Distributed Hunting for Multi USVs Based on Cyclic Estimation and Pursuit

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Abstract. In this brief, we study the hunting problem for a group of underactuated surface vehicles (USVs), in which the vehicles converge to the target as the center, as well as maintain the desired relative distance to the target when rotating around the target at the same speed. A approach based on the cyclic matrix is delivered. The overall control objectives are divided into two subobjectives, where the first is target circling that all vehicles rotate a circle around the target, and the second is that the vehicles are eventually evenly spaced on the circle. The former part is based on the cyclic estimation of target to get close to the target, the latter is designed by also cyclic pursuit strategy using the relative angle between the neighbors. An important feature of the controller is that not all vehicles know the target's position. For hunting with obstacle avoidance, artificial potential method and label's change strategy between neighbors are also applied to guarantee obstacle avoidance. Numerical simulations are given to verify the effectiveness of the proposed controller.

Keywords: Distributed hunting · Cyclic estimation and pursuit
Multi underactuated surface vehicles

1 Introduction

Recent decades witnesses the rapid development of distributed control of multi-agent systems. It is partially due to the increasing need to perform more difficult and complex tasks, where it contributes to increasing efficiency, reducing the system cost and providing the redundancy against individual failure. In particular, hunting behavior where multi agents enclose the target in a certain area,

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has attracted much attention recently. For example, In [15], a feedback control method was first taken into account to make multi robots round up the target robot into a certain area. In [11], the neural network method and the methods of dynamic alliance was studied for the pursuit and capture behavior. In [12,14], the author performed the case where the unicycles form a circular formation under all-to-all communication with unit constant velocity or nonidentical constant velocity.

Then, formation under the cyclic pursuit strategy was further studied. In [9], a cyclic controller for the desired pursuit pattern of moving target in 3D space was developed. In particular, there are many case where the target is given and known to all the vehicles. In [2], the limited visibility of the onboard sensors were took into account in the cyclic pursuit strategy. In [7], the author studied the case where a rigidity of graphs was utilized in the spaced formation of circle. A hybrid control law of the cyclic enclosing formation was introduced in [10]. It was shown in [16] that a distributed dynamic control law for circle formation of unicycles when the target is just known to one cycles was developed. In [17], the cycle formation is only based on the bearing-only measurement. Furthermore, it is some work of circle formation with more general networks. In [13], a balanced graph condition was considered in designing the dynamic controller. A controller is studied for circle formation with even a jointly connected network was proposed in [3,4]. In [5], the author developed the case of which cyclic pursuit formation with a hierarchical controller.

Compared with the existing result, the main contribution of this brief are listed in the following four aspects. First, the aforementioned results on hunting problem almost consider the case of unicycles where the target is known to all the vehicles. Our proposed controller is based on the information of estimation and neighbors via cyclic communication network $\mathbb{G}$. Second, a hierarchical structure of the controller is to solve the underactuated and nonlinear characters. Third, artificial method and label-change strategy are considered to avoid collision, space evenly and rotate around the target.

The rest of this paper is organized as follows: Sect. 2 introduces some preliminaries and gives definition of hunting problem. Section 3 presents the distributed hunting controller based on the cyclic estimation and pursuit strategy. Some computer simulation results are presented in Sect. 4. Section 5 concludes the article.

Notation: Throughout the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean Space. $\|\cdot\|$ denotes the Euclidean norms. $(\cdot)_{ij}$ denotes the element of $(\cdot)$ in row i, column j. $\lambda_{min}(\cdot)$ denotes the smallest eigenvalues of a square matrix $(\cdot)$.

2 Problem Formulation

First, consider a group of N underactuated surface vehicles represented by the dynamics found in [6] with kinematics and kinetics:

$$\begin{cases} \dot{\boldsymbol{\eta}}_i = \mathbf{J}(\psi_i)\mathbf{v}_i \\ \mathbf{M}_i\dot{\mathbf{v}}_i + \mathbf{C}_i\mathbf{v}_i + \mathbf{D}_i\mathbf{v}_i = \boldsymbol{\tau}_i \end{cases} \tag{1}$$

