Chapter 31
Counting and Basic Numerical Skills

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You're enjoying a lovely day at the park with your 3-year-old nephew. A paddling of ducks waddles by and you start a conversation, *"Hey Charlie, look at the ducks! How many are there?"*

A pretty straightforward question. Your nephew jumps at the opportunity to demonstrate his skills. Faithfully pointing to each duck, one-by-one, he responds, *"one…, two…, three…, four…, five!"*

Ah, he's brilliant. You knew as much. Let's keep this conversation going. *"That's right!"* you say. *"So, how many ducks are there?"*

He immediately responds, "*Eight!*"

Right! Wait…what?

This narrative, having played out in countless situations, is likely familiar to any caretaker or educator. Indeed, the phenomenon is well documented: while most children appear to have learned to count by the time they are 2 or 2 ½ years old (Fuson, 1988), most often, they are simply demonstrating their ability to reproduce a counting routine. Consequently, their behavior is often difficult to interpret – it is not, as we would be inclined to presume, a reliable indicator of their number knowledge. This is similar (and not unrelated) to that other pre-scholastic phenomenon of reciting the alphabet without yet having developed an understanding of orthography or phonics.

In fact, even after a successful counting routine is achieved, children continue to face several underlying challenges on their way to acquiring early number concepts and basic counting skills. One of the core challenges follows from the fact that there is an important dissociation between conceptual and procedural knowledge of counting. In early phases of number acquisition, conceptual knowledge lags far

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behind that of procedural knowledge. Our nephew in the anecdote above has clearly learned some basic counting procedures (and recognizes that the question "how many" prompts these procedures) well before he will ultimately understand how this activity reveals the correct answer to this question. In fact, only over the next couple years will his incremental advances in both procedural and conceptual knowledge culminate in the ability to form and maintain precise representations of natural number (e.g., Carey, 2010).

Number Sense

While ubiquitous in discussions of early education and mathematics, the term *number sense* is often used to refer to a variety of abilities and behaviors. Early childhood curricula and assessments often use the term to broadly describe children's "fluidity and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics… and make comparison" (e.g., Gersten & Chard, 1999). The following review, however, will adopt the term's primary definition, referring specifically to the evolutionarily primitive ability to represent non-symbolic quantity (Dantzig, 1967; Dehaene, 2011). This definition includes the ability to subitize (i.e., the ability to recognize the exact number of items in a small set without counting[1]; Kaufman, Lord, Reese, & Volkmann, 1949), which manifests from our ability to represent and track individual items (e.g., Feigenson & Carey, 2003). This definition of number sense also includes the ability to represent rough estimates of magnitude and number (e.g., Xu, 2003).

Small Number Representations

> It's time for a snack. You offer your nephew two cookies but he immediately recognizes that you have given yourself three. He raises the alarm. "*How did he know?*" you think to yourself, "*didn't we just establish that he doesn't know how to count yet?*"

We can chalk this one up to the ability to represent and visually discriminate arrays of one, two, or three items, an ability available to even very young infants (Xu, 2003). Consider the following experiment: 10- to 12-month-old infants were presented with two adjacent buckets, one containing just 1 cracker and the other containing 2 crackers. When given the opportunity, the infants in this study consistently chose (crawled to) the bucket with 2 crackers over the bucket with 1 (wouldn't you?) (Feigenson, Dehaene, & Spelke, 2004). Similarly, the infants chose the bucket with 3 crackers when the other had just 2 or 1. However, with choices of 4 vs 6, 3 vs 4, 2 vs 4, and even 1 vs 4 crackers, infants chose at random. Taken together, these

[1] The term "subitize" also enjoys many definitions across early childhood curricula and assessment. The present chapter, however, will adopt and adhere to the definition provided above.

results show that infants' preference for the greater number does not depend on the *relative* quantity, or the ratio of the two sets (infants consistently chose the bucket with 3 crackers to a bucket with 2 but seemed perfectly happy to go to either bucket when presented with a choice between 4 vs 6 crackers). Instead, their ability to make a meaningful choice is contingent upon *absolute* quantity (in this case the number of crackers), and their ability to represent these exact quantities is capped at three items. This limited (though impressive) ability has been demonstrated across a variety of experimental paradigms, each yielding similar results (e.g., Clearfield & Mix, 1999; Feigenson & Carey, 2003; Starkey & Cooper, 1980).

While greater number is generally correlated with greater continuous quantity (such as summed spatial extent or volume) in the natural world, these studies extensively control for continuous properties showing that these discriminations are based on number alone. Moreover, these representations are not limited to the visuospatial modality. Infants also assess exact quantities (up to 3) when presented with a series of temporal events and auditory sequences (e.g., puppet jumps and sounds; Wynn, 1996).

This representational system then allows us to easily identify small, exact quantities immediately, accurately, and without counting (cf., Cordes, Gelman, Gallistel, & Whalen, 2001). The signature limits of this system, however, remain relatively constant over the course of development (though older children and adults are often able to represent up to 5 or possibly 7 items in a set; Mandler & Shebo, 1982; Trick & Pylyshyn, 1993) such that subitizing does not present a viable pathway to the representation of large, exact numbers like 27 or 308.

Approximate Number Representations

> So we've righted our mistake. Both of us now have three cookies. *Phew*. Wait… your astute (and somewhat righteous) nephew notices that yours has more chocolate chips! It seems there are a gazillion chocolate chips in each cookie, so we are well beyond subitizing. And, he's not counting… Enter the Approximate Number System.

The ability to represent large approximate quantities and detect differences between two large sets is supported by the approximate number system (ANS), a cognitive resource that is also available in early infancy (e.g., Lipton & Spelke, 2003). Early access to this system is often demonstrated through the use of a habituation paradigm. For example, infants (as young as 6 months) are presented with a series of pictures, each with an array of 8 dots. Then, when presented with a picture with 16 dots, infants look longer at the novel array, showing that they discern the difference between sets of 8 and 16. While infants also respond to changes in overall spatial extent (e.g., summed area and/or contour length; Clearfield & Mix, 1999), several studies that have controlled for alternative dimensions of quantity have shown that infants are able to make judgements on numerosity alone.

