Chapter 8
Mathematical Learning and Its Difficulties: The Case of Nordic Countries

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The Nordic countries are located in the North corner of Europe and consist of Denmark, Finland, Iceland, Norway, and Sweden. They form a culturally and politically isomorphic group with tight relationships. These welfare societies share the ideology of a strong responsibility of the state on the well-being of the members of the society. The strong economies (World Bank, 2013) and high levels of taxation (see KPMG International, 2013) have been the guarantees for that the states have had the assets to organize the welfare including health, social services, and education. During the current millennium, the Nordic countries have consistently been at the top

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of different international comparisons on welfare, health, quality of living, economic competitiveness, and even happiness of the citizens (Helliwell, Layard, & Sachs, 2013). Likewise, these countries are similar in a high expenditure on education, relatively small class sizes in schools, and long academic teacher education (see OECD, 2012). Investing in education has been one of the core features of the success of these countries. Despite many similarities, there are differences how the educational systems work and how education is conceptualized. For example, Finland and Denmark have *compulsory learning*, while Iceland, Norway, and Sweden have *compulsory schooling*. Compulsory schooling means that a pupil is obliged to attend school, while compulsory learning means that the educational authorities are obliged to ensure that pupils acquire the knowledge laid down in the curriculum (Tomas, 2009).

One of the marked differences between the Nordic countries has been the results of the OECD PISA studies, where Finland since the first study in 2000 has been among the top performers in mathematics and Denmark significantly above OECD average, while the other Nordic countries have been close to the OECD average (OECD, 2013a). In PISA 2015, Finland's and Denmark's students performed equally high and significantly higher than students in the other three countries. However, in Denmark, Finland, Sweden, and Iceland, the trend of performance level since 2003 has been declining (OECD, 2016). The percentage of low performers (defined as below Level 2) was as low as 6 in Finland in 2006 but has raised to 14% in the 2015 assessment. In other Nordic countries, the percentage of low performers in mathematics has varied from 14% in Denmark in 2006 and 2015 up to 27% in Sweden 2012. The latest TIMMS studies for fourth- and eighth-grade students have shown similar trends (Mullis, Martin, Foy, & Arora, 2012) (Fig. 8.1).

Despite these differences at school age, the Nordic countries reach the world's highest levels of numeracy in adulthood. In the recent study on the numeracy proficiency in adulthood (16–65 years of age), all Nordic countries topped the list together with Japan and the Netherlands (Iceland did not participate) (OECD, 2013b).

Likewise, the participation rates in adult lifelong learning and training have been the highest in the world in the Nordic countries (Eyridice, 2012). The high levels of basic skills in adulthood may be connected to the dynamic nature of the work life.

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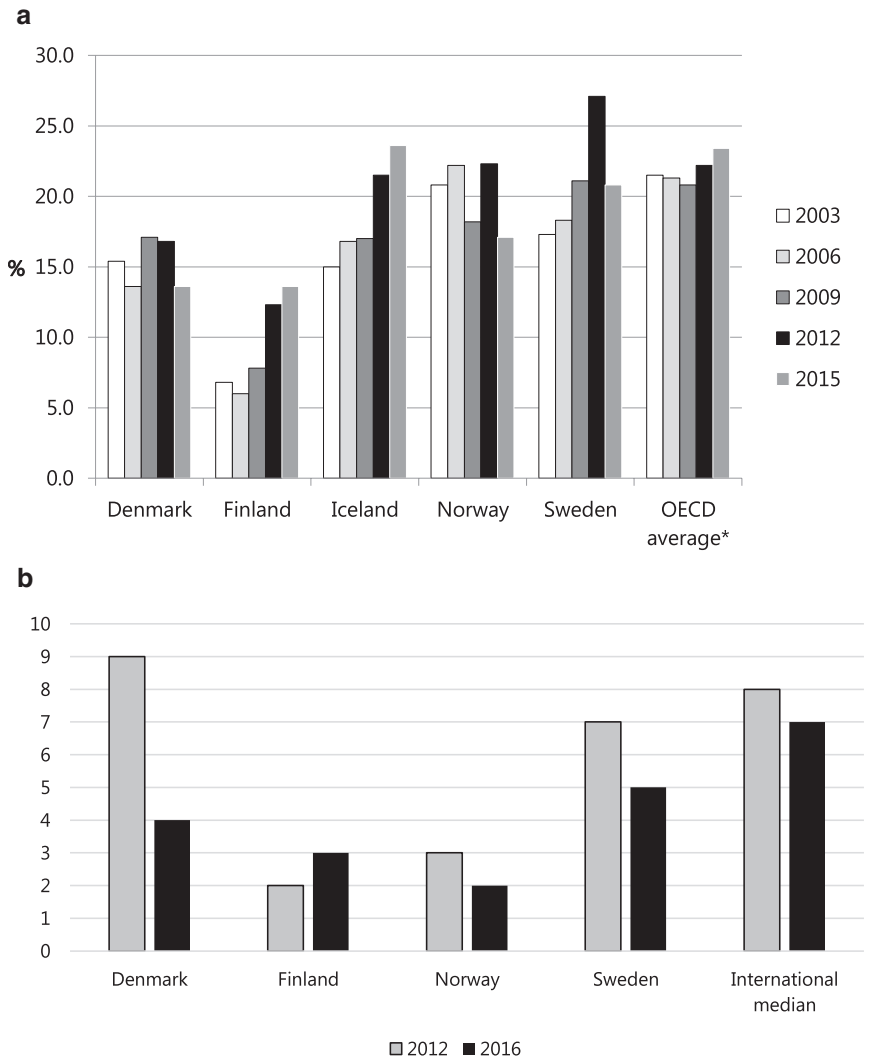

