Reliability Based Design Optimization by Using Metamodels

Niclas Strömberg(✉)

Örebro University, Örebro, Sweden
niclas.stromberg@oru.se
www.oru.se

Abstract. This paper summarize our work so far on reliability based design optimization (RBDO) by using metamodels and present some new ideas on RBDO using support vector machines. Design optimization of complex models, such as non-linear finite element models, are treated by fitting metamodels to computer experiments. A new approach for radial basis function networks (RBFN) using a priori bias is suggested and compared to established RBFN, Kriging, polynomial chaos expansion, support vector machines (SVM), support vector regression (SVR), and least square SVM and SVR. Different types of computer experiments are also investigated such as e.g. S-optimal design of experiments, Halton- and Hammersley sampling, and different adaptive sampling approaches. For instance, SVM-supported sampling is suggested in order to improve the limit surface by putting extra sampling points at the margin of the SVM. Uncertainties in design variables and parameters are included in the design optimization by FORM- and SORM-based RBDO. By establishing the most probable point (MPP) at the limit surface using a Newton method with an inexact Jacobian, Taylor expansions of the metamodels are done at the MPP using intermediate variables defined by the iso-probabilistic transformation for several density distributions such as lognormal, gamma, Gumbel and Weibull. In such manner, LP- and QP-problems are derived which are solved in sequence until convergence. The implementation of the approaches in an in-house toolbox are very robust and efficient. This is demonstrated by solving several examples for a large number of variables and reliability constraints.

Keywords: DoE · Metamodels · RBDO · FORM · SORM

1 Introduction

Over a period of many years an in-house toolbox named MetaBox for metamodel-based simulations and optimization with and without uncertainties has been develop during different research projects. Today, the toolbox contains several approaches for DoE, a bunch of metamodels packed together as an ensemble of metamodels, and methods for simulations and optmizations with uncertainties

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Design of experiments
- Linear Koshal
- Full factorial
- Face centered cubic
- Symmetrical Koshal
- Quadratic Koshal
- Spherical
- Box-Behnken
- S-optimal
- Latin hypercube sampling
- Halton sampling
- Hammersley sampling

Metamodels
- Linear regression
- Quadratic regression
- OPRM
- Kriging with LRM
- Kriging with QRM
- A priori RBN with LRM
- A priori RBN with QRM
- A posteriori RBFN with LRM
- A posteriori RBFN with QRM
- Analytical model
- Hybrid model of analytical model and RBFN
- Polynomial chaos expansion
- Support vector machines
- Support vector regression
- Least square SVM & SVR
- Optimal ensemble

Solvers
- Genetic algorithm
- SLP
- SQP
- Succesive response surface methodology
- Newton's method

Solvers
- RBDO with SLP
- RBDO with SQP
- FORM based RBDO
- SORM based RBDO
- Crude Monte Carlo
- Quasi-Monte Carlo
- Importance sampling

Distributions
- Normal
- Lognormal
- Gumbel
- Gamma
- Weibull

Fig. 1. Available options in MetaBox (www.oru.se).

such as Monte Carlo, FORM, SORM and RBDO. This paper presents the current status of the toolbox concerning metamodel-based RBDO. An overview of available options in MetaBox is given in Fig. 1.

The project of MetaBox was initiated by simulating rotary draw bending using artificial neural networks (ANN) [1]. It was concluded that several hidden layers are needed in order to get satisfactory results. Shortly after, we started to develop a successive response surface methodology using ANN [2,3]. This approach has proven to be very efficient for screening and generating proper DoEs. Methods for DoE of non-regular spaces were developed in [4,5]. In particular, the S-optimal DoE presented in this paper has proven to be very useful. Work on metamodel-based simulations with uncertainties was initiated in [6]. Then, work on metamodel-based optimization with uncertainties was started, during several years [7–10] a framework for metamodel-based RBDO was developed. Meanwhile, several metamodels were also implemented in MetaBox such as Kriging, RBFN [11–13], polynomial chaos expansion (PCE), SVM [14] and SVR.

