Chapter 6 Optimal Structure of Experiential Services: Review and Extensions



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Abstract In many consumer-intensive (B2C) services, delivering memorable customer experiences is often a source of competitive advantage. And yet, there exist few formal guidelines to design the structure of such experiences. In this chapter, we introduce a utility-based model of customer satisfaction when customers are subject to acclimation, satiation, and memory decay. We then review and extend principles for optimizing the structure of an experience to maximize customer satisfaction; specifically, we characterize the optimal sequence of activities, the optimal activity selection, and the optimal information policy about an uncertain outcome. We find that, in general, the optimal experience structure is non-monotone in service levels and makes use of breaks/intermissions to create contrasts and reset satiation levels. However, in many extreme cases, we show that a crescendo design is optimal. We then discuss the implications of our framework for quality management in services, especially as it relates to a potential gap between ex-ante expectation and ex-post satisfaction, and for monetizing customers' utilities derived while anticipating or recalling the event.

Keywords B2C services · Experiences · Behavioral operations management · Scheduling · Social psychology

6.1 Introduction

In competitive consumer-intensive (B2C) service industries (e.g., healthcare, leisure and hospitality, transportation), delivering memorable customer experiences is often a source of competitive advantage (McKinsey 2016). Experiences are indeed one of the key service differentiators once basic service outcomes are met for a given price

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point (Berry et al. 2002). Experience drives customer satisfaction, which then drives customer loyalty (Braff and DeVine 2008), which in turn drives revenue growth and profitability (Heskett et al. 1994). In fact, Pine and Gilmore (1998, pp. 97–98) propose that "from now on, leading-edge companies ... will find that the next competitive battleground lies in staging experiences."

Because experiences are ubiquitous in B2C services and can be a source of competitive advantage, Ostrom et al. (2015) identify the topic of "enhancing the service experience" as one of the 12 research priorities for service research, and they classify it within the context of value creation. Zomerdijk and Voss (2010) identify six levers that can be pulled to enhance service experiences, namely (1) the orchestration of clues (or cues) that are emitted by products, services, and the environment, within and across service encounters (see, e.g., Berry et al. 2002, Haeckel et al. 2003), (2) the design of the sensory environment using, e.g., servicescape frameworks (Bitner 1992), (3) the engagement of front-line employees with customers, (4) the dramatic structure of the experience, (5) the management of the presence of fellow customers, and (6) the coordination between the front- and backstage processes, and more generally, of processes across customer interfaces, using, e.g., service experience blueprints (Patrício et al. 2008, 2011). This chapter focuses on the fourth lever, namely the design of the dramatic structure (i.e., the sequence, progression, and duration of activities) of an experience.

We take the perspective of a service provider who seeks to optimize the structure of a service encounter to maximize customer satisfaction. Throughout the encounter, the customer is exposed to various stimuli, which can be multi-dimensional and time-varying, and she derives (instantaneous) utility from them. Her satisfaction is a summary of these instantaneous utilities, assessed at the end of the process (Oliver 2015).

In practice, experiences are built up through a collection of touchpoints in multiple phases of a customer's decision process or purchase journey (Lemon and Verhoef 2016). We focus here on one such touchpoints, i.e., a particular encounter. Accordingly, we adopt customer satisfaction as our main performance objective, and not the more holistic metric of customer experience, which is affected by factors falling outside the encounter (e.g., search, after-sale purchase), across channels, or even outside the service provider's control (e.g., influence of others); see Verhoef et al. (2009).

Because the customer is the ultimate recipient of the experience (Pullman and Gross 2004), one needs to turn to behavioral science to understand how different types of experience structures affect customer satisfaction. Building on the findings from behavioral science, Chase and Dasu (2001) formulate five experience design principles: Finish strong; Get the bad experiences out of the way early; Segment the pleasure, combine the pain; Build commitment through choice; Give people rituals and stick to them. See also DeVine and Gilson (2010). Although these principles are very sensible, there has been little guidance—until recently—as to when and how they should apply.

To fill that gap, an emerging stream of research has offered novel design insights by formally modeling a customer's utility, with specific preferences or behavioral regularities, and optimizing the structure of the experiential process to maximize that utility. The purpose of this chapter is to review that nascent literature and offer novel insights into the optimal design of the structure of service experiences by generalizing results by Das Gupta et al. (2015) and Ely et al. (2015), among others. Specifically, we propose a utility-based model of customer satisfaction when customers are subject to acclimation, satiation, and memory decay and we embed that utility model into a service design optimization model. We consider a single service encounter with fixed total duration, consisting of several activities, potentially preceded by an anticipation period and followed by a recall period. Throughout the experience, the customer is exposed to a sequence of activities, each associated with various service levels (or stimuli) from which the customer derives utility. Activities are homogenous in the sense that they are characterized by the same set of attributes, but they differ in terms of service levels on each of these attributes.

We study the following structure design decisions: How to sequence activities within the encounter? How to allocate duration to the activities? Which activities to select? How to reveal information about an uncertain state of nature to maximize suspense or surprise?

As proposed by Kahneman et al. (1997), customer satisfaction, or equivalently customers' remembered utility, may differ from their total utility derived from the service. In particular, we assume that customers are subject to memory decay (Ebbinghaus 1913); that is, when customers recall how much utility they derived from the experience, they put greater weight to the most recent events. In addition, we consider specific customer preferences, or behavioral regularities, which affect their instantaneous utilities. Specifically, we assume that customers are subject to acclimation (a.k.a., adaptation, habituation); that is, a customer's instantaneous utility from a particular activity's service level is assessed relative to a reference point, which adapts to states and reacts to changes (Hsee and Abelson 1991; Wathieu 1997). We also assume that customers are subject to satiation; in particular, a customer's instantaneous utility from a particular activity from a particular activity is a function of past consumption (Baucells and Sarin 2007). Finally, customers may exhibit decreasing marginal returns to gains (i.e., concave utilities) and loss aversion.

In practice, customer utilities may be subject to other behavioral preferences or regularities such as mental accounting or the endowment effect (see Thaler 2015 for an overview). We focus here on memory decay, acclimation, and satiation because their effect on satisfaction is intimately related to the structure of the experience (e.g., sequencing and duration of activities). In contrast, the effect of other behavioral factors (e.g., mental accounting) may be less related to the structure of the experience, but more to its framing (or marketing; e.g., communication, pricing), which falls outside the scope of this chapter. There are other behavioral factors that may be related to the structure of the experience (e.g., the primacy effect), but to the best of our knowledge, there has been no formal model characterizing the optimal experience design in the presence of these effects, and we leave it for future research to further explore those phenomena.

Throughout the analysis, we assume that customers are captive, i.e., are present from the beginning to the end of the encounter. In particular, we do not consider decisions that relate to customer engagement, such as the design of customer narratives or work allocation policies (Roels 2014; Bellos and Kavadias 2017), and leave it to future research to incorporate those into our analytical framework. We also assume that the service is not customer-routed, that is, the sequencing, duration allocation, and activity selection decisions are under the provider's control. Examples of such non-customer-routed service experiences with homogenous activities and captive customers are live performances (e.g., music concerts, magic shows, fireworks), executive education programs, conferences, massages and spa treatments, fitness classes, museum tours, and dental procedures.

The chapter is structured as follows. In the next section, we introduce a utilitybased model of customer satisfaction in the presence of acclimation, satiation, and memory decay, and we embed it within a generic optimization model of experience structure design. Sections 6.3-6.5 study three specific cases of structure design decisions: Considering a fixed set of activities with fixed duration and fixed service levels, Sect. 6.3 characterizes the optimal sequence of activities under various effects of acclimation, satiation, and memory decay. Considering fixed durations, Sect. 6.4 characterizes the optimal activity selection, subject to a budget constraint on their service levels. In Sect. 6.5, we consider a specific type of activities, namely messages that update customer beliefs about an uncertain outcome. We characterize the optimal sequence of messages, i.e., the optimal information policy, that maximizes customer satisfaction from their experienced suspense or surprise. Sections 6.6 and 6.7 expand the scope of the analysis beyond a single encounter to assess what happens before and after the experience. Specifically, Sect. 6.6 identifies a potential gap between a customer's ex-ante expectations about an experience and her ex-post satisfaction and discusses its implications for quality management; and Sect. 6.7 proposes a model of customer utility during anticipation and recall. We conclude in Sect. 6.8 with future research directions. All proofs appear in the Appendix.

6.2 Model

We consider a service encounter taking place over T discrete time periods and consisting of N activities. Each activity is characterized along K orthogonal attributes, which can be physiological (e.g., noise, smell, sweetness), cognitive (e.g., level of mathematical sophistication), or emotional (e.g., fear, joy).

For any activity i = 1, ..., N, let $x_{k, i}$ be the service level on attribute k = 1, ..., K, and $\mathbf{x}_i = (x_{1, i}, ..., x_{K, i})$ be the corresponding vector of attributes. For simplicity, we assume that the service level remains constant during the duration of an activity. (Otherwise, an activity consisting of multiple phases with different service levels could be split into multiple activities with constant service levels.) In Sect. 6.5, we interpret \mathbf{x}_i as a collection of messages.

Let \underline{d}_i and \overline{d}_i be respectively the lower and upper bounds on activity *i*'s duration. When $\underline{d}_i = \overline{d}_i$, the duration of the activity is fixed. When $\underline{d}_i = 0$, the service provider has the flexibility to spend zero time on activity *i*, i.e., to remove it from the encounter. The service provider's decision consists of choosing which activity to schedule in any period. For any t = 1, ..., T, let π_t be the activity index scheduled in period t; that is, $\pi_t = i$ if activity i is scheduled in period t. Let $\pi = (\pi_1, ..., \pi_T)$ be the service provider's decisions, constrained to belong to a feasible set $\Pi(\mathbf{x})$. In this model formulation, we keep the representation of $\Pi(\mathbf{x})$ abstract, but note that it can include many different types of constraints such as

- minimum activity durations, i.e., if Activity *i* has been scheduled to start at time *t*, then no other activity can be scheduled in periods $t, \ldots, t + \underline{d}_i$, and Activity *i* cannot be scheduled to start at an earlier or later time;
- precedence constraints, e.g., Activity *i* must precede Activity *j*;
- budget constraint on the total set of activities being scheduled, e.g., when K = 1, $\sum_{t=1}^{T} x_{\pi_t} \leq B$ for some budget *B*;
- disjunctive constraints on activity selection, e.g., either Activity *i* or Activity *j* may be scheduled in the encounter, but not both of them.

With a slight abuse of notation, we denote by $x_t = x_{\pi_t}$ the service levels of the activity scheduled in period *t*, and by $x = (x_1, ..., x_T)$ the corresponding vector. This generic framework can encompass such design decisions as activity sequencing, duration allocation, or activity selection.

The utility the customer derives from an activity scheduled in period *t* depends on three variables, namely,

- 1. the activity's service level on each attribute *k*, denoted as $x_{k, t}$, with $\mathbf{x}_t = (x_{1, t}, \dots, x_{K, t})$,
- 2. a reference level on attribute *k* at the beginning of period *t*, denoted as $r_{k, t}$, with $\mathbf{r}_t = (r_{1, t}, \dots, r_{K, t})$, and
- 3. a satiation level on attribute k at the beginning of period t, denoted as $s_{k, t}$, with $s_t = (s_{1, t}, \dots, s_{K, t})$.

We denote by $u_k(x, r, s)$ the customer's instantaneous utility associated with service level *x* on attribute *k* with a reference level *r* and a satiation level *s*. Following Baucells and Sarin (2010), we assume that

$$u_k(x, r, s) = v_k(x - r + s) - v_k(s), \tag{6.1}$$

in which $v_k(x)$ denotes the customer's instantaneous utility associated with service level x on attribute k with an initial reference level of zero and an initial satiation level of zero. We assume throughout that $v_k(x)$ is increasing and that $v_k(0) = 0$. For certain results (e.g., Propositions 3, 5, 6, and 7), we will make additional restrictions on $v_k(x)$, such as concavity or loss aversion.¹

¹Prospect theory posits that is concave for all $x \ge 0$, convex for all x < 0, and exhibits loss aversion in the sense that $-v_k(-x) \ge v_k(x) \ge 0$ for all x > 0; see, e.g., Baucells and Sarin (2010) and Kőszegi and Rabin (2006).

Hence, the utility a customer derives from a service level *x* is assessed relative to a reference point *r*, and the higher that reference point, the smaller the utility. For instance, a customer entering a store that has an ambient temperature of 70 °F will enjoy more the ambient warmth if the outside temperature is low. In addition, the utility from current consumption is a function of past consumption, i.e., of the satiation level *s*; specifically if $v_k(x)$ is concave, the higher past consumption, the lower the utility from current consumption. For instance, a customer eating steak will enjoy more utility if she is hungry than if she just had a filling appetizer.

Attributes are orthogonal in the sense that reference and satiation levels on attribute k are a function of past service levels on that particular attribute k, but independent of the past service levels on the other attributes $l \neq k$. Similar to Baucells and Sarin (2010), we consider the following state transitions:

$$r_{k,t+1} = \alpha \, x_{k,t} + (1 - \alpha) \, r_{k,t} \tag{6.2}$$

$$s_{k,t+1} = \gamma (x_{k,t} - r_{k,t} + s_{k,t}), \tag{6.3}$$

in which $\alpha \in [0, 1]$ is the rate of acclimation (a.k.a. adaptation, habituation) and $\gamma \in [0, 1]$ is the rate of decay in the satiation level. More general models could consider attribute- or activity-specific rates.

