Optimal Design of New Steel Connections

Salvatore Benfratello, Luigi Palizzolo(✉), and Pietro Tabbuso

University of Palermo, 90144 Viale delle Scienze, Ed 8, Palermo, Italy
{salvatore.benfratello, luigi.palizzolo, pietro.tabbuso}@unipa.it

Abstract. The Limited Resistance Rigid Perfectly Plastic Hinge (*LRPH*) are special steel connections mainly usable to join beam elements of plane or spatial steel frames. The fundamental characteristics of these devices are the mutual independence of their own resistance and stiffness features as well as the respect of assigned constraints related to the elastic and limit behaviour of the joined elements. Within the frame structural scheme, the device plays the role of a rigid perfectly plastic hinge, constituted by a suitably sized sandwich section. The efficient use of the *LRPH* in the relevant frame depends on the appropriate design of the device geometry. In the present paper, a new approach devoted to the optimal flexural design of the *LRPH* is presented, according with the imposed mechanical constraints as well as with further suggested technological ones. The optimization procedure is based on a genetic algorithm approach and different applications are reported confirming the good applicability of the computational method as well as the reliability of the relevant device.

Keywords: Steel device · *LRPH* · Optimal design · Behavioural features
Finite element analysis

1 Introduction

The greater part of the European standards, which regulates the structural design of civil and industrial constructions, prescribes the evaluation of different load conditions and, consequently, different limit behaviours to be imposed to the structure. Usually, two different conditions are defined: the first one, called serviceability condition, is characterized by the presence of quasi static loads (gravity loads and wind actions) and moderate intensity seismic actions; the second one, called ultimate limit condition, is characterized by the presence of suitably reduced gravity loads and full intensity seismic actions. Correspondingly, it is prescribed that the structure exhibits an elastic or a shakedown behaviour in serviceability conditions and that it does not collapse under limit load conditions, even suffering a limited amount of damage. Therefore, optimal structures must possess adequate stiffness properties in order to ensure the complete usability in serviceability conditions and good resistance and ductility features in order to respect the imposed limit conditions.

In this context, steel structures find their ideal application and, actually, they are more and more frequently utilized in new constructions as well as within restoration interventions. Moreover, the practical utilization of these structures and the evolution of

H. C. Rodrigues et al. (Eds.): *EngOpt 2018 Proceedings of the 6th International Conference on Engineering Optimization*, pp. 644–656, 2019.
https://doi.org/10.1007/978-3-319-97773-7_57

the structural standards are the result of the great effort spent by the researchers in their scientific activity. Actually, in the last decades, many papers have been proposed regarding the analysis and/or the design of steel frames subjected to static loads either within the elastic limit or beyond the same limit (see, e.g., [1–8]), and as many theoretical and computational studies have been devoted to steel structures subjected to dynamic loads (see, e.g., [9–20]).

The behaviour of the structure depends on the adopted steel profiles and on the way they are joined each other. Usually, the connections are designed to ensure very high stiffness and resistance to the nodes, so that the elastic and post elastic response of the structure is just governed by the steel element features. In other words, the elastic and the limit behaviours of the structure are depending on each other being related both to the stiffness of the utilized profiles and to their limit resistance. On the other side, in many cases of practical interest it can be required that structures exhibit an elastic and a limit behaviour independent of each other. Two examples of such cases are that when the structure capacity design must be satisfied and that of the design of a steel frame devoted to the structural restoration of an opening in a wall panel. In these cases, it is desirable to introduce appropriate resistance reduction in suitably chosen structural cross sections and, yet, it would be desirable not to introduce any elastic stiffness variation. In this topic, some studies have been published (see, e.g., [21–27]) proposing suitable modifications of the steel element geometry with the aim of obtaining a reduced resistance capacity. It has to be noticed that utilizing these approaches the mutual dependence of stiffness and resistance is still present.

Recently, a new type of connection for steel elements has been proposed by the authors [28, 29] which allows to design a structure with appropriate and independent stiffness and resistance features. This special connection is a steel device and it represents a Limited Resistance Rigid Perfectly Plastic Hinge (*LRPH*), covered by patent n. 102017000088597 at the Italian Ministry of Economic Development, constituted by two parallel bounding plates, connecting the relevant structure elements, with inside a suitably designed sandwich section. In the last referenced papers, a first design has been presented, based on a simple sequential computational approach, being the latter just aimed to prove the compliance of the proposed devices to the imposed constraints. In the present paper, an optimization procedure based on a genetic algorithm approach is utilized in order to perform the flexural optimal design of *LRPH* connections according with the prescribed mechanical, kinematical and technological constraints. The obtained results, even if obtained in the limited field of pure bending, are very encouraging and they represent a fundamental starting point for practical applications related to full stress behaviour and structures that are more complex.

