Chapter 12
Mathematical Learning and Its Difficulties in the United States: Current Issues in Screening and Intervention

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Mathematical Learning and Its Difficulties in the United States: Best Practices for Screening and Intervention

Results from recent cross-national comparative studies indicate that despite spending more per student than many other countries, the United States performs below average in mathematics, ranking in the bottom half of countries in the Organization for Economic Co-operation and Development (OECD, 2012). This result, however, does not provide a complete picture of US education. There are significant socioeconomic differences across and within states, which explain about 15% of variation in student performance (OECD, 2012).

Figures 12.1 and 12.2 depict the percentage of fourth- and eighth-grade students, respectively, who performed below the basic level on the US National Assessment of Educational Progress (NAEP), overall and broken down by selected states and public versus private schools. Although the percentage of students who are struggling has gone down since 1992, there are substantial achievement differences, depending on state and geographic region. In Massachusetts, for example, there is less poverty than in Mississippi; in 2015 80% of the students in Massachusetts met standards versus only 50–60% in Mississippi.

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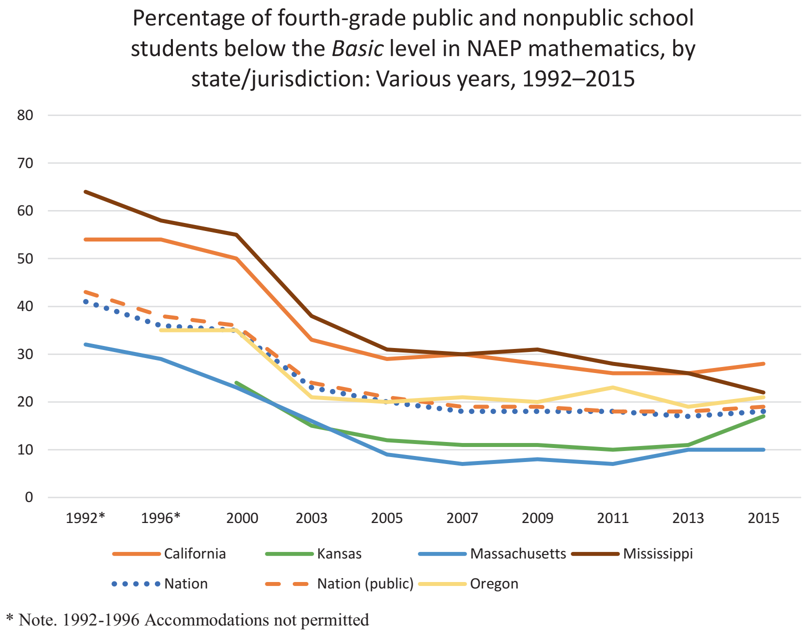

Fig. 12.1 The percentage of fourth-grade public and nonpublic school students below the *Basic* level in NAEP mathematics. *Note: 1992–1996 accommodations not permitted. (Source: The National Assessment of Educational Progress (NAEP))

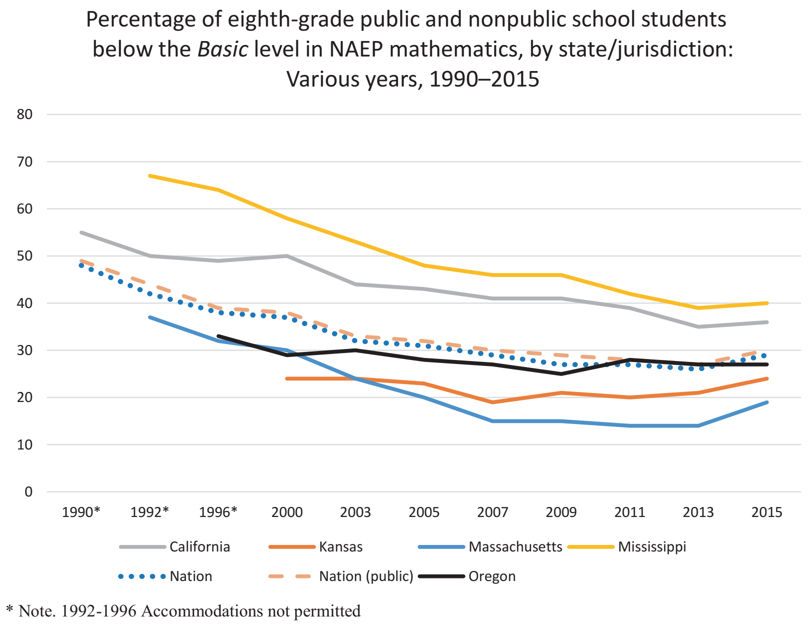

Fig. 12.2 The percentage of eighth-grade public and nonpublic school students below the *Basic* level in NAEP mathematics. *Note: 1992–1996 accommodations not permitted. (Source: The National Assessment of Educational Progress (NAEP))