Following Kőszegi and Rabin (2006), Bleichrodt et al. (2009), and Baucells and Sarin (2010), we assume that the instantaneous utility associated with a multi-attribute service level x_t , reference level r_t , and satiation level s_t , denoted as $u(x_t, r_t, s_t)$, is additively separable, i.e.,

$$u(\mathbf{x}_{t}, \mathbf{r}_{t}, \mathbf{s}_{t}) = \sum_{k=1}^{K} u_{k}(x_{k,t}, r_{k,t}, s_{k,t}).$$
(6.4)

Although the customer derives a *total* utility $U(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \sum_{t=1}^{T} u(\mathbf{x}_t, \mathbf{r}_t, \mathbf{s}_t)$ from the experience (Edgeworth 1881), the customer's *remembered* utility, which drives future purchase decisions, usually differs from the total utility (Kahneman et al. 1997). Let $S(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1)$ be the customer's remembered utility, or *satisfaction*, derived from an encounter featuring service levels \mathbf{x} , when the customer's reference level and satiation level at the beginning of the encounter are equal to \mathbf{r}_1 and \mathbf{s}_1 . A customer who is subject to memory decay (Ebbinghaus 1913), will remember more recent events than past events. With exponential memory decay, this leads to

$$S(\boldsymbol{x}, \boldsymbol{r}_1, \boldsymbol{s}_1) = \sum_{t=1}^T \delta^{T-t} u(\boldsymbol{x}_t, \boldsymbol{r}_t, \boldsymbol{s}_t), \qquad (6.5)$$

in which δ is the rate of memory decay (Das Gupta et al. 2015). More generally, serial effects such as primacy and recency could be incorporated (Karmarkar and

Karmarkar 2014), e.g., $S(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \sum_{t=1}^{T} w_t u(\mathbf{x}_t, \mathbf{r}_t, \mathbf{s}_t)$, where w_t is the weight associated with position t.^{2,3}

Alternatively, Frederickson and Kahneman (1993) suggested that customers only remember the peak and the end of an experience, i.e., that $S(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \delta \max_{t=1,...,T} u(\mathbf{x}_t, \mathbf{r}_t, \mathbf{s}_t) + (1 - \delta) u(\mathbf{x}_T, \mathbf{r}_T, \mathbf{s}_T)$. Building upon the peak-end rule, Dixon and Verma (2013) empirically find that customers' remembered utility from a concert season of a performance art center is a function of the peak, end, spread (i.e., timing of the peak), and trend. Dixon and Thompson (2016) use this objective to optimize season bundles. However, the peak-end rule is deceptive for design since it would imply that all sequences of activities with the same end would lead to identical satisfactions.

In this chapter, we consider a model with acclimation, satiation, and memory decay. Accordingly, the service provider seeks to optimize the sequence of activities, allocate duration, and/or select activities to maximize customer satisfaction,

$$\max_{\boldsymbol{\pi} \in \Pi(\boldsymbol{x})} S((\boldsymbol{x}_{\pi_1},\ldots,\boldsymbol{x}_{\pi_T}),\boldsymbol{r}_1,\boldsymbol{s}_1)$$

when the customer is subject to memory decay, i.e., (6.5), when utilities are additively separable across attributes, i.e., (6.4), and when the customer is subject to acclimation, i.e., (6.1) and (6.2), and satiation, i.e., (6.1) and (6.3).

We next characterize the optimal design in three particular cases, namely (1) the optimal sequencing of activities for a fixed set of activities with fixed duration; (2) the optimal selection of activities when there is a budget constraint on the aggregate service levels; and (3) considering activities as messages, the optimal information policy to maximize recollection of suspense or surprise.

6.3 Activity Sequencing

In this section, we consider a fixed set of activities with fixed durations (i.e., $\underline{d}_i = \overline{d}_i = d_i \ge 1$) such that $\sum_{i=1}^{N} d_i = T$ and we characterize the optimal sequence of activities to maximize customer satisfaction. In order to derive first-order structural results, we assume no precedence constraints, i.e., all permutations are possible.

In general, the optimal sequence of activities can be quite complex in the presence of the three behavioral factors of acclimation, satiation, and memory decay, and a complete characterization is beyond the scope of this chapter. Instead, we next consider several extreme cases, and we find that sequencing activities in increasing

 $^{^{2}}$ Even without explicitly modeling primacy effects, we find that a U-shape sequence may be optimal under Model (6.5). See Proposition 5.

³Baucells and Bellezza (2017) consider an even more general model with a discount factor that is period-specific and dependent on the magnitude of the utility experienced in that period.

Proposition	Memory decay	Acclimation	Satiation	Utility	Optimal design
1		No $(\alpha = 0)$	No $(\gamma = 0)$		Crescendo
2		No ($\alpha = 0$)	Full ($\gamma = 1$)		Crescendo
3	No $(\delta = 1)$	Full ($\alpha = 1$)	No $(\gamma = 0)$	Subadditive, loss	Crescendo
				aversion	
4	No $(\delta = 1)$		Full ($\gamma = 1$)		Crescendo
5				Linear	U-shaped

Table 6.1 Summary of results on optimal sequences

order of service levels, i.e., in crescendo, is often optimal. Considering singleattribute service levels (i.e., K = 1), the first two propositions study the role of memory decay with full or no decay in satiation (i.e., $\gamma = 0$ or $\gamma = 1$), and the next two propositions study the role of acclimation with full or no decay in satiation. We then consider multi-attribute service levels when utilities are linear (i.e., $v_k(x) = w_k x$ for all k). Table 6.1 offers a summary of our results on optimal sequences.

Considering single-attribute service levels, we first characterize the case with only memory decay, no acclimation, and no satiation, generalizing the result obtained by Das Gupta et al. (2015) to the case of nonlinear utility functions. Because memory decay puts greater weight on the last activities, it is optimal to schedule the activities with the highest service levels near the end of the encounter.

Proposition 1: Suppose that K = 1 and that $\underline{d}_i = \overline{d}_i = d_i \ge 1$ for all *i*. When there is no acclimation nor satiation ($\alpha = \gamma = 0$), it is optimal to sequence activities in increasing order of service level.

The next proposition complements Proposition 1 by considering the case with no decay in satiation. Similar to the case with no satiation, a crescendo is optimal when satiation never decays.

Proposition 2: Suppose that K = 1, that $\underline{d}_i = \overline{d}_i = d_i \ge 1$ for all *i*. When customers experience no decay in satiation ($\gamma = 1$) and never acclimate ($\alpha = 0$), it is optimal to sequence activities in increasing order of service level.

However, for intermediate levels of decay in satiation $(0 < \gamma < 1)$, the optimal sequence when there is no acclimation may not necessarily be a crescendo, as illustrated in Fig. 6.1. In order to yield a high utility in the last periods (which are heavily weighted due to memory decay), it is important to set the satiation level prior to the last period s_T at a low value. Because of decay in satiation, the satiation level s_T depends more on the most recent service levels than on the earlier ones. Accordingly, it may be optimal to drop the service levels in the middle of the encounter to reset the satiation level to a low value and maximize the utility derived from the subsequent activities. Effectively, one should insert a break or intermission to reduce satiation and fully enjoy the end of the encounter.



Fig. 6.1 Optimal sequence with memory decay, but no acclimation. (Note: $\delta = 0.2$, $\alpha = 0$, $\gamma = 0.5$, $v(x) = \sqrt{x}$ if $x \ge 0$ and $-\sqrt{-x}$ if x < 0, T = 5, $r_1 = 0$, $s_1 = 0$, $\mathbf{x} = (1, 2, 3, 4, 5)$. Optimal sequence identified through exhaustive search)

We next investigate the role of acclimation without memory decay (i.e., when $\delta = 1$). Das Gupta et al. (2015) show that with linear utilities, crescendos are optimal. We next generalize their result to the case with nonlinear utilities.

We first consider the case of no satiation ($\gamma = 0$) and assume full acclimation ($\alpha = 1$). We require the utility function to be such that for all x > 0, $v(x + y) \le v(x) + v$ (y), which is a weak form of subadditivity (and satisfied when v(x) is concave), and that $-v(-x) \ge v(x) \ge 0$ for all x > 0, which implies loss aversion. Under those conditions, an increasing sequence $x_1 < x_2 < x_3$ always generates greater satisfaction than a U-shaped sequence (e.g., $x_2 > x_1 < x_3$), because the disutility obtained from the initial drop in service levels will not be compensated by the utility obtained from the final increase in service levels due to loss aversion.

Proposition 3: Suppose that K = 1, that $\underline{d}_i = \overline{d}_i = d_i \ge 1$ for all *i*, that, for all x > 0, $v(x + y) \le v(x) + v(y)$, and that $-v(-x) \ge v(x) \ge 0$ for all x > 0. When customers fully acclimate ($\alpha = 1$), but experience neither memory decay ($\delta = 1$) nor satiation ($\gamma = 0$), it is optimal to sequence activities in increasing order of service level.

The next proposition complements Proposition 3 by considering the other extreme of satiation, i.e., when there is no decay in satiation level ($\gamma = 1$). Unlike Proposition 3, no condition is required on the shape of the utility function.

Proposition 4: Suppose that K = 1 and that $\underline{d}_i = \overline{d}_i = d_i \ge 1$ for all *i*. When customers experience no decay in satiation ($\gamma = 1$) and no memory decay ($\delta = 1$), it is optimal to sequence activities in increasing order of service level.

However, for intermediate levels of decay in satiation $(0 < \gamma < 1)$, the optimal sequence when there is no memory decay may not necessarily be a crescendo, as illustrated in Fig. 6.2. Decreasing the service level of the activities in the middle of the encounter indeed resets both the reference point and the satiation level to low



Fig. 6.2 Optimal sequence with acclimation, but no memory decay. (Note: $\delta = 1$, $\alpha = 0.1$, $\gamma = 0.4$, $v(x) = \sqrt{x}$ if $x \ge 0$ and $-\sqrt{-x}$ if x < 0, T = 5, $r_1 = 0$, $s_1 = 0$, $\mathbf{x} = (1, 2, 3, 4, 5)$. Optimal sequence identified through exhaustive search)

values, thereby increasing the utility from the subsequent activities. Although this initial drop in service level may potentially result in negative utility due to acclimation (especially in the presence of loss aversion), such disutility can be mitigated if the satiation levels are high already. Here the role of the break or intermission is not only to reduce satiation, but also to create contrast.

Overall, we have shown that when satiation exhibits either full or no decay, memory decay and acclimation individually lead to crescendos. A common recommendation for experience designers is indeed to "finish strong;" for instance, the tour of Guinness Storehouse ends with a highly-valued complimentary drink in a sky bar (Zomerdijk and Voss 2010).

However, as shown in Das Gupta et al. (2015), even with no satiation, combining memory decay and acclimation could lead to U-shaped optimal designs, as is illustrated in Fig. 6.3. The intuition is as follows: Together, memory decay and acclimation favor a steep gradient in service levels near the end of the encounter. To achieve a sharp increase in service levels at the end of the encounter it may be optimal to move some of the activities that are associated with a high service level at the beginning of the encounter. Although this results in negative utility when the customer experiences a drop in service levels, this carries little weight in the customer's overall assessment of the encounter given that this disutility happens at the beginning of the encounter and tends to be forgotten. U-shape sequences are in fact ubiquitous in practice, such as in music concert's sequence of songs (Baucells et al. 2016) or in arc-like structures of exposition (Zomerdijk and Voss 2010).

We next generalize the characterization obtained by Das Gupta et al. (2015) to multi-attribute service levels for linear utilities. With linear utilities, satiation has no impact. In this case, the optimal sequence is in general U-shaped in the activities' weighted average service levels, and the last two activities are sequenced in increasing order of weighted average service level.



Fig. 6.3 Optimal sequence with acclimation and memory decay, but no satiation. (Note: $\delta = .5$, $\alpha = 0.9$, $\gamma = 0$, $v(x) = \sqrt{x}$ if $x \ge 0$ and $-\sqrt{-x}$ if x < 0, T = 5, $r_1 = 0$, $s_1 = 0$, $\mathbf{x} = (1, 2, 3, 4, 5)$. Optimal sequence identified through exhaustive search)

Proposition 5: Suppose that $\underline{d}_i = \overline{d}_i \ge 1$ for all *i* and that $v_k(x) = w_k x$ for all *k*. Then, it is optimal to sequence activities in a U-shaped fashion in terms of weighted average attributes $\sum_{k=1}^{K} w_k x_{k,i}$. In particular, if Activity *i* precedes Activity *j* in the optimal sequence and

- If Activity *j* is not the last activity, then $\sum_{k=1}^{K} w_k x_{k,i} \ge \sum_{k=1}^{K} w_k x_{k,j}$ if and only if the starting time of Activity *i* is less than $t \le T+1-\$\$\frac{\ln\left(\frac{1-\delta^{-d}i-\delta^{-d}j+\delta^{-d}i-d}{1-(1-\alpha)^{-d}j+(1-\alpha)^{-d}i-d}\right)}{\ln\left(\frac{1-\delta}{2}\right)}.$
- If Activities *i* and *j* are the last two activities, then $\sum_{k=1}^{K} w_k x_{k,i} \leq \sum_{k=1}^{K} w_k x_{k,j}.$ In particular, for short encounters, i.e., when $T \leq \frac{\ln\left(\frac{1-\delta^{-d_i}-\delta^{-d_j}+\delta^{-d_i-d_j}}{1-(1-\alpha)^{-d_i}-(1-\alpha)^{-d_j}+(1-\alpha)^{-d_i-d_j}}\right)}{\ln\left(\frac{1-\alpha}{\delta}\right)}$

-1 for all possible durations (d_i, d_j) , it is optimal to sequence activities in increasing order of their weighted service level $\sum_{k=1}^{K} w_k x_{k,i}$. This condition can easily be shown to hold true when there is no memory decay $(\delta = 1)$ or no acclimation $(\alpha = 0)$, consistent with Propositions 1 and 3.

With nonlinear utilities, satiation matters, and the optimal design is in general more complex than a crescendo or a U-shape. For instance, Fig. 6.4 shows that even in the absence of memory decay ($\delta = 1$) and acclimation ($\alpha = 0$), the optimal design could consist of multiple local minima aimed at resetting the satiation level to a low value and increasing the utility from the subsequent activities. Because of satiation, it may thus be optimal to insert breaks in a performance to maximize the utility from the next segments.



