Uncertainty Quantification and Model Identification in a Bayesian and Metaheuristic Framework

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Abstract. Uncertainty quantification of identified parameters is an important feature when some quality assessment of the results of model updating procedure is necessary, or when important decisions depend upon these values. In this work, a modification of the conventional sensitivity method is tested along with a Bayesian Monte Carlo framework for identification of system parameters from experimental data, and their probability distributions. First, the updating procedure uses a metaheuristic algorithm (derivative-free) and the Euclidean norm metric. Then, a modification of Markov Chain Monte Carlo method called Transitional MCMC is applied to obtain an approximation of the mean values and probability distributions of the updated parameters based on the scattering of the experimental data. An example is presented with real structure experimental data for updating discrete mass, stiffness and damping parameters, as well as a comparison with previous results yielded by different methods, suggesting equivalent levels of agreement in the updated parameters, but with the advantage of MCMC formulation being practically independent of parameters vectors.

Keywords: System identification · Uncertainty quantification
Model updating · Modal analysis

1 Introduction

Limited available information, misinterpretation of underlying mechanisms, as well as inherent randomness have been imposing the necessity of the study of engineering systems in the presence of uncertainties. The problem of uncertainty quantification is not trivial and depends immensely on the engineer's ability to correctly choose the uncertainties to be assessed. In principle, the presented discrepancies between model and measurements can be accommodated by a set of uncertainty variables that may or may not be the right ones related to the uncertainty in the measurement values. Besides, the attributed uncertainty in the model's parameters may group more than one source of uncertainty. In any case, it is a consensus that a numerical model that behaves like, and explain experimental values obtained from a real system is of immense value for correct estimates and parametric studies.

Dynamical systems with different geometric configurations may present remarkable distinct dynamic behavior. Differently from the case of damage detection, large

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changes in geometrical configurations will produce large measurable variations in the dynamic behavior. Apart from symmetrical configurations, the source of this variability is obvious and it is easily perceived/identified in the measured mode shapes and mode frequencies.

Considering the significant levels of inaccuracy in the estimation of some important parameters, various methods have been proposed, that now are broadly available, for identification/updating of dynamic models (Friswell and Mottershead 1995; Maia and Silva 1997). The former part of this work presents an application of the Bayesian MCMC method (Patelli et al. 2017) in the updating of equivalent transverse motion stiffness of a three-storey building model using experimental data from accelerometers.

2 Methodology

2.1 Markov Chain Monte Carlo (MCMC)

Markov Chain Monte Carlo is now a well-established technique used to acquire knowledge on distribution statistics based upon measured data. Several specialized textbooks in the literature deal with the algorithm and their use in physics, engineering, biology, etc. (Brooks et al. 2011; Gamerman et al. 2006; Yuen 2010). In the Bayesian Model Updating framework, it is assumed a prior joint probability density function for the unknown parameters. Using the Bayes theorem, this prior probability is updated using an error estimation of the differences between measured output parameters and the predicted ones. Let $\boldsymbol{\theta}$ be the vector of the unknown parameters, $\boldsymbol{\varepsilon} = z_m - z_p$, the vector of the errors between the measured output parameters and the predicted ones. So, by the Bayes theorem (Yuen 2010) the conditional joint probability density function $p(\boldsymbol{\theta}|\boldsymbol{\epsilon})$ of the unknown parameters, given the known error between the measured and predicted output parameters had occurred, is given by:

$$p(\boldsymbol{\theta}|\boldsymbol{\epsilon}) = \frac{p(\boldsymbol{\epsilon}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\boldsymbol{\epsilon})}, \tag{1}$$