2 Features and Design of *LRPH*

As previously described, the Limited Resistance Rigid Perfectly Plastic Hinge (*LRPH*) is an innovative steel device, devoted to connect steel beam elements of plane and spatial frames. This device possesses two fundamental independent features [29]: (a) its yield bending moment can be suitably prefixed; (b) its elastic flexural stiffness can be appropriately designed in order to equal the elastic flexural stiffness of the connected frame elements.

In Fig. 1a, a scheme of *LRPH* connecting two typical steel profiles is represented, while in Fig. 1b and c, the *LRPH* is represented according to different views being the dimensions pointed out.

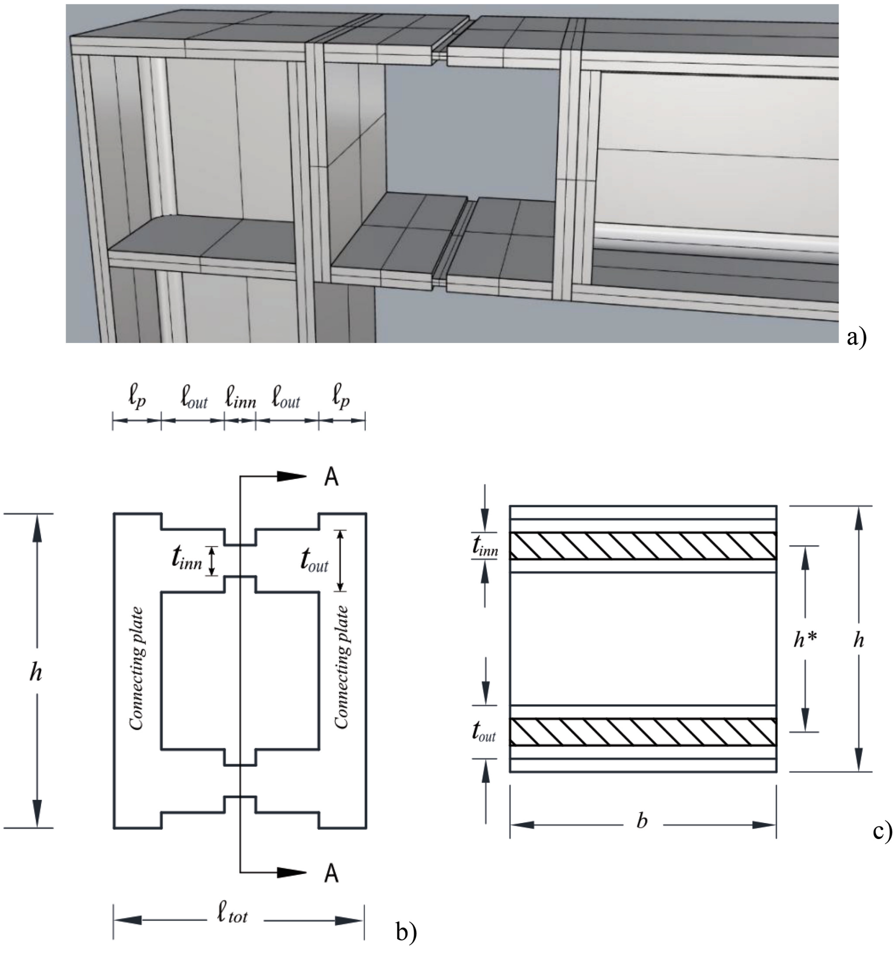

Fig. 1. (a) 3D view of *LRPH* connecting beam and pillar; (b) front view; (c) section A–A.

In particular, b and h are the plane dimensions of the connecting plates, equal to the dimensions of the joined steel profiles, and ℓ_{tot} is the total length of the device. *LRPH* can be thought as constituted by five subsequent portions:

- the first and last portions are the connection plates just devoted to ensure a perfectly rigid joint between *LRPH* and steel elements;

- the central portion is a perfect sandwich section with small thickness wings, appropriately designed in order to ensure the suitably assigned yield bending moment value;
- the remaining two symmetric portions are as many sandwich sections with appropriate thickness such that the device complies with the imposed elastic flexural stiffness constraints.

It is easy to verify that:

$$\ell_{tot} = \ell + 2\ell_p \tag{1}$$

with:

$$\ell = 2\ell_{out} + \ell_{inn} \tag{2}$$

being:

- ℓ the total length of the composed sandwich section;
- ℓ_p the thickness of the connection plates;
- ℓ_{out} the length of the outer portions of the composed sandwich section;
- ℓ_{inn} the length of the inner portion.

