A Critique of Observers Used in the Context of Feedback Control

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Abstract. One of the core tenets of feedback control is that a system's state contains all of the information necessary to predict a system's future response given future inputs. If the state is not directly measured then it can be estimated using a suitably designed observer. This is a powerful idea with widespread consequences. This paper will present a critique of the use of observers in feedback control. Benefits and drawbacks will be highlighted including fundamental design limitations. The analysis will be illustrated by several real world examples including robots executing a repetitive task, relay autotuning in the presence of broadband disturbances, power line signalling in AC microgrid power systems, Type 1 diabetes management and harmonic suppression in power electronics.

Keywords: Fundamental limitations · Harmonic suppression
Observers · Periodic disturbances · Type 1 diabetes management

1 Introduction

Observers play a central role in many fields [9–11,13,14,87,91]. They allow the current state of a system to be estimated by combining prior information (including the system model) with real-time measurements. The literature on this subject is immense. For example [86] lists over 350 core references with a history dating back to the 1700's. Our goal in the current paper is not to attempt a comprehensive survey. Instead, we focus on inherent limitations associated with observers especially when used in the context of feedback control. In the context of feedback control, the state provides the link between past behaviour and future behaviour.

Many different design tools are available, including but not limited to:

– Lumberger Observer [15,42]
– Kalman Filter [12,16,17]
– H_∞ design [80–84,86]
– Extended Kalman Filter [17,19,20,25,26,28,29,35]
– Unscented Kalman Filter [18]
– Nonlinear observer related to Luenberger observer [8]
– High Gain Observers [21–24,27]

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- Nonlinear Observers based on output injection [6,7]
- Sliding Mode Observers [1–3]
- Multiple Model Observers [4,5]
- Moving Horizon Estimators [32,34]
- Particle Filtering [30,31,33,34,36–38]
- Dual Control Methods [39]

This plethora of techniques may give the impression that the final word has already been written on this subject. However, certain issues are not yet fully resolved.

The current paper will focus on the benefits, and drawbacks, of observers with special emphasis on their use in feedback control. Observers are a key component of a feedback system designer's tool kit. Both controllable and uncontrollable states are of importance. Specifically, a simplified view of control is that the overall aim is to steer the controllable part of a system so as to cancel (as far as is feasible) two components, namely (i) the future output response arising from unanticipated changes in the initial state of the controlled part of the system and (ii) the future output response arising from the uncontrolled part of the system. In many applications, the latter aspect eclipses the former. Hence some emphasis will be placed in the current paper on estimating uncontrollable modes including sinusoidal disturbances.

Alas, observers of all forms come with a set of fundamental limitations [40] which inhibit their performance. In some scenarios, these limitations can be so severe that high performance control is simply impossible. This suggests that one has no option but to invest time and effort into developing additional physical sensors rather then relying upon a "soft sensor" or an observer.

For pedagogical reasons we will restrict our analysis to simple cases but will point to generalizations and occasional open problems.

2 A Class of Linear Observers

As a starting point, consider a linear time-invariant multi-input multi-output system described in transfer function form by

$$y^o = T^o_{yu} \cdot u + T^o_{yw} \cdot w \tag{1}$$

where $y^o \in \mathbb{R}^p$, $u \in \mathbb{R}^p$, $w \in \mathbb{R}^w$ denote output, input and process noise, respectively. Here, and in the sequel, T^o_{ba} denotes the direct linear mapping from signal a to signal b, in either continuous or discrete time, with the appropriate operator and dimensions.

Consider an auxiliary variable, $\eta \in \mathbb{R}^n$, given by

$$\eta = T^o_{\eta u} \cdot u + T^o_{\eta w} \cdot w, \tag{2}$$

having the property that $y^o = T^o_{y\eta} \cdot \eta$. The quantity η can be a performance variable, the system state, some suitable combination of the states, or some other variable of interest. We have $T^o_{yu} = T^o_{y\eta} \cdot T^o_{\eta u}$ and $T^o_{yw} = T^o_{y\eta} \cdot T^o_{\eta w}$.

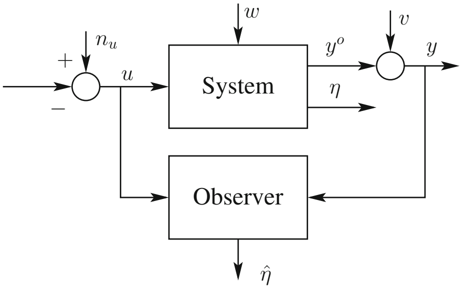

Fig. 1. Observer architecture

We restrict attention to a class of linear observers of the following generic form:

$$\hat{\eta} = F_y \cdot y + F_u \cdot u \tag{3}$$

where $\hat{\eta}$ is an estimate of $\eta, F_y,\ F_u$ are linear $(n \times p)$ stable transfer functions and where y is the measured output:

$$y = y^o + v \tag{4}$$

for some measurement noise process $v \in \mathbb{R}^p$. The general set-up is shown schematically in Fig. 1.

3 Unbiased Linear Observers

A desirable property for an observer is that it should "on average" yield the correct result, i.e., is unbiased [40].

3.1 The Standard Linear Model

For the standard linear model, (1), an unbiased observer has the property that the error between $\hat{\eta}$ and η does not depend upon the input [41]. Unbiased observers include Luenberger observers and the steady state Kalman filter as special cases. However, unbiased observers are more general since, for example, the degree of the observer may differ form that of the plant. Motivations for this could include providing robustness against modeling errors in certain frequency bands or discriminate against non-white noise. (The latter property is also an integral part of the standard Kalman filter formulation [12,16,17].)