Historically, the United States has differed from other developed nations in that control of education policy—including that related to mathematics—has tended to be highly decentralized (Woodward, 2004). There is no US national curriculum, resulting in a high level of state and local control over what is taught in school. As such, there is significant variation in instruction, both across and within states. However, 42 of the 50 states and the District of Columbia have now voluntarily adopted the national Common Core State Standards (CCSS; Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010), which specify in relative detail the mathematical content to be covered as well as standards for student learning. The CCSS, however, are controversial, and to date, their long-term impact on student achievement remains uncertain.

In terms of special education, US federal law, under the 1975 Education of all Handicapped Children Act, mandates that all children and youth with disabilities, including those with learning disabilities in mathematics, receive a free and appropriate education, including nondiscriminatory evaluation and an individual education plan. The Individuals with Disabilities Education Improvement Act (IDEIA) of 2004 eliminated the law's original requirement to consider whether children exhibit a severe discrepancy between achievement and intelligence, leading to the broad implementation of alternative response to intervention (RTI) approaches. RTI approaches screen broadly for academic problems and then provide evidence-based interventions aimed at helping individual students, tracking progress along the way to gauge effectiveness. Still, specific methods for assessment are typically established in a localized manner at school or school district levels, meaning that there is a high degree of variation in screening procedures and types of interventions provided to children with or at risk for disabilities.

In a widely cited article, Gersten, Jordan, and Flojo observed in 2005 that research on early screening for mathematics difficulties and disabilities in the United States was in its "infancy" (p. 293). In contrast, extensive research had already been conducted on early screening for reading difficulties, which produced reliable measures that could accurately predict which students would have trouble learning to read. The reading screeners helped US schools provide research-based literacy support and intervention for kindergarten and first-grade students and, to a large extent, drove the RTI movement in US special education. On the other hand, there was far less research on screening for potential mathematics difficulties and a relatively small corpus of evidence-based mathematics interventions.

Since that time, however, the field of mathematics learning difficulties in the United States has advanced significantly through various theoretical studies that identify the most powerful predictors of and influences on mathematics learning difficulties (MLD; e.g., Berch & Mazzocco, 2007; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Further, studies have validated screeners for detection of potential difficulties in mathematics (e.g., Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012), and rigorous intervention studies have helped determine best practices for young students with or at risk for MLD (e.g., Fuchs et al., 2008; Gersten, Jordan, & Flojo, 2005).

In addition to developing early screeners and interventions to help students acquire whole number competencies or number sense, recent studies have also focused on learning rational numbers (e.g., fractions) in later grades. Typically, fractions are introduced in US mathematics in third grade (Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010). Both whole number and rational number knowledge are crucial aspects of mathematics education and are necessary for later success in mathematics as well in everyday life (Gersten et al., 2009).

In the present chapter, we highlight key contributions from relatively recent studies related to whole number understanding in the early grades and fraction understanding in the intermediate grades. Although not comprehensive, the contributions reflect research-based findings related to MLD that are currently influencing educational practice in the United States.

Early Number Competencies

Early Number Competencies Predict Future Mathematics Success, and Deficiencies in Number Concepts Underlie Many Mathematical Learning Difficulties

Early mathematics skills correlate with long-term outcomes. Independent of cognitive ability and social class, kindergarten mathematics concepts predict later learning outcomes not only in mathematics but also in reading (Duncan et al., 2007). Most US benchmarks (e.g., Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010) for kindergarten and first grade primarily concern knowledge of number, including number relations and operations, forming a foundation on which later mathematics content is built (National Research Council, 2009). Mathematics delays as early as kindergarten and first grade put students at risk for difficulties in acquiring mathematics concepts in subsequent grades, including fractions and algebra (Mazzocco & Thompson, 2005; Milgram, 2005; Wu, 1999). Poor number sense also leads to dependence on rote memorization, which in turn makes it harder later on for students to develop meaningful problem-solving skills (Locuniak & Jordan, 2008; Robinson, Menchetti, & Torgesen, 2002).