Furthermore, the outer portions have the same thickness t_{out}, while t_{inn} is the thickness of the inner portion. Due to the required features, as it will be clarified later on, it will always result $t_{out} > t_{inn}$. Furthermore, always with reference to Fig. 1c, h^* is the internal lever arm common to the three sandwich sections. Consequently, the moments of inertia of the inner and outer sections are, respectively:

$$J_{inn} = \frac{bt_{inn}^3}{6} + bt_{inn}\frac{h^{*2}}{2} \tag{3a}$$

$$J_{out} = \frac{bt_{out}^3}{6} + bt_{out}\frac{h^{*2}}{2} \tag{3b}$$

and the corresponding yield bending moments are:

$$M_{inn}^0 = \sigma_y bt_{inn}h^* \tag{4a}$$

$$M_{out}^0 = \sigma_y bt_{out}h^* \tag{4b}$$

being σ_y the material yield stress.

Therefore, the yield bending moment of the device is:

$$M_{LRPH} = M_{inn}^0 < M_{out}^0 \tag{5}$$

The use of *LRPH* is substantially required when we want to introduce in a beam element a reduction of the resistance capacity at any cross-section element without modifying the original stiffness of the relevant steel profile.

As a typical example, we can consider the simple steel frame in Fig. 2, where geometry, material features and load conditions are defined.

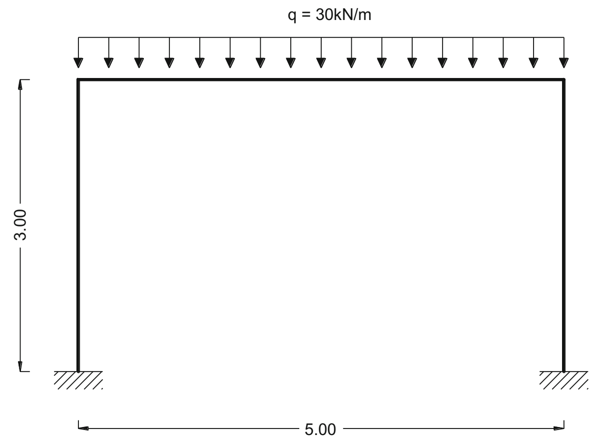

Fig. 2. Geometry and load condition of the S275 steel frame

As usual, a sufficiently reliable design can be realized by imposing resistance and displacement constraints. In particular, by evaluating the maximum bending moment acting on the beam as $M_{max} = 48.00\,\text{kNm}$, an IPE200 could be used, being

$$W^{el}_{\text{IPE200}} = 194.3\,\text{cm}^3 > \frac{M_{max}}{\sigma_y} = \frac{4,800}{27.5} = 174.55\,\text{cm}^3 \tag{6}$$

Besides the mechanical constraint in the design of such structure, it is usual to impose a displacement constraint. The latter, in the case under consideration, assumes the following expression

$$f_{lim} = \frac{\ell}{300} = 1.67\,\text{cm} \tag{7}$$

being f_{lim} the maximum allowed deflection. It can be easily deduced that the IPE200 profile is no longer adequate and that a different steel profile with suitable characteristics is necessary. After very simple analysis, it is possible to deduce that, among the on-sale profiles, the suitable one is an IPE270, for which

$$J_{\text{IPE270}} = 5,790\,\text{cm}^4 > \frac{3q\ell^4}{384Ef_{lim}} = 4,176.9\,\text{cm}^4 \tag{8}$$

In these conditions, in order to respect the structure capacity design imposing that the yield bending moment of the pillar be not lower than the corresponding bending moment of the beam, even the pillar must be constituted at least by IPE 270 profiles, with useless waste of material.

Alternatively, the proposed device can be utilized with the constraint

$$M_{LRPH} = M_{max} = 48.00\,\text{kNcm} \tag{9}$$

and designed so that its flexural stiffness equals the analogous stiffness related to the beam. Utilizing such a device, the frame can be constituted by pillars HEA180 and beam IPE270 being satisfied the required structure capacity design (Fig. 3).