The following result is immediate.

Lemma 1. *A necessary and sufficient condition for* (3) *to be unbiased, i.e.* $T^o_{\hat{\eta}u} = T^o_{\eta u}$*, is that the following identity holds:*

$$F_y \cdot T^o_{yu} + F_u = T^o_{\eta u} \tag{5}$$

Proof (See [40], Lemma 7.3.1). The transfer function from u to $\hat{\eta}$ is

$$T^o_{\hat{\eta}u} = F_u + F_y[T^o_{yu}] \tag{6}$$

For unbiasedness it is required that

$$T^o_{\hat{\eta}u} = T^o_{\eta u} \tag{7}$$

Equating (6) and (7) yields (5). □□□

We define the observer error as $\tilde{\eta} = \hat{\eta} - \eta$. Using (3), (4), and (1) for $\hat{\eta}$, and using (2) and (5) for η, it follows that

$$\tilde{\eta} = \underbrace{F_y}_{T_{\tilde{\eta}v}} \cdot v + \underbrace{\left(F_y T^o_{yw} - T^o_{\eta w}\right)}_{T_{\tilde{\eta}w}} \cdot w \tag{8}$$

3.2 Impact of Errors-in-Variables

Some caution is necessary when demanding that an observer be unbiased. As an illustration, consider the following, seemingly simple, state estimation problem:

$$\dot{x}_t = A[u_t]x_t + Bu_t + \omega_t \tag{9}$$

$$y^m_t = C[u_t]x_t + v_t \tag{10}$$

where $\{\omega_t\}, \{v_t\}$ are noise processes and where $A[u_t], C[u_t]$ denote functions of the input, u_t. If the input $\{u_t\}$ were exactly known, then the above problem would fall within the standard time-varying Kalman filter framework [85]. However, say that $\{u_t\}$ is measured in the presence of noise, i.e., the available measured input satisfies

$$u^m_t = u_t + \tilde{u}_t \tag{11}$$

where $\tilde{u}_t$ denotes a measurement error.

In this case, the above state estimation problem is of the Errors-in-Variables class [88,89] and the standard Kalman filter may be biased.

Example: To illustrate the above ideas, consider the following simple discrete-time state estimation problem:

$$x_{t+1} = x_t + \omega_t; E\left\{\omega^2_t\right\} = Q \tag{12}$$

$$y^m_t = x_t u_t + v_t;\ E\left\{v^2_t\right\} = R \tag{13}$$

$$u^m_t = u_t + \tilde{u}_t;\ E\left\{\tilde{u}^2_t\right\} = V \tag{14}$$

where $\{\omega_t\}$, $\{v_t\}$, $\{\tilde{u}_t\}$ are white noise sequences. If we simply replace $\{u_t\}$ by $\{u^m_t\}$, then the standard time-varying Kalman filter would be

$$\hat{x}_{t+1} = \hat{x}_t + J_t\left\{y^m_t - \hat{x}_t u^m_t\right\} \tag{15}$$

where the gain J_t satisfies:

$$J_t = P_t u_t^m \left(R + P_t \left(u_t^m\right)^2\right)^{-1} \tag{16}$$

$$P_{t+1} = P_t - P_t \left[R + P_t \left(u_t^m\right)^2\right]^{-1} \left(u_t^m\right)^2 P_t + Q \tag{17}$$

Note that this estimate is, in general, biased when $V \neq 0$. Indeed, the error $\tilde{x}_t = \hat{x}_t - x_t$ satisfies

$$\tilde{x}_{t+1} = J_t \left[u_t + \tilde{u}_t\right] \left\{u_t \tilde{x}_t - \hat{x}_t \tilde{u}_t\right\} \tag{18}$$

It can be seen that the error $\tilde{u}_t$ appears via a product which is the source of the bias issues. In the absence of additional information, then the best one can say is that the state lies in a range of possibilities [90]. Additional side information can, in some cases, be used to yield a unique estimate [88]. This problem has been extensively studied in the context of parameter estimation but, seemingly, less so in the context of state estimation. One possible remedy is to treat u_t^m and y_t^m both as measurements. If the noise characteristics are known, then it would be possible to utilize a nonlinear maximum likelihood estimator. This will be, at least, asymptotically unbiased.

4 Linear Observer Sensitivity Functions

We return to the standard linear model of Sect. 1.

It would be tempting to use $T_{\tilde{\eta}v}$ and $T_{\tilde{\eta}w}$, as in (8), to define observer sensitivity functions. However, the variables v and ω have different dimensions and units. We thus follow the ideas of [40] and introduce "normalized" observer sensitivity functions.

Definition 1. *The observer complementary sensitivity is defined as* $M = \left[T^o_{\eta y}\right]^{-1} \cdot T_{\tilde{\eta}v}$*, and the observer sensitivity function as* $P = I - M$.