Kindergarten number sense performance and growth, in particular, predict mathematics achievement in elementary school (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Jordan, Glutting, & Ramineni, 2010; Locuniak & Jordan, 2008). Unfortunately, many children from low-income communities in the United States enter kindergarten showing delays in core number knowledge relative to their middle-income peers (Jordan, Kaplan, Olah, & Locuniak, 2006; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Starkey, Klein, & Wakeley, 2004); additionally, they are four times more likely than middle-income children to show little to no growth in number knowledge between kindergarten and first grade (Jordan et al., 2006, 2007). Jordan et al. (2007) found that number sense performance in kindergarten and rate of number sense growth from kindergarten to early first grade accounted for about two thirds of the variance in general mathematics achievement at the end of first grade. Importantly, income status did not add explanatory variance after controlling for performance and growth in number knowledge. That is, the poor mathematics achievement of low-income learners was largely accounted for by their weak number knowledge. This finding is significant in that number competencies can potentially be changed through intervention, unlike income status, which is relatively immutable.

Core Number Competencies for Early Screening Involve Knowledge of Number, Number Relations, and Number Operations

A wide variety of number competencies have been targeted for early screening (Jordan & Dyson, 2016; Jordan, Resnick, Rodrigues, Hansen, & Dyson, 2016; Malofeeva, Day, Saco, Young, & Ciancio, 2004; National Research Council, 2009; Rittle-Johnson & Jordan, 2016). In US prekindergarten, kindergarten, and first-grade classrooms, screening has often focused on verbal number sense, that is, abilities related to the symbolic representation of numbers, as opposed to more fundamental nonsymbolic numerical representations (e.g., ANS or approximate number system), which appear to develop without much verbal input or instruction (Feigenson, Dehaene, & Spelke, 2004; Jordan & Levine, 2009). Each screening area is discussed next.

Number: Young children recognize small quantities through subitizing (Baroody, 1987; Baroody, Lai, & Mix, 2006), which involves apprehending and labeling the numerical value of two or three objects without having to count them. Counting, in turn, expands the child's quantitative understanding beyond small sets. Before formal schooling, many children can easily recite the count sequence to ten and higher. Later, children learn to enumerate sets in one-to-one correspondence with counting numbers, recognizing that the last number counted indicates the number of objects in the set (i.e., cardinality principle; Gelman & Gallistel, 1978). Children discover that they can count any set presented in any configuration, so long as they count each object once in numerical order (Gelman & Gallistel, 1978). Children also learn to recognize and produce written number symbols (Arabic numerals 1, 3, 5, etc.) (National Research Council, 2009). In kindergarten, many US children become familiar with the decade words and learn that two-digit numbers represent tens and ones. Persistent difficulties with counting are a characteristic of older children with MLD (Geary, 2004).

Number relations. Understanding the magnitudes of numbers is a key developmental achievement (Case & Griffin, 1990; Griffin, 2002, 2004; Siegler, Thompson, & Schneider, 2011). Recognizing that four objects is more than three objects—or that two objects is fewer than five—reflects understanding of magnitude relations early in development. Later in prekindergarten, children can make judgments about quantities in the absence of physical objects, through mental counting or external representations, such as the number line. Children learn that as they move to the right on the line, numbers represent larger quantities, while moving left is associated with decreasing quantities. Eventually, children learn that each number in the count list is exactly one more than the previous one. Linking abstract representations to observed numerical magnitudes is critical for the development of mathematical ability; deficits in the ability to draw such connections are associated with MLD (Rousselle & Noël, 2007).

Number operations. Many preschoolers successfully solve simple addition and subtraction problems using physical representations (Levine, Jordan, & Huttenlocher, 1992). Even children with limited counting facility can solve problems with sums or minuends of four or less (Huttenlocher, Jordan, & Levine, 1994). Early on, counting (e.g., counting fingers) is a key strategy for solving addition and subtraction problems with sums and minuends of five or more. Knowing that the next number in the count sequence is always one more than the preceding number enables children to compute the value of $n + 1$ (Baroody, Eiland, & Thompson, 2009). By the end of kindergarten, many children can count on from the first or larger addend to find the sum of two numbers (e.g., for 4 + 3, the child counts 5, 6, 7 to get 7). This approach is more efficient than counting out both addends (Baroody et al., 2006). Kindergartners who use counting principles to evaluate number combinations develop calculation fluency earlier in school (Jordan et al., 2009).