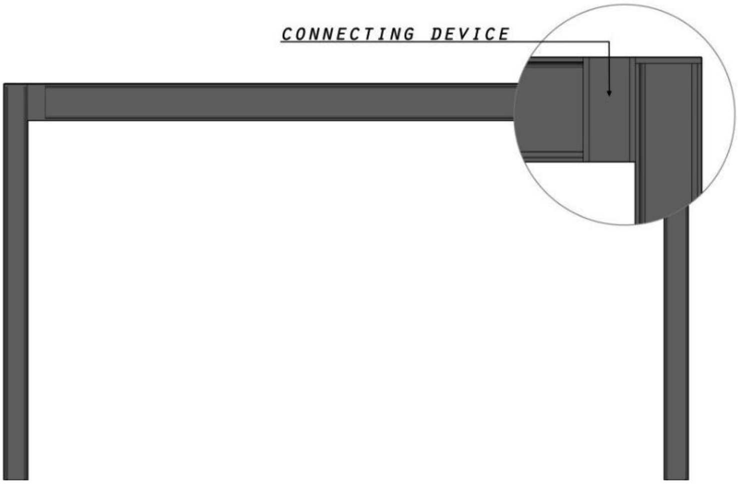

Fig. 3. Connecting device between HEA180 pillars and IPE270 beam.

Therefore, *LRPH* must be characterized by an assigned yield bending moment M^0_{ass} and by appropriate flexural stiffness, features independent of each other.

The first requirement can be satisfied imposing $M_{LRPH} = M^0_{ass}$ and designing t_{inn} and h^*. With regard to the definition of the appropriate flexural stiffness, the following approach can be utilized. The proposed device (Fig. 3) substitutes a limited portion of the beam to which it is connected. If any variation in the stiffness properties of the beam must be avoided, *LRPH* must possess a flexural stiffness equal to the one characterizing the substituted portion of the beam. This goal can be achieved by imposing the equality between the relative rotation of the bound sections of the *LRPH* device and that of the portion of substituted original beam [29] as reported in Fig. 4.

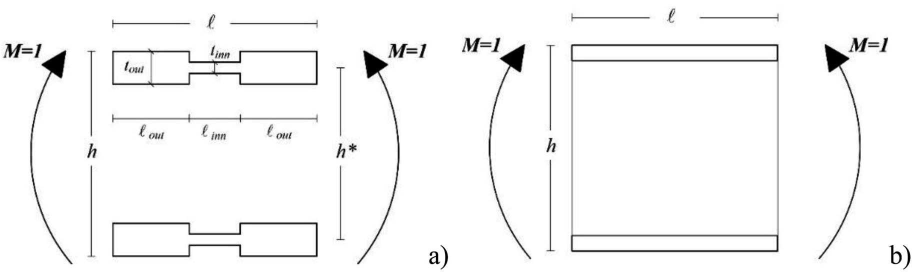

Fig. 4. Evaluation of the equivalent bending stiffness: (a) *LRPH*; (b) given profile.

In the hypothesis of small displacements, linear elasticity and by adopting an Euler-Bernoulli beam model, the following relation ensures the respect of the above described flexural stiffness constraint:

$$\frac{J_{inn}}{J_{out}}\left(\frac{J_{out}-J_p}{J_p-J_{inn}}\right)=\frac{\ell_{inn}}{\ell_{out}} \tag{10}$$

where J_p is the moment of inertia of the steel profile connected to the device.

In order to satisfy all the above requirements the following optimization problem must be solved:

$$\min_{(\boldsymbol{x})} f(\boldsymbol{x}) \tag{11a}$$

subjected to:

$$\boldsymbol{x}_{lb} \leq \boldsymbol{x} \leq \boldsymbol{x}_{ub} \tag{11b}$$

$$\boldsymbol{A}_{eq}\boldsymbol{x} = \boldsymbol{c}_{eq} \tag{11c}$$

$$\boldsymbol{A}_{in}\boldsymbol{x} \leq \boldsymbol{c}_{in} \tag{11d}$$

$$G_{eq}(\boldsymbol{x}) = 0 \tag{11e}$$

$$G_{in}(\boldsymbol{x}) \leq 0 \tag{11f}$$

where $\boldsymbol{x}$ is the design variable vector ($\boldsymbol{x}^T = |h^* \quad t_{out} \quad t_{inn} \quad \ell_{out} \quad \ell_{inn}|$) and $f(\boldsymbol{x}) = t_{out}/t_{inn}$ is the relevant objective function chosen in order to fulfill clear desirable technological requisites and to ensure a good coherence with the adopted mechanical model (Euler-Bernoulli model).

Furthermore,

$$\boldsymbol{x}_{lb}^T = \left|h/2 \quad t_p \quad 0 \quad 0 \quad 0\right| \tag{12}$$

and

$$\boldsymbol{x}_{ub}^{T} = \left| h - t_p \quad h/2 \quad h/2 \quad \ell/2 \quad \ell \right| \tag{13}$$

are the lower and upper bound vectors, respectively, related to the design variables, being t_p the wing thickness of the connected steel profiles,

$$\boldsymbol{A}_{eq} = |0 \quad 0 \quad 0 \quad 2 \quad 1| \tag{14}$$

is the row matrix of the linear equality constraint and $\boldsymbol{c}_{eq} = \ell$ is the related given constant term,

$$\boldsymbol{A}_{in} = \left| \begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & -1 \end{matrix} \right| \tag{15}$$

is the matrix of the linear inequality constraints, and

$$\boldsymbol{c}_{in}^{T} = | \, h \quad 0 \, | \tag{16}$$

is the related given constant term vector. It is worth noting that $0 < \alpha \leq \ell_{inn}/t_{inn}$ introduces a suitable technological constraint for the device.