The observer complementary sensitivity, M, captures the relative effect of the measurement noise, v, on the estimation error $\tilde{\eta}$. The observer sensitivity function captures the relative effect of process noise ω on the estimation error. These relationships can be readily seen, at least in the scalar case, by noting that

$$M = T_{\tilde{\eta}v} \left[T^o_{\eta y}\right]^{-1} \tag{19}$$

and

$$T_{\tilde{\eta}\omega} = F_y T^o_{y\omega} - T^o_{\eta\omega} \tag{20}$$

$$= \left[F_y T^o_{y\eta} - I\right] T^o_{\eta\omega} \tag{21}$$

$$= \left[M - I\right] T^o_{\eta\omega} \tag{22}$$

Hence

$$T_{\tilde{\eta}\omega} \left[T^o_{\eta\omega}\right]^{-1} = -P \tag{23}$$

5 Sensitivity Trade-Offs for Observers

5.1 Sum of Sensitivity and Complementary Sensitivity

Since $P + M = I$, it is evident that the observer error, $\tilde{\eta}$, cannot simultaneously have low sensitivity to both measurement noise and process noise but, instead, the sum of the (normalized) sensitivities must always be I (1 in the scalar case). Thus observers, no matter how designed, must always make a trade-off between sensitivity to measurement noise and sensitivity to process noise.

In practice, the situation can be worse than might be, at first sight, apparent from the trade-off mentioned above. In particular, when considered as a function of frequency, P and M are complex numbers. Hence it is entirely possible for P and M to add to 1 but have a magnitude, at any given frequency, which is much greater then 1.

This is illustrated in Fig. 2 for the scalar case.

5.2 Relative Degree Issues

A particular difficulty arises due to relative degree. Say, for example, that one wishes to design an observer when the input is unknown, is measured in the presence of large noise, or there exists large unmeasured input disturbances. In these cases, the dependence of the observer on the input can be reduced by letting $F_u \rightarrow 0$. However, it then follows from (5) that $F_y \rightarrow [T^o_{\eta u}][T^o_{yu}]^{-1}$.

Hence, if the relative degree, $\rho(T^o_{\eta u})$, is less than that of T^o_{yu}, then the observer will approach an $\bar{r}$ fold differentiator, where $\bar{r} = \rho[T^o_{yu}] - \rho[T^o_{\eta u}]$. In practice, one must use bandlimited differentiations to yield a causal observer. However, the degree of bandlimiting will necessarily impact performance. These ideas are summarized in Table 1:

Table 1. Observer extremes

M = 1: P = 0:	Full sensitivity to measurement noise No sensitivity to input errors Requires output differentiation
P = 1: M = 0:	No sensitivity to measurement noise Full sensitivity to Model and input
Intermediate values:	Somewhere between the above extremes

The fact that observers lie somewhere between a filter that differentiates the output and one that use an open loop model driven by known inputs is well known in practice, see for example [92]. Thus, "optimality"of an observer may not necessarily mean that it performs well but only that it makes the best from a bad situation.

A real world application of the impact of this trade-off will be given in Sect. 8.9.

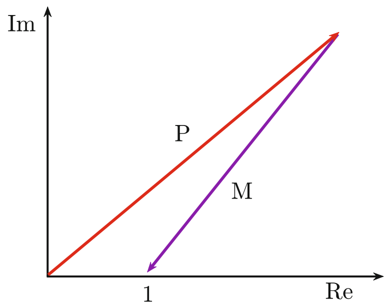

Fig. 2. Sum of observer sensitivity - complementary sensitivity

5.3 Bode Sensitivity Integrals and Impact of Non-minimum Phase Zeros

The conceptual idea illustrated in Fig. 2 leads to an associated question, namely, when might one expect that the magnitudes $|P|$ and $|M|$ are much greater than 1 yet $P + M = 1$. This is a well studied problem in the context of control [40]. Interestingly, there exist dual results which apply to the sensitivity functions of observers. For example, P and M satisfy Bode type integral constraints analogous to those that hold for the control sensitivity functions – see [40].

For example, when $T^o_{\eta u}$ is minimum phase but T^o_{yu} is non-minimum, then the following integral constraint holds for the complementary observer sensitivity function:

$$\int_0^\infty log \left| \frac{M(j\omega)}{M(0)} \right| \frac{d\omega}{\omega^2} = \frac{\pi}{2} \frac{1}{M(0)} \lim_{s \to 0} \frac{dM(s)}{ds} + \pi \sum_{i=1}^{n_q} \frac{1}{q_i} + \frac{\pi}{2}\tau \tag{24}$$

where $\{q_i : i = 1, \ldots, n_q\}$ is the set of open right half plane zeros of T^o_{yu} and τ is a pure delay. (The above result is established in [40] on p.57 save for the delay term which holds analogously to the corresponding control result.)

An immediate implication of (24) is that all observers satisfy a conservation of "sensitivity dirt" principle similar to that which applies in feedback control [43]. Thus reducing sensitivity to either measurement noise or process noise in one frequency band inevitably leads to increased sensitivity in another band.

It is also clear from (24) that "small" non-minimum phase zeros or delays in T^o_{yu} will make the sensitivity trade-off more difficult.

More will be said in Sect. 7 regarding the relationship between observer sensitivity functions and performance of output feedback controllers.

6 Estimating Sinusoidals in Noise

An interesting, and widely applicable, use of observers arises in the context of the estimation of sinusoidal components from signals having broad band noise.

Associated design issues are discussed below.

6.1 A Suitable Observer

The core idea is to model a sinusoidal signal (of frequency ω_r) by a second order system of the form:

$$\dot{x}_1(t) = x_2(t) + \omega_1(t) \tag{25}$$

$$\dot{x}_2(t) = \omega_r^2 x_1(t) + \omega_2(t) \tag{26}$$

$$y(t) = x_1(t) + v(t) \tag{27}$$

where $\omega_1(t)$, $\omega_2(t)$, $v(t)$ denote noise sources.

