Chapter 16
Part II Commentary 2: Disparities and Opportunities: Plotting a New Course for Research on Spatial Visualization and Mathematics

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Although the chapters contained in this volume focus on the singular topic of spatial visualization as it relates to mathematics, they span two distinct fields of study with different literatures and different scholarly approaches. In many ways, despite their common goals, the two sets of chapters seem worlds apart. We are reminded of Susan Carey's classic developmental psychology book, *Conceptual Change in Childhood* (1985), that discussed incommensurate ideas in science and the ways children reconcile structurally disparate conceptual systems as they grow and learn. The gist was that when one conceptual structure lacks isomorphism with another conceptual structure, it is a significant challenge. We believe the fields represented in this volume face a similar challenge. Yet, these disciplinary asymmetries can also define and stimulate fruitful new research questions, as the advances made in one discipline raise new questions for the other. In this commentary, we aim to identify such asymmetries and consider what new research directions they suggest. We organize our comments around three major questions that cut across research from both fields:

1. What is spatial visualization?
2. How does spatial visualization relate to mathematics?
3. How are these relations reflected in development and learning?

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What Is Spatial Visualization?

All of the chapters grappled with the nature of spatial visualization, but the two fields approached this issue somewhat differently. Some of the education chapters seemed willing to commit to a specific underlying process model as a starting assumption. For example, Battista invoked the construct of mental models (i.e., Johnson Laird, 1980), which are schematic representations arranged in space but without necessarily having a visual component. Gutiérrez et al. adopted Gutiérrez's (1996) mental imagery framework, which rests on the assumption that spatial processing uses visual images. In both cases, the definition of spatial visualization was presented early in the chapter as a way of contextualizing the research to follow.

In contrast, the psychology chapters tended to see the nature of spatial visualization as an open question. Although spatial visualization has certainly been defined in the psychology literature, the definitions are based on observable behaviors that seem to require similar processing, rather than a commitment to any particular representational format. For example, a common definition of spatial skill is the ability to mentally manipulate objects. Such manipulation may involve mental models or visual imagery, but need not. Indeed, pinning down the underlying structure of spatial skill has been a preoccupation of psychologists for decades (see Mix & Cheng, 2012, for a review) and this focus is clearly reflected in the psychology chapters included here. For example, Young, Levine, and Mix (this volume) focus on ways to model the underlying structure of spatial thought and how to interpret the results of different modeling approaches.

Because psychologists continue to work toward a generally accepted process model for spatial thought, it may be premature for related literatures to make strong claims regarding the underlying representational format of spatial visualization. For example, it may seem uncontroversial to claim that spatial representations are visual images (particularly because we call it "spatial visualization"), but there have been challenges to this view in the psychology literature. Research has shown that although spatial development is delayed and more error-prone in blind versus sighted children, blind children can perform tasks that require spatial visualization (Bigelow, 1996; Landau, Spelke, & Gleitman, 1984). A long debate in the psychological literature also centered on whether ordered syllogisms are solved via mental imagery or linguistic information, admitting the possibility that even tasks that seem likely to require spatial visualization may not (e.g., Clark, 1973; Huttenlocher & Higgins, 1971; Trabasso & Riley, 1975; Sternberg, 1980). Finally, there has been discussion about the level of visual detail needed for mental models to be useful. Research suggests that sparse, schematic spatial representations are better for mathematics problem solving than detailed, pictorial images (e.g., Hegarty & Koszhenikov, 1999). As Huttenlocher, Jordan and Levine (1994) pointed out, mental models may resemble physical models in some ways, but these representations need only preserve relevant critical features for problem solving situations when used for mathematics. When solving a mathematics problem involving number of pieces of fruit, for example, it is not necessary to accurately represent the color of the fruit, but only the number of pieces of fruit. If critical features are not preserved, and irrelevant features are, an erroneous answer may be obtained.

