Mathematical Modelling for an Optimal Monitoring Design in Quality Control of Traffic

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Abstract. This paper deals with the monitoring of traffic flow and air pollution on an urban road. Specifically, the location of monitoring stations is studied, looking for points where obtained measures can be representative of the surrounding areas. In order to do it, a 1D mathematical model for obtaining the traffic flow on an urban road network is combined with a 2D model for air pollution. From the numerical estimations of these parameters, the problem of designing the monitoring strategy is formulated as a Mixed Integer Multiobjective Optimization Problem (MIMOP), which is solved by an ad-hoc procedure. Finally, this technique is applied to a simplified but realistic situation in the Guadalajara Metropolitan Area (Mexico).

Keywords: Monitoring · Quality control · Optimal design
Mathematical modelling

1 Introduction

Traffic management is a serious problem in almost all cities. Moreover, in major cities, vehicular traffic is one of the main causes of air pollution, which, in turn, is considered one of the most important environmental challenges nowadays. So, the control of traffic flow and air pollution is a very important task for all municipal governments. In this process, the correct design of a monitoring system is crucial. A lot of scientific literature about the topic could be found in last decades, and still today it remains as a very hot topic (see, for instance, [6] or [10]).

Recently [2], the authors proposed a general methodology for designing a monitoring strategy in quality control process, in case that reliable estimations

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of the variables under study are available. Traffic flow on road networks can be obtained by numerical simulation [4,5], and air pollution due to vehicular traffic can be also estimated by using mathematical modelling [1,3,9]. So, in this work we combine all of these techniques to design a monitoring strategy for traffic flow and air pollution on an urban road. First, we present a mathematical model for obtaining traffic and pollution estimations on an urban domain. Next, we formulate the problem of designing the monitoring strategy as a Mixed Integer Multiobjective Optimization Problem (MIMOP), and we propose a numerical algorithm to solve it. Finally, we apply our technique to a realistic case posed in the Guadalajara Metropolitan Area (Mexico).

2 Materials and Methods

2.1 Mathematical Modelling of Traffic Flow and Air Pollution

We consider an urban domain $\Omega \subset \mathbb{R}^2$ including a road network composed of N_R unidirectional avenues (segments) meeting at a number N_J of junctions (intersections), such that the endpoints of each segment are either on the boundary of Ω or corresponds to one of the junctions. We denote by $\mathcal{I}^{in}$, $\mathcal{I}^{out} \subset \{1\ldots,N_R\}$ the sets of indices corresponding to incoming and outgoing avenues in the network, respectively. Moreover, for each junction $j = 1\ldots,N_J$, we denote $\mathcal{I}_j^{in}$, $\mathcal{I}_j^{out} \subset \{1\ldots,N_R\}$ the sets of indices corresponding to avenues incoming and outgoing in that junction, respectively. Representing each avenue $i = 1\ldots,N_R$, for a real interval $[a_i,b_i]$, the traffic flow in the road network is governed by the following system [9]: for $i = 1,\ldots,N_R$, $y \in \mathcal{I}^{in}$, $z \in \mathcal{I}^{out}$, $j = 1,\ldots,N_J$, $k \in \mathcal{I}_j^{in}$, and $l \in \mathcal{I}_j^{out}$:

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial f_i(\rho_i)}{\partial s} = 0 \quad \text{in } (a_i,b_i) \times (0,T), \tag{1}$$

$$\rho_i(.,0) = \rho_i^0 \quad \text{in } [a_i,b_i], \tag{2}$$

$$f_k(\rho_k(b_k,.)) = \sum_{l\in\mathcal{I}_j^{out}} \min\left\{\alpha_{lk}^j D_k(\rho_k(b_k,.)),\, \beta_{kl}^j S_l(\rho_l(a_l,.))\right\} \quad \text{in } (0,T), \tag{3}$$

$$f_l(\rho_l(a_l,.)) = \sum_{k\in\mathcal{I}_j^{in}} \min\left\{\alpha_{lk}^j D_k(\rho_k(b_k,.)),\, \beta_{kl}^j S_l(\rho_l(a_l,.))\right\} \quad \text{in } (0,T), \tag{4}$$

$$f_z(\rho_z(b_z,.)) = \min\{f_z^{out}, D_z(\rho_z(b_z,.))\} \quad \text{in } (0,T), \tag{5}$$

$$f_y(\rho_y(a_y,.)) = \min\{D_y^{in}(q_y,.), S_y(\rho_y(a_y,.))\} \quad \text{in } (0,T), \tag{6}$$

$$\left.\begin{aligned} &\frac{dq_y}{dt} = f_y^{in} - f_y(\rho_y(a_y,.)) \quad \text{in } (0,T), \\ &q_y(0) = q_y^0, \end{aligned}\right\} \tag{7}$$

where:

- $\rho_i : [a_i,b_i] \times [0,T] \to [0,\rho_i^{max}]$ are unknowns representing the density of cars on the avenues.