The outline of the paper is as follows: in Sect. 2 a short overview of implemented metamodels in MetaBox is given, in Sect. 3 the main ideas of FORM- and SORM-based RBDO as well as our implemented SQP-based RBDO algorithm are presented, in Sect. 4 a well-known benchmark is studied using different choices of DoEs and metamodels with and without uncertainties. Finally, some concluding remarks are given.

2 Metamodels

Let us assume that we have a set of sampling data $\{\hat{\boldsymbol{x}}^i, \hat{f}^i\}$ obtained from design of experiments. We would like to represent this set of data with a function, which we call a response surface, a surrogate model or a metamodel. One choice of such a function is the regression model given by

$$f = f(\boldsymbol{x}) = \boldsymbol{\xi}(\boldsymbol{x})^T\boldsymbol{\beta}, \tag{1}$$

where $\boldsymbol{\xi} = \boldsymbol{\xi}(\boldsymbol{x})$ is a vector of polynomials of $\boldsymbol{x}$ and $\boldsymbol{\beta}$ contains regression coefficients. By minimizing the sum of squared errors, i.e.

$$\min_{\boldsymbol{\beta}} \sum_{i=1}^{N} \left(X_{ij}\beta_j - \hat{f}^i \right)^2, \tag{2}$$

where $X_{ij} = \xi_j(\hat{\boldsymbol{x}}^i)$ and N is the number of sampling points, then we obtain optimal regression coefficients from the normal equation according to

$$\boldsymbol{\beta}^* = \left(\boldsymbol{X}^T \boldsymbol{X} \right)^{-1} \boldsymbol{X}^T \hat{\boldsymbol{f}}. \tag{3}$$

Examples of other useful metamodels are Kriging, radial basis functions, polynomial chaos expansion, support vector machines and support vector regression. The basic equations of these models as implemented in MetaBox are presented in the following.

2.1 Kriging

The Kriging model is given by

$$f(\boldsymbol{x}) = \boldsymbol{\xi}(\boldsymbol{x})^T \boldsymbol{\beta}^* + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}(\boldsymbol{\theta}^*) \left(\hat{\boldsymbol{f}} - \boldsymbol{X}\boldsymbol{\beta}^* \right), \tag{4}$$

where the first term represents the global behavior by a linear or quadratic regression model and the second term ensures that the sample data is fitted exactly. $\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{\theta}) = [R_{ij}]$, where

$$R_{ij} = R_{ij}(\boldsymbol{\theta}, \hat{\boldsymbol{x}}^i, \hat{\boldsymbol{x}}^j) = \exp\left(-\sum_{k=1}^{N} \theta_k (\hat{x}_k^i - \hat{x}_k^i)^2 \right). \tag{5}$$

Furthermore, $\boldsymbol{\theta}^*$ is obtained by maximizing the following likelihood function:

$$\frac{1}{\sigma^N \sqrt{\det(\boldsymbol{R})(2\pi)^N}} \exp\left(-\frac{(\boldsymbol{X}\boldsymbol{\beta} - \hat{\boldsymbol{f}})^T \boldsymbol{R}^{-1} (\boldsymbol{X}\boldsymbol{\beta} - \hat{\boldsymbol{f}})}{2\sigma^2} \right) \tag{6}$$

and

$$\boldsymbol{\beta}^* = \left(\boldsymbol{X}^T \boldsymbol{R}^{-1}(\boldsymbol{\theta}^*) \boldsymbol{X} \right)^{-1} \boldsymbol{X}^T \boldsymbol{R}^{-1}(\boldsymbol{\theta}^*) \hat{\boldsymbol{f}}. \tag{7}$$

2.2 Radial Basis Function Networks

For a particular signal $\hat{\boldsymbol{x}}^k$ the outcome of the radial basis function network can be written as

$$f^k = f(\hat{\boldsymbol{x}}^k) = \sum_{i=1}^{N_\Phi} A_{ki}\alpha_i + \sum_{i=1}^{N_\beta} B_{ki}\beta_i, \tag{8}$$

where N_{Φ} is the number of radial basis functions, N_{β} is the number of regression coefficients in the bias,

$$A_{ki} = \Phi_i(\hat{\boldsymbol{x}}^k) \text{ and } B_{ki} = \xi_i(\hat{\boldsymbol{x}}^k). \tag{9}$$

Both linear and quadratic regression models are used as bias. Furthermore, $\Phi_i = \Phi_i(\hat{\boldsymbol{x}}^k)$ represents the radial basis function.