A suitable observer then takes the following generic form:

$$\dot{\hat{x}}_1(t) = \hat{x}_2(t) + K_1 \left[y(t) - \hat{x}_1(t)\right] \tag{28}$$

$$\dot{\hat{x}}_2(t) = -\omega_r^2 \hat{x}_1(t) + K_2 \left[y(t) - \hat{x}_1(t)\right] \tag{29}$$

$$\hat{y}(t) = \hat{x}_1(t) \tag{30}$$

The gain $[K_1, K_2]$ can be defined by any suitable method. A simple choice is

$$[K_1, K_2] = [2\xi\omega_r, 0]\,; \; 0 < \xi < 1 \tag{31}$$

The corresponding observer transfer function is easily seen to be

$$F(s) = \frac{2\xi\omega_r s}{s^2 + 2\xi\omega_r s + \omega_r^2} \tag{32}$$

where $0 < \xi \ll 1$.

The frequency response of $F(s)$ is shown in Fig. 3 for different values of the damping ratio, ξ.

The observer (28)–(32) recursively generates the Fourier Transform at ω_r.

6.2 Sampled Data Implementation

Equation (32) assumes "continuous" implementation of the filter. In practice, one will need to use a sampled data implementation. For small ξ care is needed with numerical precision. In particular, incremental implementation is advisable, see [52–54].

Let the sampling period be Δ and the sinusoidal frequency of interest be ω_r. Then, a possible state space realisation, in incremental (or δ) form, is ([52–54])

$$\begin{aligned} x_{k+1} &= x_k + \Delta \begin{bmatrix} \frac{C-1}{\Delta} & \frac{S}{\omega_r \Delta} \\ \frac{-\omega_r S}{\Delta} & \frac{C-1}{\Delta} \end{bmatrix} x_k + \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} \left(y_k - \begin{bmatrix} 1 & 0 \end{bmatrix} x_k\right), \\ y_k^F &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_k, \end{aligned} \tag{33}$$

where

$C \doteq \cos(\omega_r \Delta)$, $S \doteq \sin(\omega_r \Delta)$, $\ell_1 = 2\xi\omega_r \Delta C$, $\ell_2 = \xi\omega_r^2 \Delta\omega_r (2C^2 - 2 + \xi\omega_r \Delta)/S$.

Note that the filter (33) has the property that for $y_k = A\cos(\omega_r k\Delta)$, then $y_k^F = A\cos(\omega_r k\Delta)$ as required.

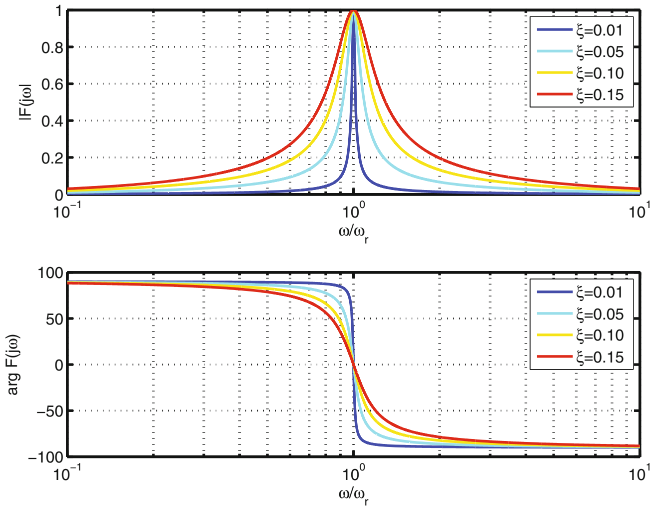

Fig. 3. Observer frequency response for estimating a sinusoid.

Also, if multiple sinusoids at different frequencies need to be estimated, then the observer should be implemented as a set of parallel second order sections.

Remark 1

1. Equations (33) correspond to a discrete steady-state Kalman Filter for the problem of estimating a sine wave buried in noise, see e.g., Example 10.7.3 in [54]. Two important caveats apply to this observation. Firstly, if one assumes a perfect sine wave in noise then the time-varying Kalman filter gain will converge to zero [88]. This effect can be avoided by assuming the presence of a small amount of process noise in the model for the sine wave. Secondly, if there is high uncertainty associated with the initial state, then the Kalman filter gain will be time varying. This could be achieved by implementing the appropriate Ricatti equation of optimal filtering [85,88].
2. Note that, as $\Delta \to 0$, the filter (33) converges to the continuous time filter (32).

6.3 Design Considerations and Performance Limitations

Key properties of the above observer are that it has unity gain at ω_r and that the frequency response is focused on a narrow frequency band. Several consequences arise from this property:

(i) Transient Time – When ξ is reduced then so is the "bandwidth" of the observer (see Fig. 3). Hence making ξ smaller gives greater frequency selectivity. However, narrow frequency selectivity comes at the cost of a greater settling time. This represents an unavoidable trade-off.

(ii) Sensitivity Dirt Trade-off – Since the observer has a narrow frequency band, then minimal reallocation of "frequency dirt" is needed. A caveat on this observation is that, when one uses a large number of such observers in parallel for multiple sinusoids, then the amount of "dirt" to be shifted can accumulate to the point where it becomes non-negligible.