- $f_i : [0, \rho_i^{max}] \to \mathbb{R}$ are known functions giving the traffic flow, in terms of density (the so-called fundamental diagram).
- D_i, $S_i : [0, \rho_i^{max}] \longrightarrow \mathbb{R}$ denote, respectively, the demand and supply functions. If C_i (*road capacity*) denotes the maximum value of f_i, and ρ_{C_i} (*critical density*) is the point where this maximum is reached, then these functions are given by:

$$D_i(\rho) = \begin{cases} f_i(\rho) & \text{if } 0 \le \rho \le \rho_{C_i}, \\ C_i & \text{if } \rho_{C_i} \le \rho \le \rho_i^{max}, \end{cases} \qquad S_i(\rho) = \begin{cases} C_i & \text{if } 0 \le \rho_i \le \rho_{C_i}, \\ f_i(\rho) & \text{if } \rho_{C_i} \le \rho \le \rho_i^{max}. \end{cases}$$

- α_{lk}^j, $\beta_{kl}^j \in [0,1]$, are known parameters giving preferences and limitations for drivers arriving to a junction. In order to guarantee the conservation of cars at junctions, they have to verify that

$$\sum_{k \in \mathcal{I}_j^{in}} \beta_{kl}^j = 1, \qquad \sum_{l \in \mathcal{I}_j^{out}} \alpha_{lk}^j = 1.$$

- $q_y : [0,T] \to [0, +\infty)$ are unknowns representing queues length (measured in number of cars) downstream the avenues $y \in \mathcal{I}^{in}$.
- ρ_i^0, q_y^0, f_y^{in}, f_z^{out} are known functions giving initial and boundary conditions.
- $D_y^{in}(q_y, t)$ represents the demand of queue q_y at time t, and it is given by:

$$D_y^{in}(q_y, t) = \begin{cases} \min\{f_y^{in}(t), C_y^{in}\} & \text{if } q_y = 0, \\ C_y^{in} & \text{if } q_y > 0. \end{cases}$$

  where C_y^{in} is the downstream road capacity, which is assumed to be known.

High levels for many air pollution indicators (for example, carbon monoxide CO) are mainly due to vehicle emissions. These emissions depend on the traffic flow and the density of cars on the network, and the CO concentration $\phi(x,t)$ can be estimated by solving the following initial/boundary value problem [3]:

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi - \nabla \cdot (\mu \nabla \phi) + \kappa \phi = \sum_{i=1}^{N_R} \xi_i \quad \text{in } \Omega \times (0,T), \tag{8}$$

$$\phi(.,0) = \phi^0 \quad \text{in } \Omega, \tag{9}$$

$$\mu \frac{\partial \phi}{\partial n} - \phi\, \mathbf{v} \cdot \mathbf{n} = 0 \quad \text{on } S^-, \tag{10}$$

$$\mu \frac{\partial \phi}{\partial n} = 0 \quad \text{on } S^+, \tag{11}$$

where $\mathbf{v}(x,t)$ represents the wind velocity field, $\mu(x,t)$ is the CO molecular diffusion coefficient, $\kappa(x,t)$ is the CO extinction rate corresponding to the (first order) reaction term, ϕ^0 is a known function giving the initial CO concentration, $\mathbf{n}$ denotes the unit outward normal vector to the boundary $\partial\Omega = S^- \cup S^+$, where $S^- = \{(x,t) \in \partial\Omega \times (0,T)$ such that $\mathbf{v} \cdot \mathbf{n} < 0\}$ represents the inflow boundary, and $S^+ = \{(x,t) \in \partial\Omega \times (0,T)$ such that $\mathbf{v} \cdot \mathbf{n} \ge 0\}$ represents the outflow boundary. Finally, for each $i = 1, \dots, N_R$, the term ξ_i stands for the source of pollution due to vehicular traffic on the corresponding avenue. So, ξ_i is a Radon measure (that is, an element of the dual of the space of continuous functions), and, if $\sigma_i : [a_i, b_i] \longrightarrow \Omega$ is a parametrization of the avenue, it is given by:

$$\langle \xi_i(t), v \rangle = \int_{a_i}^{b_i} (\gamma_i f_i(\rho_i(s,t)) + \eta_i \rho_i(s,t)) \; v(\sigma_i(s)) \, \|\sigma_i'(s)\| \, ds, \quad \forall v \in \mathcal{C}(\overline{\Omega}).$$

where ρ_i is the solution of model (1)–(7), and γ_i and η_i are weight parameters representing contamination rates.