with

$$\boldsymbol{\eta}_i = [x_i, y_i, \psi_i]^T, \mathbf{v_i} = [u_i, v_i, r_i]^T, \boldsymbol{\tau_i} = [\tau_{iu}, 0, \tau_{ir}]^T$$
$$\mathbf{M}_i = \begin{bmatrix} m_{11i} & 0 & 0 \\ 0 & m_{22i} & 0 \\ 0 & 0 & m_{33i} \end{bmatrix}, \mathbf{C}_i = \begin{bmatrix} 0 & 0 & c_{13i} \\ 0 & 0 & c_{23i} \\ c_{31i} & c_{32i} & 0 \end{bmatrix}$$
$$\mathbf{D}_i = \begin{bmatrix} d_{11i} & 0 & 0 \\ 0 & d_{22i} & 0 \\ 0 & 0 & d_{33i} \end{bmatrix}, \mathbf{J} = \begin{bmatrix} \cos(\psi_i) & -\sin(\psi_i) & 0 \\ \sin(\psi_i) & \cos(\psi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$\boldsymbol{\eta}_i \in \mathbb{R}^3$ is the position vector in the earth-fixed reference frame; $\mathbf{v}_i \in \mathbb{R}^3$ is the velocity vector in the body-fixed reference frame; $\mathbf{M}_i \in \mathbb{R}^{3\times3}$ is the inertia matrix; $\mathbf{C}_i \in \mathbb{R}^{3\times3}, \mathbf{D}_i \in \mathbb{R}^{3\times3}$ denote the coriolis and centripetal matrix damping matrix, respectively; $\boldsymbol{\tau}_i \in \mathbb{R}^3$ is control vector with τ_{iu} the surge force and τ_{ir} the yaw moment.

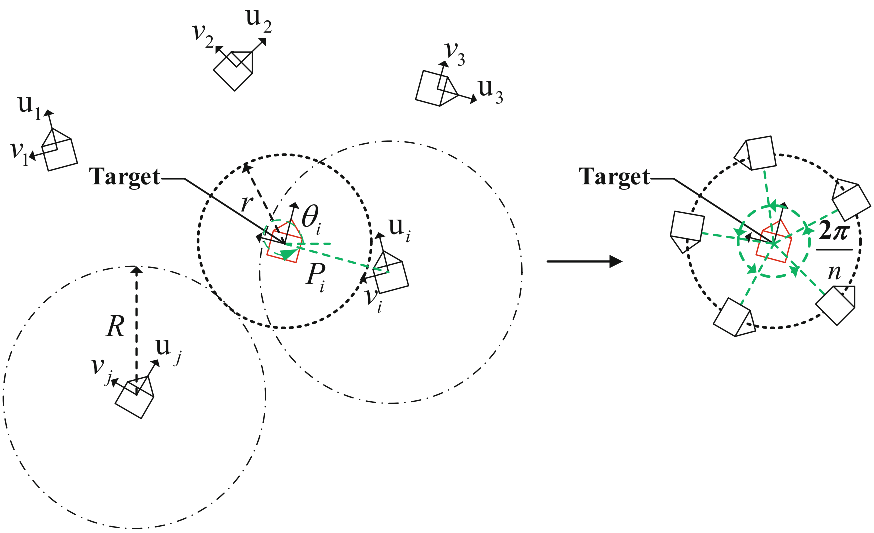

Fig. 1. Distributed hunting behavior (USVs which are initially located in plane forms a circle formation and rotate the target).

Figure 1 illustrates a group of N, underactuated surface vehicles perform the distributed hunting behavior for a preset target P^*. Given sense radius $R > 0$, if $||P_i - P_*|| \leq R$, vessel i get the position P_*. For ease of expression, we label the vehicles as follows:

Remark 1. The label are sorted first in ascending order in a counterclockwise manner based on the angle $\theta_i, 0 \leq \theta_i \leq 2\pi$ and $\theta_i \leq \theta_{i+1}, i = 1, 2, \ldots, N-1$ between their position and estimation of target.

$$\begin{aligned} \theta_i &= \mathrm{atan2}(\mathrm{x_i^D}, \mathrm{y_i^D}) \\ &= \mathrm{atan2}\big((\mathrm{x_i^Q} - \mathrm{x_i^P}), (\mathrm{y_i^Q} - \mathrm{y_i^P})\big) \end{aligned} \tag{2}$$

where $P_i(x_i^P, y_i^P)^T, Q_i(x_i^Q, y_i^Q)^T$ is the position of vessel i and the estimation i of the target. $D_i \triangleq ||P_i - Q_i||$ is the relative position from vehicle i to estimator i.