Fig. 6.4 Optimal sequence with satiation, but no acclimation and no memory decay. (Note: $\delta = 1$, $\alpha = 0$, $\gamma = 0.1$, $v(x) = \sqrt{x}$ if $x \ge 0$ and $-2\sqrt{-x}$ if x < 0, T = 5, $r_1 = 0$, $s_1 = 0$, $\mathbf{x} = (1, 2, 3, 4, 5)$. Optimal sequence identified through exhaustive search)

6.4 Activity Selection

In contrast to the previous section, which considered a given set of activities to be sequenced, we now consider how to select activities under a budget constraint on their service levels. Specifically we assume that K = 1 and set the feasible set such that $\Pi(\mathbf{x}) = \left\{ \mathbf{x} \middle| \sum_{t=1}^{T} x_t \le B \right\}$.⁴ (In addition, we can restrict service levels to be nonnegative at the expense of more cumbersome notation.) Economists and decision scientists (e.g., Samuelson 1937, Koopmans 1960) have studied how to optimize an individual's future consumption plan to maximize her expected utility subject to a budget constraint. In contrast to that literature, which optimizes a customer's ex-ante discounted utility, we optimize here a customer's ex-post satisfaction; that is, time discounting operates backward here (due to memory decay) as opposed to forward.

We first characterize the optimal activity selection when the customer is subject to both acclimation and memory decay, but not to satiation. As established in Proposition 5 and illustrated in Fig. 6.3, when the service provider controls only the sequence of activities, the optimal design may end up being U-shaped so as to induce a steep gradient in service levels near the end of the encounter, but at the expense of negative utilities in the early periods of the encounter. In contrast, when the provider is free to select which activities to schedule, this trade-off is no longer at

⁴A more general model with multi-attribute activities could consider that the intensity of each attribute moves proportionally to the budget allocated to the activity, i.e., consider attributes as rays specific to each activity.



Fig. 6.5 Optimal activity selection with satiation and memory decay, but no acclimation. (Note: $\delta = 0.97$, $\alpha = 0$, $\gamma = 0.3$, $v(x) = x^{1/4}$, T = 20, $\lambda = 1$, $s_1 = 0$)

work and it is optimal to allocate the budget so that the net service level, $x_t^* - r_t$, is increasing over time. With power utility functions, i.e., when $v(x) = x^{\beta}$, this results in a crescendo sequence.

Proposition 6: Suppose that K = 1 and $\underline{d}_i = \overline{d}_i = d_i \ge 1$ for all *i*. When customers experience no satiation ($\gamma = 0, s_1 = 0$), when v'(x) > 0, v(0) = 0, and v''(x) < 0 for all *x*, and when the service provider is free to set any service level subject to a budget constraint, i.e., when $\Pi(\mathbf{x}) = \left\{ \mathbf{x} \left| \sum_{t=1}^T x_t \le B \right\} \right\}$, it is optimal to set the service levels such that $x_{T-t}^* = r_{T-t} + \left(v' \right)^{-1} \left(\frac{\lambda(1 + t\alpha)}{\delta^t} \right)$ for all t = 0, ..., T - 1, in which $\lambda > 0$ is such that $\sum_{t=1}^T x_t^* = B$. Moreover, $x_t^* - r_t$ is increasing in *t*. If in addition $v(x) = x^\beta$ for some $0 < \beta < 1$ when $x \ge 0, x_{T-t}^* \ge x_{T-t-1}^*$ for all t = 0, ..., T - 2.

We next consider the case with satiation and memory decay, but no acclimation. Baucells and Sarin (2007) showed that, when $\delta = 1$, the optimal service levels were constant in period t = 2, ..., T - 1, and observed that an individual's optimal consumption plan (with forward discounting) was in general decreasing over time, with possible upticks in the first and last periods. Considering customer satisfaction (with backward discounting) as the objective, we complement their result by showing that the optimal service levels are in general increasing over time, with possible upticks in the first and last periods; see Fig. 6.5 for an illustration.

Proposition 7: Suppose that K = 1 and $\underline{d}_i = \overline{d}_i = d_i \ge 1$ for all *i*. When customers experience no acclimation ($\alpha = 0$), when v'(x) > 0 and $v''(x) < (\gamma^2/\delta)v''(\gamma x) < 0$ for all *x*, and when the service provider is free to set any service level subject to a budget constraint, i.e., when $\Pi(\mathbf{x}) = \left\{ \mathbf{x} \middle| \sum_{t=1}^T x_t \le B \right\}$, it is optimal to set the service levels such that $v'(x_T^* + s_T) = (1 - \gamma)\lambda$ and $v'(x_{T-t}^* + s_{T-t}) - \$\$(\gamma/\delta)v'(\gamma(x_{T-t}^* + s_{T-t}))$ $= (1 - \gamma)\lambda/\delta^t$ for all t = 1, ..., T - 1, in which $\lambda > 0$ is such that $\sum_{t=1}^T x_t^* = B$. Moreover, $x_t^* + s_t$ is decreasing in *t*. If in addition $v(x) = x^{\beta}$ for some $\ln \delta / \ln \gamma < \beta < 1$ when $x \ge 0$, then $x_{T-t}^* \ge x_{T-t-1}^*$ for all t = 0, ..., T - 3; on the other hand, it may be that $x_2^* < x_1^*$.

Comparing Propositions 6 and 7 shows that the optimal net service level $x_t^* + s_t - r_t$ is increasing in *t* with acclimation and no satiation and decreasing in *t* with satiation and no acclimation. We conjecture that, with both acclimation and satiation, the optimal net service level will evolve in a non-monotone fashion and we leave it for future research to characterize the optimal activity selection in this more general case.

6.5 Suspense and Surprise

In entertainment, games, and sports, experiences are often characterized by an uncertain outcome (e.g., the name of the murderer in a mystery novel, or the winner of a tennis game), where uncertainty is gradually resolved as the experience unfolds. In such settings, customers may derive utility from suspense and/or surprise as they update their beliefs about the outcome, based on various information signals they capture during the experience.

We consider here a particular case of the model proposed by Ely et al. (2015) and extend their results to accommodate memory decay. Specifically, we consider an experience characterized by an uncertain event (e.g., the event that a book's main character would defeat a villain, or that one favorite's tennis player wins a game) that may be true or false. We adopt a broader conceptualization of the notion of service levels introduced in Sect. 6.2 to encompass an information policy, i.e., a set of signals to send to the customer so that she can update her beliefs about the likelihood of the event under consideration.⁵

If the customer updates her beliefs in a Bayesian fashion, the sequence of her beliefs form a martingale in the sense that the best estimate for next period's belief is the customer's current belief. Ely et al. (2015, Lemma 1) show that, for any belief martingale, there exists an information policy that induces such belief martingale. Hence, from a modeling standpoint, one need not model the details of the service provider's information policy. Indeed, one may frame the service provider's decision as choosing the customer's posterior distribution of beliefs, provided that the martingale property is satisfied, i.e., that the expected value of that posterior distribution is equal to the customer's current belief.

⁵Although a book writer has complete control over the unfolding of the story, a sports event or game manager may not fully control it; yet, the rules of the sport or game may be altered to induce more or less variance in outcomes, as is currently under consideration for the game of tennis (*The Economist* 2017).

Within the framework introduced in Sect. 6.2, we model the customer's prior belief as her reference point r_t and the service provider's decision as the choice of a posterior belief distribution that respects the martingale property. Let \tilde{x}_t be the (random) posterior belief. The service provider thus needs to choose a distribution $F(\tilde{x}_t) \in \Phi(r_t)$, where $\Phi(r_t)$ is the set of probability distributions $F(\tilde{x}_t)$ such that

$$\mathbb{E}_F[\tilde{x}_t] = \int_0^1 \tilde{x}_t dF(\tilde{x}_t) = r_t \text{ and } \int_0^1 dF(\tilde{x}_t) = 1, \text{ in which } \mathbb{E}_F[.] \text{ denotes the expecta-}$$

tion operator. Hence, the provider's decision in period *t*, given state r_t , is a distribution of beliefs $F(\tilde{x}_t) \in \Phi(r_t)$, i.e., the service provider randomizes over posterior beliefs.

Because the customer's posterior belief in period *t* will become her prior belief in period t + 1, the state transition (6.2) simplifies to $r_{t+1} = x_t$, as if $\alpha = 1$. It is thus as if the service provider were randomizing between service levels (the posterior distribution) and the customer were fully adapting to the realized service level; there is no concept of satiation in this model (i.e., $\gamma = 0$).

In addition to being subject to memory decay, the customer derives (instantaneous) utility from *suspense*, i.e., from the variance in next period's beliefs relative to her current period's beliefs, and/or from *surprise*, i.e., from any jump in belief from the previous period to the current one. As in Sect. 6.2, we assume memory decay; thus customer satisfaction evaluated at the end of the encounter puts greater weight on the most recent instantaneous utilities. Formally, the satisfaction of a customer who values *suspense* is equal to $S(\mathbf{x}, r_1) = \sum_{t=1}^{T} \delta^{T-t} \sqrt{\mathbb{E}_F[(\tilde{x}_t - r_t)^2]}$, and that of a customer who values *suspense* is equal to $S(\mathbf{x}, r_1) = \sum_{t=1}^{T} \delta^{T-t} \mathbb{E}_F[|\tilde{x}_t - r_t|]$. (With a slight abuse of notation, we use here the same notation to refer to both suspense and surprise, in reference to the concept of customer satisfaction introduced in Sect. 6.2, but note that they correspond to two different objectives.) See Ely et al. (2015) for more general forms of utility, multidimensional outcome uncertainty, and trade-offs between suspense and surprise. In particular, the model can be expanded to incorporate preferences for specific outcomes (e.g., preference that one's favorite hero would survive at the end) in addition

to suspense and surprise (Ely et al. 2015). Given that the service provider adapts the signals to the customer's beliefs, the service provider's choice of signals (or equivalently, of posterior probability distributions of beliefs) can be cast as a dynamic optimization problem. Let $\delta^{T-t} W_t(r_t)$ be the expected satisfaction generated from the instantaneous utilities derived from time *t* to the end of the encounter *T*, if the customer's current belief is equal to r_t . This customer's "satisfaction-to-go" function can be defined recursively as follows:

$$W_{T+1}(r_{T+1}) = 0$$
 for all r_{T+1} , (6.6)

and if the customer values suspense:

$$W_{t}(r_{t}) = \max_{F \in \Phi(r_{t})} \sqrt{\mathbb{E}_{F} \left[\left(\tilde{x}_{t} - r_{t} \right)^{2} \right]} + \delta^{-1} \mathbb{E}_{F} \left[W_{t+1} \left(\tilde{x}_{t} \right) \right], \forall t < T,$$

$$(6.7)$$

and if she values surprise:

$$W_t(r_t) = \max_{F \in \Phi(r_t)} \mathbb{E}_F[|\tilde{x}_t - r_t|] + \delta^{-1} \mathbb{E}_F[W_{t+1}(\tilde{x}_t)], \forall t < T.$$
(6.8)

In each period the service provider's optimization problem consists in choosing a distribution subject to moment constraints, namely that $\int \tilde{x}_t dF(\tilde{x}_t) = r_t$ and

 $\int dF(\tilde{x}_t) = 1$. It turns out that, with two moments, there exists a 2-point distribu-

tion that attains the optimum (Smith 1995). Hence, it is enough to restrict the optimization to searching over 2-point distributions that satisfy the moment constraints. In particular, F is a 2-point distribution that belongs to $\Phi(r_t)$ if there exists two numbers \bar{x}_t and \underline{x}_t such that $1 \ge \bar{x}_t \ge r_t \ge \underline{x}_t \ge 0$, such that F(x) = 0 for all $x < \underline{x}_t$, $F(x) = \frac{\bar{x}_t - r_t}{\bar{x}_t - x_t}$ for all $\underline{x}_t \le x < \bar{x}_t$, and F(x) = 1 for all $x \ge \bar{x}_t$. With these observations, we next extend the results by Ely et al. (2015) to the case with memory decay. We first consider the case of suspense.

Proposition 8: When the customer values suspense and is subject to memory decay i.e., $S(\mathbf{x}, r_1) = \sum_{t=1}^{T} \delta^{T-t} \sqrt{\mathbb{E}_F \left[\left(\tilde{x}_t - r_t \right)^2 \right]}$ with $\delta < 1$, it is optimal for the service provider solving (6.6) and (6.7) to send signals such that if the customer's belief in period t < T is equal to r_t , then her posterior belief at the end of period t is equal to $\frac{1}{2} + \sqrt{\left(r_t - \frac{1}{2}\right)^2 + \frac{1 - \delta^{-2}}{1 - \delta^{-2(T-t+1)}}r_t(1 - r_t)}}$ with probability $\frac{1}{2} + \frac{(r_t - \frac{1}{2})}{2\sqrt{(r_t - \frac{1}{2})^2 + \frac{1 - \delta^{-2}}{1 - \delta^{-2}(T - t + 1)}r_t(1 - r_t)}} \text{ and to } \frac{1}{2} - \sqrt{(r_t - \frac{1}{2})^2 + \frac{1 - \delta^{-2}}{1 - \delta^{-2}(T - t + 1)}r_t(1 - r_t)}} \text{ with }$ probability $\frac{1}{2} - \frac{\left(r_t - \frac{1}{2}\right)}{2\sqrt{\left(r_t - \frac{1}{2}\right)^2 + \frac{1 - \delta^{-2}}{1 - \delta^{-2}(T - t + 1)}r_t(1 - r_t)}}$. In period *T*, full revelation is optimal, i.e.,

the customer's posterior belief at the end of period T is equal to 1 with probability r_T

and 0 with probability $1 - r_T$.

Similar to Ely et al. (2015), we find that, in order to maximize suspense under memory decay, it is optimal to fully reveal the outcome in period T, and only in period T. Figure 6.6 illustrates a typical belief sample path. In the figure, the markers



indicate the set of possible posterior beliefs $\{r_t\}$ whereas the dotted line represents a sample path. Belief updates consist of either confirmation beliefs, which reinforce the current belief (i.e., if the current belief is greater than 0.5, the next period's belief is higher than the current belief), or plot twists, which make beliefs switch from one path to the other. The sample path depicted in Fig. 6.6 depicts two plot twists, in period 4 and in period 8. As the experience unfolds, given the dependence of the probabilities on r_t , confirmation beliefs are more frequent and plot twists less frequent.