(iii) Phase Shift – The observer provides significant positive and negative phase shift relatively to small changes in amplitude. The property is evident in Fig. 3. This can be exploited to develop a relay autotuner for systems have large broadband disturbances – see Sect. 8.4.

7 State Feedback Control Versus Output Feedback Control

Here we investigate the link between observer performance and output feedback control.

For simplicity we will restrict attention to the linear SISO case. Multivariable extensions are addressed in [93].

7.1 State Feedback

We begin by considering the case when the full state is measured.

Consider a linear system having state space description

$$\dot{x} = Ax + Bu \tag{34}$$

$$y = Cx \tag{35}$$

Say we have access to the full state vector, then we can use a feedback control law of the form:

$$u = -\eta + r; \ \eta = Kx \tag{36}$$

Define

$$T^o_{yu} = C(sI - A)^{-1}B = \frac{b(s)}{a(s)} = \frac{b_{n-1}s^{n-1} + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \tag{37}$$

$$T^o_{\eta u} = K(sI - A)^{-1}B = \frac{h(s)}{a(s)} = \frac{h_{n-1}s^{n-1} + \ldots + h_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \tag{38}$$

The closed loop transfer function resulting from the use of the law (36) is

$$T^o_{yr} = \frac{T^o_{yu}}{[1 + T^o_{\eta u}]^{-1}} = \frac{b(s)}{[a(s) + h(s)]} \tag{39}$$

7.2 State Variable Feedback Sensitivity Functions

We define state variable feedback sensitivity function as follows:

Definition 2. *The state variable feedback complementary sensitivity function, T^o_{SF}, and sensitivity function, S^o_{SF}, are defined as:*

$$T^o_{SF} = (I + T^o_{\eta u})^{-1} T^o_{\eta u} \tag{40}$$

$$S^o_{SF} = (I + T^o_{\eta u})^{-1} = I - T^o_{AF} \tag{41}$$

Elementary control design considerations suggest that one should avoid non-minimum phase zeros in $T^o_{\eta u}$ if one wishes to have good closed loop performance. Further justification for this statement is obtained by noting that for high gain feedback, the closed loop poles tend to the open loop zeros of $T^o_{\eta u}$ plus other poles that tend to $-\infty$. Hence one should avoid non-minimum phase zeros in $T^o_{\eta u}$.

7.3 Output Feedback Sensitivity Functions

In the case of output feedback, Eq. (37) should be replaced by

$$u = -\hat{\eta} + r \tag{42}$$

where $\hat{\eta}$ is an estimate of η provided by an observer as in Eq. (3). This yields $u = -F_y y - F_u u + r$ or equivalently, $u = (1 + F_u)^{-1}(-F_y y + r)$.

Hence, the closed loop output feedback control complementary sensitivity and sensitivity functions are respectively

$$T_{OF} = \frac{T^o_{yu} F_y}{(1 + F_y T^o_{yu} + F_u)}; \; S_{OF} = 1 - T_{OF} \tag{43}$$

7.4 Relating Control and Observer Sensitivity Functions

An insightful connection can now be established as follows:

Lemma 2 *(see [93]). Consider T_{OF}, T^o_{SF} and M as in (43), (40) and (19) respectively. Then,*

$$T_{OF} = [T^o_{SF}][M] \tag{44}$$

Proof. From (43)

$$T_{OF} = \frac{T^o_{yu} F_y}{(1 + F_y T^o_{yu} + F_u)} \tag{45}$$

Assuming that an unbiased observer is utilized, then substituting (5) into (45) yields

$$T_{OF} = \frac{T^o_{yu} F_y}{(1 + T^o_{\eta u})} \tag{46}$$

$$= (T^o_{\eta u})^{-1} T^o_{SF} T^o_{yu} T^o_{\eta y} M \tag{47}$$

$$= T_{SF} M \tag{48}$$

□□□

As argued above, one would normally choose η so that T^{o}_{SF} is devoid of sensitivity peaks when considering state feedback control. This lead to the inevitable conclusion from (44) that the observer complementary sensitivity function is the prime source of control difficulties in output feedback control.

Thus, any limitations inherent in the observer will manifest themselves in output feedback control performance degradation. In some cases, it may be impossible to achieve suitable observer performance. In such a case, good control performance requires that new physical measurements be developed.

8 Selected Applications of Observers

To illustrate the design trade-offs inherent in observers, a summary of a number of applications carried out by the author and his colleagues will be given below:

8.1 Estimating Rotor Blade Disturbances in Helicopters

This application is a straightforward application of estimating sinusoidal components in noise. Details are given in [44]. Note that, in this application, time is measured relative to engine speed so as to give "constant frequency" sinusoids when the engine speed changes.

8.2 Eccentricity Compensation in Rolling Mills

This application is very similar to that discussed in Sect. 8.1 save that here time is measured relative to the rolling speed. This is again aimed at achieving constant frequency in the face of roll speed changes. Details are given in [45].

8.3 Control of a Robot Executing Repetitive Tasks

A repetitive (or periodic) task can (via Fourier decomposition) be thought of as being composed of a sum of harmonically related sinusoids. Hence, one can extract the error associated with executing a repetitive task by using a bank of sinusoidal observers. The output of the observers can then be used to reduce the tracking error to zero. An early publication describing this idea is [46]. Many follow up embellishments of this idea exist in the literature.