Consider the vehicles' communication networks are described by θ_i as $\mathbb{G} = (\nu, \varepsilon)$, where $\nu = \{1, 2, \ldots, N\}$ and $\varepsilon = \{(1,2), (2,3), \ldots, (N-1, N), (N, 1)\}$. It means that vehicle i only gets the information from neighbor $i+1$ that are in front of itself.

Now, we are ready to provide two problems of distributed hunting problem as below:

Problem 1: Collective Hunting Problem: Given n vessels defined as (1), design a distributed control law:

$$u_i = f(P_i, Q_i, \theta_i, \theta_{i+1}), i \in \nu$$

such that n vehicles perform the collective hunting behavior by spacing evenly on the same circle and rotating around the target, as follows.

$$\begin{cases} D_i = r, \\ (\theta_{i+1} - \theta_i) mod(2\pi) = \frac{2\pi}{n} \\ \dot{\theta}_i = \dot{\theta}_j, \end{cases} \tag{3}$$

where $i \neq j$ and $i, j \in \nu$. When $i = n, i+1 = 1$. r is defined as the hunting radius.

Note that, in real application, USV is a rigid-body system with its body and length, the control problem becomes a collective control problem with obstacle avoidance, as follows.

Problem 2: Collective Hunting with Obstacle Avoidance Problem: Given n vessels defined as (1), design a distributed control law:

$$u_i = f(P_i, P_j, Q_i, \theta_i, \theta_{i+1}),\ i \in \nu, j \in N_i$$

where $N_i = \{j, j \in \nu | (||P_i - P_j|| \leq \frac{1}{2}R)\}$. Such that n vehicles achieve not only the above objectives but also the obstacle avoidance as below.

$$P_i(t) \neq P_j(t) \neq P_*(t) \tag{4}$$

where $i \neq j, i, j \in \nu$ at any time t.

In this brief, we focus on the problem of collective hunting with obstacle avoidance. The following section will give the control law to achieve the above all objectives.

3 Cyclic Estimation and Pursuit Hunting Controller Design

From the practical viewpoint, it is important to achieve the desired global hunting behavior through only local information. Figure 2 illustrates the structure of distributed hunting controller, which consists two parts, namely, hunting behavior control and vehicle kinetic control. The immediate control signal u_i^d, ψ_i^d are only based on the estimator Q_i and relative angle θ_{i+1}, θ_i. Then discuss and prove the stability of the closed system. Finally, the surge force τ_{iu} and the sway force τ_{ir} of the hunter vehicle i are derived via the PID controller. Some assumptions are given before designing the control law.

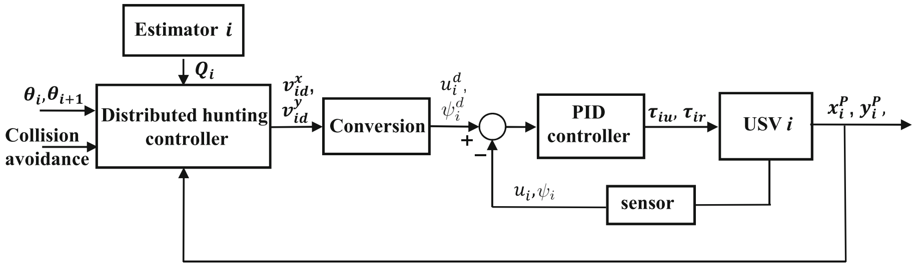

Fig. 2. Structure of distributed hunting controller

Assumption 1. $R \geq r$ and vessel i get the global position of P_* if target is in the sense radius R.

Remark 2. $R \geq r$ means that all vehicles eventually sense the position of position of target, which is a necessary condition for solving the collective hunting problem. USV equips with the differential GPS and millimeter wave radar, which can calculate the global position of P_* if $||D_i|| \leq R$.

Assumption 2. The vehicle i needs to know the initial label of its own.

Remark 3. Label-change strategy contribute to the obstacle avoidance between the vessels during the hunting. The following section give the reason.