Thus, for a set of signals, the corresponding outgoing responses $\boldsymbol{f} = \{f^i\}$ of the network can be formulated compactly as

$$\boldsymbol{f} = \boldsymbol{A}\boldsymbol{\alpha} + \boldsymbol{B}\boldsymbol{\beta}, \tag{10}$$

where $\boldsymbol{\alpha} = \{\alpha_i\}$, $\boldsymbol{\beta} = \{\beta_i\}$, $\boldsymbol{A} = [A_{ij}]$ and $\boldsymbol{B} = [B_{ij}]$. If we let $\boldsymbol{\beta}$ be given a priori by the normal equation as

$$\boldsymbol{\beta} = \left(\boldsymbol{B}^T\boldsymbol{B}\right)^{-1} \boldsymbol{B}^T\hat{\boldsymbol{f}}, \tag{11}$$

then

$$\boldsymbol{\alpha} = \boldsymbol{A}^{-1}\left(\hat{\boldsymbol{f}} - \boldsymbol{B}\hat{\boldsymbol{\beta}}\right). \tag{12}$$

Otherwise, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are established by solving

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^T & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{Bmatrix} = \begin{Bmatrix} \hat{\boldsymbol{f}} \\ \boldsymbol{0} \end{Bmatrix}. \tag{13}$$

2.3 Polynomial Chaos Expansion

Polynomial chaos expansion by using the Hermite polynomials $\varphi_n = \varphi_n(y)$ can be written as

$$f(\boldsymbol{x}) = \sum_{i=0}^{M} c_i \prod_{j=1}^{N_{\mathrm{VAR}}} \varphi_i(x_j), \tag{14}$$

where $M+1$ is the number of terms and constant coefficients c_i, and N_{VAR} is the number of variables x_i. The Hermite polynomials are defined by

$$\varphi_n = \varphi_n(y) = (-1)^n \exp\left(\frac{x^2}{2}\right) \frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(\exp\left(-\frac{x^2}{2}\right)\right). \tag{15}$$

For instance, one has

$$\varphi_0 = 1, \tag{16a}$$

$$\varphi_1 = y, \tag{16b}$$

$$\varphi_2 = y^2 - 1, \tag{16c}$$

$$\varphi_3 = y^3 - 3y, \tag{16d}$$

$$\varphi_4 = y^4 - 6y2 + 3, \tag{16e}$$

$$\varphi_5 = y^5 - 10y^3 + 15y, \tag{16f}$$

$$\varphi_6 = y^6 - 15y^4 + 45y^2 - 15, \tag{16g}$$

$$\varphi_7 = y^7 - 21y^5 + 105y^3 - 105y. \tag{16h}$$

The unknown constants c_i are then established by using the normal equation. A nice feature of the polynomial chaos expansion is that the mean of $f(\boldsymbol{X})$ in (14) for uncorrelated standard normal distributed variables X_i is simply given by

$$\mathrm{E}[f(\boldsymbol{X})] = c_0. \tag{17}$$

2.4 Support Vector Machines

Let us classify the sampling data in the following manner:

$$y^i = \begin{cases} 1 & \hat{f}^i \geq \tilde{f}, \\ -1 & \text{otherwise}, \end{cases} \tag{18}$$

where $\tilde{f}$ is a threshold value. The soft non-linear support vector machine separate sampling data of different classes by a hyper-surface on the following format:

$$\sum_{i=1}^{N} \lambda_i^* y^i k(\boldsymbol{x}^i, \boldsymbol{x}) + b^* = 0, \tag{19}$$

where $k(\boldsymbol{x}^i, \boldsymbol{x})$ is a kernel function, and λ_i^* and b^* are obtained by solving

$$\begin{cases} \min\limits_{\boldsymbol{\lambda}} \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_i \lambda_j y^i y^j k(\boldsymbol{x}^i, \boldsymbol{x}^j) - \sum_{i=1}^{N} \lambda_i \\ \text{s.t.} \begin{cases} \sum_{i=1}^{N} \lambda_i y^i = 0, \\ 0 \leq \lambda_i \leq C, \quad i = 1, \ldots, N. \end{cases} \end{cases} \tag{20}$$