8.4 Relay Autotuning in the Presence of Large Broadband Disturbances

Relay autotuning was first introduced in 1984 by Åström and Hägglund [55]. Since that time the method has been widely adopted in industry. Many associated commercial products are available. There has been substantial follow up research and embellishments of the original idea. For example, [56] discusses practical features aimed at industrial application of the method.

In its basic form, the relay autotuner is sensitivity to broadband disturbances. One idea for dealing with this issue is discussed in [47] where narrow band sinusoidal observers are added to the basic relay circuit so as to extract the relay induced oscillation in the presence of large broadband disturbances. The basic set-up is shown schematically in Fig. 4 where $F(s)$ is an observer of the type given in Eq. (32), $G_p(s)$ is the plant transfer function and where $u_{ext}, u, d_u, d_m, \hat{y}, y$ denote, respectively, an external input, the plant input, an unmeasured disturbance, a measured disturbance, a set-point and the plant output.

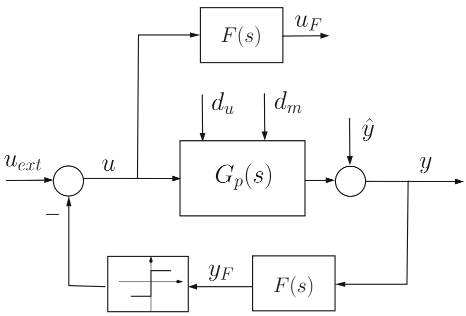

Fig. 4. Modified autotuner

The phase shift provided by $F(s)$ can be utilized to cause an oscillation to occur near the center frequency of the filter. The plant frequency response at the oscillation frequency can then be obtained by dividing the narrow-band signal y_F by the narrow band signal u_F. A full description is given in [47].

Several examples of the use of this modified relay autotuner are described in [47] including application to insulin sensitivity estimation in a Type 1 diabetes patient.

8.5 Harmonic Suppression in Power Electronic Inverters

Inverters play a central role in modern society including integration of wind and solar energy into the grid, high voltage DC power transmission and electronic motor speed control. An inverter uses high frequency switching to produce (nominally) sinusoidal voltage waveforms from a DC source. An inevitable consequence of the use of switching is that harmonics are generated. These can lead to mechanical load resonances and power supply difficulties. Thus there has been on-going research aimed at minimizing harmonics at specific frequencies. A valuable strategy, in this context, is to utilize a set of observers to extract the harmonic content at particular frequencies of interest. A suitable observer is a parallel combination of observers of the form of (33). As remarked in Sect. 6.3, since the observers have narrow bandwidth, their impact on "sensitivity dirt" is relatively small unless many frequencies are considered simultaneously. The estimated harmonic components can then be fed-back (with loop gain 1) so as to suppress these components from the waveform. Various practical embellishments are needed to ensure that this scheme operates satisfactorily in practice including attention to numerical issues and non-linear delay compensation in the feedback path – see discussion in [50].

8.6 Power Line Communication

The capability of easily estimating sinusoidal components from a signal leads to an interesting possibility for Power Line Communication, see [51]. The basic idea is that a required message can be coded into a binary representation and then used to turn-on or -off harmonic components via switching electronics (e.g., three frequencies would allow one to code eight alternatives). The presence, or otherwise, of the harmonics can then be detected at some remote point using appropriate sinewave observers. The message can then be decoded. Other harmonics can similarly be used for onward communication. An application of this idea to an AC Microgrid is described in detail in [51].

8.7 Control of Highly Resonant Systems

Another application of narrowly focused observers is for the control of highly resonant systems. A sinusoidal observer can be used to focus a controller on a particular frequency band. Two such applications are:

- Micro gyroscope manufacture [48]
- Control of atomic force microscopes [49]

8.8 Inverted Pendulum State Estimation

An inverted pendulum is often used to illustrate state feedback and output feedback control, see e.g., [75]. Our interest here focuses on the estimation of the pendulum cart angle when one measures only the pendulum cart position. A source of difficulty in this problem is that the transfer function from input to cart position contains a non-minimum phase zero. Hence, the Bode sensitivity constraint given in (24), implies that observer sensitivity issues are inevitable. Reference [40] on p.194, considers this problem in detail. It is shown that estimating the angle of the pendulum from the cart position is necessarily associated with large sensitivity peaks. For example, consider a pendulum where the ratio of the mass at the end of pendulum to the mass of the cart is 0.1. Sensitivity peaks, at the order of 50:1 are a consequence of the plant non-minimum phase zero. On the other hand, angle information is critical to achieving satisfactory control. Hence, in practice, it is essentially mandatory to provide a direct physical measurement of the angle rather than attempting to estimate it via an observer.

8.9 Type 1 Diabetes Management

Type 1 diabetes is an autoimmune disease in which the pancreas is unable to produce the insulin necessary to control Blood Glucose Levels (BGL) [57]. Excessively high BGL has long term health consequences, including blindness and limb amputation. Excessively low BGL has short term health consequences including dizziness, coma and even death in extreme cases. Type 1 diabetes can manifest itself at any age but is particularly devastating for young people. Its causes are unknown. Dramatic improvements in survival have occurred since the discovery of the beneficial impact of injected insulin.

Because of its importance, there has been an enormous amount of work devoted to improving diabetes management. Of relevance to the current paper is work based on using some form of feedback (i.e., a closed loop) to link measured blood glucose levels (obtained from a continuous glucose sensor) to insulin infusion (via a pump) [57–69].