In contrast to Ely et al. (2015), who show that, in the absence of memory decay (i.e., when $\delta = 1$), the variance in beliefs $\sqrt{\mathbb{E}_F \left[\left(\tilde{x}_t - r_t \right)^2 \right]}$ remains constant across periods under the optimal policy; Proposition 8 shows that with memory decay, a crescendo in variance in beliefs is optimal. That is, the variance in beliefs should be increasing over time since a customer who is subject to memory decay will put higher value to suspense that happens at the end of the encounter. Figure 6.7 shows that, as the intensity of memory decay increases (i.e., as δ decreases), the feasible set of the beliefs becomes more narrow and evolves more sharply near the end of the encounter. (Here, we connected the markers depicting the feasible sets, but a sample path may alternate between the boundaries of the feasible set, similar to Fig. 6.6.) As a result, memory decay induces more stable beliefs throughout most of the experience, but greater uncertainty about the final outcome near the end of the experience.

We next consider the combined effect of surprise and memory decay, generalizing the result obtained by Ely et al. (2015). As Ely et al. (2015), we only consider three periods and leave it for future research to analytically characterize the optimal solution when T > 3.

Proposition 9: When the customer values surprise and is subject to memory decay, i.e., $S(\mathbf{x}, r_1) = \sum_{t=1}^{T} \delta^{T-t} \mathbb{E}_F[|\tilde{x}_t - r_t|]$, if T = 3 and $r_1 \in [\delta(1 + \delta)/4, 1 - \delta(1 + \delta)/4]$, it is optimal for the service provider solving (6.6) and (6.8) to send signals such that the customer's posterior belief at the end of period *t* is equal to $r_t + \delta^{T-t}/4$ with probability $\frac{1}{2}$ and to $r_t - \delta^{T-t}/4$ with probability $\frac{1}{2}$. In period *T*, full revelation is optimal, i.e., the customer's posterior belief at the end of period *T* is equal to 1 with probability r_T and 0 with probability $1 - r_T$.

Figure 6.8 illustrates Proposition 9 when $r_1 = 1/2$. The markers represent the feasible set of posterior beliefs and the lines denote possible belief trajectories. As in the case with suspense, it is optimal to fully reveal the outcome in the last period; however, unlike the case with suspense, it may be optimal to do so before the last period if $r_1 \notin [\delta(1 + \delta)/4, 1 - \delta(1 + \delta)/4]$. (This latter case is not depicted in the figure since it is assumed that $r_1 = 1/2$.) In case of early resolution of uncertainty, the customer's utility in the last periods is equal to zero, given that no surprise is generated once the time the state of the event is revealed. Although it may seem counterintuitive to fully reveal the state before the end of the encounter, the possibility of such sample paths enriches the overall environment and makes the other sample paths more surprising. For instance, if every mystery novel always followed the same story template, e.g., always revealed the name of the murderer in the last chapter, there would be little room for surprise as the reader would then give no credibility to any early suspicion on identifying the murderer.

In addition, sample paths of beliefs under surprise are much spikier than sample paths under suspense. While beliefs under suspense are mostly confirming, with occasional twist plots that become less frequent as time goes by, beliefs under surprise go up and down by small increments with equal probability. Until the uncertainty is fully resolved, beliefs evolve as a random walk in which the magnitude of the steps in either direction increases over time, but the probability of updating beliefs upwards or downwards remains constant at 50%.



Fig. 6.8 Posterior belief feasible sets with surprise and memory decay. (Note: T = 3, $\delta = 0.95$, $r_1 = 0.5$)



Because of memory decay, a crescendo in surprise is optimal. Specifically, as illustrated in Fig. 6.9, as memory decay increases (i.e., as δ decreases), the funnel of belief sample paths become narrower up to the next-to-last-period, creating more room for a high (and memorable) surprise in the last period. In particular, full revelation of the outcome before the last period becomes less likely with greater memory decay. Hence, similar to its effect on suspense, memory decay tends to prolong a high degree of uncertainty about the final outcome when the customer values surprise. Moreover, the greater the memory decay, the smaller the belief updates from one period to the next. However, in contrast to its effect on suspense, memory decay does not affect the likelihood of revising upwards or downwards one's beliefs, which remains constant at 50% until uncertainty is fully resolved.

6.6 Gap Model: Satisfaction and Expectation

Our discussion has so far consisted in maximizing customer's *satisfaction*, evaluated ex-post, i.e., $S(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \sum_{\tau=1}^{T} \delta^{T-\tau} u(\mathbf{x}_{\tau}, \mathbf{r}_{\tau}, \mathbf{s}_{\tau})$, when the customer is subject to memory decay with decay rate δ . In contrast, a customer discounting time at rate θ , consistent with the economics literature (Samuelson 1937; Koopmans 1960), would value ex-ante the total utility she expects to receive from the experience as $E(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \sum_{\tau=1}^{T} \theta^{\tau-1} u(\mathbf{x}_{\tau}, \mathbf{r}_{\tau}, \mathbf{s}_{\tau})$. (The term "expectation" refers to the ex-ante nature of the assessment, and not to the stochastic nature of the experience, unlike Sect. 6.5.)

There may be a discrepancy between the customer's ex-ante expectations from the service and the overall ex-post satisfaction because consumption is discounted forward in the former and backward in the latter. As a result, customers' perceived service quality, which generally stems from comparing what they feel the service firm should offer with their perceptions of the performance of the firm (Parasuraman et al. 1988; Oliver 2015), i.e., from $S(x, r_1, s_1) - E(x, r_1, s_1)$, could be affected by that discrepancy.

In particular, a customer with reservation utility \overline{U} may choose to join a service priced at p if she expects to obtain a positive surplus from the transaction, i.e., if

 $E(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) - p \ge \overline{U}$; see, e.g., Aflaki and Popescu (2013) and Bellos and Kavadias (2017). The service provider, in turn, sets its price to capture the entire customer surplus, i.e., to $p = E(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) - \overline{U}$. Accordingly, when the customer evaluates her relative satisfaction from the service at the end of the encounter (e.g., to consider patronizing the service in the future), she will compare her overall satisfaction from the service to the price she paid, i.e.,

$$S(\mathbf{x}, \mathbf{r}_{1}, \mathbf{s}_{1}) - p = S(\mathbf{x}, \mathbf{r}_{1}, \mathbf{s}_{1}) - E(\mathbf{x}, \mathbf{r}_{1}, \mathbf{s}_{1}) + \bar{U} = \sum_{t=1}^{T} \left(\delta^{T-t} - \theta^{t-1} \right) u(\mathbf{x}_{t}, \mathbf{r}_{t}, \mathbf{s}_{t}) + \bar{U}.$$
(6.9)

In (6.9), each instantaneous utility is weighted by $(\delta^{T-t} - \theta^{t-1})$. Although this difference in discount factors is increasing over time (similar to memory decay), the optimal design may change. For instance, with linear utility functions, the sequence that maximizes customer satisfaction is U-shaped (Proposition 6), but the one that maximizes (6.9), i.e., the gap between customer satisfaction and expectations, may have an interior local maximum, as shown in Fig. 6.10.

In particular, when the customer's perceived quality is a function of the gap between her ex-post satisfaction and her ex-ante satisfaction, it may be optimal for the service provider, if her objective is to maximize the customer's perceived quality, to set low expectations (provided of course, that the customer is captive) so as to increase that gap. In Fig. 6.10, swapping the order between the first activity (x = 1) and the second activity (x = 4) would result in higher expectations (because of the immediacy of the consumption of the high service level x = 4), which would then negatively affect the gap between satisfaction and expectations.

With uncertainty in the delivery of the service levels and misaligned communication, the gap could be even larger. To illustrate this, suppose that service levels Xare random (e.g., due to lack of process conformance and heterogeneity in customer's inputs) with realization x. Suppose also that the customer, from what she heard about the service or past experience, expects to receive (random) service levels Y. With these constructs, the total gap between the satisfaction the customer derives from the service and her expectation of utility prior to the experience, is equal to

$$S(\mathbf{x},\mathbf{r}_1,\mathbf{s}_1) - \mathbb{E}[E(\mathbf{Y},\mathbf{r}_1,\mathbf{s}_1)].$$

Fig. 6.10 Sequence that maximizes gap between satisfaction and expectation. (Note: $\theta = 0.2$, $\delta = 0.8$, $\alpha = 1$, v(x) = x, x = (1, 2, 3, 4, 5))



Similar to Karmarkar and Roels (2015), this gap can be broken down into the following subcomponents:

- A process conformance gap, S(x, r₁, s₁) E[S(X, r₁, s₁)], which measures the expected gap between a customer's actual satisfaction and the expected satisfaction the process is supposed to deliver; the discrepancy here lies in the randomness in service levels;
- A customer's quality perception gap, $\mathbb{E}[S(X, r_1, s_1)] \mathbb{E}[E(X, r_1, s_1)]$, which measures the gap between a customer's expected ex-post satisfaction (given the random service levels) and her ex-ante expectations; the discrepancy here lies in the way the customer aggregates the sum of individual utilities to set her expectations (forward discounting) and to assess her satisfaction (backward discounting);
- A communication gap, E[E(X, r₁, s₁)] E[E(Y, r₁, s₁)], which measures the gap between a customer's expectations from the service, if she knew ahead of time the sequence of activities (and the variations in service levels) X relative to her expectations based on what she anticipates to receive Y.

In principle, nothing precludes these gaps to be negative, in which case they would be quality-enhancing. For instance, in case the customer values suspense or surprise, it may be optimal to introduce some degree of variability in the service levels X, as discussed in Sect. 6.5.

Assuming positive gaps, this gap decomposition highlights three possible levers to improve quality:

- To improve process conformance by reducing the variability in inputs (customeror server-related) and in process execution;
- To align customers' ex-ante and ex-post assessment methods of how much utility they derive; for instance, memory decay from a vacation can be reduced by keeping a log of the most memorable events;
- To improve the relevance of marketing campaigns to create more realistic expectations about the service delivery.

Naturally, additional gaps could exist if the service provider misunderstands the value customers derive from service levels (i.e., their utility function v(x)), their extent of memory decay (δ), acclimation (α), and satiation (γ), or the way they discount future consumption (θ).

6.7 Anticipation and Recall

We next extend the scope of our analysis to include periods of anticipation (before the encounter) and recall (after the encounter). Customers indeed derive utility from anticipating an event (Jevons 1905), and that utility can be positive (savoring) or negative (dread), depending on the nature of the event (Lowenstein 1987). Similarly, customers may derive utility from recalling the event (Baucells and Bellezza 2017).

Similar to Baucells and Bellezza (2017), we distinguish three phases: anticipation, event, and recall. Let t_a be the time at which anticipation starts, t_b be the time at which service begins, t_e be the time at which service ends, i.e., $t_e = t_b + T$, and t_r be the time at which recall ends.

During the anticipation phase, the customer looks forward to the forthcoming events, but discounts them as they are far in the future (Jevons 1905). To formalize this growing anticipation, let β be the anticipation discount factor, which may not necessarily be equal to the discount rate used to discount future consumption (Lowenstein 1987). At time t_a , the anticipated utility from the experience is thus equal to $E(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \sum_{\tau=t_b}^{t_e} \beta^{\tau-t_b} u(\mathbf{x}_{\tau}, \mathbf{r}_{\tau}, \mathbf{s}_{\tau})$. We denote by $u_A(E, t)$ the utility derived in period t, $t_a \leq t < t_b$, from anticipated utility is thus a function of both the intensity and the duration of the event (Jevons 1905).

The anticipated utility could include time discounting, e.g., $u_A(E, t) = k_A \beta^{t_b-t} E$ = $k_A \sum_{\tau=t_b}^{t_e} \beta^{\tau-t} u(\mathbf{x}_{\tau}, \mathbf{r}_{\tau}, \mathbf{s}_{\tau})$ (Lowenstein 1987). It could also include reference effects (Baucells and Bellezza 2017). For instance, let us denote by $\rho_{A, t}$ be the reference point at time *t* on the total anticipated utility and by α_A the acclimation rate in the anticipation phase. With acclimation and time discounting, the utility derived at time *t* from anticipating an experience generating a utility of *E* can be defined as $u_A(E, \rho_{A,t}, t) = \beta^{t_b-t}v_A((E - \rho_{A,t}))$, where $\rho_{A, t+1} = \alpha_A E + (1 - \alpha_A) \rho_{A, t}$ for all *t*, $t_a \leq t < t_b$, and $\rho_{A,t_a} = 0$, $v'_A(x) \geq 0$, and $v_A(0) = 0$.

Similarly, utilities during the recall phase depend on the total satisfaction $S(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \sum_{\tau=t_b}^{t_e} \delta^{t_e-\tau} u(\mathbf{x}_{\tau}, \mathbf{r}_{\tau}, \mathbf{s}_{\tau})$, which discounts utilities backward due to memory decay. We denote $u_R(S, t)$ as the utility derived in period t, $t_e < t \leq t_r$, from recalling a total utility S. The recalled utility could include time discounting, e.g., $u_R(S, t) = k_R \, \delta^{t-t_e} \, S = k_R \sum_{\tau=t_b}^{t_e} \delta^{t-\tau} u(\mathbf{x}_{\tau}, \mathbf{r}_{\tau}, \mathbf{s}_{\tau})$, but it could also include reference effects (Baucells and Bellezza 2017). For instance, let us denote $\rho_{R, t}$ as the reference point at time t on the total recalled utility and by α_R the acclimation rate in the recall phase. With acclimation and memory decay, the utility at time t from recalling an experience generating satisfaction S can be defined as $u_R(S, \rho_{R,t}, t) = \delta^{t-t_e}v_R((S - \rho_{R,t}))$, where $\rho_{R, t+1} = \alpha_R S + (1 - \alpha_R) \, \rho_{R, t}$ for all t, $t_e < t \leq t_r$, and $\rho_{R,t_e} = 0$, $v'_R(x) \geq 0$, and $v_R(0) = 0$.

In principle, the reference points during anticipation and recall, i.e., $\rho_{A, t}$ and $\rho_{R, t}$, may be different constructs from the reference points during the experience itself, i.e., r_t , given that the objects of utility during anticipation and recall, namely the total expectation *E* and the total satisfaction *S*, are different from the objects of utility during the experience, namely the service levels x_t . Alternatively, one may assume that the reference point evolves continuously throughout the different phases of anticipation, experience, and recall (Baucells and Bellezza 2017).