where $p(\boldsymbol{\theta})$ is the prior joint probability density function of the parameters, that should be known *a priori*. The conditional bayesian probability $p(\boldsymbol{\theta}|\boldsymbol{\epsilon})$ is most frequently called the *posterior probability*, and will be referred to in this manner in what follows. Usually, in the absence of any information related to this probability, the first approach is to assume a uniform multidimensional probability density function $U(\boldsymbol{\theta})$ (all combinations of parameters are equally likely). $p(\boldsymbol{\epsilon})$ is the probability of the errors. $p(\boldsymbol{\epsilon}|\boldsymbol{\theta})$ means the joint probability density function of the errors given that the parameters $\boldsymbol{\theta}$ had occurred (sampled). It is also known as the likelihood. This last probability is assumed as a Gaussian, zero-mean joint probability, as:

$$P(\boldsymbol{\epsilon}|\boldsymbol{\theta}) = \frac{1}{(2\pi)^{1/np}||\boldsymbol{S}_{\varepsilon}||^{1/2}} e^{\left[\boldsymbol{\epsilon}^T \boldsymbol{S}_{\epsilon}^{-1} \boldsymbol{\epsilon}\right]} \tag{2}$$

where $\boldsymbol{S}_\varepsilon$ is the auto-correlation matrix between the measured output parameters and is the probability of the errors, also known as the normalization factor, such that:

$$P(\epsilon) = \int P(\epsilon|\boldsymbol{\theta})P(\boldsymbol{\theta})d\boldsymbol{\theta} \tag{3}$$

It should be noticed that $P(\epsilon|\boldsymbol{\theta})P(\boldsymbol{\theta})$ can assume any shape in the multidimensional space of the unknown parameters, differently from $p(\epsilon|\boldsymbol{\theta})$ that is assumed as zero mean Gaussian probability. Once the Posterior probability is obtained, a sampling technique should be used in order to get the statistics of the unknown parameters. One way is using a Markov Chain Monte Carlo (MCMC) simulation that is an improvement of the Metropolis Hastings Algorithm to generate samples based on an unknown joint probability density function.

Briefly, MCMC follows the main steps of the Metropolis Hastings algorithm (Yuen 2010). Let's say that $p(\boldsymbol{\theta}|\epsilon)$ is available after a Bayesian Updating. For a given number of samples to be generated, it starts with an initial vector of the unknown parameters $\boldsymbol{\theta}_0$ and then random perturbations are given for this guess point $\boldsymbol{\theta_p} = \boldsymbol{\theta}_0 + \boldsymbol{\delta\theta}$. An acceptance criterion is stated as:

Algorithm 1. Metropolis Hastings algorithm.

1. Set $j = 0$
2. Guess $\boldsymbol{\theta}_0$ such that $P(\theta_0|\boldsymbol{\varepsilon})$ exists
3. Repeat until desired number os samples are met
4. Evaluate an attempt for the next element of the chain $\boldsymbol{\theta_p} = \boldsymbol{\theta_0} + \boldsymbol{\delta\theta}$
5. Evaluate $\alpha = min\,[1, P(\boldsymbol{\theta}_{j-1}|\boldsymbol{\varepsilon})/P(\boldsymbol{\theta}_p|\boldsymbol{\varepsilon})]$ or $max\,[0, P(\boldsymbol{\theta}_{j-1}|\boldsymbol{\varepsilon})/P(\boldsymbol{\theta}_p|\boldsymbol{\varepsilon})]$
6. Evaluate $r = rand(0,1)$
7. If $r < \alpha$, then accept $\boldsymbol{\theta}_p$ as a new element of the chain $\boldsymbol{\theta}_j = \boldsymbol{\theta}_p$ otherwise set the new element of the chain as $\boldsymbol{\theta}_j = \boldsymbol{\theta}_{j-1}$
8. Update $j = j + 1$ and return to 3.

So, at the end, the will exist a chain of parameters $\boldsymbol{\Theta} = \boldsymbol{\theta}_0, \boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_n$ that represents the unknown joint probability density function. Statistics of the parameters, such as mean, standard deviation, histograms (pdf estimate), c.d.f., skewness, etc. can be obtained based on the values of the chains, discarding some very early initial values when the chains had not converged to a stationary distribution. More details can be found in (Ching and Chen 2007). The MCMC should contain a large number of elements in the chain in order to obtain the reliable statistics to evaluate mean values, cumulative distributions and even covariance matrix (Silva et al. 2016).