Fig. 8.1 (**a**) Percentage of low performers (below Level 2) in Nordic countries in PISA studies 2003–2015, (**b**) percentage of low performers in fourth grade in TIMSS 2012 and 2016

The Nordic countries had the highest percentage of workers who reported changes that affected their work environment (substantial restructuring or reorganization and an introduction of new processes or technologies) in their current workplace during the previous 3 years (OECD, 2013b). Continual changes in work life require continuous training of employees to be successful. At the same time, the workers need to have strong basic skills to be able to assimilate new skills and ways to work and to adjust to the changes what digitalization and automatization bring. Those with low achievement

Table 8.1 Comparison of the countries in different features of education and education policy

	DK	SE	FI	NO	IS
Compulsory education (age)					
	6–16	7–16	7–16	6–16	6–16
Public expenditure on education, % of GDP (2007, source: Eurostat, UOE) (EU average 4,98%)					
	7,83	6,69	5,91	6,76	7,36
Ratio of pupils to teachers in ISCED 1 (2008, source: Eurostat, UOE)(EU average 16)					
	10,1	12,2	14,4	10,8	10,0
Decision-making authorities involved in developing and approving the principal steering documents for mathematics teaching					
Curriculum	(a)	Central	Central	Central	Central
Guidelines for teachers	Central	Central	Regional/ school	All levels	–
School plans	Schools	Schools	Schools	Regional/ schools	Schools
Evaluation of the effectiveness of curriculum implementation					
External	Yes	Yes	No	No	No
School self-evaluation	No	Yes	Yes	Yes	(b)
Assessment criteria prescribed					
Learning objectives/outcomes	Yes/yes	Yes/yes	Yes/no	Yes/no	Yes/yes
Recommended minimum taught time compared to total time					
Primary	15,3	13,5	17,5	17,2	15,1
Compulsory secondary	12,9	13,5	11,8	11,0	13,5
Total time primary, estimation for (9 years)	1200	900 (from 2016 1125	912	1092	1200
Textbooks					
Autonomy	Yes	Yes	Yes	Yes	No
Monitoring of consistency	No	No	No	No	No
Central level guidelines for teaching methods					
Prescribed or recommended	Yes	No	Yes	Yes	No
Types of grouping	Yes	No	No	No	No
Low achievement					
Surveys or report on low achievement	Yes	Yes	No	Yes	No
Central level support	Yes (c)	No	Yes (d)	Yes (e)	No
Differentiation of curriculum content according to ability	No (f)	No	Yes	Yes	No (f)
Support for low achievers					
Standardized tests	Yes	No	Yes	Yes	No
Intervention of a specialized teacher			No	Yes	
Small group tuition			Yes	No	
Compulsory diagnostic tests at grades	Third, sixth, ninth	Third, sixth, ninth		Second	

(continued)

Table 8.1 (continued)

	DK	SE	FI	NO	IS
National surveys on motivation	Yes	No	Yes	Yes	No
Strategy to increase motivation	No	Yes	Yes	Yes	No
Lack of qualified teachers in upper primary education (% reported by principles)	2	2,6	2,9	17,8	7,6
Teacher training (advocated by central authorities)					
Differentiating teaching for pupils with different abilities and motivation levels	No	No	Yes	Yes	No
Detecting and tackling pupils' difficulties in mathematics	No	No	Yes	No	Yes

Source: Eurydice (2011)

(a) Denmark: National authorities develop and publish a document entitled Fælles Mål which includes central guidelines and objectives for mathematics teaching, but this is not defined as a curriculum in national regulations

(b) Iceland: School self-evaluation is obligatory, but schools do not have to focus on the curriculum