Children must also learn that whole numbers can be decomposed into sets of smaller numbers. For example, 4 can be broken into either 1 and 3 or 2 and 2. Along with quantity discrimination, number line estimation, counting, and number word comprehension, kindergartners' ability to identify different combinations that equal a given sum predicts growth in mathematics achievement from kindergarten through second grade (Fuhs, Hornburg, & McNeil, 2016). Children with strong mathematics skills use their knowledge of number sets to derive solutions for new combinations (e.g., if 1 + 3 = 4, then 2 + 3 = 5). However, young children with or at risk for mathematics difficulties have trouble counting on from a number, decomposing numbers, and deriving solutions from known combinations to help them calculate totals of 5 or more. These difficulties lead to poor addition and subtraction skills (Jordan et al., 2006).

Deficits in Number Sense Can Be Reliably Identified Through Early Screening, and Interventions Based on Screening Lead to Improved Mathematics Achievement in School

Gersten et al. (2012) evaluated the predictive validity of early number screeners developed by researchers. Screeners assessing number relations (e.g., Clarke, Baker, Smolkowski, & Chard, 2008; Jordan et al., 2008; Seethaler & Fuchs, 2010) and number operations (e.g., Jordan et al., 2010; Seethaler & Fuchs, 2010) have been especially effective in predicting later mathematics performance. These screening measures demonstrate high classification accuracy (Geary, Bailey, & Hoard, 2009; Jordan et al., 2010; Seethaler & Fuchs, 2010), accurately identifying children who will later need additional help in mathematics (Gersten et al., 2012). Moreover, measures assessing numerical magnitudes are sensitive diagnostic tools for identifying children with dyscalculia, a severe form of MLD (Reigosa-Crespo et al., 2012).

Importantly, there is clear evidence that core number competencies can be improved in most US children (Frye et al., 2013). At the prekindergarten level, experimental studies reveal meaningful effects for interventions that emphasize number sense (Baroody et al., 2009; Clements & Sarama, 2007, 2008; Dobbs, Doctoroff, & Fisher, 2003; Klein, Starkey, Sarama, Clements, & Iyer, 2008). Jordan and colleagues (Dyson, Jordan, & Glutting, 2011; Jordan & Dyson, 2016; Jordan et al., 2012) developed and tested a kindergarten number sense intervention that specifically targets skills with number, relations, and operations—competencies that underlie mathematics difficulties, as described in the previous section. Study participants were at-risk kindergartners who were from low-income communities and/or performed poorly on a number screener. Results from a series of randomized experiments showed that children in the intervention group consistently exhibited greater improvement in terms of both a proximal measure of number sense and a general mathematics achievement test compared to control children who received a language intervention or business-as-usual instruction (Jordan & Dyson, 2016). Of particular significance was the finding that many of the intervention gains held over time, and the achievement gap between intervention children and their normally achieving counterparts decreased substantially. Clarke et al. (2016) report comparable findings from a kindergarten intervention focused on whole number knowledge.

In sum, recent research has highlighted the importance of early number competencies or number sense for future mathematics success and has identified useful targets for intervention that are being used in US schools, such as skill with number relations and operations. Compared to basic cognitive abilities or socioeconomic status, number sense appears to be relatively malleable, and interventions targeting children identified through early screening lead to improved mathematics achievement. Current US practices in early mathematics education are continuing to be revised in concert with what researchers have learned about the sources of early

MLD risk and the effectiveness of early screening measures and interventions. Many research-based early number interventions are being incorporated under RTI models for assessment and intervention.

Fractions

Fraction Knowledge in the Intermediate Grades Predicts Algebra Success in Secondary School, and Weaknesses with Fractions Characterize Middle School Students with Mathematical Learning Difficulties

Whereas having a good sense for whole numbers is central in primary mathematics education, competency with fractions is the hallmark mathematics achievement in intermediate grades in the United States (Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010). Fraction knowledge in middle school predicts subsequent performance in algebra, over and above socioeconomic status, IQ, and whole number abilities (Siegler et al., 2012). Relative to research on whole number knowledge, however, few studies have focused on the development of fraction competencies until recently.