Figure 6.11 depicts the evolution of a customer's instantaneous utility during the phases of anticipation, event, and recall, when the event consists of a constant service level of 1, starting from period 5 to period 10, in the presence of acclimation



Fig. 6.11 Instantaneous utilities under anticipation, experience, and recall. (Note: $t_a = 1, t_b = 5$, $t_e = 10, t_r = 13, \alpha = \alpha_A = \alpha_R = 0.3, \beta = 0.4, \gamma = 0.4, \delta = 0.7, \rho_{A, 1} = \rho_{R, 11} = r_5 = 0, s_5 = 0, v(x) = v_A(x) = v_R(x) = \sqrt{x}$ if $x \ge 0$ and $-2\sqrt{-x}$ if x < 0)

during all phases and satiation during the event. During the event, the customer experiences a burst of utility at the beginning (t = 5) because the service level (x = 1) is higher than her reference point $(r_5 = 0)$ and because her level of satiation is low $(s_5 = 0)$. As the event progresses, the customer's instantaneous utility declines as she acclimates to the high service level and starts satiating.

In the anticipation phase $(1 \le t \le 4)$, because the anticipation time discount factor $(\beta = 0.4)$ is relatively small, utilities increase as time gets closer to the actual start of the event. However, should the customer discount time less (i.e., higher β) or acclimate more quickly (i.e., higher α_R), utilities could be decreasing as the time-distance to the event would matter less and the customer would acclimate more quickly to the prospect of the event. With a more intricate model of time discounting, Baucells and Bellezza (2017) show that utilities during the anticipation phase could even be U-shaped.

At the beginning of the recall phase (t = 11), the customer experiences another burst of utility from the comparison between the total satisfaction and the recall reference point (set to zero). As the distance from the event increases ($11 < t \le 13$), this utility gradually declines over time as memory fades away and the reference point adjusts to the satisfaction level. Unlike the anticipation phase, the effects of time discounting and acclimation are aligned during the recall phase and we expect recall utilities to be always trending towards zero.

Although it is well documented that customers derive utilities from anticipation and recall, it is unclear how service firms could capitalize on them since the experience either has not started or has been completed. Lowenstein (1987) observed that the total discounted utility, assessed in period t_a ,

$$\begin{split} \sum_{t=t_a}^{t_b} & \theta^{t-t_a} \ u_A\big(E, \rho_{A,t}, t\big) + \sum_{t=t_b}^{t_b+T} \theta^{t-t_a} \ u(\boldsymbol{x}, r_{t_b}, s_{t_b}) \\ & + \sum_{t=t_b+T}^{t_r} \theta^{t-t_a} \ u_R\big(S, \rho_{R,t}, t\big), \end{split}$$

in which θ is the regular rate at which customers discount future consumptions, may be unimodal in t_b . In that case, it may be optimal to delay or advance the consumption of the experience. In fact, Baucells and Bellezza (2017) demonstrate that in some cases, it may be optimal to advance the event to the point that there is no anticipation, i.e., a surprise. Hence, if a firm has control on when to start an experience, after they engaged with a customer, they may want to optimize the starting time of the experience to maximize the customer's total discounted utility.

Similarly, in the example used to build Fig. 6.11, we observed that the total discounted utility is unimodal in t_a ; that is, when $\theta = .9$, setting $t_a = 2$ instead of $t_a = 1$ (but keeping t_b , t_e , and t_r unchanged) improves the total discounted utility from 1.87 to 2.05. Hence, even for experiences that have been scheduled on particular dates (fixed t_b), service firms could potentially optimize the time they reach out to customers so that they can start anticipating the event. For instance, marathon organizers typically send participants emails in anticipation to the marathon. In particular, and consistent with our discussion of the gap model in Sect. 6.6, a customer may be willing to pay the highest price for an experience when her total discounted utility (including anticipation and recall) is the highest. By strategically timing its engagement with its customers, a service firm may then be able to charge a higher price for its service (and the anticipation and recall thereof).

6.8 Conclusions

In this chapter, we reviewed and extended existing results to design the structure of experiential services when customers are subject to acclimation, satiation, and memory decay. In particular, we considered how to sequence a given set of activities, how to select activities subject to a budget constraint on the activities' service levels, and how to disclose information about an uncertain event to maximize a customer's ex-post satisfaction, i.e., a customer's remembered utility from the service. We also discussed the design implications on service quality, specifically on the potential gap between a customer's ex-ante expectations and ex-post satisfaction, and on customer's anticipation and recall from the experience.

One may think that, in order to deliver outstanding experiences, one needs to achieve outstanding service in every activity of an encounter. Although this would certainly be a costly strategy, as argued by the design firm IDEO (Zomerdijk and Voss 2010), we showed here that this could even be counterproductive: There is indeed value creating contrast (because of acclimation) and interruptions (because of satiation). Rather than striving to excel on every activity, for a given structure of experience (e.g., sequence, activity selection and duration, information policy), one may create higher customer satisfaction by keeping the activities' service levels fixed, but changing the overall structure of the experience.

We demonstrated that crescendo designs often turn out to be optimal. Hence, despite their simplicity, they should not be underappreciated. A common design recommendation is indeed to "finish strong." While this is a robust recommendation, we identified the mechanisms under which this design is optimal. Specifically, when the satiation level either never or fully decays (Propositions 1–4) or when the customer's utility is linear (Proposition 5), and the customer is subject to either only memory decay or only acclimation, the optimal sequence of activities is a crescendo. Similarly, when activities need to be selected, the optimal design tends to a crescendo in service levels, with the exception perhaps of the first activity (Propositions 6 and 7), although the *net* service levels (i.e., relative to the reference point and satiation level) may not be monotone. Finally, when customers value suspense or surprise, memory decay leads to an information policy that increases the level of suspense or surprise over time (Propositions 8 and 9).

In general, however, the optimal design may be more complex, potentially U-shaped (Proposition 5), but also potentially with many "breaks" (Fig. 6.4). Inserting breaks resets satiation levels, creates more contrasts, and may make the subsequent activities more enjoyable. With memory decay, the potential disutility arising from an early break may be quickly forgotten, but the boost in utility in the last activities arising from the resetting satiation levels and creating contrast will tend to be the most memorable.

On potential caveat of this research is that since customers are all different, they may respond differently to particular structures of experience. However, even if different customers derive different levels of satisfaction from crescendo or U-shaped designs, they may still prefer these structures over alternative designs. Moreover, these designs tend to be relatively robust (Das Gupta et al. 2015); that is, even if there is a loss of optimality, it tends to be small. Finally, we note that the development of information technology potentially enables real-time customization of experiences (Rust and Oliver 2000); thus, if customer preferences are properly elicited, there is an opportunity to customize the experience to maximize every individual customer's satisfaction.

The stream of research on the design of structure of experiences is emerging and the opportunities for analytical extensions are numerous. Some of the potential opportunities are:

- Incorporate other behavioral factors in the customer utility model such as preferences for specific sequences, timing of peak, trend, etc. (Karmarkar and Karmarkar 2014; Dixon and Thompson 2016) and hyperbolic discounting (Plambeck and Wang 2013).
- Incorporate other "stock" variables (besides satiation), such as moods and trust (Dasu and Chase 2010), which can be affected by reputation (Gebbia 2016).
- Capture the notion of customer engagement or control (Dasu and Chase 2010), perhaps due to customer participation (a.k.a., the IKEA effect, see Norton et al. 2012), which would require incorporating a model of joint production (Roels 2014; Rahmani et al. 2017; Bellos and Kavadias 2017).
- Relax the assumption that customers are captive and test the robustness of the crescendo design in that case.
- Leverage group dynamics, such as social comparisons (Roels and Su 2013) and learning (Acemoglu et al. 2011), in case the experience involves a group of customers.

Enlarge the scope of the analysis beyond the single encounter to encompass the anticipation and recall phases and what drives customer retention across encounters (Aflaki and Popescu 2013), and more generally, what drives customer experience throughout their journey (Verhoef et al. 2009).

In addition to these analytical extensions, further empirical evidence is needed to estimate the parameters of the model (acclimation rate, satiation decay rate, memory decay rate) and validate the model predictions about customers' preferences for specific designs. This analytical work built upon findings from psychology and behavioral science, but it may now be time to go back to the lab or the field, validate (or not) the model predictions, inform future analytical developments, and improve the accuracy of their predictions and the relevance of their prescriptions. In the spirit of design thinking, it would be valuable to actively engage customers in the design process, leverage technological advances to come up with novel service designs, fertilize multi-disciplinary research, and derive design principles through iterative hypothesis testing and prototyping; see Patrício et al. (2018) for an outline of a research agenda along those directions.

Finally, from a practical standpoint, service providers operating customer-routed services (e.g. theme parks, online experiences) should investigate how to guide customers to choose a sequence of activities that maximize their ex-post satisfaction (e.g., through recommendations), which may differ from the sequence they may choose ex-ante. In addition, service providers should investigate how to monetize customer utilities derived in the anticipation and recall periods, perhaps by targeting customers at the time their ex-ante expectations about their total utility is the highest. In an era where experiences can be engineered by computers,⁶ we believe the time is indeed ripe for deriving more formal guidelines for designing the structure of experiences.

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Appendix

Lemma 1: For any $p, q \ge 1$ and $x \in [0, 1], 1 - x^{-p} - x^{-q} + x^{-p} - q \ge 0$.

Proof: The derivative of the function $1 - x^{-p} - x^{-q} + x^{-p} - q$ with respect to p equals $x^{-p} \ln x (1 - x^{-q}) \ge 0$, and similarly for the derivative with respect to q. Hence, for any $p, q \ge 1, 1 - x^{-p} - x^{-q} + x^{-p} - q \ge 1 - x^{-1} - x^{-1} + x^{-2} = (1 - x^{-1})^2 \ge 0$.

⁶For instance, the trailer of the movie Morgan was compiled by IBM's Watson; see https://www.ibm.com/blogs/think/2016/08/cognitive-movie-trailer/

Lemma 2: For any $p, q \ge 1$ and $x \in [0, 1], 1 - x^{-p} - x^{-q} + x^{-p-q}$ is decreasing in x.

Proof: The derivative of the function $1 - x^{-p} - x^{-q} + x^{-p} - q$ with respect to x equals $(p+q) x^{-p-q-1} \left(\frac{p}{p+q} x^q + \frac{q}{p+q} x^p - 1 \right)$, and it is negative given that $x^q \le 1$ and $x^p \le 1$.

Proof of Proposition 1: The proof uses an interchange argument. Throughout the proof, since K = 1, we omit the subscript k. Because $\alpha = \gamma = 0$, $S(\mathbf{x}, r_1, s_1) = \sum_{\tau=1}^{T} \delta^{T-\tau} v(x_{\tau} - r_1) - v(s_1)$. Suppose that, in the optimal sequence, Activity *i* starts in time period *t* and immediately precedes Activity *j*. In that case, because $x_{\tau} = x_i$ whenever $\pi_{\tau} = i$, we obtain

$$S(\mathbf{x}, r_1, s_1) = \sum_{\tau=1}^{t-1} \delta^{T-\tau} v(x_{\tau} - r_1) + \sum_{\tau=t}^{t+d_i-1} \delta^{T-\tau} v(x_i - r_1) + \sum_{\tau=t+d_i}^{t+d_i+d_j-1} \delta^{T-\tau} v(x_j - r_1) + \sum_{\tau=t+d_i+d_j}^{T} \delta^{T-\tau} v(x_{\tau} - r_1) - v(s_1).$$

Consider a suboptimal sequence \tilde{x} in which Activities *i* and *j* have been permuted. Because x is optimal, we must have that $S(x, r_1, s_1) - S(\tilde{x}, r_1, s_1) \ge 0$, i.e.,

$$\begin{split} S(\mathbf{x}, r_1, s_1) &- S(\tilde{\mathbf{x}}, r_1, s_1) \\ &= \sum_{\tau=t}^{t+d_i-1} \delta^{T-\tau} v(x_i - r_1) + \sum_{\tau=t+d_i}^{t+d_i+d_j-1} \delta^{T-\tau} v(x_j - r_1) \\ &- \sum_{\tau=t}^{t+d_j-1} \delta^{T-\tau} v(x_j - r_1) - \sum_{\tau=t+d_j}^{t+d_i+d_j-1} \delta^{T-\tau} v(x_i - r_1) \\ &= v(x_i - r_1) \left(\sum_{\tau=t}^{t+d_i-1} \delta^{T-\tau} - \sum_{\tau=t+d_j}^{t+d_i+d_j-1} \delta^{T-\tau} \right) \\ &+ v(x_j - r_1) \left(\sum_{\tau=t+d_i}^{t+d_i+d_j-1} \delta^{T-\tau} - \sum_{\tau=t}^{t+d_j-1} \delta^{T-\tau} \right) \\ &= (v(x_i - r_1) \\ &- v(x_j - r_1)) \delta^{T-t} \frac{1 - \delta^{-d_i} - \delta^{-d_j} + \delta^{-d_i-d_j}}{1 - \delta^{-1}} \ge 0, \end{split}$$

which implies, by Lemma 1, that $v(x_i - r_1) \le v(x_j - r_1)$. Because v(x) is increasing, this implies that $x_i \le x_j$.