(c) In Denmark, the Ministry of Education has produced a special document that contains several recommendations on how to address learning difficulties in mathematics. It recommends that mathematics teachers carefully observe low achievers, engage in a dialogue with them, and focus on what they can do, rather than on what they cannot do. Beyond assigning such students easier tasks, teachers should also guide them toward new strategies to cope with their difficulties

(d) In Finland, the core curriculum contains guidelines on general support for students. The most common approach is early detection and support. The Ministry of Education organizes targeted in-service teacher training and maintains a website (10) with information on the most common learning problems in mathematics in the early school years. The site provides access to computer-assisted instruction methods for mathematics (Number Race, Ekapeli-Matikka, and Neure). In addition, specific tests for the diagnosis of learning problems are available for purchase from private companies

(e) In Norway, the main elements of the national policy to reduce low achievement are based on early intervention, national tests and mapping (diagnostic) tests, and the integration of basic mathematics skills in all subject curricula. The national strategy, Science for the future: Strategy for strengthening mathematics, science and technology (MST) 2010–2014, and the National Centre for Mathematics Education (see Annex) are important agents in promoting mathematics education

(f) Same content but at different levels of difficulty

in mathematics are vulnerable in work life and often among the first to suffer from economic turbulences, or as Parsons and Bynner (2005, p. 7) state it, "Poor numeracy skills makes it difficult to function effectively in all areas of modern life."

There are large differences how the educational systems in the Nordic countries (see Table 8.1) have responded to low achievement in mathematics at school age and how the educational system provides support both to the children with low achievement and to the teachers in the schools to work with these children. Therefore, it is reasonable to look at the similarities and differences how low achievement in mathematics is treated in these countries. To enlighten the similarities and differences in the educational support systems on learning disorders in mathematics between the Nordic countries, we presented five questions.

1. How are special needs in mathematics education (mathematical learning disabilities, MLD) defined?
2. What kind of support do children get at school for severe MLD?
3. Who gives the support, and what qualifications they have for this work?
4. Are the evidence-based assessment tools and intervention methods available?
5. What are the key issues and current trends in MLD at the moment?

Sweden

In Sweden, the legislative text does not use the terms MLD, "dyscalculia," "math disabilities," or even special needs in math education. Instead, the legislative text says that all children that are in risk of not attaining the national knowledge goals in a school subject have the right to special support, that is, some form of special education. The legislative text says nothing about what kind of or how much support the children have the right to receive. It only says that the schools must have a competence for special education. It is the responsibility of a school to provide the child individually adapted and adequate support.

In practice, there is large variability how support is organized at the school level. For example, some municipalities/schools require that the child has an ICD-10-based (KSH, 2011; WHO, 2005) medical diagnosis for mathematical disabilities to receive any special support, whereas other municipalities/schools focus on the functional level, which is in line with the legislation. Unfortunately, not all children receive the support that they need and have the right for it according to the law, because the schools do not have the required financial resources or the special education competence. In principle, there are two types of support: individual support with special needs teacher (one-to-one teaching) and level-groupings in small groups (2–5 children) with special needs teacher or regular class teacher.

An additional problem in Sweden is that not all schools acknowledge the concept or term mathematical disabilities (dyscalculia). Accordingly, it is very difficult to estimate the prevalence of children with mathematical disabilities. About 15% of the students usually do not get a pass on the national test in mathematics for the final grade (Skolverket, 2013). Immigrant students or students whose parents have low levels of education are overrepresented among those who do not reach the goals. There is great variation between schools and municipalities in performance levels. There are suburban schools in metropolitan areas with large numbers of immigrant students where the majority of students do not get a pass on the national test.

In Sweden, there is a considerable lack of special needs teachers on mathematical difficulties, because the university-based special needs teacher program started as late as 2008. This program is a 1.5-year-long training program for teachers. Therefore, most of the teachers responsible for helping children with MLD are not specialized to these pedagogical questions. The legislative text does not specify that the schools should have special needs teachers and/or special education teachers.

It only says that the schools must have special educational competence in some form. Likewise, there are no organized systems for continuing education or further training for special needs teachers or special education teachers.

The schools do not use any evidence-based assessment tools or intervention methods because there are none available. Furthermore, there are very few experts in Sweden who do assessments on MLD. However, at Danderyds Hospital in Stockholm, which is one of the few places where this kind of assessments is done, they use the British Dyscalculia Screener (Butterworth, 2003), and recently they have started to use the Panamath test (Halberda, Mazzocco, & Feigenson, 2008).

The new Swedish Education Act from 2010 stipulates that the education and instructions used in Swedish schools must be founded on scientific evidence and established experience. Thus, in the future, the Swedish school authorities will probably put more emphasis on matters regarding evidence-based teaching methods and evidence-based assessment tools. There is, however, some skepticism about the "evidence movement" developed in Anglo-Saxon countries.