2.2 Optimal Monitoring Strategy: A Mixed Integer Multiobjective Optimization Problem

Solving the coupled model (1)–(11) with different data (for example different boundary conditions for traffic model, or different wind velocity fields for pollution model), we obtain estimations of traffic density and CO concentrations in different situations (different *scenarios*). So, first of all, we define the N_S scenarios to be considered (for example, a weekday and a holiday combined with two typical wind velocity fields lead to $N_S = 2$ different scenarios for traffic and $N_S = 4$ for pollution). Next, we fix the road R to be monitored and denote by $I = [a, b]$ the real interval representing it (for simplicity, hereinafter the subscript i corresponding to the avenue is deleted). For each scenario $m = 1, \dots, N_S$, we solve the full system (1)–(11) for the corresponding data, and obtain estimations of the following indicators on $[a, b] \times [0, T]$: traffic density $\rho^m(s,t)$, traffic flow $q^m(s,t) = f(\rho^m(s,t))$, and CO concentration $\phi^m(s,t) \equiv \phi^m(\sigma(s),t)$.

Our objective is related to designing a monitoring strategy for quality control of one of these indicators. We are interested in a strategy that allow us, from the measures of the chosen indicator at several specific points, to extrapolate its behaviour on the whole road. Following the methodology developed in [2], we propose to divide the interval $[a, b]$ in a number $N \in \mathbb{N}$ of subintervals $[c_{n-1}, c_n]$, and measure the indicator at one point $p_n \in [c_{n-1}, c_n]$, with the final aim that the measure at that point gives a global idea of the values of the indicator in the entire subinterval. So, if we define $c_0 = a$, $c_N = b$, and take $\mathbf{c} = (c_1, \dots, c_{N-1})$ and $\mathbf{p} = (p_1, \dots, p_N)$, the optimal monitoring strategy is given by the solution of the following Mixed-Integer Multiobjective Optimization Problem (see [2]):

$$\begin{array}{l} \text{minimize} \;\; \mathbf{J}(N, \mathbf{c}, \mathbf{p}) = (f(N), J_\sigma(\mathbf{c}), J_d(\mathbf{c}, \mathbf{p})), \\ \text{subject to } c_{n-1} \le p_n \le c_n, \quad n = 1 \dots, N, \end{array} \tag{12}$$

where:

- $f(N) = wN$ gives the economic costs of the monitoring system (w is the price of each monitoring station).
- $J_\sigma(\mathbf{c})$ indicates how good (*representative*) are the mean values of the indicator in the different subintervals.

– $J_d(\mathbf{c}, \mathbf{p})$ indicates how good are points p_n to capture the mean values in subintervals $[c_{n-1}, c_n]$.

For example, if the indicator to be monitored is the traffic density, $J_\sigma(\mathbf{c})$ and $J_d(\mathbf{c}, \mathbf{p})$ are defined in the following way (if the indicator is traffic flow or CO concentration, ρ should be replaced by q or ϕ below):

1. For $n = 1 \ldots, N$ we define:
   - The mean value at subinterval $[c_{n-1}, c_n]$ for the scenario m:

$$\bar{\rho}_n^m(t) = \frac{\displaystyle\int_{c_{n-1}}^{c_n} \rho^m(s,t)\, ds}{c_n - c_{n-1}}.$$

   - The deviation from the mean value (averaged for different scenarios):

$$\sigma_n = \frac{1}{N_S} \sum_{m=1}^{N_S} \frac{1}{T} \int_0^T \sqrt{\int_{c_{n-1}}^{c_i} \left(\rho^m(s,t) - \bar{\rho}_n^m(t)\right)^2 ds}\, dt. \tag{13}$$

   - The difference of the indicator at point p_n from the mean value in the corresponding subinterval (averaged for different scenarios):

$$d_n = \frac{1}{N_S} \sum_{m=1}^{N_S} \frac{1}{T} \int_0^T \left(\rho^m(p_n,t) - \bar{\rho}_n^m(t)\right)^2 \, dt. \tag{14}$$