One new direction suggested by this contrast may be to use the paradigms offered by the mathematics education studies to more precisely determine the nature of the underlying representations. For example, the diagrams and visual supports used by Gutiérrez et al. or Herbst et al. could be manipulated to be more or less schematic. It might also be possible to see whether students' self-generated physical supports change over time, perhaps becoming less detailed and visual as they master a particular task. Another new direction would be to reframe some of the mathematics education work using current psychological theory. The mental models literature of the 1980s and 1990s provided part of the foundation for what has become the literature related to embodied cognition, or the idea that abstract thought is grounded in bodily movement, perception, and action (Barsalou, 2008; Clark, 1998, 2008; Glenberg, 2008, 2010; Lakoff & Nunez, 2000; Novack, Congdon, Hemani-Lopez, & Goldin-Meadow, 2014; Schöner & Spencer, 2015; Thelen & Smith, 1996). This literature may offer a stronger mechanistic explanation for phenomena such as those identified by Battista, and it would be interesting to see what new questions arise from a thorough meshing of the two. Sinclair et al.'s chapter provides a nice example of this (i.e., integrating mathematics education with theories of embodied cognition). Lowrie and Logan also provided a detailed and rich integration of current psychological theory regarding the nature of spatial thought and children's understanding of geometry. Perhaps extending this approach to other areas of mathematics content would be fruitful.

How Does Spatial Visualization Relate to Mathematics?

The central thesis of this book is that spatial visualization relates to mathematics, so all the chapters work from this premise. However, there are many potential characterizations of this relation and the chapters differed along several lines.

One salient distinction has to do with how inherent spatial processing is to mathematical thought. At one extreme, space can be seen as the representational medium for abstract thought and is thus inherently engaged whenever people perform mathematical tasks. This perspective is exemplified, to some extent, in all of the psychology chapters. For example, Jirout and Newcombe point to evidence that the meanings of numbers are represented as relative quantities using spatial scaling. Congdon et al. emphasize the role of measurement units in conceptualizing quantity across a range of tasks from counting whole objects to ordering fractions. On this view, individual differences in mathematics performance could be construed as individual differences in spatial processing. This strong view was not as evident in the mathematics education chapters. This is particularly surprising given that all focused on geometry—a mathematics topic for which, if any, spatial reasoning is arguably most inherent. Yet, only Battista's chapter made a strong argument along these lines. It was also interesting that none of the mathematics education chapters focused on the inherent nature of spatial processing in numerical thinking more broadly, though this was mentioned by Lowrie and Logan.

An alternative to this strong view is one in which spatial skill is not inherent to mathematical thought, but rather is an optional aid that may be recruited to ground concepts or support reasoning. This seems to be the view of Herbst et al. in that their spatial visualization activity is constructed to ground geometric problem solving in a real world context. Similarly, the explicit aim of Gutiérrez et al. is to develop spatial skills that may be recruited when children are reading diagrams used in mathematics, and Sinclair et al. demonstrate the benefit of figure drawing in children's understanding of geometry. In these studies, the idea seems to be that practice in a spatial skill will transfer back to the more spatial aspects of mathematics in a supportive way.

An interesting question following from this perspective is whether children spontaneously recognize how spatial reasoning can help them in mathematics and the extent to which teacher direction is needed to utilize spatial reasoning. The education chapters allude to the mapping between mental representations, real world, and symbols (including vocabulary) without taking advantage of the recent developments in structure mapping theory, relational learning, or perception-action/embodied cognition. These literatures may help in the design of instructional approaches that avoid the problems of lack of transfer or encapsulated learning. Indeed, by bringing the education and psychological literatures into closer alignment, it is likely that real advances in understanding the relation of spatial and mathematical learning could be made, with beneficial consequences for instructional approaches and children's learning outcomes.

Another contrast was in the level of multidimensionality acknowledged for either spatial skill, mathematics, or both. Nuances in spatial representation are very salient to psychologists, whose research centers more squarely on underlying cognitive processes. Attention to the multidimensionality of spatial thought permeated these chapters. There was also attention paid to the multidimensionality of mathematical thought but this was relatively impoverished and rough compared to what could be said about the multidimensionality of spatial thinking.