Judgments supported by the ANS, however, are imprecise, and the threshold for a just noticeable difference follows Weber's law, such that numerical discrimination is a function of the ratio between the two magnitudes under comparison, and not their absolute difference (e.g., Halberda & Feigenson, 2008). Importantly, and unlike the small number representation system discussed above, ANS precision improves over the course of development (Halberda & Feigenson, 2008; Odic, Libertus, Feigenson, & Halberda, 2013). On average, 6-month-olds can reliably discriminate 1:2 ratios (such as was presented in the example above; Lipton & Spelke, 2003), 9-month-olds can discriminate 2:3 ratios (Xu & Spelke, 2000), 3-year-olds discriminate 3:4 ratios, 4-year-olds discriminate 4:5 ratios, and 5-year-olds discriminate 5:6 ratios (Odic et al., 2013); and adults can discriminate 10:11 ratios (Halberda & Feigenson, 2008).

Notably, individual differences in ANS acuity within these age groups are associated with math achievement. In fact, several studies have shown that individuals with more precise ANS acuity perform better on tests of formal mathematics (Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, & Halberda, 2012; Lyons & Beilock, 2011). In one study, performance on the Test of Early Math Ability (TEMA-3; Ginsburg & Baroody, 2003) could be predicted from ANS acuity measured at 6 months (Libertus et al., 2011). In another, numerical acuity measured in 14-year-olds correlated with their performance on standardized math tests as far back as kindergarten (Halberda, Mazzocco, & Feigenson, 2008). Furthermore, there is evidence to suggest that ANS acuity is malleable and may be influenced by environmental factors (Tosto et al., 2014) and formal instruction (Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza, Pica, Izard, Spelke, & Dehaene, 2013).

Summary

Together, these two systems are considered core cognitive resources that serve as a foundation for the construction of natural-number concepts (Carey, 2010). Each is clearly necessary for the development of counting and basic number skills; however, neither is sufficient. The following sections will review how children's developing understanding of the verbal count list (e.g., individual number words such as *one, two,* and *three*) ultimately allows for the construction of natural-number concepts (i.e., the ability to represent exactly 27 or 308).

Number Language

As discussed above, the ability to represent small, exact numbers and large, approximate numerosity is available in early infancy, but mapping these representations to symbolic representations of number (e.g., number words) is no small feat. Whereas children as young as 2 years old have little difficulty mapping approximate quantifiers (such as *more* and *a lot*) to representations of quantity (Dale & Fenson, 1996), children can spend upward of 2 years sequentially assigning meaning to individual number words and figuring out how the verbal count list works.

While a long and protracted process, the acquisition of number language is a crucial milestone in children's quantitative development (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1990, 1992). As the following section will discuss, the language system itself is largely responsible for the ability to represent large exact number. In fact, children who experience significant language barriers, such as those born deaf to hearing parents, show delays not only in their acquisition of individual number words but also in later math achievement (Kritzer, 2009). Moreover, individuals who grow into adulthood without learning to count proficiently demonstrate poorer performance on tasks assessing representations of exact number and cardinality (Frank, Everett, Fedorenko, & Gibson, 2008; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011).

Knower Levels

> "*The kid's really put one over on me,*" you think. When it comes to cookies, he clearly knows what he's talking about (*three* cookies is more than *two*, and don't even think about saving the cookie with more chocolate chips for yourself!). But you're not entirely satisfied so you decide to put it to the test…
>
> You give him the whole bag of cookies, but ask him if you can have just *one*. He happily obliges. One cookie, no problem. "*Can you give me two cookies?*" you ask. Sure, he hands you two. One last time for good measure – this time you ask for *three* cookies. "*Sure!*" he says as he hands over as many as he can grab. Not *three*, not *two*, but an entire handful!

While seemingly inconsistent and unpredictable, it turns out that our nephew's response is not unusual for a 3-year-old. In fact, it often takes 2 or more years to learn even a subset of number words, during which time children work out the cardinal meanings of each number word one at a time and in order (Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Lee, 2009; Wynn, 1990, 1992). Interestingly, as they go through this process, children appear to traverse a predictable series of knowledge states, or "knower" levels (see Sarnecka, Goldman, & Slusser, 2014 for review).

This incremental progression shows up on assessments such as the Give-N (or Give-a-Number) task in which children are asked to create sets in response to specific prompts (e.g., "Can you give *three* bananas to the puppet?") (Wynn, 1992; see Fig. 31.1). In such tasks 2- to 4-year-olds, who can generally recite the count list up to 10 or so without error, are often unable to give the correct number of items when asked for those same numbers in the Give-N task. In response to a Give-N trial asking for *six* bananas, for example, these children may simply grab a handful of items without counting, even when prompted to count or check their response (e.g., "Can you count and make sure you gave the puppet *six* bananas?" or "Can you fix it so that the puppet gets *six* bananas?") (e.g., Le Corre et al., 2006).