Norway

The Norwegian educational policy is founded on the principles of inclusion and adapted education. However, to develop educational practices that achieve these overarching principles is a continuous challenge (Haug, 2010; Mathisen & Vedøy, 2012).

Laws and regulations in Norway do not define or apply the terms dyscalculia and mathematical disabilities. The term learning difficulties is used. According to the Educational Act, the focus is on pupils who do not benefit satisfactorily from ordinary teaching and thereby have the right to be assessed for being in some kind of special needs (See section "A Lack of Certain Arithmetical Abilities or a Certain Way of Doing Arithmetic?" in Chap. 6). Pupils should be referred to educational and psychological counseling service (EPS) for an expert assessment. The expert assessment shall consider and determine the following:

- The pupil's learning outcome from the ordinary educational provisions
- Learning difficulties the pupil has and other special conditions of importance to education
- Realistic educational objectives for the pupil
- Whether it is possible to provide help for the pupil's difficulties within the ordinary educational provisions
- What kind of instruction is appropriate to provide (See section "Evidence on the Impact of Instructional Efforts Focused on Non-counting Strategies" in Chap. 6)

In 2013 an amendment became in force that describes more details about administrative procedures in connection with decisions concerning special education. "Before an expert assessment is undertaken, the school must have considered and tested out, if relevant, measures within the ordinary education facilities that might

make the pupil benefit satisfactorily" (See section "Overcoming Computing by Counting as a Didactic Challenge" in Chap. 6).

This can be interpreted as pointing toward a more systematic problem-solving approach in line with recent response to intervention models (Glover & Vaughn, 2010). Further descriptions or guidelines regarding how to assess satisfactory learning outcome and/or the substance of the local schools' investigations are not provided. However, obligatory standardized national test (grades 2, 5, 8, and 9) is a part of the assessment of the children's mathematics at school. The tests aim to be a tool for the teachers to adapt the teaching to each child.

An emerging use of the term dyscalculia is taking place in Norway, and related diagnostic practices evolve. There is, however, no unified and agreed upon definition overall related to mathematics difficulties. On this grounding, it is not straightforward to find the extent of pupils with MLD. If difficulties are defined as getting a low grade in mathematics in school (low achievers), the results from the exam of Norwegian 15-year-old pupils show that 35–40% got 1 or 2 in mathematics (the grading system 6–1, with 6 as the highest). In 2012–2013 the percentage of pupils with individual decisions about special needs education was 8.6% in total (The Ministry of Education, 2013). How many of them with special needs in mathematics is not known. In research, e.g., Ostad (1997) used the term mathematical disable for the lowest performing 10% of children in Norwegian schools and found this level of low performance to be stable through all school years.

The support provided by schools varies. Lessons can be given in smaller groups or individually, outside or inside the regular classes and classrooms. The quality of the support also varies in line with the helper's background, from adequate support from a teacher in special needs with competence in mathematics to an assistant without teacher training at all. The use of assistants in special education increased from 2001 to 2008 (Bonesrønning, Iversen, & Pettersen, 2010).

Due to a lack of research-based knowledge about what goes on in segregated and inclusive special education in Norway, a joint research project was carried out from 2012 to 2015. The project had as main research questions: "What special education is about, and what is its function?" (http://www.hivolda.no/speed).

One of the main points from the research is to build education for all on a professional ground, to understand the complexity of the challenges, and to make institutions responsible, not only individuals (Haug, 2016).

Laws and regulations in education put emphasis on identifying pedagogical needs and developing supportive actions. Categorizing students or groups by diagnostic labeling is subordinate. However, this question of diagnosis and labeling causes a tension in the public and is a constant topic of the educational debate.

New practices of assessment in contexts (Nielsen, 2013) are being developed and tried out by Statped and EPS (Daland & Dalvang, 2009, 2016). It adopts a stance toward curiosity on how mathematical learning situations can be understood and further developed. This assessment approach seeks to investigate and analyze relations between three main dimensions: developing as a person, learning mathematics, and participating in learning communities.

Iceland

Like in other Nordic countries, laws and regulations in Iceland do not define difficulties with mathematics or dyscalculia at any school level. Schools set their own targets of competency in mathematics in coherence with the national curriculum guidelines, and pupils are offered support based on them, as well as on outcomes from standardized testing in mathematics in grades 4, 7, and 10. On those tests, between 17% and 24% of pupils score 0–22 points on a normal scale and fall into the category of poor performance (Sverrisson & Skúlason, 2012).