Finally,

$$G_{eq}(\boldsymbol{x}) = \frac{b t_{inn} \sigma_y h^*}{M_{LRPH}} - 1 \tag{17}$$

is the nonlinear equality constraint function and

$$G_{in}(\boldsymbol{x}) = \frac{\ell_{inn}}{\ell_{out}} - 2 \frac{t_{inn}\left[t_{inn}^2 + 3h^{*2}\right]}{t_{out}\left[t_{out}^2 + 3h^{*2}\right]} \left\{ \frac{b t_{out}\left[t_{out}^2 + 3h^{*2}\right] - 6J_p}{6J_p - b t_{inn}\left[t_{inn}^2 + 3h^{*2}\right]} \right\} \tag{18}$$

is the nonlinear inequality constraint function.

Therefore, in order to solve the above-reported optimization problem, once the geometrical features (b, h, t_p and J_p) of the steel profile to be connected and the related material yield stress σ_y are known, the following quantities must be suitably assigned: ℓ_{tot}, ℓ_p (and as a consequence ℓ), α and M_{LRPH}. These choices will be explained in the following application section.

3 Applications

As first application, we shall design the same *LRPH* device considered in [29]. Therefore, let us consider a S275 steel profile HEA240. The required goal is the design of the optimal device characterized by a yield bending moment $M_{LRPH} = 0.40$ $M_{p,y}^0 = 75.948$ kNm, being $M_{p,y}^0$ the relevant yield bending moment of the steel profile, and such that it also respects the above described stiffness constraints. Basing on a simple computational approach, as utilized in [29], imposing $\ell_{tot} = h = 230\,\text{mm}$ and $\ell_p = 24\,\text{mm}$, a sub-optimal solution is reached. It results (Table 1).

Table 1. Geometry features of the sub-optimal device connecting HEA240 profile.

h^*	t_{out}	t_{inn}	ℓ_{out}	ℓ_{inn}	J_{out}	J_{inn}	t_{out}/t_{inn}	ℓ_{inn}/ℓ_{out}
190	40	6.056	70.817	40.366	17,584	2,624.35	6.60	0.57

An appropriate finite element analysis has been effected for the obtained device and the results confirm its goodness. Actually, it respects all the imposed constraints in terms of resistance and elastic stiffness behaviour, but the high ratio between outer and inner thickness produces an undesired stress concentration in correspondence of the two common sections between the thinner and the thicker sections (Fig. 5).

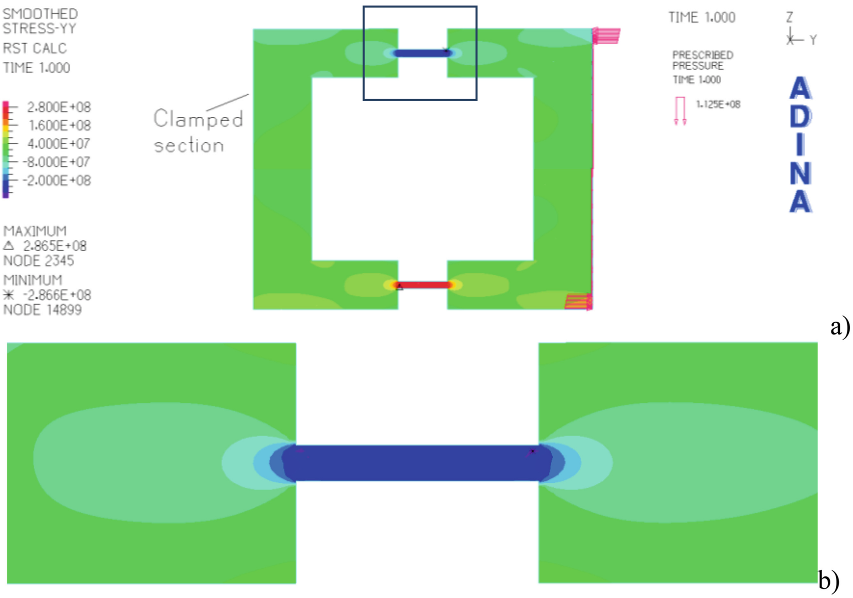

Fig. 5. (a) Sub-optimal *LRPH* connecting HEA240 profile; (b) Stress distribution between thinner and the thicker sections.