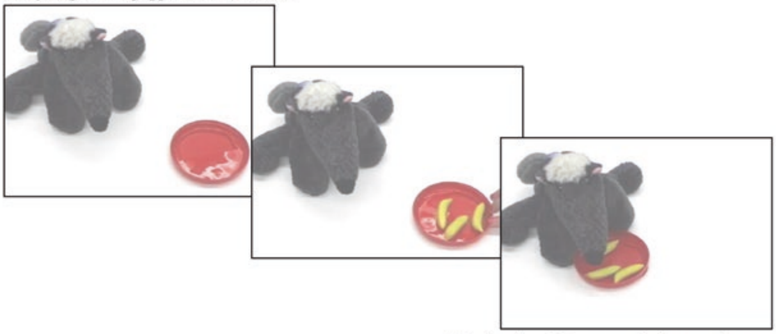

Fig. 31.1 The Give-a-Number task can be used to assess children's number-knower levels (e.g., Wynn, 1992). For this task, children are typically asked to create set sizes of 1 to 6 items. Children are given the opportunity to check and fix their responses after each trial

At the earliest knower level (often referred to as the "preknower" level; e.g., Slusser, Ditta, & Sarnecka, 2013), children's responses to any given prompt are generally unrelated to the number of items requested. These children may give just one item, or even a handful of items, regardless of the specific prompt. At the next level, children reliably give 1 item when asked for *one* but give 2 or more items when asked for any other number. Note that their responses seem to be simple guesses, not counting or estimation errors (Sarnecka & Lee, 2009), and these children appear to understand that number words that they do know are not used to refer to sets of any other size (i.e., they will not offer 1 item when asked for any number other than *one*; Wynn, 1990, 1992). The one-knower level is followed by the "two-knower" level, then the "three-knower" level, and sometimes the "four-knower" level. At each *N*-knower level, children demonstrate predictable and accurate performance up to *but not* beyond *N*. Eventually, around the time they reach the three- or four-knower level (often 2 years after they first entered the one-knower level), children realize that the final number word in their count sequence refers to the cardinal value of the set they are enumerating. At this point they may be said to have induced the "cardinality principle" (Gelman & Gallistel, 1978) and can henceforth employ counting procedures felicitously to create any set size within their count list (Sarnecka & Carey, 2008; Wynn, 1990; cf. Davidson, Eng, & Barner, 2012). It has been argued that, as children progress through these individual knower levels, they are incrementally assigning each of the first three or four number words to their representations of small, exact sets (Carey, 2010). Numbers exceeding the set size limit of 3 or 4 items must then be represented through counting. For this reason, we don't typically see children who would be characterized as "five-," "six-," or "seven-knowers" (cf. Wagner & Johnson, 2011).

The one- through four-knower levels are found not only for speakers of English but also for speakers of Japanese (Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007), Mandarin Chinese (Li, Le Corre, Shui, Jia, & Carey, 2003), and Russian (Sarnecka et al., 2007). Furthermore, bilingual children who have memorized the counting lists in both of their languages before learning the exact meanings of these words in either language show the same or similar knower-levels in both languages (Goldman, Negen, & Sarnecka, 2014).

There is, however, a notable variability across children with different learning backgrounds and experiences. For example, while children from relatively high socioeconomic backgrounds typically reach an understanding of cardinality sometime between 3 and 4 years old (see Sarnecka & Lee, 2009), children from less privileged backgrounds often do not reach this level of understanding until well after their fourth birthday (e.g., Dowker, 2008; Jordan & Levine, 2009).

While the cardinality induction is often recognized as a major conceptual achievement, we will put this aside for now (but revisit it in the "Counting Principles" section below). The following sections will instead explore what subset-knowers (a term used to describe children at the one-, two-, three-, and four-knower levels; Le Corre et al., 2006) know and have yet to learn about number.

Discrete Quantification

One piece of knowledge that is integral to understanding natural-number concepts is the idea that number is a property of sets and that sets are comprised of discrete individuals. Indeed, a conceptual dissociation between continuous substances (such as water and sand) and discrete objects (such as blocks and coins) is available in infancy (Hespos, Ferry, & Rips, 2009), and as children acquire language, they reflect this distinction through their appropriate use of linguistic morphology (i.e., the English singular/plural marking) to denote the difference between mass and count nouns (e.g., Barner, Thalwitz, Wood, & Carey, 2007).

To determine whether children with an incomplete understanding of number words (i.e., subset-knowers) understand that number words, in general, are used to refer only to sets of discrete individuals, we invited a group of subset-knowers (2–4 years old) to complete the Blocks and Water task (Slusser, Ditta, & Sarnecka, 2013; see Fig. 31.2). For this task, children watched as an experimenter placed five objects (e.g., blocks) in one cup and five scoops of a continuous substance (e.g., water) in another cup. Four trials asked children about a number word outside the range of numbers known by any subset-knower (e.g., "Which cup has five?"), and another four trials asked about a quantifier (e.g., "Which cup has more?").[2] For half of the trials, the cup with discrete objects was full; for the other half, the cup with the continuous substance was full. Results showed that, while children correctly chose the full cup when asked which cup has "more," they had to have reached the three-knower level before reliably choosing the cup with discrete objects as an example of "five." A series of follow-up experiments seem to indicate that one- and two-knowers have an emergent but tenuous understanding of this constraint but are,

[2] Note that approximate quantifiers such as "more" and "a lot" can take a wide range of referents, with few constraints, while number words refer only to collections of discrete individuals.

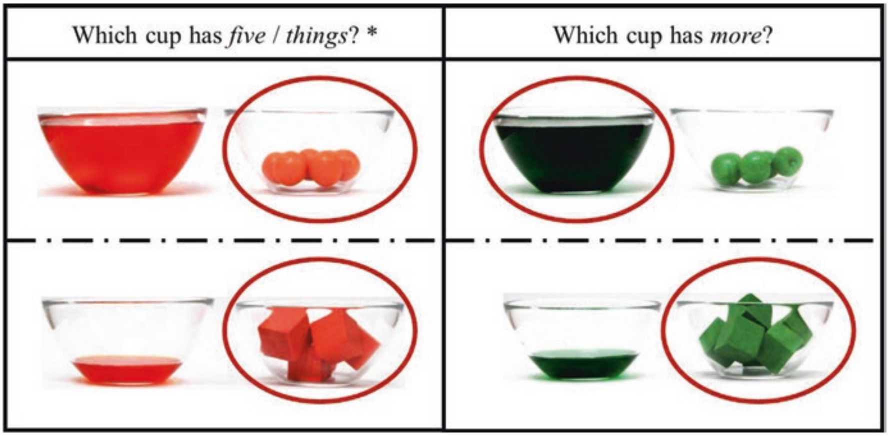

Fig. 31.2 The Blocks and Water task was used to determine whether and when children understand that number words reference discrete sets (Slusser, Ditta, & Sarnecka, 2013) and whether linguistic context (in the form of a count noun + plural marking in English or the general noun classifier, 個 [*ge*], in Mandarin) facilitates this understanding (Slusser, 2010) (Figure adapted from Slusser, Ditta, & Sarnecka, 2013). (* Prompt differed according to the experiment and trial type. *Note:* The cup with continuous substance is full for half of the trials. Red circles indicate the correct response)

in general, as likely to extend the word "five" to continuous substances as to sets of discrete objects.