3 Numerical and Experimental Model

The three-storey building used in the experiments is composed of three polymer blocks rigidly attached to two metal beams, one at each side, and the beams fixed to another block serving as a base to whole structure. Figure 1 shows a sketch of the model with main dimensions and variables.

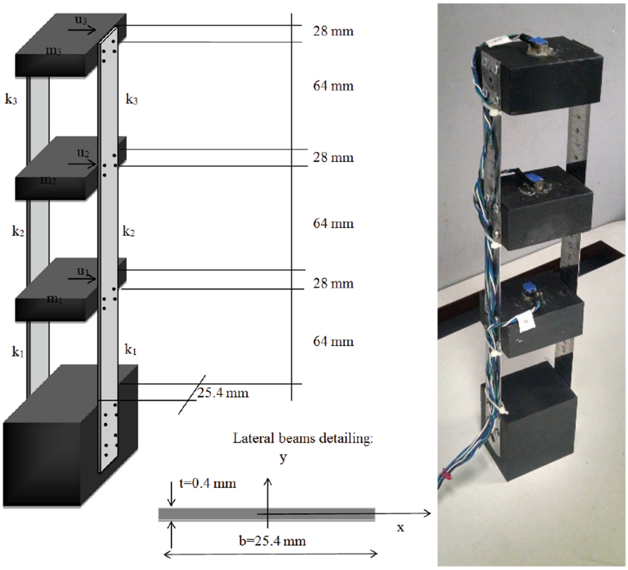

Fig. 1. Main dimensions and variables for the three-storey building example.

Due to the two lateral beams, the rotational degree of freedom is suppressed, remaining only the lateral one. The nominal values for the masses are $m_1 = m_2 = m_3 = 112$ g. The lateral beams are made of steel and have a nominal Young's modulus E = 196 GPa, and so the equivalent stiffness of the relative transverse motion between the blocks was calculated considering clamped-clamped Euler–Bernoulli beams, giving 1.215 N/m as a first estimate to be used in the iterative numerical procedure. With these values, mass and stiffness matrices were assembled, as written below, and the predicted natural frequencies were calculated considering the undamped case.

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{m}_1 & 0 & 0 \\ 0 & \boldsymbol{m}_2 & 0 \\ 0 & 0 & \boldsymbol{m}_3 \end{bmatrix}, \quad \boldsymbol{K} = \begin{bmatrix} 2\boldsymbol{k}_1 + 2\boldsymbol{k}_2 & -2\boldsymbol{k}_2 & 0 \\ -2\boldsymbol{k}_2 & 2\boldsymbol{k}_2 + 2\boldsymbol{k}_3 & -2\boldsymbol{k}_3 \\ 0 & -2\boldsymbol{k}_3 & 2\boldsymbol{k}_3 \end{bmatrix} \tag{4}$$

The corresponding (undamped) eigenvalue/eigenvector system is defined, as usual, as:

$$\boldsymbol{K}\boldsymbol{\phi}_i = \lambda_i \boldsymbol{M}\boldsymbol{\phi}_i \tag{5}$$

where $\boldsymbol{\phi}_i$ is the i-th eigenvector and the i-th eigenvalue, λ_i, is the square of the i-th natural frequency, ω_i, measured in rad/s ($\lambda_i = \omega_i^2$).

Then, the nominal natural frequencies for the model are evaluated solving Eq. (5), giving: f_1 = 10.43 Hz, f_2 = 29.24 Hz, f_3 = 42.25 Hz.