3.1 Cyclic Estimation of the Target for Balanced Hunting

In this section, a decentralized estimator of the target's position Q_i is designed for vehicle i to get close to the target. The vehicles exchange their estimation via the cyclic communication graph $\mathbb{G}$. Before giving the form of estimator, a lemma of circular matrix is introduced.

Lemma 1. *Every circular matrix* $\mathbf{C} \in \mathbb{R}^{n\times n}$ *can be represented as in [9]:*

$$\begin{aligned}\mathbf{C} &= circ(c_1, c_2, \ldots, c_n)\\ &= c_1 I_n + c_2 \Pi_n + c_3 \Pi_n^2 + \cdots + c_n \Pi_n^{n-1}\end{aligned} \tag{5}$$

where $\Pi_n = circ(0, 1, 0 \ldots, 0) \in \mathbb{R}^{n\times n}$*. Further, the circulant's representer is defined as:*

$$p_c(\lambda) = c_1 + c_2\lambda + c_3\lambda^2 + \cdots + c_n\lambda^{n-1} \tag{6}$$

Since $\mathbf{C} = p_c(\Pi_n)$*. Then, the eigenvalues of* $\mathbf{C}$ *are* $\lambda_i = p_c(\omega^{n-1})$*, where* $\omega = e^{j2\pi/n}$ *with* $j = \sqrt{-1}$ *and* $i = 1, 2, \ldots, n$*.* ■

From the lemma above, we will have the dynamic equation of estimator Q as follows:

$$\dot{Q}_i = k_1((Q_{i+1} - Q_i) + sgn^+(\Delta_i)(P^* - Q_i)) \quad (7)$$

where Q_i is vehicle i's estimation of target when $i = 1, 2, \ldots, n-1$. When $i = n$, estimator Q_{i+1} is equal to Q_1. $k_1 > 0$ is a positive constant, which determines the convergence speed of estimators. $\Delta_i = R - \|D_i\|_2$ represents difference between the sense radius R and relative distance $\|D_i\|_2$ to the target. $sgn^+(\cdot)$ is a sign function defined as follows:

$$sgn^+(x) = \begin{cases} 1, x \geq 0 \\ 0, x < 0 \end{cases}$$

Further, the initial value of the Q_i is based on the sense radius R of the equipment, just like radar, which is calculated as follows:

$$Q_i(0) = sgn^+(\Delta_i)P^*(0) \quad (8)$$

for all i $\in$ n. $P^*(0)$ denotes the initial position of the target.

In order to analysis the overall estimation of the multi underactuated surface vehicles, we will rewrite the (7) in the following vector form:

$$\dot{Q}(t) = AQ(t) + B(t)(\mathbf{1} \otimes P^* - Q(t)) \quad (9)$$

with $A = circ(-k_1, k_1, 0, \ldots, 0) \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n}$ is a matrix where the diagonal terms $b_{ii} = 0, or, k_1$, the other term $b_{ij} = 0, i \neq j$. Where $Q = [Q_1, Q_2, \ldots, Q_n]^T \in \mathbb{R}^n$ and cir denotes the circular matrix. $\mathbf{1}$ represents a column vector $[1, 1, \ldots, 1]^T$. Then we will give the theorem for the estimation.

Theorem 1. *Suppose that estimation Q of the target based on the cyclic communication network $\mathbb{G}$ uses the form of (9) with the initial conditions (8), one has $lim_{t \to \infty}(Q(t) - \mathbf{1} \otimes P^*) = 0, \forall i \in n$.* ■

Proof: First, we introduce the estimation error $\mathbf{e}^Q = Q - \mathbf{1} \otimes P^*$, which is also a column represented $\mathbf{e}^Q = [e_1^Q, e_2^Q, \ldots, e_n^Q]$. Take the derivative of $\mathbf{e}^Q$ and substitute the (9), we will have the following dynamics:

$$\begin{aligned} \dot{\mathbf{e}}^Q(t) &= \dot{Q} \\ &= AQ(t) + B(\mathbf{1} \otimes P^* - Q(t)) \\ &= (A - B)(\mathbf{e^Q} + \mathbf{1} \otimes P^*) + B(\mathbf{1} \otimes P^*) \\ &= (A - B)\mathbf{e}^Q + A(\mathbf{1} \otimes P^*) - B(\mathbf{1} \otimes P^*) + B(\mathbf{1} \otimes P^*) \\ &= (A - B)\mathbf{e}^Q \end{aligned} \quad (10)$$