Thus, it seems that children come to understand that number words are used for discrete quantification only after learning the precise meanings of at least a subset of number words. It is possible then that children use their understanding of the number words "one" and "two" to draw inferential connection between number words and discrete objects. Alternatively, children may use the linguistic context that generally occurs in natural speech to form this connection (Bloom & Wynn, 1997). This argument arises from the observation that number words reference nouns morphologically coded according to their conceptual category (i.e., count vs mass) – that is to say, count nouns take the plural marking, "-s," whereas mass nouns do not. After first confirming that number words are in fact most often accompanied by an adjacent count noun and plural marking (e.g., "Look, five *ducks*!") in both child and child-directed speech (Slusser, 2010), we tested whether children use this information to establish that number words reference count nouns, and consequently collections of discrete objects.

The 2- to 4-year-old children in this study completed the Blocks and Water task above, but in this iteration each test question was presented within a syntactically "rich" linguistic context (Slusser, 2010; see Fig. 31.2). For example, children were asked, "Which cup has five *things*?" rather than "Which cup has five?" Results show that English-speaking children connect number words to discrete quantification before learning the specific meaning of any number words *so long as* the number word is paired with an adjacent count noun and plural marking.

Similarly, Mandarin-speaking children demonstrate similar learning trajectories when presented with a number word in isolation and when accompanied by the noun classifier 個 (pronounced "*ge*").

Overall, this series of experiments shows that children use their emerging understanding of number words as well as linguistic cues that occur in natural speech to connect number words to discrete quantification. Moreover, these data constrain future hypotheses on how children learn number words: the fact that this process may involve generalization from certain exemplars and surrounding language provides evidence that number word knowledge is not entirely built upon a priori principles.

Numerosity

Connecting number words to discrete quantification is only one step in acquiring an understanding of natural numbers. Children must also understand that number words denote numerosity (and not, for example, some other characteristic of set, such as total volume or spatial extent). Setting out to address this question, Sarnecka and Gelman (2004) invited 2- to 5-year-old subset- and CP-knowers to complete the Transform-Sets task. For this task, the experimenter placed a certain number of objects in a box while saying (e.g.), "I'm putting *six* buttons in this box." The experimenter then performed some action with the box (either shaking it, turning it around, adding one object, or removing one object). The children were then asked (e.g.), "Now how many buttons are in the box? *Five* or *six*?" Results show that subset-knowers (and CP-knowers) do indeed understand that the number word should change when an item has been added or removed from the box (and that the number word does not change when a non-numerical transformation takes place, such as when the experimenter simply shakes the box). It seems that, while they still do not understand the precise meanings of the number words *five* and *six* (as illustrated through their performance on the Give-N task), subset-knowers do understand something about these number words – that they denote some aspect of quantity.

Note the use of the term *quantity*, not *numerosity*. Upon careful inspection, we see that the Transform-Sets task does not disambiguate number or numerosity from the broader dimension of quantity. Remember, children's intuitive number sense supports representations of both numerosity and continuous spatial extent (see section on "Approximate Number Representations" above). In the Transform-Sets task described above, the number of items in the box changed, but so did other dimensions of quantity (i.e., area, volume, weight). While subset-knowers clearly associate number words with quantity, it is not entirely clear whether they understand that number words refer specifically to numerosity.

To address this specific confound, we developed a Match-to-Sample task with careful controls and manipulations of continuous spatial extent (either summed area or contour length, depending on the trial) so as to pit dimensions of quantity directly against numerosity (Slusser & Sarnecka, 2011; see Fig. 31.3). For this task, children

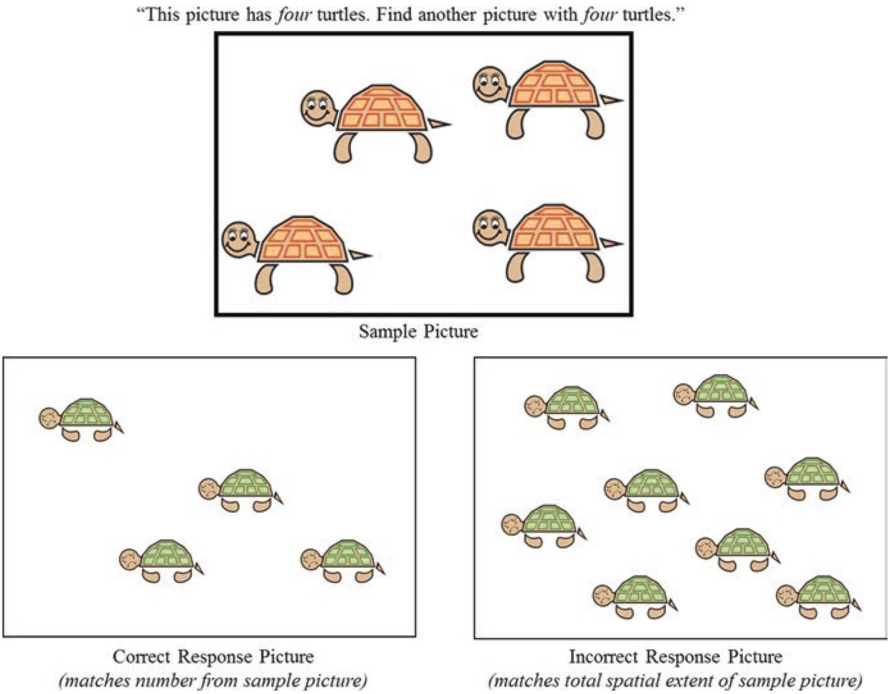

Fig. 31.3 A Match-to-Sample task was used to determine whether children understand that number words denote numerosity, rather than some other dimensions of quantity (e.g., summed spatial extent) (Slusser & Sarnecka, 2011) (Figure adapted from Slusser & Sarnecka, 2011). (*Note:* On this particular trial, there is no possible match on the characteristics of the individuals comprising the set (e.g., the color or mood of the turtles))