The base of the block was rigidly fixed and each of the storeys was excited by manual tapping. The acceleration of each story block was measured with an Analog

Devices ADXL 203 accelerometer, with sensitivity of 970 mV/g and saturation of ± 1.7 V, and the signals were acquired with a Measurement Computing 12 bit USB acquisition board at a sampling rate of 500 Hz. The period of acquisition was defined as 15 s, so that the frequency resolution of the discrete Fourier transform was approximately 0.066 Hz, and the natural frequencies were obtained by simple peak picking. Some additional treatments were performed on the experimental data. The pick peaking process for natural frequencies determination was improved by tracing splines over some few points around the picks in the acceleration spectrum. Moreover, Chauvenet's criterion was applied to detect and remove experimental recordings most likely contaminated with gross human errors during the manual tapping process. The experimental data (resonance frequencies and damping ratios) were then determined again from the set of remaining observations, and the obtained mean values and standard deviations are presented further, together with the outputs predicted by the updated model.

Applying this procedure, the measured statistical data of damped frequencies, summarized by Table 1, for a set of 130 samples, were obtained independently. The corresponding correlation coefficients of the measured data are presented in Table 2.

Table 1. Statistical data of measured damped frequencies and identified damped frequencies by MCMC using only stiffness and mass parameters.

	Experimental		Identified		Difference
	Mean (Hz)	Std. Deviation (Hz)	Mean (Hz)	Std. Deviation (Hz)	(Hz)
f_1	10.754	0.01730	10.758	0.01751	−0.004
f_2	28.806	0.03525	28.814	0.03551	−0.008
f_3	42.909	0.01292	42.921	0.01308	−0.012

4 Numerical Results

4.1 Model Updating of Stiffness and Mass Parameters Only

In this example, only stiffness and mass parameters were updated using MCMC, leaving for a posterior analysis, the damping updating. It was allowed a range of 1% to 100% of variation in the initial nominal values for k_1, k_2, k_3, m_1, m_2 and m_3. It was used 200 updates for the MCMC method and 200 chains with an acceptance rate of 75% in the metropolis Hastings algorithm and a rate of increment of 1%. Figure 2 shows the scatter plot for the sample of 130 for three damped frequencies and those obtained with the updated model. Tables 1 and 2 also shows the obtained frequencies and corresponding correlation coefficient after MCMC model updating process (Fig. 3).

For the updated parameter, Fig. 4 shows the convergence of the MCMC algorithm for the six parameters used in the model updating, where a burn in of 30% of the data was applied. The statistics for the updated parameters are presented in Table 3.

Table 2. Correlation coefficient for the measured damped frequencies and those from the updated model.

	Experimental			Identified		
ρ	f_1	f_2	f_3	f_1	f_2	f_3
f_1	1	0.6185	0.7290	1	0.6333	0.7402
f_2	0.6185	1	0.4576	0.6333	1	0.4902
f_3	0.7290	0.4576	1	0.7402	0.4902	1

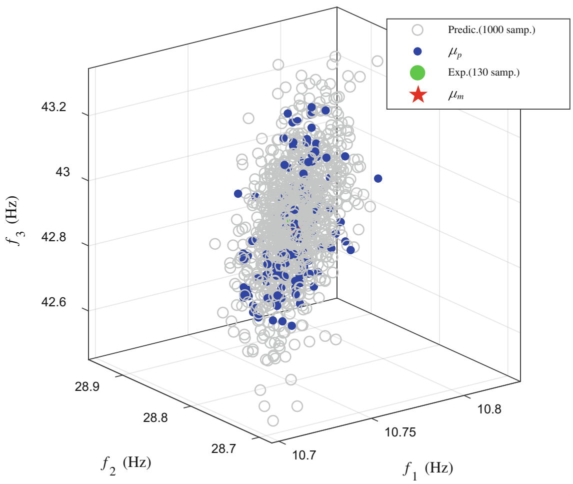

Fig. 2. Scatter cloud data for the experimentally measured three damped frequencies and that obtained with the updated numerical model.