Support in schools for pupils with difficulties in mathematics is either in the hands of special education needs (SEN) teachers or mathematics (or other) teachers. In a survey from 2010 (Óskarsdóttir, 2011), different approaches to grouping and teaching were evident. In some schools, the tradition is that the SEN teachers work with pupils that need support in small groups of two to four pupils two to four lessons a week usually in a separate room. In other schools, pupils in the same year group are tracked into groups in mathematical classes depending on their level of performance, and the low-performing pupils work in small groups often with a mathematics teacher (or other experienced teachers) up to six lessons a week. In a minority of schools, SEN teachers or mathematics teachers go into classrooms and assist pupils that need support.

SEN teachers, according to the survey, map pupils' abilities before they begin working with them and tend to work with tailor-made assignments. They use manipulatives and physical models in their teaching and do not necessarily follow the textbook that is used for mainstream mathematics teaching. The focus in their teaching is on how to learn algorithms as a means of solving problems and to establish ways of working with mathematics. Mathematics teachers on the other hand use the textbooks and other teaching materials used by the year group and tend to follow the curriculum guidelines. The emphasis in teaching is placed on basic algorithms, teaching pupils how to calculate but less on how to use manipulatives other than computers and calculators.

In Iceland teachers and SEN teachers have a university degree. There is one course aimed at preparing SEN teachers to teach mathematics, and it is called "Mathematics for all." The focus of this course is on mathematics learning and how children develop mathematical thinking. The participants of the course also work to develop their own understanding of mathematics and discuss their different ways of approaching mathematical problems. The aim is to be able to understand children's diverse ways of developing mathematical thinking. The main goal of the course is to prepare teachers to map pupils' abilities and to learn how to support children to overcome their difficulties in learning. Also, there is a discussion about diverse pupils' difficulties and how SEN teachers need to collaborate with mathematics teachers in assisting pupils. The course is based on research on how children learn mathematics as well as on research on learning difficulties in mathematics and teacher development in teaching mathematics in inclusive settings (Guðjónsdóttir, Kristinsdóttir, & Óskarsdóttir, 2007, 2009, 2010).

One standard-based assessment tool is available to SEN teachers as well as mathematics teachers. This test, Talnalykill (Guðmundsson & Arnkelsson, 1998), is standardized and criterion-referenced in Iceland. Those who want to use it must be licensed. The test is made up of two main test components, written group tests, and individual oral testing. Some schools in Reykjavik and other places have used the written part of the test to screen third grades for mathematics difficulties. The test has been criticized for focusing mainly on children's fluency in using traditional algorithms and not screening for other mathematical competencies such as the ability to deal with mathematical language and tools. Many teachers in schools also find it too time-consuming, both regarding assessing the pupil and the time it takes to calculate the results. School psychologists also assess pupils for difficulties with mathematics using tests such as WISC-IV (Guðmundsson, Skúlason, & Salvarsdóttir, 2006), which has been standardized and localized for the Icelandic context.

In the new national curriculum (Ministry of Education, Science and Culture, 2011), the emphasis is on equal opportunities for all pupils regardless of their abilities or circumstances. At the compulsory school level, all pupils have the right to compulsory education in their inclusive neighborhood school. The focus in the mathematics chapter is on the right of all children to develop their mathematical thinking and get the support they need to develop mathematical competencies (Mennta-og menningarmálaráðuneytið, 2013).

Finland

The Finnish educational system is state governed and funded but municipally organized. The private school sector is practically non-existing. The leading principle of the educational policy has been to offer free, high-quality education to all in local schools. There are no standardized or national assessments in primary education, but every school and teacher have a freedom to decide how they monitor the development and learning of their pupils. Typically, teachers use a lot of formal and informal exams to follow the progress of their students.

The number of pupils in special education increased rapidly in Finland during the last two decades from less than 3% up to 8.5% in 2010. At the same time, the number of children receiving part-time special education peaked at 23.3% (The Finnish Centre for Statistics, 2013) resulting in about one-third of children at the early-grade education to receive some individualized support. Even though supporting reading skills was clearly the largest subject, special needs education in mathematics showed the largest growth (Räsänen & Koponen, 2011). In 2010 about one-fourth of part-time special education was devoted to mathematics. All these figures were world records at their time.

This unexpected growth in special education caused the Finnish special education system to be reformed. It started to be a too expensive solution for treating individual differences in learning. The changes in the Basic Education Act were

passed in 2010. The new strategy emphasized inclusion over segregation and stressed the importance of a pedagogical approach over medical and psychological approaches. The aim was to change the old diagnostic terminology to a more pedagogically oriented language. According to the "new educational talk," medical or psychological terms like mathematical learning disorders or dyscalculia were not recommended. Instead, the focus should be given to identifying pedagogical needs and taking supportive measures (Thuneberg et al., 2013).