were presented with a sample picture as the experimenter said (e.g.), "This picture has *four* turtles." The experimenter then presented two additional pictures and said (e.g.), "Find another picture with *four* turtles." One picture had the same number of items as the sample but different overall spatial extent (e.g., 4 small turtles). The other had a different number of items, but the same overall spatial extent (e.g., 8 small turtles). Results showed that while CP-knowers understand that two sets of the same numerosity should be labeled with the same number word, subset-knowers are as likely to extend that number word (e.g., *four*) to other dimensions of continuous quantity (by, in this case, selecting a picture of 8 small turtles).

Summary

Taken together, these findings reveal that subset-knowers' understanding of numbers matures as they acquire the meanings of individual number words. In addition to enriching our understanding of how children's understanding develops over time, these studies highlight a series of additional conceptual and linguistic challenges that are often overlooked in the development of early childhood curricula and assessments.

Counting Principles

The previous section discusses how children learn each of the number words in their count list one-by-one and in order. The process appears to take upward of 2 years, and as they do this, they learn some of the fundamental properties of number (i.e., number words refer only to discrete sets and are used to denote numerosity, not continuous quantity). Whereas the counting routine, in and of itself, does not appear to be integral to this process, children are certainly gaining experience and learning about counting procedures over this period of time.

As Gelman and Gallistel (1978) pointed out in their seminal work on *Young Children's Understanding of Numbers*, in order to count productively, children (and adults) must at the very least (1) recite the count list in the same sequence every time (e.g., *one, two, three, four* and not *one, four, three, two*), (2) count each object in a set without skipping or double-counting, (3) understand that they can count the objects in any order (e.g., counting from left to right yields the same answer as when counting from right to left), and (4) understand that the last number word recited in the counting routine indicates the total number of items in the set. While the first three rules seem to unfold with experience and practice, the following sections will focus on the final counting principle in this list – the cardinality principle.

Cardinality Principle

After your little experiment with the cookies, you think back to your conversation about the ducks in the park. Your nephew *did* recite the count list in order; he *did* count each duck in one-to-one correspondence, and he didn't seem too concerned with the order or arrangement of the ducks. But wait… there's just one thing missing. He did *not* seem to understand that the last word in his count list should indicate the total number of ducks. Well, jeez, that seems simple enough…

When considered a part of Gelman and Gallistel's (1978) list of counting principles, the cardinality principle (or "last word rule") simply stipulates that the last number word in a count sequence represents the cardinal value of that set. In reality, however, it seems children's understanding of this specific procedure is contingent upon a crucial conceptual induction – often referred to as the cardinality principle induction (Carey, 2010). As mentioned previously (section "Knower Levels"), prior to this induction, children progress through a series of intermediate knowledge states (knower levels), during which time they do not seem to understand how counting is used to generate or identify specific set sizes (e.g., Le Corre et al., 2006). Importantly, children who understand the cardinality principle (i.e., CP-knowers) perform differently from subset-knowers on a variety of tasks assessing early number knowledge. Some of these tasks explicitly involve counting. For example, on the Give-N task, CP-knowers use counting to generate specific set sizes and can fix their answers when they make mistakes. While subset-knowers often engage in counting behaviors (extensively abiding by the counting principles outlined above), they fail to use counting to generate specific set sizes. Some tasks, however, do not explicitly involve counting. Examples of these include the Blocks and Water and Match-to-Sample tasks discussed above, which reveal that subset-knowers do not yet understand the fundamental properties of number words (i.e., that they are used for discrete quantification and denote exact numerosities).

Another notable difference between subset- and CP-knowers is that only CP-knowers understand that any set with *N* items can be put into one-to-one correspondence with any other set labeled with the same number word (*N*) – an idea referred to as "equinumerosity" (Muldoon, Lewis, & Freeman, 2009; Sarnecka & Wright, 2013). Like many of the skills outlined above, children's understanding of equinumerosity seems to align closely with their induction of the cardinality principle. For example, if one child were to have a handful of grapes for a snack and the other were offered the same (both snacks are recognized to be "just the same" through one-to-one correspondence), then each snack should also be labeled with the same number word. Results on a task that evaluated children's understanding of this concept show that only CP-knowers know that sets that are "just the same" are labeled with the same number word (and if the sets are not the same, then a different number word should be used) (Sarnecka & Wright, 2013).

Furthermore, there is emerging evidence to suggest that children tap into ANS representations as they learn how counting represents number (Carey, Shusterman, Haward, & Distefano, 2017; Chu, van Marle, & Geary, 2015; Shusterman, Slusser, Halberda, & Odic, 2016; van Marle, Chu, Li, & Geary, 2014). One such study tracked 2- to 4-year-old's understanding of individual number words and counting procedures (through the Give-N task) as well as their ANS acuity over a 6-month period (Shusterman et al., 2016). Results show that children's acquisition of the cardinality principle is tightly linked to marked improvement in ANS acuity and that there is little evidence to suggest that ANS representations underlie advancements across subset-knower levels (e.g., moving from the one-knower to two-knower level) (see Fig. 31.4). These findings provide further evidence for the notion that the cardinality principle is not just a counting rule – it is essential to the creation and representation of natural-number concepts.