Since A is circular matrix, its representer is $p_A(\lambda) = -k_1(1 + \lambda)$ and the eigenvalues are given as $\lambda_i = p_A(\omega^{i-1}), i = 1, 2, \ldots, n$, from Lemma 1. Then, the eigenvalues can be rewritten in complex form as:

$$\lambda_i = k_1[\cos(\tfrac{2\pi(i-1)}{n}) - 1] + jk_1 \sin(\tfrac{2\pi(i-1)}{n})$$

where $i = 1, 2, \ldots, n$. Since $k_1 > 0$. The matrix A always has a zero eigenvalue, λ_1, while the remaining $n-1$ eigenvalue $\lambda_i, i = 2, 3, \ldots, n$, lies in the left-half complex plane. Based on the Gersǧorin disk theorem [8], all the eigenvalues of $A = [a_{ij}]$ are located in the disks as follows:

$$D_i = \{z \in \mathbb{C} : |z - a_{ii}| \leq \sum_{j \in A, j \neq i} a_{ij}\} \tag{11}$$

Then the eigenvalues of matrix A are all located in the circle centered at $(-k_1, 0)$ with the radius of k_1. Since matrix B only changes the center of the disk for A. The eigenvalues of matrix $(A - B)$ are also located in the disk as follows.

$$D'_i = \{z \in \mathbb{C} : |z - a_{ii} - b_{ii}| \leq \sum_{j \in A, j \neq i} a_{ij}\} \tag{12}$$

where b_{ii} is equal to 0 or k_1, which is determined by whether the vehicle i can sense the target's position or not. So the eigenvalues of $(A - B)$ are located in the circle D', where the radius is the same as A and center of the disk may moves left a distance of k_1.

Therefore, $Q(t)$ converges to the stable and invariant. Based on (9), it is easy to get that $Q_1(t_1) = Q_2(t_1) = \cdots = Q_n(t_1)$ when $t = t_1$. From *Assumption* 1, if the estimation Q converge stable in the circle but not the center of the circle, which means that there is s static error between the estimation Q_i and the target P^*. There must be at least one vehicle can sense the target's position, which means that the feedback of the target's position for estimation. Then estimation exchange their information via cyclic communication network to finally eliminate the error between estimation and target. Finally, *Theorem* 1 is proved. ■

3.2 Rotation and Cyclic Pursuit

In this section, we design the hierarchical controller for the hunting problem with obstacle avoidance via the information of estimation of target and neighborhood. The first step is to derive the immediate signal to get close to the target and rotate around the target.

$$\mathbf{u} = \mathbf{u}^r + \mathbf{u}^\theta \tag{13}$$

where $\mathbf{u} \in \mathbb{R}^n$ is a column vector controller defined as $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_n]^T$. It is consists of two parts: $\mathbf{u}^r \in \mathbb{R}^n$ is a feedback control law determining D. The second part $\mathbf{u}^\theta \in \mathbb{R}^n$ denotes the cyclic pursuit law determining θ, which is to space evenly and rotate around the target.

Then, Q_i and D_i are seen as the center of the circle and radius respectively, the immediate signal directly determine D_i and θ_i, given the kinematics between the vehicle i and estimator i as below.

$$\begin{cases} \dot{D}_i = \mathbf{u}_i^r \\ \dot{\theta}_i = \mathbf{u}_i^\theta \end{cases} \tag{14}$$

Then we give the detailed control law for $\mathbf{u}^r$ and $\mathbf{u}^\theta$ to control D and θ.

$$\mathbf{u}^r = k_2(\mathbf{1} \otimes r - D); \tag{15}$$

where k_2 is the positive constant. $D = [D_1, D_2, D_3, \ldots, D_n]^T$ is a column denoting the relative distance from vehicles to the estimation.