Proof of Proposition 2: The proof uses an interchange argument. Throughout the proof, since K = 1, we omit the subscript k. Without loss of generality, we set $r_1 = 0$. Because $\alpha = 0$, $S(\mathbf{x}, r_1, s_1) = \sum_{\tau=1}^{T} \delta^{T-\tau} (v(x_{\tau} + s_{\tau}) - v(s_{\tau}))$. Suppose that, in the optimal sequence, Activity i starts in time period t and immediately precedes Activity j. In that case, because $x_{\tau} = x_i$ whenever $\pi_{\tau} = i$, we obtain

$$\begin{split} S(\mathbf{x}, r_1, s_1) &= \sum_{\tau=1}^{t-1} \delta^{T-\tau} \left(v(x_\tau + s_\tau) - v(s_\tau) \right) \\ &+ \sum_{\tau=t}^{t+d_i-1} \delta^{T-\tau} \left(v((\tau - t + 1) x_i + s_t) \right) \\ &- v((\tau - t) x_i + s_t) \right) \\ &+ \sum_{\tau=t+d_i}^{t+d_i+d_j-1} \delta^{T-\tau} \left(v((\tau - t - d_i + 1) x_j + d_i x_i + s_t) \right) \\ &- v((\tau - t - d_i) x_j + d_i x_i + s_t) \right) \\ &+ \sum_{\tau=t+d_i+d_j}^{T} \delta^{T-\tau} \left(v(x_\tau + s_\tau) - v(s_\tau) \right) \\ &= \sum_{\tau=1}^{t-1} \delta^{T-\tau} \left(v(x_\tau + s_\tau) - v(s_\tau) \right) - v(s_t) \delta^{T-t} \\ &+ \sum_{\tau=t+d_i}^{t+d_i-1} \delta^{T-\tau-1} v((\tau - t + 1) x_i + s_t) (\delta - 1) \\ &+ \sum_{\tau=t+d_i}^{t+d_i-1} \delta^{T-\tau-1} v((\tau - t - d_i + 1) x_j + d_i x_i \\ &+ s_t) (\delta - 1) + \delta^{T-t-d_i-d_j} v(d_j x_j + d_i x_i + s_t) \\ &+ \sum_{\tau=t+d_i+d_j+1}^{T} \delta^{T-\tau} \left(v(x_\tau + s_\tau) - v(s_\tau) \right). \end{split}$$

Consider a suboptimal sequence \tilde{x} in which Activities *i* and *j* have been permuted. Because x is optimal, we must have that $S(x, r_1, s_1) - S(\tilde{x}, r_1, s_1) \ge 0$. Because the terms associated with activities scheduled before *t* or after $t + d_i + d_j + 1$ are identical across both expressions, we must thus have that

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$$S(\mathbf{x}, r_1, s_1) - S(\tilde{\mathbf{x}}, r_1, s_1) = \sum_{\substack{\tau=t \\ \tau=t}}^{t+d_i-1} \delta^{T-\tau-1} v((\tau - t + 1) x_i + s_t)(\delta - 1)$$

$$= \sum_{\substack{\tau=t+d_i \\ \tau=t+d_i}}^{t+d_i+d_j-1} \delta^{T-\tau-1} v((\tau - t - d_i + 1) x_j + d_i x_i)$$

$$+ s_t)(\delta - 1)$$

$$- \sum_{\substack{\tau=t \\ \tau+d_i+d_j-1 \\ \tau=t+d_j}}^{t+d_i-1} \delta^{T-\tau-1} v((\tau - t + 1) x_j + s_t)(\delta - 1)$$

$$- \sum_{\substack{\tau=t+d_j \\ \tau=t+d_j}}^{t+d_j-1} \delta^{T-\tau-1} v((\tau - t - d_j + 1) x_i + d_j x_j)$$

$$+ s_t)(\delta - 1) \ge 0,$$

which implies, since v(x) is increasing, that $x_i \leq x_j$.

Proof of Proposition 3: The proof uses an interchange argument. Throughout the proof, since K = 1, we omit the subscript k. Because $\delta = 1$ and $\gamma = 0$, $S(\mathbf{x}, r_1, s_1) = \sum_{\tau=1}^{T} v(x_{\tau} - r_{\tau}) - v(s_1)$. Suppose first that Activities i and j are not the last ones. Specifically, suppose that, in the optimal sequence, Activity i starts in time period t and immediately precedes Activity j, and that Activity j precedes Activity l. In that case, because $x_{\tau} = x_i$ whenever $\pi_{\tau} = i$, we obtain

$$S(\mathbf{x}, r_1, s_1) = \sum_{\tau=1}^{t-1} v(x_{\tau} - r_{\tau}) + v(x_i - r_t) + v(x_j - x_i) + v(x_l - x_j) + \sum_{\tau=t+d_i+d_j+d_i}^{T} v(x_{\tau} - r_{\tau}) - v(s_1).$$

Consider a suboptimal sequence \tilde{x} in which Activities *i* and *j* have been permuted. Because x is optimal, we must have that $S(x, r_1, s_1) - S(\tilde{x}, r_1, s_1) \ge 0$. Because the terms associated with activities scheduled before *t* or after $t + d_i + d_j + d_i$ are identical across both expressions, we obtain that

$$S(\mathbf{x}, r_1, s_1) - S(\tilde{\mathbf{x}}, r_1, s_1) = v(x_i - r_i) + v(x_j - x_i) + v(x_l - x_j) - v(x_j - r_l) - v(x_i - x_j) - v(x_l - x_i) \ge 0.$$

We next show that this inequality holds if only if $x_i \le x_j$. Suppose first that $x_i \le x_j$. Because v(x) is subadditive, $v(x_j - r_t) \le v(x_j - x_i) + v(x_i - r_t)$. Similarly, $v(x_l - x_i) \le v(x_l - x_j) + v(x_j - x_i)$. Moreover, because of loss aversion, when $x_i \le x_j$, $v(x_j - x_i) \le -v(x_i - x_j)$. Combining these inequalities yields the desired inequality. Conversely, suppose that $x_i > x_j$. Because v(x) is subadditive, $v(x_i - r_t) \le v(x_i - x_t) \le v(x_i - x_t)$. $(x_i - x_j) + v(x_j - r_i)$. Similarly, $v(x_l - x_j) \le v(x_l - x_i) + v(x_i - x_j)$. Moreover, because of loss aversion, when $x_i > x_j$, $v(x_j - x_i) < -v(x_i - x_j)$. Combining these inequalities yields the opposite inequality. Hence, $S(\mathbf{x}, r_1, s_1) - S(\tilde{\mathbf{x}}, r_1, s_1) \ge 0$ implies that $x_i \le x_i$.

Next, suppose that Activities *i* and *j* are the last ones. Then,

$$S(\mathbf{x}, r_1, s_1) = \sum_{\tau=1}^{t-1} v(x_{\tau} - r_{\tau}) + v(x_i - r_t) + v(x_j - x_i) - v(s_1).$$

Consider a suboptimal sequence \tilde{x} in which Activities *i* and *j* have been permuted. Because x is optimal, we must have that

$$S(\mathbf{x}, r_1, s_1) - S(\tilde{\mathbf{x}}, r_1, s_1) = v(x_i - r_t) + v(x_j - x_i) - v(x_j - r_t) - v(x_i - x_j) \\ \ge 0.$$

Similar to the argument above for the case where Activities *i* and *j* are not the last ones, we can show that this inequality holds if and only if $x_i \le x_j$. Hence, $S(\mathbf{x}, r_1, s_1) - S(\tilde{\mathbf{x}}, r_1, s_1) \ge 0$ implies that $x_i \le x_j$.

Proof of Proposition 4: The proof uses an interchange argument. Throughout the proof, since K = 1, we omit the subscript k. Because $\delta = \gamma = 1$, $S(\mathbf{x}, r_1, s_1) = v$ $(x_T - r_T + s_T) - v(s_1)$. Suppose first that Activities *i* and *j* are not the last ones. Specifically, suppose that, in the optimal sequence, Activity *i* starts in time period *t* and immediately precedes Activity *j*, and that Activity *j* precedes Activity *l*. In that case, because $x_{\tau} = x_i$ whenever $\pi_{\tau} = i$, we obtain

$$x_{T} - r_{T} + s_{T} = x_{T} - r_{t} + s_{t} + \sum_{\tau=t}^{T-1} (1 - \alpha)^{T-\tau} (x_{i} - r_{t}) + \sum_{\tau=t+d_{i}}^{T-1} (1 - \alpha)^{T-\tau} (x_{j} - x_{i}) + \sum_{\tau=t+d_{i}+d_{j}}^{T-1} (1 - \alpha)^{T-\tau} (x_{l} - x_{j}) + \dots$$

Consider a suboptimal sequence \tilde{x} in which Activities *i* and *j* have been permuted. Because x is optimal, we must have that $S(x, r_1, s_1) - S(\tilde{x}, r_1, s_1) \ge 0$. Because the function v(x) is increasing, this implies that $x_T - r_T + s_T \ge \tilde{x}_T - \tilde{r}_T + \tilde{s}_T$, in which \tilde{r}_T and \tilde{s}_T are the reference point and satiation level in period *T* corresponding to sequence \tilde{x} . Because the terms associated with activities scheduled before *t* or after $t + d_i + d_j + d_l$ are identical across both expressions, as well as r_t and x_l , we must thus have that

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$$(x_{T} - r_{T} + s_{T}) - (\tilde{x}_{T} - \tilde{r}_{T} + \tilde{s}_{T}) = (x_{i} - x_{j}) \times (\sum_{\tau=t}^{T-1} (1 - \alpha)^{T-\tau} - \sum_{\tau=t+d_{i}}^{T-1} (1 - \alpha)^{T-\tau} - \sum_{\tau=t+d_{j}}^{T-1} (1 - \alpha)^{T-\tau} + \sum_{\tau=t+d_{i}+d_{j}}^{T-1} (1 - \alpha)^{T-\tau}) \ge 0.$$

After expanding the series, we obtain that

$$(x_i - x_j) \times \frac{(1 - \alpha)^{T - t + 1}}{\alpha} \times (-1 + (1 - \alpha)^{-d_i} + (1 - \alpha)^{-d_j} - (1 - \alpha)^{-d_i - d_j}) \ge 0$$

Using Lemma 1, we obtain that the second term in parentheses is always negative, which implies that $x_i \le x_j$.

When Activities *i* and *j* are the last two activities, we obtain, using a similar logic, that

$$(x_i - x_j) \times \left(\sum_{\tau=t}^{T-1} (1 - \alpha)^{T-\tau} - \sum_{\tau=t+d_i}^{T-1} (1 - \alpha)^{T-\tau} - \sum_{\tau=t+d_j}^{T-1} (1 - \alpha)^{T-\tau} \right)$$

$$\ge 0.$$

After expanding the series using the fact that $T - t = d_i + d_j$, we obtain

$$(x_i - x_j) \times \frac{(1 - \alpha)^{d_i + d_j + 1}}{\alpha} \\ \times \left(-1 + (1 - \alpha)^{-d_i} + (1 - \alpha)^{-d_j} - (1 - \alpha)^{-d_i - d_j} \right) \ge 0$$

Using Lemma 1, we obtain that the second term in parentheses is always negative, which implies that $x_i \le x_j$.

Proof of Proposition 5: The proof uses an interchange argument. Because $v_k(x) = w_k x$ for all k, $S(\mathbf{x}, \mathbf{r}_1, \mathbf{s}_1) = \sum_{\tau=1}^{T} \sum_{k=1}^{K} w_k \delta^{T-\tau} (x_{k,\tau} - r_{k,\tau})$, i.e., the terms in s_{τ} cancel each other. Suppose first that Activities *i* and *j* are not the last ones. Specifically, suppose that, in the optimal sequence, Activity *i* starts in time period *t* and immediately precedes Activity *j*, and that Activity *j* precedes Activity *l*. In that case, because $x_{k,\tau} = x_{k,i}$ whenever $\pi_{\tau} = i$, we obtain

$$S(\mathbf{x}, \mathbf{r}_{1}, \mathbf{s}_{1}) = \sum_{\tau=1}^{t-1} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} \left(x_{k,\tau} - r_{k,\tau} \right) + \sum_{\tau=t}^{T} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} (1-\alpha)^{\tau-t} \left(x_{k,i} - r_{k,i} \right) \\ + \sum_{\tau=t+d_{i},k=1}^{T} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} (1-\alpha)^{\tau-t-d_{i}} \left(x_{k,j} - x_{k,i} \right) \\ + \sum_{\tau=t+d_{i}+d_{j}}^{T} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} (1-\alpha)^{\tau-t-d_{i}-d_{j}} \left(x_{k,l} - x_{k,j} \right) \\ + \dots$$

Consider a suboptimal sequence \tilde{x} in which Activities *i* and *j* have been permuted. Because x is optimal, we must have that $S(x, r_1, s_1) - S(\tilde{x}, r_1, s_1) \ge 0$. Because the terms associated with activities scheduled before *t* or after $t + d_i + d_j + d_l$ are identical across both expressions, as well as $r_{k, t}$ and $x_{k, l}$, we thus obtain that

$$S(\mathbf{x}, \mathbf{r}_{1}, \mathbf{s}_{1}) - S(\tilde{\mathbf{x}}, \mathbf{r}_{1}, \mathbf{s}_{1}) = \sum_{\tau=t}^{T} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} (1-\alpha)^{\tau-t} \left(x_{k,i} - x_{k,j} \right) \\ + \sum_{\tau=t+d_{i}, k=1}^{T} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} (1-\alpha)^{\tau-t-d_{i}} \left(x_{k,j} - x_{k,i} \right) \\ + \sum_{\tau=t+d_{j}, k=1}^{T} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} (1-\alpha)^{\tau-t-d_{j}} \left(x_{k,j} - x_{k,i} \right) \\ + \sum_{\tau=t+d_{i}+d_{j}, k=1}^{T} \sum_{k=1}^{K} w_{k} \, \delta^{T-\tau} (1-\alpha)^{\tau-t-d_{i}-d_{j}} \left(x_{k,i} - x_{k,j} \right) \\ \geq 0.$$

Equivalently,

$$egin{aligned} &\left(\sum_{k=1}^K w_k \left(x_{k,i}-x_{k,j}
ight)
ight) \ & imes \left(\sum_{ au=t}^T \delta^{T- au}(1-lpha)^{ au-t}-\sum_{ au=t+d_i}^T \delta^{T- au}(1-lpha)^{ au-t-d_i} \ &-\sum_{ au=t+d_j}^T \delta^{T- au}(1-lpha)^{ au-t-d_j} \ &+\sum_{ au=t+d_i+d_j}^T \delta^{T- au}(1-lpha)^{ au-t-d_i-d_j}
ight) \geq 0. \end{aligned}$$

After expanding the series, we obtain

6 Optimal Structure of Experiential Services: Review and Extensions

$$\begin{split} \left(\sum_{k=1}^{K} w_{k}\left(x_{k,\bar{x}_{k,\bar{j}}}\right)\right) \times &\frac{1}{\delta - (1 - \alpha)} \\ \times \left(-(1 - \alpha)^{T - t + 1} + (1 - \alpha)^{T - t - d_{i} + 1}\right) \\ &+ (1 - \alpha)^{T - t - d_{j} + 1} - (1 - \alpha)^{T - t - d_{i} - d_{j} + 1} + \delta^{T - t + 1} \\ &- \delta^{T - t - d_{i} + 1} - \delta^{T - t - d_{j} + 1} + \delta^{T - t - d_{i} - d_{j} + 1}\right) \ge 0. \end{split}$$