The least-square support vector machine is established by solving

$$\begin{bmatrix} 0 & -\boldsymbol{y}^T \\ \boldsymbol{y} & \boldsymbol{A} + \gamma \boldsymbol{I} \end{bmatrix} \begin{Bmatrix} b \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{1} \end{Bmatrix}, \tag{21}$$

where $\boldsymbol{y} = \{y^1; \ldots; y^N\}$, $\gamma = 1/C$, $\boldsymbol{1} = \{1; \ldots; 1\}$ and

$$\boldsymbol{A} = [A_{ij}], \quad A_{ij} = y^i y^j k(\boldsymbol{x}^i, \boldsymbol{x}^j). \tag{22}$$

2.5 Support Vector Regression

The soft non-linear support vector regression model reads

$$f(\boldsymbol{x}) = \sum_{i=1}^{N} \lambda^i k(\boldsymbol{x}^i, \boldsymbol{x}) - \sum_{i=1}^{N} \hat{\lambda}^i k(\boldsymbol{x}^i, \boldsymbol{x}) + b^*, \tag{23}$$

where λ^i, $\hat{\lambda}^i$ and b^* are established by solving

$$\begin{cases} \min\limits_{(\boldsymbol{\lambda},\hat{\boldsymbol{\lambda}})} \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\lambda^i-\hat{\lambda}^i)(\lambda^j-\hat{\lambda}^j)k(\boldsymbol{x}^i,\boldsymbol{x}^j)+\sum_{j=1}^{N}(\hat{\lambda}^i-\lambda^i)\hat{f}^i+\delta\sum_{j=1}^{N}(\lambda^i+\hat{\lambda}^i) \\ \text{s.t.} \begin{cases} \sum_{j=1}^{N}(\lambda^i-\hat{\lambda}^i)=0, \\ 0\le\lambda^i,\hat{\lambda}^i\le C, \qquad i=1,\ldots,N. \end{cases} \end{cases} \tag{24}$$

Finally, the corresponding least square support vector regression model is established by solving

$$\begin{bmatrix} 0 & -\mathbf{1}^T & \mathbf{1} \\ \mathbf{1} & \boldsymbol{B}+\gamma\boldsymbol{I} & -\boldsymbol{B} \\ -\mathbf{1} & -\boldsymbol{B} & \boldsymbol{B}+\gamma\boldsymbol{I} \end{bmatrix} \begin{Bmatrix} b \\ \boldsymbol{\lambda} \\ \hat{\boldsymbol{\lambda}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \hat{\boldsymbol{f}}-\delta\mathbf{1} \\ -\hat{\boldsymbol{f}}-\delta\mathbf{1} \end{Bmatrix}, \tag{25}$$

where $\gamma = 1/C$, $\mathbf{1} = \{1;\ldots;1\}$ and

$$\boldsymbol{B} = [B_{ij}], \qquad B_{i,j} = \varphi(\boldsymbol{x}^i)\varphi(\boldsymbol{x}^j) = k(\boldsymbol{x}^i,\boldsymbol{x}^j) \tag{26}$$

is a matrix containing kernel values.

3 Reliability Based Design Optimization

By using the metamodels presented in the previous section it is straight-forward to set up any design optimization problem as

$$\begin{cases} \min\limits_{\boldsymbol{x}} f(\boldsymbol{x}) \\ \text{s.t. } g(\boldsymbol{x}) \le 0. \end{cases} \tag{27}$$

For instance the metamodel $f = f(\boldsymbol{x})$ might represent the mass of a design and $g = g(\boldsymbol{x})$ is a metamodel-based limit surface for the stresses obtained by finite element analysis.