Fractions typically afford students their first opportunity to learn about numbers with properties that differ from those of whole numbers (Siegler & Pyke, 2013). Many US students, especially those with MLD, struggle with basic knowledge of fractions (e.g., Bailey, Hoard, Nugent, & Geary, 2012; Ni & Zhou, 2005; Hansen, Jordan, & Rodrigues, 2017). These difficulties extend past the intermediate grades—students in middle and high school—and even some college students have trouble with basic fractions tasks, such as ordering simple fractions from least to greatest and estimating sums of two fractions (Siegler & Pyke, 2013). For example, when asked to estimate the sum of 12/13 + 7/8 from the response options 1, 2, 19, and 21, 15% of college students at a major US university estimated the sum to be either 19 or 21 (Lewis & Hubbard, 2015). That is, students tended to add together either the numerators or denominators of the fraction, overgeneralizing whole number properties to fractions. Despite errors such as these, whole number knowledge is helpful for learning about fractions. In fact, many students who struggle with fractions have concomitant difficulties with whole numbers, particularly with respect to judging numerical magnitudes (Jordan et al., 2016). Understanding numerical magnitudes with whole numbers provides a foundational structure for thinking about fractions in terms of magnitudes (Case & Okamoto, 1996; Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). As such, effective whole number sense interventions, such as those described previously, may be crucial for building a general understanding of numerical magnitudes that can later be applied to fractions.

Fractions Are Especially Hard for Children with MLD

As noted, many students with or at risk for MLD have poorly developed fraction knowledge (Fuchs et al., 2013). Because children with MLD tend to lack a sound understanding of number magnitudes, many are unable to move beyond the erroneous assumption that properties of whole numbers are true for all numbers in general (Ni & Zhou, 2005; Jordan, Rodrigues, Hansen, Resnick, & Dyson, 2017; Siegler et al., 2011). In contrast to whole numbers, which each directly correspond to one and only one magnitude, have unique successors, and are expressed as a single symbol, different fractions may have the same magnitude and therefore refer to the same location on a number line (1/4 is the same as 2/8 or 4/16). The magnitudes of fractions do not always change in consistent ways with the absolute values of their numerators and denominators (Schneider & Siegler, 2010). For example, 4 is greater than 2, and 12 is greater than 4, but 4/12 is a smaller fraction than 2/4.

When children first start learning fractions, a common misconception is that larger numbers produce larger fraction values in all cases, regardless of whether they appear in the numerator or the denominator (Rinne, Ye, & Jordan, 2017). For example, a child may erroneously think that 1/12 is larger than 1/5 because 12 is larger than 5. Instruction leads some students to develop a partial misconception that smaller values in both denominators *and* numerators decrease fraction magnitudes, but this is usually just a stepping stone on the way to a normative understanding. Eventually, successful students come to understand that numeral values can be inversely related to fraction magnitudes, but this is only true for the denominator. However, Rinne et al. further showed that children who come to fraction instruction with a poor understanding of whole number magnitudes are much less likely to move beyond the simple view that larger numerals always lead to larger magnitudes. Thus, for children with MLD, a lack of whole number magnitude understanding impedes the ability to grasp fraction concepts.

Further problems arise when struggling children begin to learn about fraction operations. For example, multiplication of two fractions may yield a product smaller than either multiplicand, while multiplication of whole numbers greater than one always produces a larger product. A failure to understand numerical magnitudes also produces fraction operation errors that *do not* appear to derive from overgeneralizations of whole number properties. For example, students often mistakenly apply the procedure for fraction addition to fraction multiplication problems and leave the denominator unchanged rather than multiplying across both the numerator and denominator (Siegler & Pyke, 2013). Significantly, the one property that bridges whole numbers with fractions—and might thereby serve as a touchstone for helping students overcome such difficulties—is that both fractions and whole numbers have magnitudes that can be represented on a number line (Case & Okamoto, 1996; Siegler & Lortie-Forgues, 2014; Siegler et al., 2011).