The new support system is divided into three levels of intensity. *General support*, targeted to all children, is for temporary needs in learning. The second level, concerning about 20% of children with needs for more regular support, is referred to a pedagogical assessment and to an *intensified support* with a time limit. Main tools are part-time special needs education, individual guidance counseling, and use of flexible teaching groups, as well as home-school cooperation. The third level, targeted to about 5% of the children, *special support*, is provided for those who cannot adequately achieve their growth, development, or learning objectives through other support measures. The most serious cases, defined in the previous system, as having severe mathematical learning disorders, go through a broad pedagogical evaluation and if needed may study according to an individual learning plan (ILP). The pedagogical evaluation is coordinated by the school teachers and typically contains a consultation of a child psychologist who has many options for standardized tests of mathematical achievement to be used as part of the assessment.

Even though the system reminds the descriptive conventions of the response to intervention (RtI) model (Fuchs & Fuchs, 2007), it was not the foundation for the new model in Finland. The key differences between the RtI and the Finnish models are the absence of standardized assessments and structured evidence-based interventions in the Finnish model. In the Finnish model, the teachers are at the helm, and they are given freedom and responsibility to tailor the needed processes to support each child. This requires a well-organized system at the school level and continuous further training for teachers. In larger cities, there are "Mathlands," which are support and learning centers for teachers. Likewise, there is a government-funded web service (lukimat.fi) run by Niilo Mäki Institute, a research center on learning disorders. The service offers information and free tools for early interventions and assessment of reading and mathematical difficulties in early primary education.

In Finland, practically every school has qualified special education teachers with a university degree. The majority of them give part-time special education in collaboration with the classroom and subject teachers. Likewise, every school has a student welfare group for multi-professional collaboration. However, even though the school system offers a lot of individualized support, there are still a lot of challenges to meet. According to the two recent analyses from the national assessments on mathematical achievement from sixth (Räsänen, Närhi, & Aunio, 2010) and ninth grades (Räsänen & Närhi, 2013), close to half of the children identified having a low achievement in mathematics (about 5–6% in total) get only a little attention from the school or teachers.

These results were a surprise because the criterion for low achievement in these evaluations was a combination of assessment and teacher's identification. Therefore, the reason for ignoring these children with low achievement from support was not due to non-identification. The biggest challenge in Finland is not whether the pupils will be identified having mathematical learning disabilities but how to guarantee that they all are offered the support and care they need.

Denmark

The Danish educational system is free and publicly funded. Even private schools get public funding for as much as 73% of the amount given to public schools, while the rest is paid by the parents (per private school student around 130 Euro per month). Private schools are getting more popular. While in 2000 the percentage was 12% in private schools, in 2016 the percentage was 18%.

All public schools prepare the students for national exam at the end of grade 9. From 2017, national compulsory assessments also include 14 digital, adaptive tests from grades 2 to 8, including 3 mathematics tests in grades 3, 6, and 8. Most private schools offer these national tests, too. For teachers, the aims of the national testing program are to provide a tool for teachers' own formative assessment of their students' progress and a tool for monitoring their own teaching. Nevertheless, many teachers find it difficult to use the national tests according to these aims. Other assessment tools are provided by publishers or the teachers themselves, and every school has the freedom to decide how to act upon test results. Besides, some schools and adult learning centers use the British Dyscalculia Screener (Butterworth, 2003).

In the present national curriculum guidelines for mathematics (Common Goals, 2016), no student characteristics (i.e., special needs students) are described. But for some specific skills and knowledge, eight "attention points" are described: they refer to the level of basic skills that are a prerequisite in order to acquire sufficient skills later on.

The political agreement in the Parliament June 2013 on improving Danish school children's performance in school subjects included initiatives for "students with dyscalculia." On this background, a test for dyscalculia for grade 4 in Danish schools, guidelines for test takers, and ideas for follow-up assistance are being developed in 2015–2018. A proposed definition of dyscalculia serves as a starting point: "Dyscalculia is an impairment that may influence education and work. Weak calculation skills are not matched by corresponding weak skills in other fields" (SFI, 2013). Expected percentage of students to be identified by this future dyscalculia test is as low as 1%. Many more students than 1% are facing mathematics difficulties and in need for focused support, either just in mathematics or also in other subjects, drawing on social, psychological, physical, and didactical perspectives. Support in mathematics is needed for students in segregated special schools and classes as well as in regular school and classes.

Since the Salamanca Declaration (UNESCO, 1994), professionals and politicians have argued for increasing efforts for inclusion. The number of students in special education and the costs of special education have been steadily growing in Denmark. Data from public schools showed that in 2008–2009 support organized in special classes and special schools was provided for 5,6% of the students, while special education in ordinary classes was provided for another 8,7%. When these and other data were brought up and analyzed (Finansministeriet et al., 2010), political efforts were intensified to include more students in ordinary classes and schools and to replace special education with another instrument. Economistic arguments were put forward but also humanistic arguments for better learning and well-being when "special students" would be more along with "regular students."