One of the continuing debates in this area is whether or not one can avoid "food announcements" i.e., whether pure feedback control is adequate – see e.g., [76–78]. We argue below that, given the current limitations of insulin delivery and blood glucose measurements, it is necessary to use food announcements to achieve satisfactory blood glucose regulation. Indeed, this is consistent with the recent development of, so called, hybrid closed loop algorithms [70, 74]. Many different models have been proposed for use in this area [71–73]. We will use a simplified model shown in Fig. 5, where $T(s)$ and $F(s)$ denote linear transfer functions, f denotes food consumption rate (g/min), u denotes injected insulin flow (units/min), b_e denotes endogenous glucose production (assumed constant) y denotes BGL (mmol/L). Note that f is often modeled as an impulse to approximately describe a meal consumed over a short period of time.

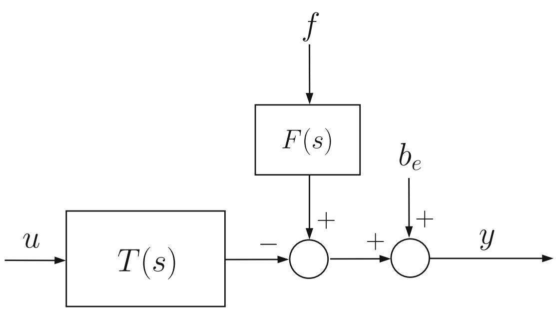

Fig. 5. Blood glucose model

Ignoring modelling errors, the one sided nature of the control and other relevant constraints (all of which have the potential to make the following conclusions worse), then all stabilizing feedback/feedforward control laws [75] are as in Fig. 6 where y^* is the BGL set point. In this figure, $T(s)$ and $T^m(s)$ denote the true "plant" and the model respectively. In the sequel, we ignore model errors, i.e., we take $T^m(s) = T(s)$.

From Fig. 6 it is easily seen that the BGL error, e, satisfies

$$e = [1 - H(s)T(s)][F(s)f - D(s)T(s)f + b_e - y^*] \tag{49}$$

We note that having $H(s)T(s) = 1$ at zero frequency removes any steady state error. This automatically yields integral action [75].

Two scenarios are considered below, namely (i) when meal announcements are available and (ii) when only the BGL response is measured. Further details are given in [94–96].

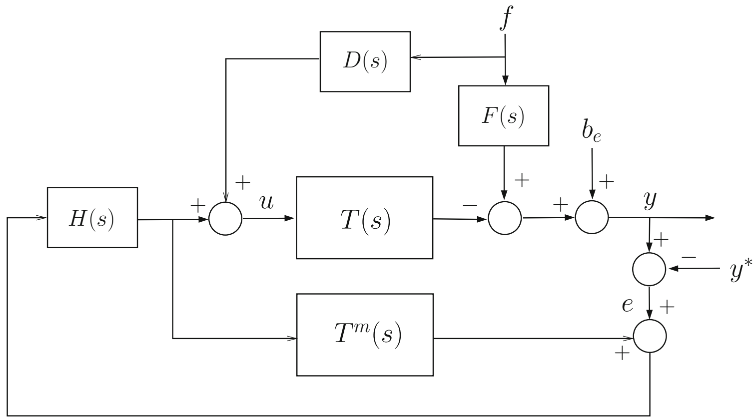

Fig. 6. Feedback and feedforward loops

With Meal Announcement. When a meal announcements are available, then this is equivalent to being able to measure the variable f in Fig. 6. In this case, the BGL error, e, can be made small by choosing the feedforward transfer function, $D(s)$, as

$$D(s) = F(s)T(s)^{-1} \tag{50}$$

This is relatively easy to achieve since $F(s)$ and $T(s)$ typically have the same relative degree. (In practice, $T(s)$ often contains pure delays of the order of 15 min to 20 min. This can be accounted for provided the meal announcement occurs prior to meal ingestion. Indeed many diabetes educators suggest that insulin be given 15 min prior to consuming a meal thus negating the delay.)

Figure 7 shows real data collected from a patient. The upper trace shows the BGL response (mmol/L) to a high fat/high protein meal (20 g CHo, 40 g fat, 50 g protein) whilst the lower trace shows the BGL response to a pulse of insulin. The smooth curve shows a model fit using finite dimensional transfer functions of the form shown in Fig. 5.

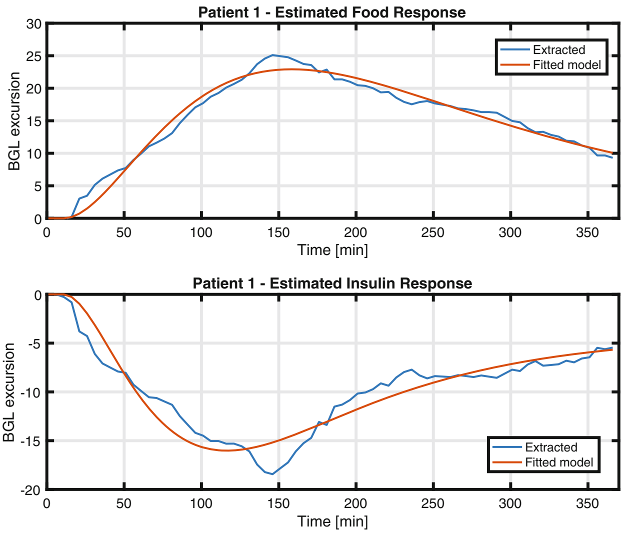

Fig. 7. BGL response to a high fat/high protein meal and to a bolus of insulin

The optimal insulin delivery profile was evaluated. The resulting strategy required 1.66 times the amount of insulin normally associated with the patient's Insulin to Carbohydrate Ratio, (ICR). Of this amount, 50% was delivered as a pulse (called a bolus) via the feedforward term i.e., $D(s)$ in Fig. 6. The remaining 50% was delivered over a subsequent period of two hours. The peak BGL (above set-point) predicted by the model was evaluated to be 0.55 mmol/L.