Theorem 2. *Suppose that the control part* $\mathbf{u}^r$ *based on the estimation of target uses the form of (15), all vehicles asymptotically converge to the circular obits of the target.* ■

Proof: Define a *Lyapunov* function $V = \sum_{i=1}^{n} \frac{1}{2}(r - D_i)^2$, take derivative of V and substitute (14) and (15):

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^{n}(r - D_i)\dot{D}_i \\ &= -(\mathbf{1} \otimes r - D)^T \dot{D} \\ &= -k_2(\mathbf{1} \otimes r - D)^T(\mathbf{1} \otimes r - D) \\ &\leq 0 \end{aligned} \tag{16}$$

where it notes that V is equal to zero only when $D_1, D_2, D_3, \ldots, D_n$ is equal to r. Since $V_i = \frac{1}{2}(r - D_i)^2 \geq 0$, so from the *Lyapunov* theorem, It is easily to see that D asymptotically converges to $\mathbf{1} \otimes r$ with control law $\mathbf{u}^r$. ■

The second step is to make vehicles rotate around the target and space evenly by the control law $\mathbf{u}^\theta$, which is in the form:

$$\mathbf{u}^{\boldsymbol{\theta}} = k_3(C\boldsymbol{\theta} + P) \tag{17}$$

where $C = circ(-k_3, k_3) \in \mathbb{R}^{n \times n}$, $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^n, P = [0, 0, \ldots, 2\pi]^T \in \mathbb{R}^n$.

Theorem 3. *Suppose that the control part* $\mathbf{u}^\theta$ *based on cyclic pursuit strategy uses the form of (17), all vehicles asymptotically space evenly and rotate around the target.*

Proof: The theorem is achieved based on a similar idea used in [9]. The previous parts implies $\dot{\boldsymbol{\theta}} = \mathbf{u}^\theta$. Then substitute (17) and take the derivative of $\dot{\boldsymbol{\theta}}$, then we have $\ddot{\boldsymbol{\theta}} = C\dot{\boldsymbol{\theta}}$. Since C is circular matrix, its representer is $p_C(\lambda) = -k_3(1 + \lambda)$ and the eigenvalues are given as $\lambda_i = p_C(\omega^{i-1}), i = 1, 2, \ldots, n$, from Lemma 1. Then, the eigenvalues can be rewritten in complex form as matrix A. Therefore, the matrix C always has a zero eigenvalue, λ_1, while the remaining $n - 1$ eigenvalue $\lambda_i, i = 2, 3, \ldots, n$, lies in the left-half complex plane, which means that $\dot{\boldsymbol{\theta}}$ converges to the null space $\{\boldsymbol{\sigma} | \sigma I_n, \sigma \in \mathbb{R}\}$, $I_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$, which corresponds to the $\lambda_1 = 0$; i.e.. $\dot{\boldsymbol{\theta}}(t_1) = [\dot{\theta}_1(t_1), \dot{\theta}_2(t_1), \ldots, \dot{\theta}_n(t_1)]^T$ satisfies $\dot{\theta}_1(t_1) = \dot{\theta}_2(t_1) = \cdots = \dot{\theta}_n(t_1) = \sigma$ in the steady state. Furthermore, we have:

$$\sum_{i=1}^{n} \dot{\theta}_i(t) = 2k_3\pi \quad for\ all\ t \geq 0 \tag{18}$$

Therefore, we have $\dot{\theta}_i(t_1) = \frac{2k_3\pi}{n} = k_3(\theta_{i+1}(t_1) - \theta_i(t_1))$. From (17), we will derive that:

$$\begin{cases} \theta_{i+1}(t_1) - \theta_i(t_1) = \frac{2\pi}{n}; & i = 1, 2, \ldots, n-1 \\ \theta_1(t_1) - \theta_i(t_1) + 2\pi = \frac{2\pi}{n}; & i = n \end{cases} \tag{19}$$

Therefore, we proof the *Theorem* 3. Finally, the circular formation of vehicles and space evenly and rotation around the target are formed.