As several special schools have been closed the last years, also some students with diagnoses as autism, Tourette syndrome, conduct disorders, or general learning difficulties are now being included in regular classes and schools. However, it has in many cases proven to be problematic, as several teachers have not been trained, are not knowledgeable, or are not given sufficient resources to create the needed inclusive learning environment.

After the law "No 379 – 28 April 2012," less than 9 specialized lessons of 60 min (equivalent to 12 lessons of 45 min) per week are not seen or regulated as a special education program. Support less than nine lessons is given as part of mainstream education. In instruction, can be used, inter alia, differentiated teaching, tracking for shorter periods, two teachers in class, teaching assistants who can both help each student and the class as a whole, or supplementary teaching and other kinds of support (www.uvm.dk 2015). Some programs for supplementary teaching are developed and used as an early intervention in mathematics; see, for instance, Lindenskov and Weng (2014).

Available data on mathematics in special education in special and regular schools is extremely sparse (Lindenskov, 2012). Nevertheless, the interest in special needs in mathematics has been growing since 2000 among school teachers in public and private schools, adult educators, high school teachers, school psychologists, special education teachers, consultants, teacher educators, and researchers. To increase the overall quality of school mathematics, 10 years ago a diploma program was set up for mathematics teachers in service to become "math tutors." The 1-year program includes six modules, and one module focuses on students in mathematics difficulties. Several seeds have been sowed for continuous interest and for development projects and initiatives at school and municipality levels. The educated math tutors have organized a national network covering about 1000 tutors spread over 800 out of 1400 schools.

In 2010, the association DanSMa (Danish Special Mathematics) was founded as a common meeting place for these professionals to discuss typical issues concerning people with special needs in mathematics in order to improve offers for children, adolescents, and adults. DanSMa initiates public debates, disseminates the latest research on the character and background of mathematics difficulties, as well as on identification and interventions, and arranges seminars with invited speakers (dansma.dk).

Summing Up

We presented five questions to analyze the similarities and differences between the Nordic countries how children with MLD are recognized and how their learning is supported. To summarize our findings, we go through the replies question by question.

The first question concerned how special needs in mathematics education and mathematical learning disabilities (MLD) are recognized and defined in each country. In all countries, the legislations recognize low achievement as a special question, but none of those take any stance on ICD or other clinical diagnostic systems. There are no commonly accepted criteria for diagnosing MLD. The assessment procedures used in Iceland, Denmark, and Norway are rather close to those defined in ICD, namely, combining standardized achievement tests and cognitive assessment. In Finland, standardized tests and a psychological assessment are a common practice in a case with persistent learning disabilities, but giving a diagnostic label for MLD is exceptional. The educational reform in 2010 pushes Finland closer to the Swedish approach where there is a strong aim to avoid assessments and diagnostic labeling and to concentrate on methods of inclusive education.

The Finnish and Icelandic schools have been extremely sensitive to define a child as having special needs in education (SNE). In Iceland, about 24% of children are defined as having special needs, while in Finland about 8% of children are defined as pupils with SNE, and an additional 20% receive a part-time special education. Denmark and Norway are in the middle, but a striking contrast is Sweden, where only 1.5% of children are defined having SNE (see Table 8.1, NESSE, 2012).

We can also contrast the Nordic models against the response-to-instruction models of special education. In the RtI models the extremes of a continuum could be called as "a standard protocol" at one and "a problem-solving approach" at the other end (Fuchs, Fuchs, & Stecker, 2010). In the standard protocol, assessment means an evidence-based intervention with standardized measures of improvement before and after the intervention to be able to define those with MLD and needs for more intensified and individualized special educational intervention. The problem-solving approach sees the assessments as a tool for a non-categorical evaluation of skills mastered and yet to be mastered and is used primarily to inform classroom instruction, rather than to guide decision-making on a diagnosis or for a more individualized intervention. In other words, while the first stresses the importance of special education as a separate process, the latter sees that the special education should be blurred inside the regular instruction (for more about this discussion, see, e.g., Fuchs, Fuchs, & Compton, 2012).

If we try to put the Nordic models into this discussion and continuum, none of the countries follow the standards approach. The success of the RtI model in the USA has not attracted the policy-makers in Nordic countries to formalize the support systems or increase the usage of standardized tests. The general discussion has been more about how to develop inclusive models and lessen the needs for separate special needs education (e.g., Statped model in Norway). Finland is the only country where SNE has been formally structured to levels of support with defined procedures

how the evaluation should be done when moving between the levels. This mimics vaguely the standards approach with pedagogical evaluations, but without specifications of assessment procedures. At the same time, there is an aim to push forward the inclusive problem-solving RTI model. Sweden has been an extreme on its reluctance toward assessments and diagnostics with a strong inclusive ideology and aims to apply the problem-solving approach.