2. We look for intervals $[c_{n-1}, c_n]$ with similar values of σ_n, and all of them with a value of σ_n as small as possible. So, we define $J_\sigma(\mathbf{c})$ as a linear combination of the maximum norm $\|.\|_\infty$ (related to the first purpose) and the Euclidean norm $\|.\|_2$ (related to the second one) of the vector $\sigma(\mathbf{c}) = (\sigma_1, \ldots, \sigma_N)$ defined by (13):

$$J_\sigma(\mathbf{c}) = r\|\sigma(\mathbf{c})\|_\infty + (1-r)\|\sigma(\mathbf{c})\|_2, \tag{15}$$

   where $r \in [0, 1]$ is a weight parameter.
3. We also want that at each subinterval the mean value is well captured. As before, it leads us to define:

$$J_d(\mathbf{c}, \mathbf{p}) = r\|d(\mathbf{c}, \mathbf{p})\|_\infty + (1-r)\|d(\mathbf{c}, \mathbf{p})\|_2, \tag{16}$$

   where $d(\mathbf{c}, \mathbf{p}) = (d_1, \ldots, d_N)$ is given by (14).

2.3 Numerical Solution

With respect to numerical solution of system (1)–(11), we proceed in two steps: First, the traffic model (1)–(7) is solved by combining a classical first order numerical method for (1)–(6), with the forward Euler scheme for (7), (see [9] for further details). Next, once functions $\rho_i(s, t)$ are known and the source terms

ξ_i can be computed, the pollution model (8)–(11) is solved by combining the method of characteristics for the time discretization, with a Lagrange P_1 finite element method for the space discretization (see [1]).

Obtaining a monitoring strategy consists of determining the number N of monitoring stations, the points $\mathbf{p}$ where these stations should be located, and the subintervals $\mathbf{c}$ where measures can be extrapolated. These optimal parameters are given by a solution of the multiobjective optimization problem (12). To obtain an efficient and satisfactory solution of problem (12), we assume that it is desired a monitoring system with the smallest possible number of stations, but ensuring that, in all subintervals, the measured values are sufficiently representative. Thus, we use the following algorithm:

Algorithm 1

- *Step 1. For each $N = 1, 2, \ldots,$ solve the problem*

$$\begin{cases} \min J_\sigma(\mathbf{c}) \\ \textit{subject to} \quad c_n - c_{n-1} \geq \delta > 0, \quad n = 1, \ldots, N, \end{cases} \tag{17}$$

  and show the decision maker the minimum values of J_σ obtained in each case.
- *Step 2. Ask the decision maker, by virtue of the information obtained in previous step, to choose the maximum number N_{max} of stations, and to set a maximum threshold for the value of J_σ (denoted by J_{max}).*
- *Step 3. For each $N = 1, 2, \ldots, N_{max}$, solve the problems*

$$\begin{cases} \min J_d(\mathbf{c}, \mathbf{p}) \\ \textit{subject to} \; c_{n-1} \leq p_n \leq c_i, \, n = 1, \ldots, N, \\ \qquad\qquad\quad c_n - c_{n-1} \geq \delta, \quad n = 1, \ldots, N, \\ \qquad\qquad\quad J_\sigma(\mathbf{c}) \leq J_{max}, \end{cases} \tag{18}$$

  and show the best solution to the decision maker.
- *Step 4. Ask the decision maker if the solution obtained is satisfactory. If it is, STOP. If not, return to Step 2 and increase the number of measurements to take (N_{max}) and/or increase the maximum threshold for J_σ (J_{max}).*

To solve problem (17) we propose the classical method of Nelder-Mead [7], using a penalty function to deal with linear constraints. To solve problem (18) we propose an ad-hoc method, based in splitting the problem in two lower-dimension problems (see [2] for full details).