4.2 Model Updating of Mass, Stiffness and Damping Parameters

In order to also update the mass and damping matrices, and at the same time evaluate the uncertainties in these parameters, the procedure was performed again in two sequential steps: (a) first with the components of the mass and stiffness matrices taken as input parameters and the undamped natural frequencies as outputs; and (b) with only the components of the damping matrix as the input parameters and damping ratios as outputs (mass and stiffness matrices remaining fixed in the values achieved in the previous step). In this last step the predicted values of the damping ratios were obtained from polynomial eigensolution (Tisseur and Meerbergen 2001), while experimental ones were extracted from measured time-domain data using exponential decrement technique and Hilbert transform to obtain response envelopes (Feldman 2011).

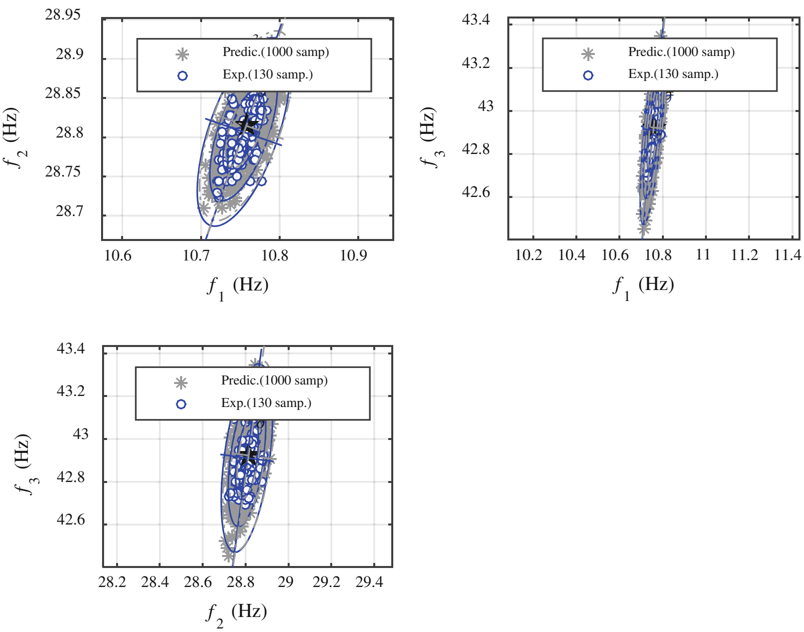

Fig. 3. Confidence ellipses for the experimentally measured three damped frequencies and those obtained using the updated model.

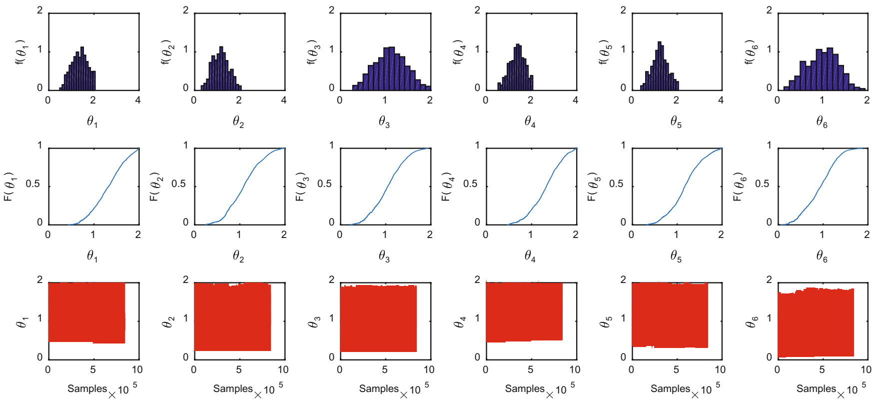

Fig. 4. Probability density functions and cumulative density function for each of the 6 updated parameters (normalized) and the corresponding convergence of Markov Chain Monte Carlo series (a burn in of 30% was applied).

Table 3. Updating factors for input parameters in the case of stiffness and mass updating.