As a consequence, problem (11) has been solved by means of a suitable genetic algorithm (see, e.g., [30–32]) with the following data: $\ell_{tot} = h = 230\,\text{mm}$, $\ell_p = 24\,\text{mm}$, $\alpha = 2$, $M_{LRPH} = 0.40 M^0_{p,y} = 75.948\,\text{kNm}$. It results (Table 2).

Table 2. Geometry features of the optimal device connecting HEA240 profile.

h^*	t_{out}	t_{inn}	ℓ_{out}	ℓ_{inn}	J_{out}	J_{inn}	t_{out}/t_{inn}	ℓ_{inn}/ℓ_{out}
214.355	15.645	5.368	85.632	10.737	8,641.61	2,960.41	2.91	0.125

The optimal device (Fig. 6) yet confirms the goodness of the device and shows that the obtained reduced optimal ratio between outer and inner thickness allows to achieve a definitely better behaviour with respect to the stress distribution.

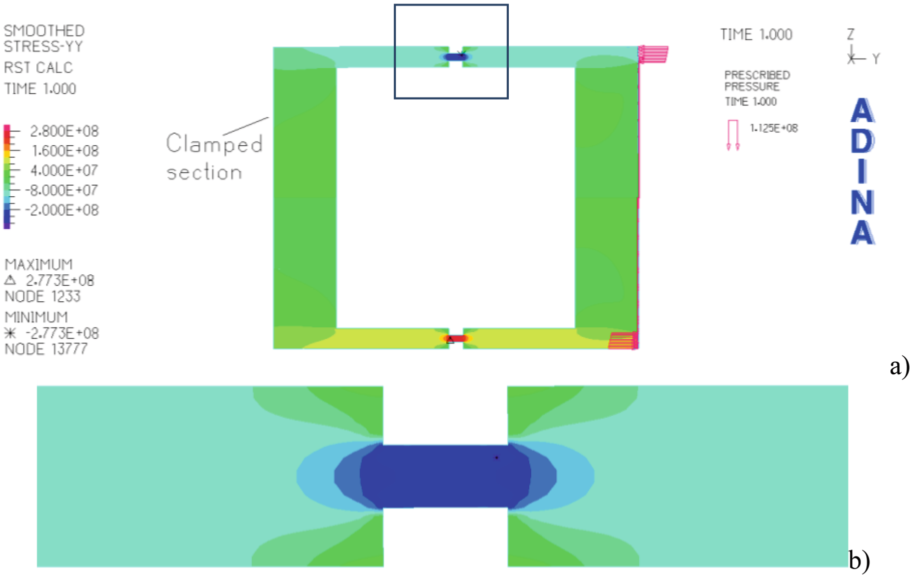

Fig. 6. (a) Optimal *LRPH* connecting HEA240 profile; (b) Stress distribution between thinner and the thicker sections.

As previously stated, one of the parameters which it is necessary to be chosen is the device total length ℓ_{tot}. It is usually assumed equal to the smaller dimension of the connected steel profile. Therefore, it was considered as a useful application the solution of the proposed optimization problem for a steel profile (S275) characterized by a more stretched cross section, and, in particular, the IPE 270 already designed in the previous section has been considered. We want to design the optimal device characterized by a yield bending moment $M_{LRPH} = 0.36M^0_{p,y} = 48\,\text{kNm}$ and respecting the prescribed stiffness constraints. The solution to problem (11), with $\ell_{tot} = b = 135\,\text{mm}$, $\ell_p = 20.4\,\text{mm}$ and $\alpha = 2$ provides (Table 3).

Table 3. Geometry features of the optimal device connecting IPE270 profile.

h^*	t_{out}	t_{inn}	ℓ_{out}	ℓ_{inn}	J_{out}	J_{inn}	t_{out}/t_{inn}	ℓ_{inn}/ℓ_{out}
253.39	16.608	5.094	42.006	10.187	7208.1	2,207.9	3.26	0.2425

The results obtained for this last optimal device by a finite element analysis (Fig. 7) are definitely coherent with the theoretical expectation.