3 Results and Discussion

3.1 Case Study

We center our attention in the Guadajara Metropolitan Area (GMA), located in the state of Jalisco (Mexico). This is the second largest metropolitan area in Mexico, it has a population of around four and a half million inhabitants and

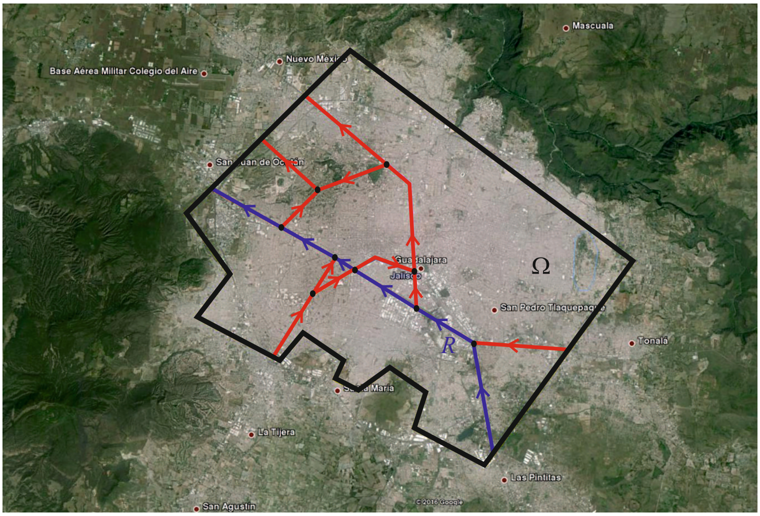

Fig. 1. Satellite photo of Guadalajara Metropolitan Area, where can be seen the domain Ω and the road network considered for numerical simulation. The road R to be monitored is highlighted in blue.

more than two million vehicles (a significant portion of them with more than ten years of usage and lack of maintenance), and it suffers recurrent episodes of pollution, with high CO levels [8]. In order to exemplify above methodology, we take the domain Ω depicted in Fig. 1, where we consider a road network consisting of $N_R = 17$ avenues, with $N_J = 9$ junctions. In this network, we center our attention on the main road R highlighted in blue in Fig. 1, formed by 6 avenues, $L = 22.07$ km long, and represented by the real interval $I = [0, L]$ given by:

$$I = [0, 5.89] \cup [5.89, 9.39] \cup [9.39, 13.33] \cup [13.33, 14.53] \cup [14.53, 17.58] \cup [17.58, L].$$

We consider two different scenarios for the traffic model, defined by boundary conditions corresponding to a typical weekday and a typical holiday. Moreover, for the pollution model, they are combined with other two scenarios defined by two different (weak and hard) wind velocity fields, which are usual in this region. In this situation, we apply previous technique to design two monitoring strategies (the former for traffic flow and the latter for CO concentration) on the road R.

3.2 Numerical Results

First of all, we solve the traffic model (1)–(7) for the two different scenarios (weekday and holiday) and the pollution model (8)–(11) for the four data set (weekday & weak wind, weekday & hard wind, holiday & weak wind, and holiday & hard wind). For instance, Fig. 2(a) shows, for a weekday ($T = 24$ hours), the

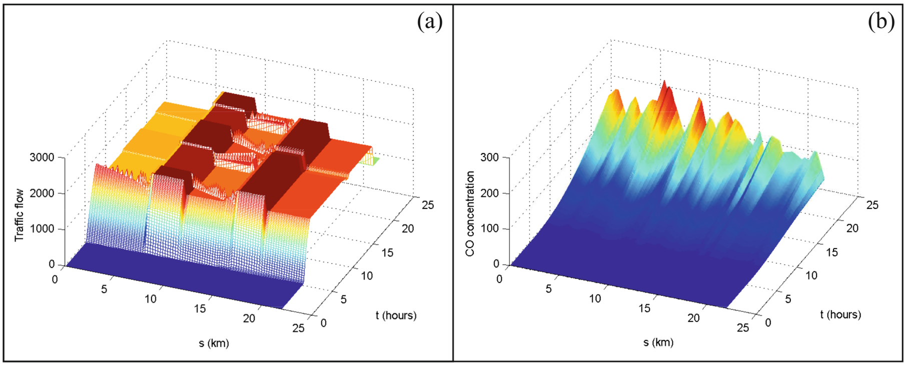

Fig. 2. (a) Estimated values of traffic flow on the road R, for a weekday. (b) Estimated values of CO concentration on the road R, for a weekday with a weak wind velocity field.

Table 1. Solutions of problem (17) for traffic flow and CO monitoring.

N	J_σ (minimal value) for traffic flow monitoring	J_σ (minimal value) for CO monitoring
2	715.2310	9.0262
3	441.1187	6.0762
4	340.0294	5.4051
5	193.6916	4.7951
6	103.8867	4.1210
7	94.6130	3.9467

estimated traffic flow on the road R. The CO concentrations for a weekday & weak wind scenario is shown in Fig. 2(b).