A possible draw-back with the formulation in (27) is that it is not obvious how to include a margin of safety. For instance, what is the optimal safety factor to be included in g? An alternative formulation that includes a margin of safety is

$$\begin{cases} \min\limits_{\boldsymbol{\mu}} \quad \mathrm{E}[f(\boldsymbol{X})] \\ \text{s.t.} \quad \Pr[g(\boldsymbol{X}) \le 0] \ge P_s, \end{cases} \tag{28}$$

where $\boldsymbol{X}$ now is treated as a random variable, $\mathrm{E}[\cdot]$ designates the expected value of the function f, and $\Pr[\cdot]$ is the probability that the constraint $g \le 0$ being true. P_s is the target of reliability that must be satisfied.

3.1 FORM

An established invariant approach for estimating the reliability is the first order reliability method (FORM) suggested by Hasofer and Lind. The basic idea is to transform the reliability constraint from the physical space to a space of uncorrelated standard Gaussian variables and then find the closest point to the limit surface from the origin. This point is known as the most probable point (MPP) of failure. The distance from the origin to the MPP defines the Hasofer-Lind reliability index β_{HL}, which in turn is used to approximate the probability of failure as

$$\Pr[g \leq 0] \approx \Phi(-\beta_{\mathrm{HL}}). \tag{29}$$

Assuming that $\boldsymbol{X}$ is normal distributed with means collected in $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ containing standard deviations, the MPP is obtained by solving

$$\begin{cases} \min\limits_{\boldsymbol{x}} \beta_{\mathrm{HL}} = \sqrt{\sum_{i=1}^{N} \left(\dfrac{x_i - \mu_i}{\sigma_i} \right)^2} \\ \text{s.t. } g(\boldsymbol{x}) = 0. \end{cases} \tag{30}$$

3.2 SORM

The approximation in (29) is derived by performing a first order Taylor expansion at the MPP and then evaluating the probability. Second order reliability methods (SORM) is obtained by also including the second order terms in the Taylor expansion. Based on these higher order terms the FORM approximation of the reliability is corrected.

For instance, by letting λ_i denoting the principle curvatures of a second order Taylor expansion of g, we can correct (29) by using e.g. Tvedt's formula, i.e.

$$\begin{aligned} &\Pr[g \leq 0] \approx P_1 + P_2 + P_3, \\ &P_1 = \Phi(-\beta_{\mathrm{HL}}) \prod_{i=1}^{N-1} \frac{1}{\sqrt{1 + 2\beta_{\mathrm{HL}}\lambda_i}}, \\ &P_2 = (\beta_{\mathrm{HL}}\Phi(-\beta_{\mathrm{HL}}) - \phi(-\beta_{\mathrm{HL}})) \left(\prod_{i=1}^{N-1} \frac{1}{\sqrt{1 + 2\beta_{\mathrm{HL}}\lambda_i}} \right. \\ &\qquad \left. - \prod_{i=1}^{N-1} \frac{1}{\sqrt{1 + 2(\beta_{\mathrm{HL}} + 1)\lambda_i}} \right), \\ &P_2 = (\beta_{\mathrm{HL}} + 1)(\beta_{\mathrm{HL}}\Phi(-\beta_{\mathrm{HL}}) - \phi(-\beta_{\mathrm{HL}})) \left(\prod_{i=1}^{N-1} \frac{1}{\sqrt{1 + 2\beta_{\mathrm{HL}}\lambda_i}} \right. \\ &\qquad \left. - \mathrm{Re} \left[\prod_{i=1}^{N-1} \frac{1}{\sqrt{1 + 2(\beta_{\mathrm{HL}} + i)\lambda_i}} \right] \right). \end{aligned} \tag{31}$$

