now know that it is a brain disorder, rooted in biology like diabetes or cancer. But when Nash got sick psychiatry was relatively primitive and so were the available treatment. Psychoanalysis, which has since been discredited as an effective treatment for schizophrenia, was in vogue. Psychotic illnesses were supposed to be the fault of bad mothers.

Many of Nash's colleagues and students were appalled by Alicia's decision to have Nash hospitalized. They feared the effects of treatment and confinement on the beautiful mind. Others, however, were shocked by his condition. One recalled his last visit:

"Robert Lowell, the poet, walked in, manic as hell. There's Mrs. Nash, sitting there, pregnant as hell. [Lowell] looks at her and starts quoting the begat sequences in the Bible… And there was John, very quiet and almost not moving. He wasn't even listening. He was totally withdrawn. I focused mostly on his wife and the coming child. I've had that picture in my mind for years. "It's all over for him," I thought."

For a very, very long time, it looked as if it was all over for Nash.

O, what a noble mind is here o'erthrown!

Act Two of Nash's life is the all too common story of a life wrecked by a chronic disease for which there is no adequate treatment, much less cure.

At times Nash believed he was the Prince of Peace, at others a Palestinian refugee. He heard voices and sensed divine revelation. He abandoned mathematics for numerology and prophecy. He wrote letters compulsively to government officials, newspapers and former colleagues. He scribbled mysterious messages on blackboards. He was obsessed with complicated calculations such as converting Nelson Rockefeller's name into base 26 and factoring the result.

He was repeatedly hospitalized, always involuntarily. He was subjected to extreme and futile treatments like insulin shock therapy. He resigned from MIT in order to pursue a quest to give up his US citizenship to become a citizen of the world.

Yet for several years, during temporary remissions, he continued to do mathematics.. "Le problème de Cauchy pour les équations différentielles d'une fluide générale," which appeared in 1962, is described as "basic and noteworthy" by *The Encyclopedic Dictionary of Mathematics* and inspired a good deal of subsequent work by others. He continued to tackle new subjects. Heisuke Hironaka, an algebraic geometer at Harvard and Fields medalist, eventually wrote up a 1964 conjecture as "Nash Blowing Up." In 1966, Nash published "Analyticity of Solutions of Implicit Function Problems with Analytic Data," which pursued his ideas about partial differential equations to their natural conclusion. And in 1967 he completed a much-cited draft, "Arc Structure of Singularities," that was eventually published in a 1995 special issue of the *Duke Journal of Mathematics*.

By the time Nash turned 40, an age at which most mathematicians are at their most productive, almost everything that had once made his life worthwhile was lost. He couldn't work. He had virtually no income. His health suffered. Before long, his front teeth were rotted down nearly to the gums. Old acquaintances avoided him on the street. He was shooed out of stores and coffee shops. Outside Princeton, scholars who built on his work didn't realize he was still alive.

But as Nash sank deeper into obscurity, his ideas were becoming more and more influential. While he was lost in his dreams, his name surfaced more and more often in journals and textbooks in fields as far-flung as economics and biology, mathematics and political science: "Nash equilibrium," "Nash bargaining solution," "Nash program," "De Georgi–Nash," "Nash embedding," "Nash–Moser theorem," "Nash blowing up."

Nash's contributions to pure mathematics—embedding of Riemannian manifolds, existence of solutions of parabolic and elliptic partial differential equations—paved the way for important new developments. By the 1980s, his early work in game theory had permeated economics and helped create new fields within the discipline, including experimental economics. Philosophers, biologists, and political scientists adopted his insights. The growing impact of his ideas was not limited to academe. Advised by game theorists, governments around the world began to auction "public" goods from oil drilling rights to radio spectra, reorganize markets for electricity, and devise systems for matching doctors and hospitals. In business schools, game theory was becoming a staple of management training.

During Nash's "lost years," the brilliant ideas Nash had in his twenties about conflict and cooperation had been widely adopted in the world of economics... Nash published only four game theory papers, but had a bigger impact on economics than any other game theorist. Before Nash, economists could analyze only two kinds of market environments, neither representative: monopolies or markets with so many buyers and sellers that no single individual or firm can affect the behavior of competitors. Most modern markets—cars, oil, airlines, utilities, pharma, housing, healthcare, social media—fall somewhere in between these extremes. Because players must take each others' strategies into account, predicting how they will behave is more complicated. The Nash equilibrium made it possible to cut through the infinite I think therefore he thinks that I think that he thinks... hence the game theory revolution of the 1970s. The impact wasn't confined to economics either but extended to political science, psychology, sociology, and biology.

The contrast between the influential ideas and the bleak reality of Nash's existence was extreme. The usual honors passed him by. He wasn't affiliated with a university. He had virtually no income. He haunted the Princeton campus, in the thrall of a delusion that he was “a religious figure of great, but secret importance.”

I shall not look upon his like again.

Then, after three decades, something extraordinary happened. Act Three began. Freeman Dyson told me later, “It was beautiful. Slowly, he just somehow woke up.”