Using Lemmas 1 and 2, that the last two terms are nonnegative if and only if $t \le T + 1 - \frac{\ln\left(\frac{1-\delta^{-d_i}-\delta^{-d_j}+\delta^{-d_i-d_j}}{1-(1-\alpha)^{-d_i}-(1-\alpha)^{-d_i}-d_j}\right)}{\ln\left(\frac{1-\delta}{\delta}\right)}.$

When Activities *i* and *j* are the last two activities, we obtain, using a similar logic, that

$$\begin{split} \left(\sum_{k=1}^{K} w_k \left(x_{k,i} - x_{k,j}\right)\right) \\ & \times \left(\sum_{\tau=t}^{T} \delta^{T-\tau} (1-\alpha)^{\tau-t} - \sum_{\tau=t+d_i}^{T} \delta^{T-\tau} (1-\alpha)^{\tau-t-d_i} \right) \\ & - \sum_{\tau=t+d_j}^{T} \delta^{T-\tau} (1-\alpha)^{\tau-t-d_j}\right) \ge 0. \end{split}$$

After expanding the series, it can be checked that the second term in parentheses is nonnegative if and only if $t \ge T + 1 - \frac{\ln\left(\frac{1-\delta^{-d_i}-\delta^{-d_j}}{1-(1-\alpha)^{-d_i}-(1-\alpha)^{-d_j}}\right)}{\ln\left(\frac{1-\alpha}{\delta}\right)}$. Because, for any p, $q \ge 1$, the function $1 - x^{-p} - x^{-q}$ is increasing in x, $1-\delta^{-d_i} - \delta^{-d_j} \ge 1 - (1-\alpha)^{-d_i} - (1-\alpha)^{-d_j}$ if and only if $\delta \ge 1 - \alpha$. Hence, the term $\frac{\ln\left(\frac{1-\delta^{-d_i}-\delta^{-d_j}}{\ln\left(\frac{1-\alpha}{\delta}\right)}\right)}{\ln\left(\frac{1-\alpha}{\delta}\right)}$ is always negative, and therefore, $t < T + 1 - \frac{\ln\left(\frac{1-\delta^{-d_i}-\delta^{-d_j}}{1-(1-\alpha)^{-d_i}-(1-\alpha)^{-d_j}}\right)}{\ln\left(\frac{1-\alpha}{\delta}\right)}$ for all Activities i and j. As a result, we must have that $\sum_{k=1}^{K} w_k (x_{k,i} - x_{k,j}) \le 0$ if Activities i and j are the last ones and if Activity i precedes Activity j in the optimal sequence. *Proof of Proposition* 6: We first show by induction that $x_{T-k} = r_{T-k} + (v')^{-1} \left(\frac{\lambda(1+k\alpha)}{\delta^k}\right)$. Consider the Lagrangean function $L((x_1, \dots, x_T), r_1, 0) =$ $S((x_1, \dots, x_T), r_1, 0) - \lambda \left(\sum_{t=1}^T x_t - B\right)$. Because the second-order optimality condition associated with x_T , i.e., $\frac{\partial^2 L((x_1, \dots, x_T), r_1, 0)}{\partial x_{\tau}^2} = v''(x_T - r_T) < 0$, is always

tion associated with x_T , i.e., $\frac{\partial - v(x_T, v(x_T, v_T), v_T)}{\partial x_T^2} = v''(x_T - r_T) < 0$, is always satisfied by concavity of v(x), every stationary point defines a global maximum.

Because $x_T = r_T + (v')^{-1}(\lambda)$ satisfies the first-order optimality condition associated with x_T , i.e., $\frac{\partial L((x_1,...,x_T),r_1,0)}{\partial x_T} = v'(x_T - r_T) - \lambda = 0$, it is optimal to set $x_T^* = r_T + (v')^{-1}(\lambda)$.

Fix k > 0 and suppose that it is optimal to set $x_{T-l}^* = r_{T-l} + (v')^{-1} \left(\frac{\lambda(1+l\alpha)}{\delta^l}\right)$ for l = 0, ..., k - 1. Using the induction hypothesis, we obtain that

$$\frac{\partial L((x_1, \dots, x_{T-k}, x_{T-k+1}^*, \dots, x_T^*), r_1, 0)}{\partial x_{T-k}} = \delta^k v'(x_{T-k} - r_{T-k}) \\
- \alpha \sum_{l=0}^{k-1} \delta^l (1-\alpha)^{k-1-l} v'(x_{T-l}^* - r_{T-l}) - \lambda \\
= \delta^k v'(x_{T-k} - r_{T-k}) - \alpha \sum_{l=0}^{k-1} (1-\alpha)^{k-1-l} (\lambda(1+l\alpha)) \\
- \lambda = \delta^k v'(x_{T-k} - r_{T-k}) - \lambda(1+k\alpha),$$

because

$$\begin{split} \sum_{l=0}^{k-1} (1-\alpha)^{-l} (1+l\,\alpha) \\ &= \frac{1-(1-\alpha)^{-k}}{1-(1-\alpha)^{-1}} \\ &+ \alpha (1-\alpha)^{-1} \frac{1-k \left(1-\alpha\right)^{-(k-1)} + (k-1)(1-\alpha)^{-k}}{\left(1-(1-\alpha)^{-1}\right)^2} \\ &= \frac{1-\alpha}{\alpha} \left(-1 + (1-\alpha)^{-k} + 1-k \left(1-\alpha\right)^{-(k-1)} \right) \\ &+ (k-1)(1-\alpha)^{-k} \right) \\ &= \frac{1-\alpha}{\alpha} k \left(1-\alpha\right)^{-k} \left(-(1-\alpha)+1\right) = k \left(1-\alpha\right)^{1-k}. \end{split}$$
Because $\frac{\partial}{\partial x_{T-k}} \left(\frac{\partial L((x_1, \dots, x_{T-k}, x_{T-k+1}^*, \dots, x_T^*), r_1, 0)}{\partial x_{T-k}}\right) = \delta^k v''(x_{T-k} - r_{T-k}) \\ <0 \quad \text{and because} \quad x_{T-k} = r_{T-k} + \left(v'\right)^{-1} \left(\frac{\lambda(1+k\alpha)}{\delta^k}\right) \quad \text{solves} \\ \frac{\partial L((x_1, \dots, x_{T-k}, x_{T-k+1}^*), r_1, 0)}{\partial x_{T-k}} = 0, \quad \text{it is optimal to set} \\ x_{T-k}^* = r_{T-k} + \left(v'\right)^{-1} \left(\frac{\lambda(1+k\alpha)}{\delta^k}\right). \text{ This completes the induction step.} \\ \text{Because } v''(x) < 0, x_{T-k}^* - r_{T-k} = \left(v'\right)^{-1} \left(\frac{\lambda(1+k\alpha)}{\delta^k}\right) \text{ is decreasing in } k. \end{split}$

Finally, suppose that
$$v(x) = x^{\rho}$$
 for some $0 < \beta < 1$ when $x \ge 0$. In that case, x_{T-k}^{*}
 $\ge x_{T-k-1}^{*}$ if and only if $r_{T-k} + \left(\frac{\lambda(1+k\alpha)}{\beta \delta^{k}}\right)^{\frac{1}{p-1}} \ge x_{T-k-1}^{*}$, i.e., if and only if $\left(\frac{\lambda(1+k\alpha)}{\beta \delta^{k}}\right)^{\frac{1}{p-1}} \ge (1-\alpha) \left(\frac{\lambda(1+(k+1)\alpha)}{\beta \delta^{k+1}}\right)^{\frac{1}{p-1}}$, i.e., if and only if $\left(\frac{\lambda(1+k\alpha)}{\beta \delta^{k+1}}\right) \le (1-\alpha)^{\beta-1}$, i.e., if and only if $\left(\frac{\lambda(1+k\alpha)}{\beta \delta^{k+1}}\right) \le (1-\alpha)^{\beta-1}$. The left-hand side is increasing in k , whereas the right-hand side is constant, so there is at most one crossing. Because the left-hand side is equal to $\left(\frac{\delta}{(1+k\alpha)}\right)$ when $k = 0$ and to δ when $k \to \infty$, and that both values are smaller than $(1-\alpha)^{\beta-1}$, we conclude that $x_{T-k}^{*} \ge x_{T-k-1}^{*}$ for all k .
Proof of Proposition 7: Without loss of generality, we set $r_1 = 0$. Consider the Lagrangean function $L((x_1, \dots, x_T), 0, s_1) = S((x_1, \dots, x_T), 0, s_1) - \lambda \left(\sum_{t=1}^{T} x_t - B\right)$. Because the second-order optimality condition associated with x_T , i.e., $\frac{\partial^2 L((x_1, \dots, x_T), 0, s_1)}{\partial x_T^2} = v''(x_T + s_T) < 0$, is always satisfied by concavity of v (x), every stationary point defines a global maximum. Because $x_T = -s_T + (v)^{-1}$

(λ) satisfies the first-order optimality condition associated with x_T , i.e. $\frac{\partial L((x_1,...,x_T),0,s_1)}{\partial x_T} = v'(x_T + s_T) - \lambda = 0$, it is optimal to set $x_T^* = -s_T + (v')^{-1}(\lambda)$.

We next show by induction that $v'(x_{T-k}^* + s_{T-k}) - (\gamma/\delta) v'(\gamma(x_{T-k}^* + s_{T-k})) = (1 - \gamma)\lambda/\delta^k$ for all k = 1, ..., T - 1. Because

$$\frac{\partial L((x_1, \dots, x_{T-1}, x_T^*), 0, s_1)}{\partial x_{T-1}} = \delta v'(x_{T-1} + s_{T-1}) + \gamma (v'(x_T^* + s_T) - v'(s_T)) - \lambda = \delta v'(x_{T-1} + s_{T-1}) + \gamma (\lambda - v'(\gamma (x_{T-1} + s_{T-1}))) - \lambda,$$

we obtain that

$$\frac{\partial}{\partial x_{T-1}} \left(\frac{\partial L((x_1, \dots, x_{T-1}, x_T^*), 0, s_1))}{\partial x_{T-1}} \right) \\ = \delta v''(x_{T-1} + s_{T-1}) - \gamma^2 v''(\gamma (x_{T-1} + s_{T-1})) < 0,$$

by assumption, and it is thus optimal to set x_{T-1}^* such that $\frac{\partial L((x_1, \dots, x_{T-1}, x_T^*), 0, s_1)}{\partial x_{T-1}} = \delta v'(x_{T-1} + s_{T-1}) + \gamma (\lambda - v'(\gamma (x_{T-1} + s_{T-1}))) - \lambda = 0.$

Fix k > 0 and suppose that it is optimal to set x_{T-l}^* such that $v'(x_{T-l}^* + s_{T-l}) - (\gamma/\delta) v'(\gamma(x_{T-l}^* + s_{T-l})) = (1 - \gamma)\lambda/\delta^l$ for all l = 1, ..., k. Using the induction hypothesis, we obtain that

$$\begin{aligned} \frac{\partial L((x_1, \dots, x_{T-k}, x_{T-k+1}^*, \dots, x_T^*), r_1, s_1)}{\partial x_{T-k}} \\ &= \delta^k \ v'(x_{T-k} + s_{T-k}) \\ &+ \sum_{l=0}^{k-1} \delta^l \ \gamma^{k-l} \left(v'(x_{T-l}^* + s_{T-l}) - v'(s_{T-l}) \right) - \lambda \\ &= \delta^k \ v'(x_{T-k} + s_{T-k}) - \delta^{k-1} \ \gamma \ v'(s_{T-k+1}) \\ &+ \sum_{l=1}^{k-1} \delta^l \ \gamma^{k-l} \left(v'(x_{T-l}^* + s_{T-l}) - \left(\frac{\gamma}{\delta} \right) \ v'(s_{T-l+1}) \right) \\ &+ \gamma^k v'(x_T^* + s_T) - \lambda \\ &= \delta^k \ v'(x_{T-k} + s_{T-k}) - \delta^{k-1} \ \gamma \ v'(s_{T-k+1}) \\ &+ \sum_{l=1}^{k-1} \gamma^{k-l} \lambda \ (1 - \gamma) + \gamma^k \lambda - \lambda \\ &= \delta^k \ v'(x_{T-k} + s_{T-k}) - \delta^{k-1} \ \gamma \ v'(s_{T-k+1}) + \gamma \ \lambda - \lambda. \end{aligned}$$

Hence,

$$\frac{\partial}{\partial x_{T-k}} \left(\frac{\partial L((x_1, \dots, x_{T-k}, x_{T-k+1}^*, \dots, x_T^*), r_1, s_1))}{\partial x_{T-1}} \right)$$
$$= \delta^k v''(x_{T-k} + s_{T-k}) - \delta^{k-1} \gamma^2 v'(\gamma (x_{T-k} + s_{T-k}))$$
$$< 0,$$

and it therefore is optimal to set x_{T-k}^* such that $\frac{\partial L((x_1, \dots, x_{T-k}, x_{T-k+1}^*, \dots, x_T^*), r_1, s_1)}{\partial x_{T-k}} = 0.$ This completes the induction step. Because $v'(x_{T-k}^* + s_{T-k}) - \left(\frac{\gamma}{\delta}\right)v'(\gamma(x_{T-k}^* + s_{T-k})) = \frac{(1-\gamma)\lambda}{\delta^*} < \frac{(1-\gamma)\lambda}{\delta^{k+1}} = v'(x_{T-k-1}^* + s_{T-k-1}) - \left(\frac{\gamma}{\delta}\right)v'(\gamma(x_{T-k-1}^* + s_{T-k-1}))$ and because the function $v'(x) - \left(\frac{\gamma}{\delta}\right)v'(\gamma x)$ is decreasing by assumption, we obtain that $x_{T-k}^* + s_{T-k} \ge x_{T-k-1}^* + s_{T-k-1}$ for any $k = 1, \dots, T-1$. Moreover, because $(1-\gamma)v'(x_T^* + s_T) = (1-\gamma)\lambda = \delta v'(x_{T-1}^* + s_{T-1}) - \gamma v'$ $(\gamma(x_{T-1}^* + s_{T-1})) < (\delta - \gamma)v'(x_{T-1}^* + s_{T-1}) \le (1-\gamma)v'(x_{T-1}^* + s_{T-1}),$ since v'(x) > 0, v''(x) > 0, and $\delta \le 1$; therefore, $x_T^* + s_T > x_{T-1}^* + s_{T-1}$. Finally, suppose that $v(x) = x^{\beta}$ for some $\ln\delta/\ln\gamma < \beta < 1$ when $x \ge 0$. In that case,

$$x_{T-k}^* = -s_{T-k} + \left(\frac{\delta^{k-1}\beta(\delta - \gamma^{\beta})}{\lambda(1-\gamma)}\right)^{\frac{1}{1-\beta}} \quad \text{for} \quad k = 1, \dots, T - 1 \quad \text{and}$$

$$x_{T-k}^* = -s_{T-k} + \left(\frac{\beta}{\lambda(1-\gamma)}\right)^{\frac{1}{1-\beta}} \quad \text{Suppose that } T > 3 \quad \text{Then} \quad x_{T}^* > x_{T-k}^* = \frac{1}{1-\beta} \quad \text{for and only if}$$