In contrast, the mathematics education chapters were striking in their fidelity to the mathematics underlying their phenomena of study, as well as the use of conceptual distinctions that arise purely from consideration of mathematics itself. Through careful analysis of the eventual learning outcomes and potential conceptual pitfalls along the way, these authors identified specific mathematical constructs or misconceptions that might benefit from spatial supports. This orientation is beautifully illustrated in Battista's chapter, in which he examined children's error patterns during geometry proofs. Interestingly, although Battista acknowledged the potential multidimensionality of spatial concepts as well (particularly in reference to Newcombe & Shipley's 2015 framework), this multidimensionality did not play a major role in the mechanism of change. The role of spatial grounding was clearly acknowledged, but the specific nature of that grounding seemed less important.

In terms of next steps, there seems to be great potential in a synthesis of these two approaches—that is, a rich, detailed account of underlying cognition married with a rich, detailed account of underlying mathematics. Moves in that direction are likely to reveal an entire host of new research questions and insights that emerge from drilling down to a deeper, more specific level of shared processing and bidirectional influence. Each of the mathematics education chapters offers a slice of mathematical development that is already fleshed out at this level. One approach may be to revisit these accounts with an eye toward achieving equally nuanced explanations of the same specific phenomena based on cognitive processing, and testing these explanations empirically.

How Are These Relations Reflected in Development and Learning?

A final question all of the chapters addressed was how relations between spatial skill and mathematics play out as children learn and change over developmental time. The chapters presented an interesting contrast between those describing the stages of development and those identifying the mechanisms that propel children through these stages. Historically, psychologists such as Jerome Bruner and Jean Piaget attempted to achieve both aims—to describe developmental stages and identify broad mechanisms of change. In the present volume, the two aims seemed to separate along the disciplinary lines.

The mathematics education chapters tended to offer detailed, carefully articulated stage theories. For example, though less rigid than a classical stage theory, Battista's learning trajectories seem very much like a Piagetian description of development, with movement from holistic to decomposed concepts, and from concrete to logic-based reasoning. In terms of learning mechanisms, the education chapters were more focused on the potential benefits of various spatial activities. For example, Gutiérrez et al. described the impact of training on a perspective-taking task. Herbst et al. sought to improve geometric reasoning via practice modeling three-dimensional space. Sinclair et al. discussed the role of drawing in understanding geometry. These are creative instructional approaches that show promise; however, they beg a host of questions related to the underlying processes. What process model can explain why they work? What is the active ingredient in these approaches that propels change?

For the psychologists' part, there was a strong emphasis on the mechanisms of change and less attention paid to typical developmental or learning trajectories. For example, Cipora et al. point to the correlations between spatial skill and mathematics performance, and raise the question of whether variation in spatial skill is driving the correlation, or perhaps the reverse (i.e., variation is arithmetic understanding leads to more sophisticated and accurate spatial representations). Increasing understanding of the mechanisms that drive the strong relation between spatial and mathematical thinking is an important goal for successfully incorporating spatially rich instructional strategies into the mathematics curriculum.

This is another dimension for which circling back and integrating the two literatures may be beneficial. For example, future research might focus on questions such as whether the mechanisms of change identified by psychologists can be manipulated experimentally so as to yield the various stages identified by the mathematics education researchers. Alternatively, a review of the detailed shifts identified by the mathematics education chapters may suggest new or revised mechanisms of change that have not been recognized previously. A bidirectional analysis and program of research such as this has the potential to yield exceptionally strong instructional approaches that may be missed by taking only one approach or the other.

In summary, the chapters in this volume make exciting strides toward understanding the relations between spatial skill and mathematics, but often do so in very different ways or from perspectives that are not easily aligned. By integrating these differing orientations, there is potential to increase our understanding and design more effective instructional interventions.

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