People ask how Nash could recover from an illness almost universally regarded as a life sentence. Was it with the help of “the modern drugs,” as Russell Crowe says in the movie? It was not. Like one in ten individuals who suffer from chronic schizophrenia, typically for decades, Nash recovered thanks to the natural chemistry of aging. He also attributed his remission to his own struggle against his delusions and hallucinations which he referred to as “going on a diet of the mind,” and the support of a few people who refused to give up on him.

In 1994, Nash’s extraordinary story was about to become public with the announcement of the Nobel Prize in economics.

Incidentally, Nash was almost denied the Nobel. One hour before the prize was scheduled to be announced, it was nearly voted down in an unprecedented refusal of many members of the Swedish Academy of Sciences to affirm the prize committee’s choice. They feared that giving the prize to a “madman” would sully the Nobel “brand” and spoil the televised prize ceremony hosted by the King and Queen of Sweden in December. Ultimately, those who insisted that a mental illness ought not be a greater bar to the prize than, say, cancer or heart disease, prevailed, but only narrowly.

A small band of contemporaries had always recognized the importance of Nash’s work. By the late 1980s, their ranks were swelled by younger scholars who launched a fight to get Nash long-overdue recognition. The prize, that Nash shared with game theorists and experimental economists Reinhard Selten of the University of Bonn and John Harsanyi of the University of California at Berkeley was more than an intellectual triumph. A Nobel rarely changes winners’ lives profoundly. Nash was an exception. “We helped lift him into daylight,” said Assar Lindbeck, chairman of the Nobel prize committee. “We resurrected him in a way.”

When Nash met Russell Crowe for the first time, he told the actor, “You’re going to have to go through all these transformations.” But the transformation in Nash’s own life was as remarkable as any the actor portrayed on the screen. He could not, of course, recover the lost years. He could however repair broken ties with his sister Martha, and his older son John David, travel to conferences, have dinner with friends, see his first Broadway play. He could enjoy the thrill of having a passport, and a drivers license again, of getting a credit card. Then there were the little things like being able to afford a $2 latte at Starbucks. “Lots of academics do that,” he told me. “If I was really poor, I couldn’t.”

To get your life back is a marvelous thing, he told an audience at the world psychiatry conference, but he could never recover the lost years of creativity. Still, he was able to get a grant from the National Science Foundation to develop a new “evolutionary” solution concept for cooperative games. He worked with some

John, Russell Crowe and Ron Howard. (Photo: C. J. Mozzochi)

graduate students. He published papers on ideal money and coalition formation in experimental games.

Most Nobel laureates, while celebrated within their disciplines, remain invisible to the public at large. Recognition not only redeemed the man—bringing him back to society and mathematics—but turned Nash into something of a cultural hero. Since winning the Nobel, the mathematician who spent his life "thinking, always thinking" has been mobbed by reporters and fans from Boston to Mumbai to Beijing.

His story particularly appealed to young people. One of my favorite letters was this one:

> Dear Mr. Nash,
> Hi! I am 9 years old. My name is Ellie Stilson. I am a girl. I really admire you. You are my roll (sic) model for a lot of things. I think you are the smartest person who ever lived. I really wish to be like you. I would love to study math. The only problem with that is that I am not very good at math. I can do it. I like it. I am just not good at it. Was that what it was like for you when you were a kid? Please write back. Love, Ellie P.S. I LOVE your name.

The most unforgettable, though, was addressed to me, arrived in a dirty envelope with no return address and it was scrawled on neon orange paper. It was signed "Berkeley Baby." It would never have made it past the New York Times mailroom after the anthrax scare.

The sender turned out to be the former night rewrite editor on the metro desk, a rising young star at the New York Times in the mid-1970s before he, too, was diagnosed with paranoid schizophrenia. Since then, he had adopted the name Berkeley Baby and lived on the streets of Berkeley, California near the university, a forlorn figure not unlike the Phantom of Fine Hall. He wrote, "John Nash's story give me hope that one day the world will come back to me too." Reading that line always made me cry.

Extraordinary things happen when individuals make extraordinary choices. That is why I dedicated the biography to Alicia Nash. To me, she is very much the hero of Nash's life.

She set out to marry a golden boy who she was convinced was a genius who would be famous one day. Only a few months after the wedding, however, Alicia's girlish notions of romance were shattered by her husband's illness. She acted courageously—and with great compassion. But half a dozen years after Nash got sick, when the husband she was trying to help began to regard her, because of his paranoia, as his worst enemy—she determined to raise their son on her own and got a divorce.

But she never let him go. Five years after they separated, when Nash had no one on earth left to whom he could turn, he wrote to Alicia from a state hospital in Virginia. I beg you "to save me from future hospitalizations and from homelessness." Thirty five and still lovely with most of her life still ahead, she took him in.