Parameters	Mean (dimensionless)	SD (dimensionless)
k_1	1.2923	0.00360
k_2	1.0797	0.00364
k_3	0.9744	0.00323
m_1	1.2209	0.00358
m_2	1.1561	0.00360
m_3	0.9223	0.00362

For the identification of the damping parameters, a viscous non-proportional model was assumed and the physical system was idealized as a translational mass-spring-damper oscillator, as described by the matrix below:

$$\boldsymbol{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 + c_4 \end{bmatrix} \tag{6}$$

More details about this model can be found in Löw and Gomes (2017). Figure 5 shows the scatter plot of the measured damping ratios and those obtained with the updated model (Fig. 6).

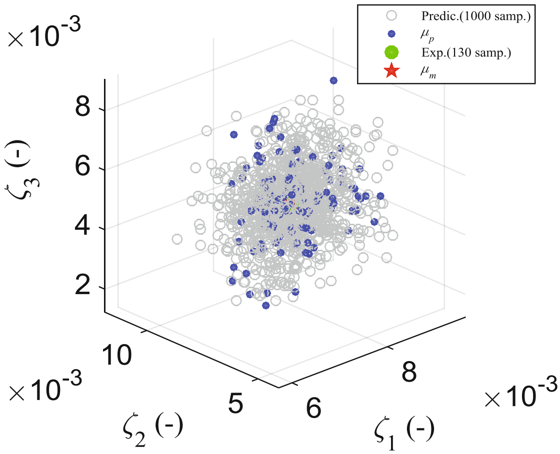

Fig. 5. Scatter cloud data for the experimentally measured three damping ratio and that obtained with the updated numerical model.

The mean updating dimensionless parameters for damping ratio obtained were: 1.63559, 0.01548, 1.66917 and 1.42906, with standard deviations of 0.17116, 0.41118, 0.23719 and 0.14540. Figure 7 shows the converged Markov Chains for the dimensionless damping coefficients.

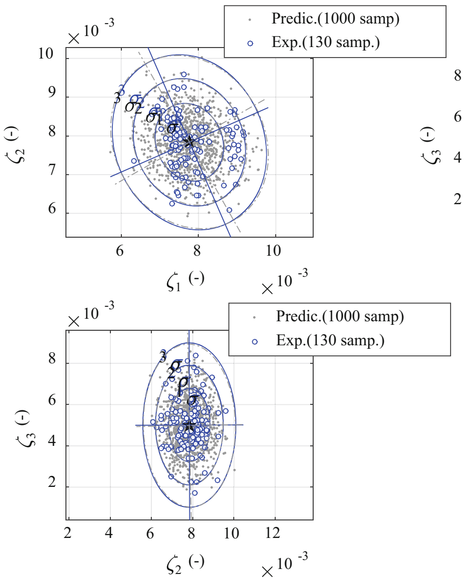

Fig. 6. Confidence ellipses for the experimentally measured damping ratios and those obtained using the updated model.