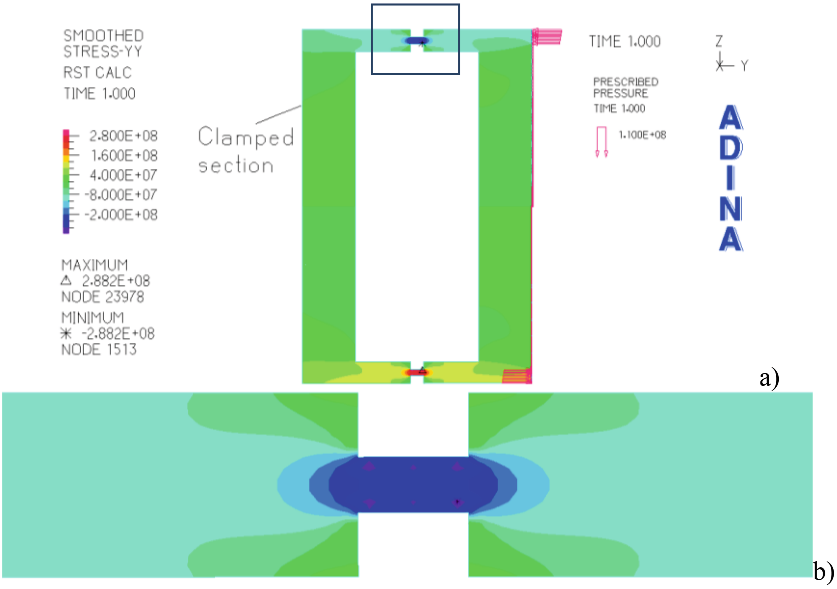

Fig. 7. (a) Optimal *LRPH* connecting IPE270 profile; (b) Stress distribution between thinner and the thicker sections.

4 Conclusions

The present paper has been devoted to the study of an innovative steel beam element connection named Limited Resistance Rigid Perfectly Plastic Hinge (*LRPH*). In particular, a specific optimization problem, devoted to the design of the optimal shape of these devices according to suitably imposed constraints, has been proposed. Actually, the proposed *LRPH* must possess two fundamental and independent characteristics: the relevant yield bending moment is given and it is lower than the yield bending moment of the connected beam elements, the elastic flexural stiffness is equal to the one of the connected beam elements. The device consists in a special sandwich section with wing thickness variable in a discrete manner. It is characterized by two identical outer sandwich section with greater thickness and an inner sandwich section with lower thickness. The proposed objective function of the relevant minimum optimal design problem is the ratio between the outer and the inner wing thickness. The optimization of this parameter is related to clear technological constraints as well as to the respect of usual beam models. Different applications are reported and they show the complete reliability of device with respect to the imposed constraints, as well as the effected FEM analyses related to the optimal *LRPH* provide encouraging results with reference to the utilized model.