What made Alicia do it? It wasn't, I think, masochism, as some suggested. It was love. Not the romantic kind of love, but down to earth, grown up love. She couldn't bear to turn him away. It was "a pretty lean life," her sister-in-law Martha told me. For years, Alicia got up at 4:30 in the morning and commuted 2 h into Manhattan. She did it to support John and their son Johnny, who, at age 15, was diagnosed with the same illness that afflicted his father. She did it to keep her small family together.

Alicia understood—years before research confirmed her intuition—that Nash's only hope lay in living at home in a community where at least a few people knew who he'd been. Nash may have all but disappeared from the world, but Alicia never lost sight of who he was. She saw past the mismatched clothes and expressionless demeanor. For her, Nash was always "a very fine man," someone who had made great contributions, someone for whom "something extraordinary" was always possible.

Recognition is a cure for many ills, but love gave Nash something to come back to: a home, family, a reason to live after his grandiose delusions faded. Alicia was the rock on which he rebuilt his life. Together they experienced the extremes of human existence: genius and madness, sickness and health, obscurity and fame. Together they cared for their disabled son, renewed family ties and friendships, savored what Joan Didion, in her New York Review of Books piece on Nash, called "life's bright pennies."

In 2001, after a nearly 40 year gap in their marriage, John and Alicia said "I do" a second time. "The divorce shouldn't have happened," Nash said. Alicia added,"We saw this as a kind of retraction of that. After all we've been together most of our lives." When the mayor of Princeton Junction pronounced them man and wife, I asked Nash to kiss his bride again for the camera. He looked up, grinning: "A second take? Just like the movies!"

It was Alicia who wanted Nash's story to be told. He was more ambivalent. A friend once asked him about Alicia's whereabouts. "Having dinner with Sylvia," he answered. After a pause he added without much conviction, "I hope they aren't talking about me." Well, 20 years later, people are still talking about him and no doubt will be for a very long time to come.

In 2015 Nash received an honor that meant even more to him than the economics Nobel, the Niels Henrik Abel's Prize in Mathematics. He shared it, as I mentioned, with an old friend from the Courant Institute, Louis Nirenberg. After the ceremony in Oslo, that Nash's older son, John David, was able to attend, Louis, John and Alicia traveled back to the U.S. together. Their flight was cancelled and they were booked on a later one. When they arrived at Newark airport, the Nashes discovered that the driver who usually picked them up had already left. After bidding Louis goodbye, they took one of the cabs lined up outside of the arrivals terminal. Princeton Junction is less than an hour from Newark, but they never made it home. On the New Jersey turnpike, their taxi crashed into the guard rail at high speed, hitting another car. Nash and his wife were both pronounced dead at the scene. He was 86 years old. Alicia was 82.

John Nash's life was tragic, sublime and, now, suddenly, over. The third act shouldn't have ended the way it did. Nonetheless that act, like the whole drama, was truly grand. We will not see the like of him again, but his story belongs to the ages.

References

This essay draws on *A Beautiful Mind*, *The Essential John Nash*, and various articles and lectures . . . For the descriptions of Nash's contributions to mathematics, I borrowed heavily from pieces by Avinash Dixit, Barry Nalebuff, Harold Kuhn, Eric Maskin, John Milnor, Roger Myerson, Al Roth and Ariel Rubinstein. All errors, of course, are mine.

1. Dixit, A., and B. Nalebuff. *Thinking Strategically*. New York: W.W. Norton, 1991.
2. Kuhn, H.W., "Introduction," to A Celebration of John F. Nash, Jr., ed. H.W. Kuhn, L. Nirenberg, and P. Sarnak, pp. i–v. In *Duke Mathematical Journal*, 81, nos. 1 and 2 (1995).
3. Kuhn, H.W., "Foreword." In *Classics in Game Theory*, ed. H. W. Kuhn, pp. ix–x. Princeton: Princeton University Press, 1997.
4. Maskin, E., "John Nash's Contributions to Economics and Game Theory," talk at Nash Memorial Celebration, Princeton University, October 24, 2015.
5. Milnor, J. "A Nobel Prize for John Nash." *The Mathematical Intelligencer* 17, no. 3 (1995): 11–17.
6. Myerson, R. B. "Nash Equilibrium and the History of Economic Theory." *Journal of Economic Literature* 37 (1999): 1067–82.
7. Nasar, S., "John Nash, the Life," talk at Nash Memorial Celebration, Princeton University, October 24, 2015.
8. Nasar, S. "Introduction," Harold Kuhn and Sylvia Nasar, eds., *The Essential John Nash*, Princeton University Press, 2001.

9. Nasar, S., "The Lost Years of the Nobel Laureate," *New York Times*, November 13, 1994, sec. F, pp. 1, 8.
10. Nasar, S., *A Beautiful Mind*. New York: Simon & Schuster, 1998.
11. Roth, A. "Game Theory as a Tool for Market Design" (1999). Available at http://web.stanford.edu/~alroth/design.pdf.
12. Rubinstein, A. "John Nash: The Master of Economic Modeling." *The Scandinavian Journal of Economics* 97, no. 1 (1995): 9–13.