 $x_T^* = -s_T + \left(\frac{p}{\lambda}\right)^{\frac{1}{1-\beta}}$. Suppose that $T \ge 3$. Then, $x_T^* \ge x_{T-1}^*$ if and only if $\left(\frac{\beta}{\lambda}\right)^{\frac{1}{1-\beta}} \ge x_{T-1}^* + s_T = x_{T-1}^* + \gamma \left(x_{T-1}^* + s_{T-1}\right) = (1+\gamma) \left(x_{T-1}^* + s_{T-1}\right) - s_{T-1} = (1+\gamma) \left$

$$(1+\gamma)\left(x_{T-1}^{*}+s_{T-1}\right)-\gamma\left(x_{T-2}^{*}+s_{T-2}\right)=(1+\gamma)\left(\frac{\beta(\delta-\gamma^{\beta})}{\lambda(1-\gamma)}\right)^{\frac{1}{1-\beta}}-\gamma(\delta\beta\frac{(\delta-\gamma^{\beta})}{\lambda(1-\gamma))^{\frac{1}{1-\beta}}}$$

Hence, $x_T^* \ge x_{T-1}^*$ if and only if $1 \ge \left((1+\gamma) - \gamma \delta^{\frac{1}{1-\beta}}\right) \left(\frac{(\delta-\gamma^{\beta})}{(1-\gamma)}\right)^{1-\beta}$, which is always true since $\left((1+\gamma) - \gamma \delta^{\frac{1}{1-\beta}}\right) \left(\frac{(\delta-\gamma^{\beta})}{(1-\gamma)}\right)^{\frac{1}{1-\beta}} \le ((1+\gamma) - \gamma) \left(\frac{(1-\gamma^{\beta})}{(1-\gamma)}\right)^{\frac{1}{1-\beta}} \le 1$. Similarly, suppose that $T - k \ge 3$. Then, $x_{T-k}^* \ge x_{T-k-1}^*$ if and only if $\left(\frac{\delta^{k-1}\beta\left(\delta-\gamma^{\beta}\right)}{\lambda\left(1-\gamma\right)}\right)^{\frac{1}{1-\beta}} \ge x_{T-k-1}^* + s_{T-k} = x_{T-k-1}^* + \gamma\left(x_{T-k-1}^* + s_{T-k-1}\right) = (1+\gamma)$ $\left(x_{T-k-1}^* + s_{T-k-1}\right) - s_{T-k-1} = (1+\gamma)\left(x_{T-k-1}^* + s_{T-k-1}\right) - \gamma\left(x_{T-k-2}^* + s_{T-k-2}\right) = (1+\gamma)\left(\frac{\delta^k\beta\left(\delta-\gamma^{\beta}\right)}{\lambda\left(1-\gamma\right)}\right)^{\frac{1}{1-\beta}} - \gamma\left(\frac{\delta^{k+1}\beta\left(\delta-\gamma^{\beta}\right)}{\lambda\left(1-\gamma\right)}\right)^{\frac{1}{1-\beta}}$. Hence, $x_{T-k}^* \ge x_{T-k-1}^*$ if and only if $1 \ge \delta^{\frac{1}{1-\beta}}\left((1+\gamma) - \gamma\delta^{\frac{1}{1-\beta}}\right)$, which is always true.

Proof of Proposition 8: The proof proceeds by backward induction by showing that
$$W_t(r_t) = \sqrt{\left(1 + \sum_{\tau=1}^{T-t} \delta^{-2\tau}\right) r_t (1 - r_t)} \text{ for all } t. \text{ To initialize the induction step,}$$

we have
$$W_T(r_T) = \max_{\bar{x}_T \ge r_T \ge \underline{x}_T} \sqrt{\frac{r_T - \underline{x}_T}{\bar{x}_T - \underline{x}_T}} \$ \$ (\bar{x}_T - r_T)^2 + \frac{\bar{x}_T - r_T}{\bar{x}_T - \underline{x}_T} (r_T - \underline{x}_T)^2 =$$

 $\sqrt{(\bar{x}_T - r_T)(r_T - \underline{x}_T)}$. Because the objective function is increasing in \bar{x}_T and decreasing in \underline{x}_T , it is optimal to set $\bar{x}_T = 1$ and $\underline{x}_T = 0$. Hence, $W_T(r_T) = \sqrt{r_T (1 - r_T)}$. Fix t < T and suppose that $W_{t+1}(r_{t+1}) = \sqrt{(1 + \sum_{\tau=1}^{T-t-1} \delta^{-2\tau}) r_{t+1} (1 - r_{t+1})}$. In that case,

$$\begin{split} W_t(r_t) &= \max_{\bar{x}_t \ge r_t \ge \underline{x}_t} \sqrt{\frac{r_t - \underline{x}_t}{\bar{x}_t - \underline{x}_t} \left(\bar{x}_t - r_t \right)^2 + \frac{\bar{x}_t - r_t}{\bar{x}_t - \underline{x}_t} \left(r_t - \underline{x}_t \right)^2} \\ &+ \delta^{-1} \frac{r_t - \underline{x}_t}{\bar{x}_t - \underline{x}_t} W_{t+1}(\bar{x}_t) + \delta^{-1} \frac{\bar{x}_t - r_t}{\bar{x}_t - \underline{x}_t} W_{t+1}(\underline{x}_t) \\ &= \max_{\bar{x}_t \ge r_t \ge \underline{x}_t} \sqrt{\left(\bar{x}_t - r_t \right) \left(r_t - \underline{x}_t \right)} \\ &+ \frac{r_t - \underline{x}_t}{\bar{x}_t - \underline{x}_t} \sqrt{\left(\sum_{\tau=1}^{T-t} \delta^{-2\tau} \right) \bar{x}_t \left(1 - \bar{x}_t \right)} \\ &+ \frac{\bar{x}_t - r_t}{\bar{x}_t - \underline{x}_t} \sqrt{\left(\sum_{\tau=1}^{T-t} \delta^{-2\tau} \right) \underline{x}_t \left(1 - \underline{x}_t \right)}. \end{split}$$

1

Taking the first-order optimality conditions yields that

$$\bar{x}_t = \frac{1}{2} + \sqrt{\left(r_t - \frac{1}{2}\right)^2 + \frac{1}{1 + \left(\sum_{t=1}^{T-t} \delta^{-2\tau}\right)} r_t(1 - r_t)} \quad \text{and} \quad \underline{x}_t = \frac{1}{2} - \frac{1}{2}$$

 $\sqrt{\left(r_t - \frac{1}{2}\right)^2 + \frac{1}{1 + \left(\sum_{\tau=1}^{T-t} \delta^{-2\tau}\right)} r_t(1 - r_t)}$. Substituting these values into the objective function yields:

function yields:

$$\begin{split} W_{t}(r_{t}) &= \sqrt{\frac{1}{1 + \left(\sum_{\tau=1}^{T-t} \delta^{-2\tau}\right)} r_{t}(1 - r_{t})} \\ &+ \sqrt{\left(\sum_{\tau=1}^{T-t} \delta^{-2\tau}\right) \left(\frac{1}{4} - \left(r_{t} - \frac{1}{2}\right)^{2} - \frac{1}{1 + \left(\sum_{\tau=1}^{T-t} \delta^{-2\tau}\right)} r_{t}(1 - r_{t})\right)} \\ &= \sqrt{\left(1 + \left(\sum_{\tau=1}^{T-t} \delta^{-2\tau}\right)\right) r_{t}(1 - r_{t})}, \end{split}$$

completing the induction step. To see that this solution is indeed optimal, note that $W_t(r_t)$ corresponds to the optimal solution of the problem of allocation a total variance $r_t(1 - r_t)$ across the T - t + 1 remaining periods so as to maximize $\sum_{\tau=t}^{T} \delta^{t-\tau} \sqrt{v_{\tau}}$ subject to $\sum_{\tau=t}^{T} v_{\tau} \leq r_t(1 - r_t)$ (Ely et al. 2015, online appendix). *Proof of Proposition 9:* The proof proceeds by backward induction. In period *T*, $W_T(r_T) = \max_{\bar{x}_T \geq r_T \geq \underline{x}_T} \frac{r_T - \underline{x}_T}{\bar{x}_T - \underline{x}_T} (\bar{x}_T - r_T) + \frac{\bar{x}_T - r_T}{\bar{x}_T - \underline{x}_T} (r_T - \underline{x}_T)$. Because the objective function is increasing in \bar{x}_T and decreasing in \underline{x}_T , it is optimal to set $\bar{x}_T = 1$ and $\underline{x}_T = 0$. Hence, $W_T(r_T) = 2r_T(1 - r_T)$. Therefore, in period T - 1,

$$W_{T-1}(r_{T-1}) = \max_{\bar{x}_{T-1} \ge r_{T-1} \ge \underline{x}_{T-1}} \frac{r_{T-1} - \underline{x}_{T-1}}{\bar{x}_{T-1} - \underline{x}_{T-1}} \left(\bar{x}_{T-1} - r_{T-1} \right) + \frac{\bar{x}_{T-1} - r_{T-1}}{\bar{x}_{T-1} - \underline{x}_{T-1}} \left(r_{T-1} - \underline{x}_{T-1} \right) + \delta^{-1} \frac{r_{T-1} - \underline{x}_{T-1}}{\bar{x}_{T-1} - \bar{x}_{T-1}} W_T(\bar{x}_{T-1}) + \delta^{-1} \frac{\bar{x}_{T-1} - r_{T-1}}{\bar{x}_{T-1} - \bar{x}_{T-1}} W_T(\underline{x}_{T-1}) = \max_{\bar{x}_{T-1} \ge r_{T-1} \ge \underline{x}_{T-1}} 2 \frac{r_{T-1} - \underline{x}_{T-1}}{\bar{x}_{T-1} - \underline{x}_{T-1}} \left(\bar{x}_{T-1} - r_{T-1} \right) + 2 \delta^{-1} \left(\frac{r_{T-1} - \underline{x}_{T-1}}{\bar{x}_{T-1} - \underline{x}_{T-1}} \overline{x}_{T-1} \left(1 - \bar{x}_{T-1} \right) + \frac{\bar{x}_{T-1} - r_{T-1}}{\bar{x}_{T-1} - \underline{x}_{T-1}} \underline{x}_{T-1} \left(1 - \bar{x}_{T-1} \right) \right).$$

Taking the first-order optimality conditions with respect to \underline{x}_{T-1} and \overline{x}_{T-1} (and ignoring the suboptimal non-informative solutions) yields the following solution: $\underline{x}_{T-1} = r_{T-1} - \delta/4$ and $\overline{x}_{T-1} = r_{T-1} + \delta/4$ if $r_{T-1} \in [\frac{\delta}{4}, 1 - \frac{\delta}{4}]$, $\underline{x}_{T-1} = 0$ and $\overline{x}_{T-1} = \sqrt{\delta r_{T-1}}$ if $r_{T-1} \in [0, \frac{\delta}{4}]$, and $\underline{x}_{T-1} = 1 - \sqrt{\delta (1 - r_{T-1})}$ and $\overline{x}_{T-1} = 1$ if $r_{T-1} \in [1 - \frac{\delta}{4}, 1]$. Hence,

$$W_{T-1}(r_{T-1}) = \begin{cases} 2r_{T-1}(1+\delta) - 4r_{T-1}\sqrt{\delta r_{T-1}} & \text{if } r_{T-1} \in \left[0, \frac{\delta}{4}\right] \\ \delta^{-1}2r_{T-1}(1-r_{T-1}) + \frac{\delta}{8} & \text{if } r_{T-1} \in \left[\frac{\delta}{4}, 1 - \frac{\delta}{4}\right] \\ 2(1-r_{T-1})(1+\delta) - 4(1-r_{T-1})\sqrt{\delta (1-r_{T-1})} & \text{if } r_{T-1} \in \left[1 - \frac{\delta}{4}, 1\right]. \end{cases}$$

Consider next period T - 2:

$$W_{T-2}(r_{T-2}) = \max_{\bar{x}_{T-2} \ge r_{T-2} \ge \underline{X}_{T-2}} \frac{r_{T-2} - \underline{x}_{T-2}}{\bar{x}_{T-2} - \underline{x}_{T-2}} (\bar{x}_{T-2} - r_{T-2}) + \frac{\bar{x}_{T-2} - r_{T-2}}{\bar{x}_{T-2} - \underline{x}_{T-2}} (r_{T-2} - \underline{x}_{T-2}) + \delta^{-1} \frac{r_{T-2} - \underline{x}_{T-2}}{\bar{x}_{T-2} - \underline{x}_{T-2}} W_{T-1}(\bar{x}_{T-2}) + \delta^{-1} \frac{\bar{x}_{T-2} - \bar{x}_{T-2}}{\bar{x}_{T-2} - \bar{x}_{T-2}} W_{T-1}(\underline{x}_{T-2}).$$

When $r_{T-2} \in \left[\frac{\delta+\delta^2}{4}, 1-\frac{\delta+\delta^2}{4}\right]$, the function is maximized at $\underline{x}_{T-2} = r_{T-2} - \delta^2/4$ and $\overline{x}_{T-2} = r_{T-2} + \delta^2/4$.

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