Importantly, children did not have an opportunity to count when completing any of the tasks introduced above (including the Block and Water and Match-to-Sample tasks discussed above), showing that children who understand the cardinality principle know more than the route counting procedures – they have developed deeper insight about numbers and number words. Thus the promotion from subset- to CP-knower seems to be far more profound than it initially appears.

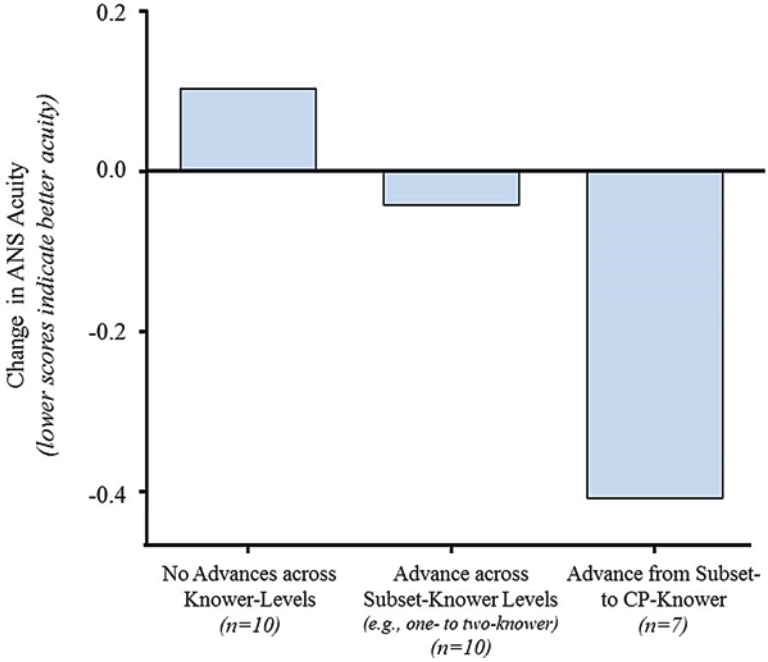

Fig. 31.4 A 6-month longitudinal study evaluating children's developing number knowledge, counting skills, and ANS acuity shows that the acquisition of the cardinality principle is tightly linked to notable increases in ANS acuity (Shusterman et al., 2016). Note that ANS acuity is not clearly linked to advances across number-knower levels. (Figure adapted from Shusterman et al., 2016)

Successor Function

With the cardinality principle comes an understanding of the successor function, which reflects another fundamental property of number – with each additional item in a set, we advance one step (i.e., word) along the verbal count list. In conjunction with the cardinality principle, an understanding of the successor function allows children to represent the cardinal meanings of every word in their count list (Sarnecka et al., 2014).

To explore children's understanding of the successor function, Sarnecka and Carey (2008) showed a group of 2- to 4-year-old children a box with 5 items inside. Similar to the Transform-Sets task described above, experimenters explained (e.g.), "There are *five* apples in this box," and then added an item to the box. In this task, however, the experimenter asked (e.g.), "Now how many are in the box? *Six* or *seven*?" As with the tasks reviewed above, only the CP-knowers seemed to understand that adding 1 item to a set moves the total count one step (word) forward along the count list (and adding 2 items moves the count two steps forward).

Together, children's understanding of the cardinality principle and successor function is often considered to be "the final piece of the puzzle" (Sarnecka et al., 2014) – the last thing that children must figure out in order to use counting to construct natural-number concepts.

Summary

While your 3-year-old nephew at the beginning of this chapter has clearly memorized several words in the verbal count list and has acquired at least some of Gelman and Gallistel's (1978) counting principles, it seems that this routine serves no meaningful purpose other than offering the expected response to the question "how many?". Gradually, however, over the next several months or years, he will come to realize that counting is used to determine the exact number of items in a set and that cardinality changes with each additional item.

Facilitating the Acquisition of Exact Number Concepts

The sections above outline several challenges that children inevitably face as they develop counting and basic numerical skills while presenting the argument that children must confront and conquer these challenges in order to construct and represent exact number concepts. Moreover, recent research has identified these achievements as central to children's eventual success in school (Aunio & Niemivirta, 2010; Bartelet, Vaessen, Blomert, & Ansari, 2014; Duncan et al., 2007; Göbel, Watson, Lervåg, & Hulme, 2014), with the unfortunate caveat that children who start school without these fundamental number concepts are at a serious disadvantage, both in the short and long term (Dowker, 2008; Jordan, Kaplan, Ramineni, & Locuniak, 2009):

> Even though you realize that your simple "judgement calls" on who has more chocolate chips will have to be supported with clear empirical evidence from here on out, you nevertheless decide to help your nephew out (that's what family's for, right?). Lucky for you, researchers' evaluations of both small- and broad-scale interventions have culminated in a collection of best practices that can be easily implemented even in informal settings.

Facilitating the Acquisition of Individual Number Words

In addition to the four counting principles outlined in the section "Counting Principles" above, Gelman and Gallistel (1978) noted that children must also understand abstraction – the idea that number is an inherent property of any set of discrete items and that a set of 10 apples, for example, shares something in common with a set of 10 oranges (who said that we can't compare apples and oranges?). Unfortunately (though interestingly), many researchers who have attempted to teach children the meaning of a new number word (e.g., teach a two-knower the exact meaning of the word *three*) find limited success. Whereas these children may come to recognize that the new number word can be used to label a set of, e.g., three marbles, they often do not understand that the word *three* can be applied or generalized to other sets of 3 (e.g., 3 blocks, 3 buttons, 3 meals) (Carey et al., 2017; Huang, Spelke, & Snedeker, 2010; Mix, Huttenlocher, & Levine, 2002).