References

1. König, J.A., Maier, G.: Shakedown analysis of elastoplastic structures: a review of recent developments. Nucl. Eng. Des. **66**(1), 81–95 (1981)
2. Polizzotto, C.: A unified treatment of shakedown theory and related bounding techniques. Solid Mech. Arch. **7**, 19–75 (1982)
3. Giambanco, F., Palizzolo, L., Caffarelli, A.: Bounds on plastic strains for elastic plastic structures in plastic shakedown conditions. Struct. Eng. Mech. **25**(1), 107–126 (2007)
4. Gokhfeld, D.A., Cherniavsky, D.F.: Limit Analysis of Structures at Thermal Cycling. Sijthoff & Noordhoff, Alphen aan den Rijn (1980)
5. Kaliszky, S., Lógó, J.: Optimal design of elasto-plastic structures subjected to normal and extreme loads. Comput. Struct. **84**(28), 1770–1779 (2006)
6. Benfratello, S., Giambanco, F., Palizzolo, L., Tabbuso, P.: Optimal design of steel frames accounting for buckling. Meccanica **48**(9), 2281–2298 (2013)
7. Palizzolo, L., Caffarelli, A., Tabbuso, P.: Minimum volume design of structures with constraints on ductility and stability. Eng. Struct. **68**(1), 47–56 (2014)
8. Marti, K.: Limit load and shakedown analysis of plastic structures under stochastic uncertainty. Comput. Methods Appl. Mech. Eng. **198**(1), 42–51 (2008)
9. Benfratello, S., Giambanco, F., Palizzolo, L.: Shakedown design of structures under dynamic loading. In: Advances and Trends in Structural Engineering, Proceedings of the 4th International Conference on Structural Engineering, Mechanics and Computation (SEMC 2010), Cape Town (ZA), pp. 1089–1092 (2010)
10. Benfratello, S., Palizzolo L., Tabbuso, P.: Dynamic shakedown design of structures under repeated seismic loads. In: Research and Applications in Structural Engineering, Mechanics and Computation, Proceedings of the 5th International Conference on Structural Engineering, Mechanics and Computation, SEMC 2013, Cape Town (ZA), pp. 241–246 (2013)
11. Benfratello, S., Palizzolo, L., Tabbuso, P.: Optimal design of elastic plastic frames accounting for seismic protection devices. Struct. Multidiscip. Optim. **49**(1), 93–106 (2014)
12. Palizzolo, L., Benfratello, S., Tabbuso, P.: Discrete variable design of frames subjected to seismic actions accounting for element slenderness. Comput. Struct. **147**, 147–158 (2015)
13. Benfratello, S., Palizzolo, L., Tabbuso, P.: Optimization of structures with unrestricted dynamic shakedown constraints. Struct. Multidiscip. Optim. **52**(3), 431–445 (2015)
14. Spence, S.M.J., Chuang, W.C., Tabbuso, P., Bernardini, E., Kareem, A., Palizzolo, L., Pirrotta, A.: Performance-based engineering of wind excited structures: a general methodology. In: Geotechnical and Structural Engineering Congress, Proceedings of the Joint Geotechnical and Structural Engineering Congress, Phoenix, USA, pp. 1269–1282 (2016)
15. Benfratello, S., Di Paola, M., Palizzolo, L., Tabbuso, P.: Evaluation of the shakedown limit load multiplier for stochastic seismic actions. Meccanica **52**(11–12), 2735–2750 (2017)
16. Corigliano, A., Maier, G., Pycko, S.: Dynamic shakedown analysis and bounds for elastoplastic structures with nonassociative, internal variable constitutive laws. Int. J. Solids Struct. **32**(21), 3145–3166 (1995)
17. Ceradini, G.: Dynamic shakedown in elastic-plastic bodies. J. Eng. Mech. Div. **106**(3), 481–499 (1980)
18. Maier, G., Novati, G.: Dynamic shakedown and bounding theory for a class of nonlinear hardening discrete structural models. Int. J. Plast. **6**(5), 551–572 (1990)
19. Duc Chinh, P.: Dynamic shakedown and a reduced kinematic theorem. Int. J. Plast. **12**(8), 1055–1068 (1996)
20. Maier, G., Vitiello, E.: Bounds on plastic strains and displacements in dynamic shakedown of work-hardening structures. J. Appl. Mech. **41**(2), 434–440 (1974)

21. Shen, J., Kitjasateanphun, T., Srivanich, W.: Seismic performance of steel moment frames with reduced beam sections. Eng. Struct. **22**(8), 968–983 (2000)
22. Jin, J., El-Tawil, S.: Seismic performance of steel frames with reduced beam section connections. J. Constr. Steel Res. **61**(4), 453–471 (2005)
23. Chou, C.C., Wu, C.C.: Performance evaluation of steel reduced flange plate moment connections. Earthq. Eng. Struct. Dyn. **36**(14), 2083–2097 (2007)
24. Sofias, C.E., Kalfas, C.N., Pachoumis, D.T.: Experimental and FEM analysis of reduced beam section moment endplate connections under cyclic loading. Eng. Struct. **59**, 320–329 (2014)
25. Kulkarni, S.A., Vesmawala, G.: Study of steel moment connection with and without reduced beam section. Case Stud. Struct. Eng. **1**, 26–31 (2014)
26. Plumier, A.: The dogbone: back to the future. Eng. J. **34**(2), 61–67 (1997)
27. Plumier, A.: Behaviour of connections. J. Constr. Steel Res. **29**(1–3), 95–119 (1994)
28. Benfratello, S., Cucchiara, C., Palizzolo, L., Tabbuso, P.: Fixed strength and stiffness hinges for steel frames. In: Ascione, L., Berardi, V., Feo, L., Fraternali, F., Tralli, A.M. (eds.) The XXIII Conference of the Italian Association of Theoretical and Applied Mechanics (AIMETA 2017), vol. 3, pp. 1287–1296. Gechi Edizioni, Milano (2017)
29. Benfratello, S., Palizzolo, L.: Limited resistance rigid perfectly plastic hinges for steel frames. Int. Rev. Civ. Eng. **8**(6), 286–298 (2017)
30. Benfratello, S., Palizzolo, L., Tabbuso, P.: Seismic shakedown design of frames based on a probabilistic approach. WIT Trans. Built Environ. **137**, 359–370 (2014)
31. Tabbuso, P., Spence, S.M.J., Palizzolo, L., Pirrotta, A., Kareem, A.: An efficient framework for the elasto-plastic reliability assessment of uncertain wind excited systems. Struct. Saf. **58**, 69–78 (2016)
32. Benfratello, S., Palizzolo, L., Tabbuso, P.: Probabilistic evaluation of the adaptation time for structures under seismic loads. Procedia Eng. **161**, 434–438 (2016)