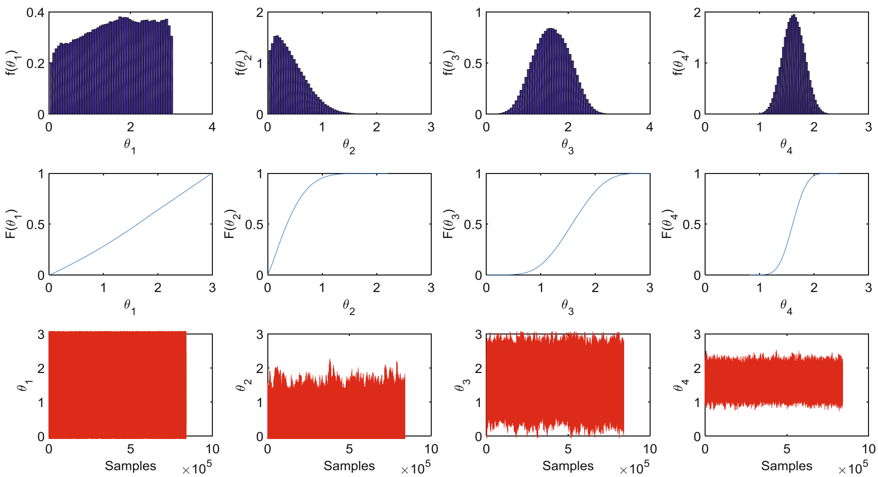

Fig. 7. Probability density functions and cumulative density function for each of the 4 updated dimensionless damping coefficient and the corresponding converged Markov Chains Monte Carlo series (a burn in of 30% was applied).

5 Conclusions

Bayesian model updating with uncertainty evaluation was applied to a shear frame three dof-like structure for which a large set of experimental acceleration observations was available. The procedure was first applied in the case that the whole uncertainty of the system is considered as concentrated only in the stiffness and mass matrices parameters. Fine agreement was achieved for mean values with obtained correction factors in the order of $\pm 10\%$. Confidence ellipses also suggest a good agreement, as well in the size of their semi-axes as in the in-plane rotation.

Latter, fixing the mass and stiffness matrices in the updated values, damping parameters were updated by matching predicted damping ratios with measured ones. Extremely low error values and well fitted confidence ellipses suggest that the resultant fitted model can reliably be used to predict dynamic behaviour. Finally, even though fairly well fitted models were obtained in both cases, an inverse relation between sensitivity and uncertainty level for the damping parameters could be perceived with the comparison of the results. It was noted that damped natural frequencies were almost insensitive to damping perturbations, but the uncertainty levels observed in measured damping ratios, and consequently in the fitted damping parameters too, are much greater than those of mass and stiffness components. Then, it was noticed that a more robust way of prescribing uncertainty allocation among the parameters that describe the physical system is needed for further developments of uncertainty quantification with sensitivity analysis.

References

Ching, J., Chen, Y.-C.: Transitional Markov Chain Monte Carlo method for Bayesian model updating, model class selection, and model averaging. J. Eng. Mech. **33**(7), 816–832 (2007)

Brooks, S., Gelman, A., Jones, G.L., Meng, X.-L.: Handbook of Markov Chain Monte Carlo. CRC Press, Boca Raton (2011)

Gamerman, D., Lopes, Hedibert, F.: Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference, 2nd edn. Chapman and Hall/CRC, New York (2006). Editors: Carlin, B.P., Chatfield, C., Tanner, M., Zidek, J.

Yuen, K.-V.: Bayesian Method for Structural Dynamics and Civil Engineering. Wiley, Hoboken (2010)

Löw, A.M., Gomes, H.M.: Parameters identification with evaluation of uncertainties using sensitivity method and covariance matrix analysis. In: Proceedings of 24th ABCM International Congress of Mechanical Engineering, Curitiba, PR, Brazil, 3–8 December 2017

Tisseur, F., Meerbergen, K.: The quadratic eigenvalue problem. SIAM Rev. **43**, 235–286 (2001)

Feldman, M.: Hilbert transform in vibration analysis. Mech. Syst. Signal Process. **25**, 735–802 (2011)

Patelli, E., Govers, Y., Broggi, M., Gomes, H.M., Link, M., Mottershead, J.E.: "Sensitivity or Bayesian model updating: a comparison of techniques using the DLR AIRMOD test data. Arch. Appl. Mech. (2017). https://doi.org/10.1007/s00419-017-1233-1

Friswell, M.I., Mottershead, J.E.: Finite Element Model Updating in Structural Dynamics. Solid Mechanics and Its Applications. Kluwer, Boston (1995)

Maia, M.M., Silva, J.M.M. (eds.): Theoretical and Experimental Modal Analysis. Research Studies Press, Baldock (1997)

Silva, T.A.N., Maia, N.M.M., Link, M., Mottershead, J.E.: Parameter selection and covariance updating. Mech. Syst. Signal Process. **70–71**, 269–283 (2016)