To explore this phenomenon further, we introduced a group of two-knowers to the word *three* (Slusser, Stoop, Lo, & Shusterman, 2017) through one of three training conditions (Fig. 31.5). Children randomly assigned to the Number Word Only condition were presented with several pictures of 3 animals and were told, "This picture has *three*." Children in the Count Noun condition were presented with this same series of pictures but were told, (e.g.) "This picture has *three* dogs." And children in the Superordinate Category condition were told, "This picture has *three* animals." Following training trials with corrective feedback, two-knowers in the Count Noun and Superordinate Category conditions failed to extend the new number word (*three*) to sets of new animals (e.g., lions) or objects (e.g., shoes), while children in the Number Word Only condition succeeded. These findings suggest that the specificity of the linguistic context in which a number word is introduced influences children's ability to generalize newly acquired number words. Thus, while a rich linguistic context seems to facilitate children's understanding of number word semantics (see "Discrete Quantification"), when introducing a specific number word, it seems adults and educators should provide varied input and avoid coupling a number word with a specific noun or category label unnecessarily.

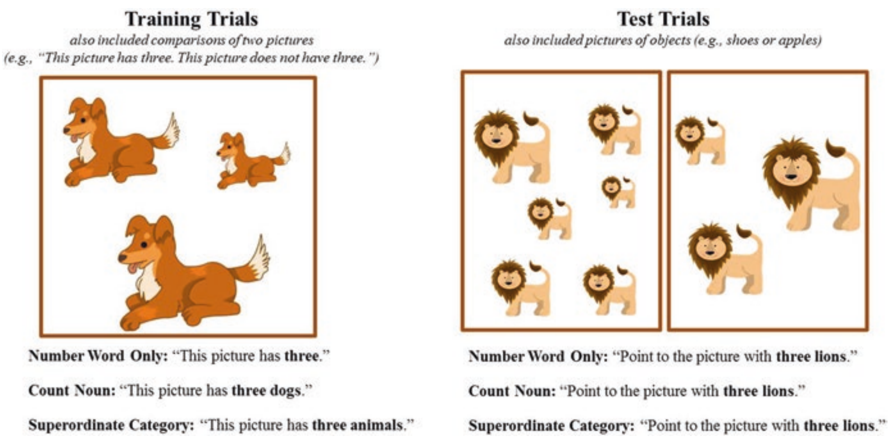

Fig. 31.5 Examples of training and test trials: To evaluate the role of linguistic context in children's acquisition of individual number words, we designed 3 training conditions. Children who were trained with the Number Word Only were more likely to generalize the newly acquired number word to new sets than children assigned to the Count Noun or Superordinate Category conditions

Facilitating the Acquisition of the Cardinality Principle

Efforts to teach children the cardinality principle over a short period of time have also been met with mixed success (e.g., Mix, Sandhofer, Moore, & Russell, 2012). Nevertheless, it seems there is growing evidence that adults can effectively scaffold children's understanding of the cardinality principle by presenting the counting routine in close temporal contiguity with an appropriate label of cardinality. Most recently, Paliwal and Baroody (2017) found that modeling a counting procedure that emphasizes the total number of items in a set facilitates children's understanding of the cardinality principle. For this study, 3- to 5-year-olds were randomly assigned to one of the three training groups. Children practiced counting 1–6 items with an experimenter several times over a 6-week period. Upon posttest (which included a measure similar to the Give-N task described above), children who practiced counting using a procedure that emphasized the total number of items in a set (e.g., "One, two, three. *Three*. There are *three* elephants!") outperformed children who simply counted the items (e.g., one, two, three) without repeating or emphasizing the cardinal value of the set.

Notably, however, adults often do not approach counting activities in this way (Mix et al., 2012). While they may count or provide a cardinal label, they do not often do both. This coupled with the observation that number talk, in general, is relatively rare in everyday interactions (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010) suggests that many children are not, on a daily basis, exposed to input that facilitates this understanding.

Broad-Scale Intervention

Following participation in "broad-scale" mathematics intervention programs (meaning that they include a multitude of both classroom- and home-based activities), children from low and middle socioeconomic backgrounds have consistently demonstrated improved performance on composite mathematical assessments (e.g., Arnold, Fisher, Doctoroff, & Dobbs, 2002; Starkey, Klein, & Wakeley, 2004). Not only do children's math scores improve, but other numerically related skills, such as measurement and problem-solving, also improve.

One notable demonstration of these benefits follows Greenes, Ginsburg, and Balfanz's (2004) evaluation of their Big Math for Little Kids program. This curriculum, designed to increase mathematical competency among 4- to 5-year-old children, includes a series of engaging number-based games that encourage and facilitate critical thinking related to number. The studies presented in the following two sections, however, suggest that meaningful experience and intervention need not take the form of established curriculum. Instead, it seems that parents and educators can facilitate children's counting and basic numerical skills by simply offering or creating numerically based games and toys and by incorporating "number talk" into daily conversations.

Numerically Based Toys

Over the last several years, researchers have begun to study the direct cognitive benefits associated with children's play with numerically based toys. One study linked cognitive benefits of play with numbered board games in preschoolers from low-income backgrounds (Siegler & Ramani, 2008). Children (ages 4–5) completed 4 sessions of play using a board game with squares labeled 1–10. Even though they initially struggled with math-related tasks as compared to their more affluent peers at pretest, these children consistently demonstrated improvements at posttest, suggesting that numerically based play can have profound effects on mathematical cognition.

More recently, in a study funded by the toy manufacturing giant Mattel©, 3- and 4-year-old children were randomly assigned to one of the four conditions, each with a specific toy predicted to support development within a particular cognitive domain (Slusser et al., 2013). Children in the Number Condition were given a set of ten small race cars (think Hot Wheels™) and a parking garage. Each car was labeled with a numeral from 1 to 10, and the parking garage included a series of parking spaces, each with an array of 1–10 dots. After a 1-month period (during which time children were encouraged to play with the toy but received no other specific instruction from the researchers), children's counting and basic numerical skills increased dramatically, significantly more than children assigned to any other condition[3] (see Fig. 31.6). Thus, simply playing with numbered toys appears to promote improvement in numerical understanding.