3.3 Collision Avoidance and Change the Label Between Vehicles

In the previous part, we achieve the objectives of the hunting problem. Note that during the forming of the circular formation, the label of vehicles is set by the initial different estimations of the target, it maybe break rule of Remark 1. For example, the label of vehicles maybe first set as $1, 2, 4, 3$, but the actual label around the target should be $1, 2, 3, 4$. It may cause serious problems such as the communication network $\mathbb{G}$ is not cyclic network again. Here, we propose the artificial potential method and the strategy of changing label between neighbors. The artificial potential methods are stated as follows:

$$\zeta_i = \begin{cases} (\frac{1}{2}R - D_i)^2, & D_i \leq \frac{1}{2}R \\ 0, & D_i > \frac{1}{2}R \end{cases} \tag{20}$$

where ζ_i is a potential function. R is the sense radius of the vehicle. ζ_i is set as repulsive function added to the immediate signal.

Combining the kinetics and kinematics of USV (1), the desired surge velocity u_i^d and sway velocity v_i^d by the immediate signal. Transform it into the earth framework, we can finally get the desired velocity of vehicle i:

$$\begin{cases} v_{id}^x = \mathbf{u_i^r} \cos\theta_i - \mathbf{u_i^\theta} \sin\theta_i + \sum\limits_{j\in N_i}^{n} \zeta_j \cos\varrho_j \\ v_{id}^y = \mathbf{u_i^r} \sin\theta_i - \mathbf{u_i^\theta} \cos\theta_i + \sum\limits_{j\in N_i}^{n} \zeta_j \sin\varrho_j \end{cases} \tag{21}$$

where v_{id}^x, v_{id}^y is the desired velocity in the earth framework. ϱ_j denotes the vehicles whose angle from vehicle i to vehicle j, which is in the following form:

$$\varrho_i = \mathrm{atan2}((\mathrm{y_i^P} - \mathrm{y_j^P}), (\mathrm{x_i^P} - \mathrm{x_j^P}))$$

For the USV vehicle, the surge velocity u_i^d and ψ_i^d can be derived from (refControl.20),

$$u_i^d = \sqrt{v_{id}^x \times v_{id}^x + v_{id}^y \times v_{id}^x}, \ \psi_i^d = \mathrm{atan2}(\mathrm{v_{id}^y}, \mathrm{v_{id}^x}) \tag{22}$$

3.4 Vehicle Kinetic Control

The following work is to design a control law τ_{iu} and τ_{ir} to make the surge velocity u_i and the sway velocity v_i of the hunter i converges to the desired immediate control signal u_i^d and v_i^d. Analyse the kinetic equation of the under-actuated hunter vehicles, the surge velocity u_i and the heading angle ψ_i are mainly influenced by control input τ_{iu} and τ_{ir} respectively. So for simplicity of achievement, a PID controller is proposed to make the goal. Define the surge velocity error e_i^u and the phase angle error e_i^ψ as:

$$\begin{cases} e_i^u = u_i - u_i^d \\ e_i^\psi = \psi_i - \psi_i^d \end{cases} \tag{23}$$

where e_i^u and e_i^ψ are the surge velocity error and heading angle error between vehicle states and the desired immediate control signal. Based on the PID controller scheme, the control law of hunter vehicle i is obtained as follows:

$$\begin{cases} \tau_{iu} = k_i^{pu} e_i^u + k_i^{iu} \int_0^t e_i^u \, dt + k_i^{du} \dot{e}_i^u \\ \tau_{ir} = k_i^{pp} e_i^\psi + k_i^{ip} \int_0^t e_i^\psi \, dt + k_i^{dp} \dot{e}_i^\psi \end{cases} \tag{24}$$

where $(k_i^{pu}, k_i^{iu}, k_i^{du})$ and $(k_i^{pp}, k_i^{ip}, k_i^{dp})$ are positive constant, which represent proportionality coefficient, integral coefficient and differential coefficient of the surge velocity and phase angle of hunter vehicle i.

4 Simulation

In this section, we carry out some numerical simulations to demonstrate the performance of the controller (13) for hunting problem. Consider an underactuated surface vehicle with model parameters just as in [1]: $m_{11} = 1.956, m_{22} = 2.405, m_{33} = 0.043, d_{11} = 2.436, d_{22} = 12.992, d_{33} = 0.0564$. For the purpose of comparisons, we suppose the controller is in two cases, which contains label-change strategy and which is not.