3.3 SQP-based RBDO Approach

Recently, a FORM-based SQP approach for RBDO with SORM corrections was proposed in [10]. For non-Gaussian variables, we derive the following FORM-based QP-problem in the standard normal space:

$$\begin{cases} \min\limits_{\eta_i} & f(\boldsymbol{\eta}) \\ \text{s.t.} & \begin{cases} \mu_{\tilde{g}} \leq -\beta_t \sigma_{\tilde{g}}, \\ -\epsilon \leq \eta_i \leq \epsilon, \end{cases} \end{cases} \tag{32}$$

where

$$\begin{aligned} f(\boldsymbol{\eta}) &= \sum_{i=1}^{N_{\mathrm{VAR}}} \left.\frac{\partial f}{\partial X_i}\right|_{\boldsymbol{X}=\boldsymbol{\mu}^k} \frac{\phi(Y_i^k)}{\rho_i(\mu_i^k;\boldsymbol{\theta}_i^k)}\eta_i + \frac{1}{2}\sum_{i=1}^{N_{\mathrm{VAR}}}\sum_{j=1}^{N_{\mathrm{VAR}}} \tilde{H}_{ij}\eta_i\eta_j, \\ \tilde{H}_{ij} &= \left.\frac{\partial^2 f}{\partial X_i \partial X_j}\right|_{\boldsymbol{X}=\boldsymbol{\mu}^k} \frac{\phi(Y_i^k)}{\rho_i(\mu_i^k;\boldsymbol{\theta}_i^k)} \frac{\phi(Y_j^k)}{\rho_j(\mu_j^k;\boldsymbol{\theta}_j^k)}, \\ \mu_{\tilde{g}} &= \sum_{i=1}^{N_{\mathrm{VAR}}} \left.\frac{\partial g}{\partial X_i}\right|_{\boldsymbol{X}=\boldsymbol{x}^{\mathrm{MPP}}} \frac{\phi(y_i^{\mathrm{MPP}})}{\rho_i(x_i^{\mathrm{MPP}};\boldsymbol{\theta}_i^k)} \left(\eta_i - y_i^{\mathrm{MPP}}\right), \\ \sigma_{\tilde{g}} &= \sqrt{\sum_{i=1}^{N_{\mathrm{VAR}}} \left(\left.\frac{\partial g}{\partial X_i}\right|_{\boldsymbol{X}=\boldsymbol{x}^{\mathrm{MPP}}} \frac{\phi(y_i^{\mathrm{MPP}})}{\rho_i(x_i^{\mathrm{MPP}};\boldsymbol{\theta}_i^k)} \right)^2}. \end{aligned} \tag{33}$$

Here, $\beta_t = \Phi^{-1}(P_s)$ is the target reliability index which can be corrected by a SORM approach as presented above or any Monte Carlo approach. The optimal solution to (32), denoted η_i^*, is mapped back from the standard normal space to the physical space using

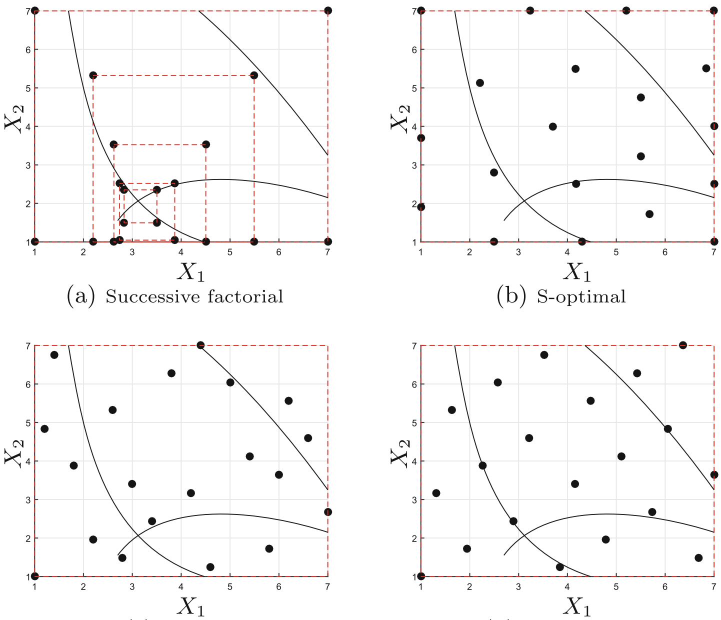
$$\mu_i^{k+1} \approx \mu_i^k + \frac{\Phi(Y_i^k)}{\rho_i(\mu_i^k;\boldsymbol{\theta}_i^k)}\eta_i^*.$$

Then, a new QP-problem is generated around $\boldsymbol{\mu}^{k+1}$ and this procedure continues in sequence until convergence is obtained. The QP-problem in (32) is solved using `quadprog.m` in Matlab.