One of the largest differences between the Nordic countries lies in the details how children's progress in learning is monitored. In Norway, Denmark, and Iceland, there are standardized assessments at specific grade levels, which are absent from the Swedish and Finnish systems. In Finland, the evaluation is totally in the hands of the teachers, who typically use a lot of formal and informal examinations to monitor the children's development in their own classroom. The specific feature in the Swedish discussion on education has been the reservations against assessments, especially standardized assessments and the evidence-based, "quantitative" approaches.

The second and third questions were: what kind of support do children with MLD get at school, and what are the qualifications for the support personnel? In all countries, the importance of inclusive education is stressed, but still, a common way of dealing with MLD is still taking the child out of a classroom to individualized or to a selected small group receiving special education. In none of the countries, there are officially recommended or recognized intervention programs to be used. In Finland, there are research centers on learning disorders, which have developed widely used programs on learning disorders. According to a recent analysis (Sabel, Saxenian, Miettinen, Kristensen, & Hautamäki, 2011), these research centers have had a large role in shaping the Finnish special education. In Norway, a state-funded Statped is developing models for special education. However, their aim is not to produce evidence-based intervention programs but to guide teachers in professional development (cf. problem-solving approach in RtI). In Denmark, the development work is concentrated around the large network of diploma-trained teachers.

In Sweden, there has been a lack of specialized teachers, and the university training of special educators started as late as in 2008, while in Finland it started in 1959, and nowadays the majority of the Finnish universities have units of special education offering studies up to the Ph.D. level. Therefore, it is not a surprise that from Nordic countries, what kind of, and who gives extra support to children with MLD, varies the most in Sweden. The Swedish educational office (Skolverket, 2009) has also raised concerns over the influence of increasing segregation in the Swedish school system after it transformed itself from one of the most centralized school systems into one of the most decentralized (Tomas, 2009). Even though the variance between schools in mathematics has increased in Sweden, the Nordic countries still have the smallest between schools variance in mathematics achievement in the world (Gaber, Cankar, Umek, & Tasner, 2012).

Our fourth question concerned the role of research and evidence-based approaches in interventions on MLD. Following the international trends, research interest toward MLD has been raising in all Nordic countries. There is a biannual Nordic Congress on special needs education in mathematics (NORSMA, The Nordic Research Network on Special Needs Education in Mathematics) where experiences on different types of assessment and interventions and on the effectiveness of special education are

shared. However, none of the educational systems require that special educational approaches should be evidence-based. Therefore, research-based tools, even though welcomed at schools, are not a standard, and it depends on teacher's own activity, if they apply any of the models or instruments.

In all Nordic countries, an increasing number of researchers are pushing toward more research-based assessment and intervention procedures. The increasing understanding of the dyscalculic brain and changes in the diagnostic definitions encourage the researchers. At the same time, new questions emerge for the interplay between research and educational practice. The new competency-based curriculums redefine the learning aims and bring new colors to the practices at school and new challenges and research questions for studies on learning disabilities. It seems that the gap between everyday activities and aims in classrooms and the neuroscientific research is not getting narrower in the near future.

Our last question was about the future challenges. We can see a perennial battle between different views on the role of individualized special needs education and inclusive education. The puzzle how to teach the whole classroom effectively but at the same time individualize education within and outside of the classroom is an open question asking for scientific efforts. Neuroscientific research on learning and learning disorders gets the headlines (e.g., Coughlan, 2014) but still gives a little to the actual educational practices in classrooms. A lot of different views are presented, and the only thing where all parties agree is the lack of scientific evidence for any of the opinions.

According to the latest TIMMS study (Mullis et al., 2012), low motivation toward mathematics learning is more apparent and concerning feature of current Nordic students than low achievement. However, in the international assessments, there has been interesting feature: Within each participating country, there is a positive correlation between students' learning motivation and achievement; but when aggregating the data at a country level, the correlation between motivation and achievement becomes negative (He & Van de Vijver, 2016). High-performing countries show lower averages in motivation than lower-performing countries. From the Nordic countries particularly Finland, together with the many Asian top performing countries, they show this strong achievement paradox of high achievement and low motivation. Despite high general well-being of youth in Nordic countries, enjoyment of learning mathematics, especially in the upper primary education, has not been a part of it. The equation how to combine efficient learning, self-efficacy, and motivation in mathematics education is a big challenge for both research and practice to solve.

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