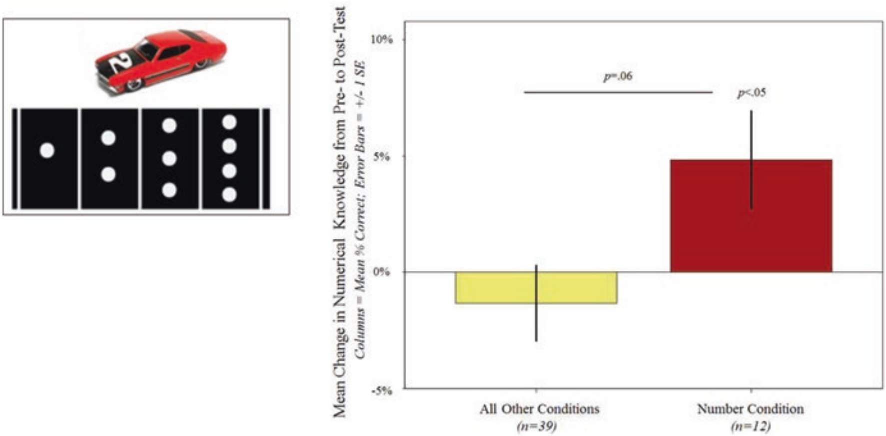

Fig. 31.6 Children's independent play with numerically based toys (left) over a 1-month period promotes their numerical understanding (right) (Slusser, Chase, et al., 2013)

[3] Children in the other conditions received either a set of ethnically diverse dolls, dress-up clothes, or wooden blocks.

Number Language

Even without the use of games or toys, recent research has shown that exposure to number language facilitates children's acquisition of number word meanings. In fact, children's knower levels can be predicted by the quality and quantity of number-specific language at home (Gunderson & Levine, 2011; Levine et al., 2010), and interventions that help parents engage in meaningful number talk can facilitate children's progress toward understanding cardinality (Berkowitz et al., 2015).

This important link between number knowledge and early language exposure is further demonstrated through a recent study that evaluates and models the influence of parent education, general vocabulary, ANS acuity, and number word knowledge on children's early math achievement (Ribner, Shusterman, & Slusser, 2015). For this study, we first evaluated the receptive vocabulary, number-knower level, and ANS acuity of a diverse group of 3- to 5-year-old preschoolers. We then administered the TEMA-3 approximately 1 year later, as they entered kindergarten. We found that children's early language (general vocabulary and number word knowledge) fully mediates the relationship between parent education and math ability. Additionally, number word knowledge mediates the noted relationship between ANS acuity and early math (see Fig. 31.7).

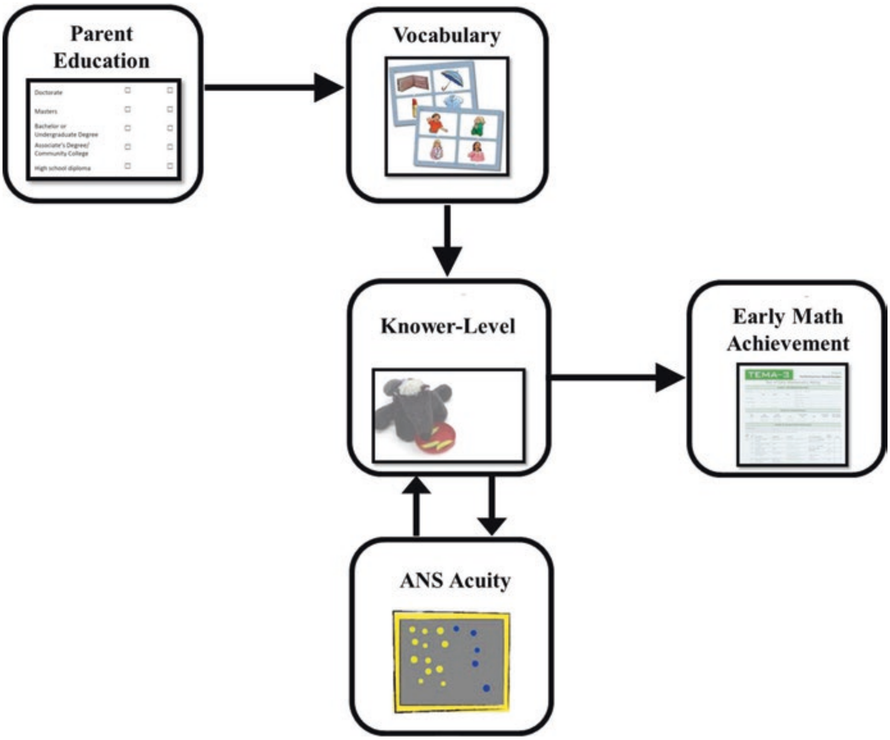

Fig. 31.7 A diagram that illustrates the relationship of parent education and early math. Results from a 1-year longitudinal study following preschoolers through kindergarten show that early language skills are linked to number word knowledge and these factors fully mediate the relationship between parent education and math ability (Ribner et al., 2015)

Even with a clear need for additional research, these findings carry implications for early education and intervention. For example, while proposals for early intervention to support children's developing number sense (ANS acuity; e.g., Wang, Odic, Halberda, & Feigenson, 2016) remain justified, these findings suggest that an increased focus on number language and general vocabulary may help to minimize disparities in math ability as children enter kindergarten.

Summary

In sum, a sampling of research across various disciplines (including early education and instruction, child development, psychology, and cognitive science) shows that children's intuitive number sense, their understanding of individual number words, and their procedural and conceptual counting knowledge serve as key building blocks for future math ability. While idiosyncrasies in each result in predictable developmental outcomes, researchers have identified a series of effective, low-cost, and practical interventions that can be easily adopted by parents and practitioners alike.

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