For simulation use, we define one target and five hunters, making the sense radius is $R = 8$ m and hunting radius is $r = 5$ m. Then we give the initial conditions for the hunting problem as follows: the target position is set $P_* = (-5, -3)^T$, the hunter vehicles' positions and states are set without loss of generality, only one vessel initially knows the position of target:

$$\mathbf{P} = \begin{bmatrix} 2 & 8 \\ 8 & -2 \\ -2 & 10 \\ -2 & -7.5 \\ 4 & -10 \end{bmatrix}, \psi = \begin{bmatrix} \arctan 4 \\ \arctan \frac{1}{7} \\ \arctan \frac{1}{5} \\ \arctan \frac{1}{5} \\ \arctan \frac{1}{5} \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The control parameter of hunter vehicle i are taken as $k_1 = 0.5, k_2 = 1, k_3 = 0.15, k_i^{pu} = 12, k_i^{iu} = 0.7, k_i^{du} = 0, k_i^{pp} = 0.1, k_i^{ip} = 0, k_i^{du} = 5$. Simulation results are shown in the following figures, and the simulation time is set 50 s.

Figures 3 and 4 show the distributed hunting behavior with label changing between neighbors. Figure 3 shows that vehicles finally surround and rotate

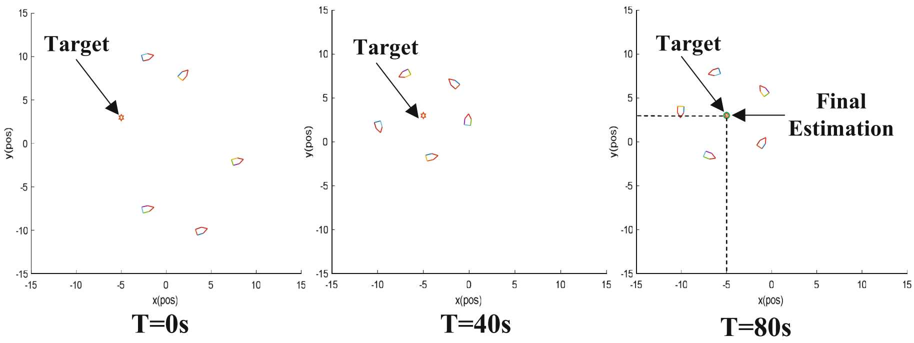

Fig. 3. Distributed hunting behavior where label can change between neighbors.

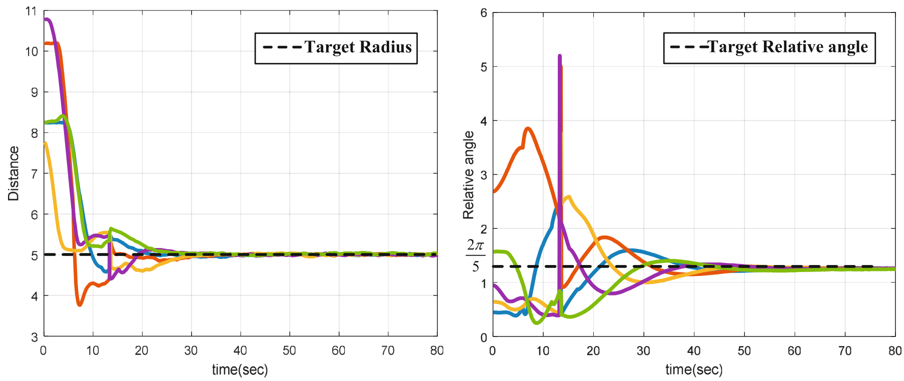

Fig. 4. Distance and relative angle between the angle where label can change between neighbors

around the target, and the vehicles space evenly around the target. Figure 4 indicates that distance from target and relative angle have oscillations at the beginning, because vehicles change its label between neighbors. But it can asymptotically converges to the desired value. One can conclude the effectiveness of the controller for distributed hunting by multi USVs.

5 Conclusions

In this paper, we have studied collective hunting problem with obstacle avoidance for a group of USVs. The problem includes two subobjectives of hunting target, for which each vehicle maintains the desired distance from target, spaces evenly and rotates around the target. The controller are based on the cyclic estimation of target and feedback control law to get close to the target. It is designed by also cyclic pursuit strategy using the relative angle between the neighbors. Artificial potential method and strategy of changing label between neighbors are also applied to guarantee obstacle avoidance and achieve the goal. It is worth to mention that the control law guarantee almost collision during the forming the formation.

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