Without Meal Announcement. In this case, it is assumed that the food signal, f, is not available. We must then rely upon the term $[1 - H(s)T(s)]$ in Eq. (49) to reduce the BGL excursion. Note that $H(s)$, in part, plays the role of an observer to estimate the states of the disturbance model, namely $F(s)$. This allows the controllable part of the system, namely $T(s)$, to be steered so as to cancel the future disturbance response as foreshadowed in Sect. 1 of the paper. It follows from Eq. (49) that, the best one can do is to choose $H(s) = T(s)^{-1}$. However, two issues mitigate against this solution, namely

(i) The delay in $T(s)$ cannot be pre-compensated in a feedback solution since meal anticipation is impossible under these conditions.
(ii) The high relative degree of $F(s)$ means that any attempt to estimate the states of the disturbance model from the measured BGL response implicity involves multiple differentiation – see discussion in Sect. 5.2.

In practice, $H(s)$ can only be chosen as a bandlimited differentiator. The achievable BGL response is then a direct function of this bandwidth. To illustrate we choose

$$T(s)H(s) = \frac{e^{-s\tau}}{(bs+1)^3} \tag{51}$$

where $\tau = 15$ min, $b = 10, 50, 100$ min. The predicted BGL peaks (mmol/L) are as follows:

$b = 10$	Peak BGL deviation = 10.67
$b = 50$	Peak BGL deviation = 19.73
$b = 100$	Peak BGL deviation = 21.95

Of these, $b = 10$ is impractical since it depends upon an approximate three fold differentiation of the BGL signal. This would result in large magnification of any measurement noise by the observer. This leaves the $b = 50$ or $b = 100$ results. The BGL deviation is then predicted to be 19.73 or 21.95. (Recall that meal announcement gave a predicted peak of 0.55 mmol/L.) These results have been confirmed on the real patient where a BGL rise of 2 mmol/L was obtained with a dual policy including a feedforward bolus delivered 15 min prior to meal.

The predicted responses for the hybrid solution (including feedforward) and for the feedback solution with $b = 10$ (feedback(ideal)) and for $b = 100$ (feedback(realistic)) are shown in Fig. 8.

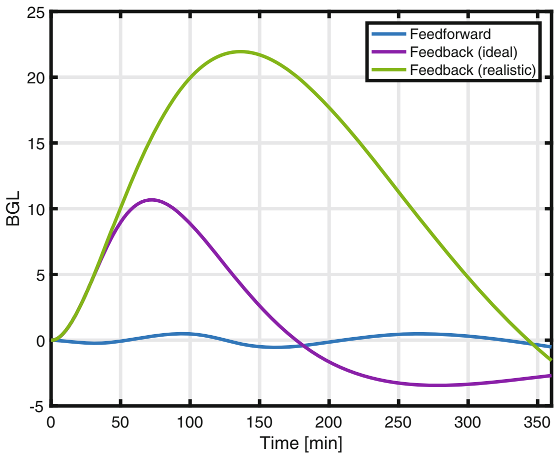

Fig. 8. Predicted BGL response with feedback and feedforward control

The results described above lead to the following conclusions, (i) it appears to be impractical to use feedback alone due to limitations arising from the delay and relative degree of the disturbance transfer function, $F(s)$. (ii) it is thus highly desirable to use both a feedforward bolus driven by a measurement of the meal characteristics (denoted f in Figs. 5 and 6) together with some continuous insulin flow (possibly delivered by feedback). The above results support the recent emphasis on hybrid closed loop algorithms for blood glucose regulation [70,73,79].

9 Conclusions and Recommendation for Future Work

This paper has presented a critique of the use of observers in output feedback control. It has been argued that observers play a pivotal role in control. On the other hand, observers have fundamental limitations which can, in some scenarios, unduly limit the achievable performance. Indeed, in some cases, the limitations can be so severe that acceptable performance can only be achieved if new direct measurements are made. For pedagogical reasons, the discussion in the paper has been limited to simple cases. Many open problems remain, including:

- Investigating performance limitations (as in Sect. 5.3) arising from the use of observers for nonlinear systems.
- Further investigation of Errors-in-Variables problems (as in Sect. 3.2) for observers in state-space models.
- A more thorough understanding of the limitations (as in Sect. 5) associated with the use of observers and the tracking of uncontrolled disturbances in optimization based control laws such as Model Predictive Control.
- Extension of the ideas (as in Sect. 6) to the multivariable case.
- Extension of the analysis of Sect. 8.9 to include the fact the insulin has a one-sided action ($u \geq 0$) and to include nonlinear dynamics. (It is hypothesized that these constraints will actually worsen the trade-offs discussed herein.)

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