4 Numerical Examples

In order to demonstrate the metamodel-based RBDO approach presented above, we consider the following well-known benchmark:

$$\begin{cases} \min\limits_{\mu_i} & (\mu_1+\mu_2)^2 \\ \text{s.t.} & \begin{cases} \Pr[g_1 = 20 - X_1^2 X_2 \leq 0] \geq \Phi(3), \\ \Pr\left[\begin{array}{l} g_2 = 1 - \dfrac{(X_1+X_2-5)^2}{30} \\ \quad - \dfrac{(X_1-X_2-12)^2}{120} \leq 0 \end{array} \right] \geq \Phi(3), \\ \Pr[g_3 = X_1^2 + 8X_2 - 75 \leq 0] \geq \Phi(3), \\ 1 \leq \mu_i \leq 7, \end{cases} \end{cases} \tag{34}$$



(a) Successive factorial (b) S-optimal (c) Halton (d) Hammersley

Fig. 2. Different design of experiments.

where $\text{VAR}[X_i] = 0.3^2$. A small modification of the original problem is done by taking the square of the objective function. This problem was recently considered in [10] by solving (32) sequentially. The example was in that work also generalized to 50 variables and 75 constraints for five different distributions simultaneously (normal, lognormal, Gumbel, gamma and Weibull). The analytical solution for two variables with normal distribution is obtained to be (3.4525, 3.2758) 45.2702 with our RBDO algorithm. The corresponding deterministic solution is (3.1139, 2.0626) 26.7965.

Now, we will consider (34) to be a “black-box”, which we treat by setting up design of experiments and metamodels. The quality of the metamodel is dependent on the choice of DoE. Figure 2 presents four useful strategies of DoEs. In Fig. 2a we adopt successive screening to set up the DoE, in Fig. 2b we use space-filling with a genetic algorithm by maximizing the distance to the closest point, and in the other two figures we apply Halton and Hammersley sampling. For these four sets of sampling data, we set up 12 different metamodels automatically and then find the deterministic solution for each metamodel-based problem of our “black-box”. The solutions are presented in Table 1. One concludes that the solutions depend on the choice of DoE and metamodel. For many combinations the corresponding “black-box” solution is very close to the analytical one.

The choices of DoE strategy and metamodel is even more pronounced when we perform metamodel-based RBDO. In Fig. 3, we set up the DoE for RBDO by first performing successive screening with Halton sampling, then this DoE

Table 1. Deterministic solutions for different DoEs and metamodels.

Successive factorial DoE				S-optimal DoE		
Metamodel	x	y	Objective	x	y	Objective
Quadratic	3.248886	2.171088	29.37613	0.999117	0.999117	3.992936
Kriging lin	3.117074	2.078969	26.92165	2.982927	1.763172	24.45783
Kriging quad	3.11152	2.060563	26.75044	3.099117	2.049533	26.50859
RBFN-pri lin	3.113471	2.062579	26.79215	3.113773	1.950322	26.24357
RBFN-pri quad	3.114144	2.062872	26.8015	3.111254	2.060329	26.74527
RBFN-post lin	3.113685	2.061552	26.78975	3.091999	1.948156	25.93791
RBFN-post quad	3.11443	2.063122	26.80704	3.097684	2.048247	26.48061
PCE	3.113883	2.062643	26.79654	3.108187	2.064329	26.7493
SVM	3.369155	2.145621	30.41275	2.82942	3.13867	35.61809
SVR	3.114132	2.061862	26.80731	3.055823	2.079891	26.35382
LS-SVM	3.18866	2.244347	29.51756	3.192832	2.747977	35.29321
LS-SVR	3.113104	2.059959	26.79126	3.054317	2.078361	26.36171
Halton DoE				Hammersley DoE		
Quadratic	3.067359	2.383666	29.71367	3.182894	2.305653	30.12416
Kriging lin	3.13906	2.131108	27.46664	3.151871	2.10432	27.50435
Kriging quad	3.09325	2.044249	26.3939	3.071617	2.024385	25.96924
RBFN-pri lin	3.12314	2.083498	27.03036	3.112407	2.050659	26.68024
RBFN-pri quad	3.113361	2.062185	26.78627	3.087332	2.038875	26.278
RBFN-post lin	3.123084	2.082355	27.02246	3.111633	2.062253	26.72894
RBFN-post quad	3.113447	2.06226	26.78794	3.084073	2.035896	26.21409
PCE	3.113601	2.063178	26.79957	3.134014	2.057725	27.06369
SVM	3.113337	2.025036	26.40287	2.734435	2.019023	22.59537
SVR	3.112977	2.088567	26.96864	3.142191	2.091771	26.98728
LS-SVM	2.821295	2.031644	23.55102	1.367785	1.794423	9.999563
LS-SVR	3.11431	2.08876	26.97181	3.144679	2.092451	26.99588