Next, following Algorithm 1, we solve the problem (17) for $N = 1, 2, \ldots$, in order to obtain the minimal values of J_σ if N monitoring stations are used (these values can be seen in Table 1). Assuming, for example, that 6 stations are used, and taking $J_{max} = 180$ as maximal threshold for J_σ if we are monitoring traffic flow, and $J_{max} = 4.5$ if it is CO concentration, the optimal locations of monitoring stations can be seen in Table 2. In this table the reader can also find the intervals in which data obtained by each station can be extrapolated. In this case, the number of stations are equal to the number of avenues which form the road R. For traffic flow monitoring, as could be expected, the subintervals given by this methodology practically coincide with the avenues. On the contrary, if we are monitoring the CO concentration, the subintervals are very different from the avenues. This fact could be also expected, since we have to take into account that CO concentration on a road is not only due to traffic in this road, but also due to traffic in the roads nearby.

Table 2. Optimal locations p_n of monitoring stations, and subintervals $[c_n - 1, c_n]$ in which obtained data can be extrapolated. For the sake of comparison, real intervals $[a_i, b_i]$ representing the avenues that form the road R are also shown.

Traffic flow monitoring			CO monitoring	
p_n	$[c_{n-1}, c_n]$	$[a_i, b_i]$	p_n	$[c_{n-1}, c_n]$
2.706	[0, 5.797]	[0,5.89]	1.389	[0, 4.702]
7.640	[5.797, 9.309]	[5.89,9.39]	5.386	[4.702, 7.385]
11.118	[9.309, 13.466]	[9.39,13.33]	8.858	[7.385, 9.675]
14.046	[13.466, 14.466]	[13.33,14.53]	11.586	[9.675, 13.284]
16.055	[14.466, 17.517]	[14.53,17.58]	13.989	[13.284, 16.327]
19.733	[17.517, L]	[17.58,L]	21.345	[16.327, L]

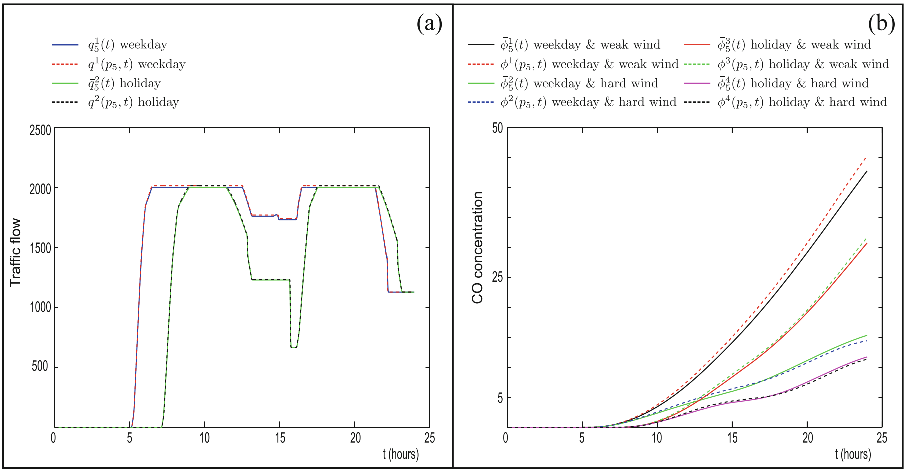

Fig. 3. Comparison, in all possible scenarios, between mean values of (a) traffic flow and (b) CO concentration, and the estimated values of these indicators at optimal locations. In both cases, depicted results correspond to the subinterval n = 5 (as given in Table 2).

Finally, to show the goodness of the location of monitoring stations, we compare traffic flow and CO concentration mean values ($\bar{q}_n^m(.)$ and $\bar{\phi}_n^m(.)$), with the estimated values of these indicators at optimal locations ($q^m(p_n,.)$ and $\phi^m(p_n,.)$). For instance, Fig. 3 shows the results in the $n = 5$ subinterval. As can be noticed, the mean values are very well captured at optimal locations.

4 Conclusions

In this paper, a general methodology previously developed by the authors in [2] has been applied to design a monitoring strategy of traffic flow and air pollution on an urban road. The results obtained in the Guadalajara Metropolitan Area for a simplified case show that this methodology can be a useful tool for designing monitoring techniques, not only in traffic pollution problems, but also in other quality control problems (in particular, in any problem where estimates of variables to be monitored can be available).

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