is augmented by adding the sampling points (green dots) based on the optimal solutions (deterministic and RBDO solution) and the active most probable points as well as points (red dots) along the limits surface based on a global SVM [14]. The corresponding metamodel-based RBDO solutions are presented in Table 2 as well as the corresponding solutions for the successive factorial DoE presented in Table 1. One concludes that the choices of augmented successive Halton sampling and RBFN, PCE or SVR produce the best performance. For these choices the metamodel-based "black-box" solution is close to the analytical one.

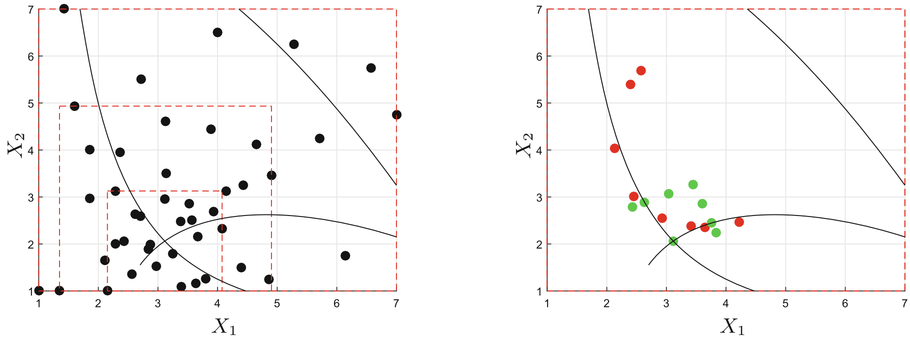


Fig. 3. Left: successive screening using Halton sampling, right: adaptive sampling using MPP and SVM.

Table 2. RBDO solutions for different DoEs and metamodels.

Successive factorial DoE				Succ. halton DoE		
Metamodel	x	y	Objective	x	y	Objective
Quadratic	3.639523	3.339185	48.70235	3.502828	3.292923	46.18223
Kriging lin	3.252461	2.716889	35.51783	3.258552	2.712108	35.6467
Kriging quad	3.248491	2.705792	35.45348	3.262384	2.713804	35.71483
RBFN-pri lin	3.497476	3.16193	44.2991	3.439621	3.286656	45.24157
RBFN-pri quad	3.307214	3.216166	42.55449	3.452365	3.257389	45.02079
RBFN-post lin	3.477641	3.204944	44.54576	3.439076	3.286574	45.23413
RBFN-post quad	3.412853	3.260071	44.52793	3.439088	3.286576	45.23457
PCE	3.45374	3.274712	45.27401	3.453746	3.274719	45.27224
SVM	3.771719	3.070926	46.82181	3.509136	3.332413	46.80679
SVR	3.467335	3.259169	44.79304	3.438362	3.286197	45.20416
LS-SVM	2.907908	6.991473	97.9982	2.756372	1.71142	19.96117
LS-SVR	3.466073	3.24107	44.55684	3.438703	3.286368	45.21783

5 Concluding Remarks

A framework for metamodel-based RBDO has been developed and implemented in MetaBox. Several options of DoEs, metamodels and optimization approaches are possible. In a near future, optimal ensemble of metamodels will also be available as an option.

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