Because they Lack Magnitude Understanding, Students with MLD Struggle to Place Fractions on a Number Line

The implications of poor magnitude understanding are also evident in research on fraction number line estimation. Resnick et al. (2016) examined the development of fraction number line estimation on 0–1 and 0–2 number lines between fourth and sixth grade, uncovering three distinct growth trajectory classes: (1) students who are highly accurate from the start and became even more accurate, (2) students who initially are inaccurate but show steep growth, and (3) students who initially are inaccurate and show minimal growth. Growth class membership accurately predicted subsequent performance on a standardized mathematics achievement test at the end of sixth grade, even after controlling for mathematics-specific abilities, domain-general cognitive abilities, and demographic variables. Students falling into the minimal growth class tended to place both proper and improper fractions below one on a number line, suggesting they do not effectively consider the relation between numerator and denominator. Multiplication fluency, classroom attention, and whole number line estimation acuity at the start of the study predicted class membership, indicating these areas make important contributions to learning fractions, and deficits in these areas may impede learning.

Fraction Difficulties Can Be Reliably Identified by Fourth Grade

Rodrigues et al. (2016) evaluated the diagnostic accuracy of mathematics screening measures (starting in fourth grade) for predicting MLD at the end of sixth grade. Receiver operating characteristic (ROC) curve analyses showed that of a broad group of fraction and general mathematics ability measures, fraction number line estimation acuity and knowledge of fraction concepts emerged as the strongest predictors of who would go on to fail a mathematics achievement test at the end of sixth grade. These measures were significantly more accurate predictors of sixth-grade mathematics failure than were measures of fraction procedures and multiplication fluency, both of which typically receive much more attention in instructional settings.

Fraction Difficulties Can Be Improved Through Meaningful Interventions that Center on the Number Line

Referring to current mathematics instruction in the United States, Gersten and Jordan (2016) observe the following, despite between- and within-state variation in the United States:

> Perhaps the most profound change in contemporary mathematics instruction for students in the elementary grades has been a strong emphasis on mastery of concepts involving fractions. This change, reflected in virtually all contemporary state standards, involves not only a shift in the amount of time dedicated to teaching fractions but also a shift in emphasis. Mathematics instruction is now making fraction concepts, most notably fraction magnitude, take priority over fraction procedures (p. 1).

This change is having a significant effect on instruction for students with MLD, as evidenced by new research showing that interventions that focus on representing fraction magnitudes on number lines lead to improved mathematics outcomes. Fraction number line activities require students to think about proportionality and to reason multiplicatively; both skills represent important underpinnings of fraction conceptual knowledge (Hansen et al., 2015; Vukovic et al., 2014). Until recently, a part-whole interpretation of fractions has been a pervasive influence in the US mathematics curriculum (Siegler, Fuchs, Jordan, Gersten, & Ochsendorf, 2015). However, in a series of experimental studies that used the number line as a basis for helping students evaluate magnitude (sometimes referred to as a measurement approach), Fuchs et al. (2016) showed that low-performing fourth graders can learn to determine the magnitudes of fractions, and this knowledge transfers to other fraction skills, including arithmetic.

To date, our research team (Dyson, Jordan, Rodrigues, Barbieri, & Rinne, in preparation; Jordan et al., 2016; Rodrigues, Dyson, Hansen, & Jordan, 2017) has conducted several experimental trials of an intervention for sixth and seventh graders who persistently struggle with fractions even after several years of typical classroom instruction. Our "fraction sense" intervention, which is centered on the number line, aims to build fundamental understandings of (1) the meaning of a fraction (how the numerator and the denominator work together to determine a fraction's magnitude), (2) fraction relations (how the magnitudes of fractions are ordered on the number line), and (3) fraction operations (how fractions are added, subtracted, multiplied, and divided). Thus, this model of instruction is partly analogous to the whole number sense model described earlier in this chapter.

To develop core fraction knowledge based on just few key ideas, the three topics described above are taught using fractions with a narrow range of denominators. For example, we start with denominators of 2, 4, and 8 and gradually expand to include denominators of 3, 6, and 12. In addition, the intervention anchors ideas in a meaningful story line to help struggling learners think about fraction concepts in a more concrete way (Bottge et al., 2014). Specifically, instruction takes place in the context of a "color run" race for charity during which runners have colored powder thrown at them at regular intervals during the race. The race context facilitates thinking about fraction magnitudes using a measurement interpretation (e.g., finding fractions of a mile), and the number line helps students see relations between fractions with both different and equivalent magnitudes. Children are asked to compare the relative sizes of numerators and denominators and to think about fractions as being close to 0, close to 1, equal to 1, or greater than 1. The intervention also applies general learning principles from cognitive science by incorporating gestures that guide students' attention (Alibali, Spencer, Knox, & Kita, 2011), side-by-side comparisons of solution methods (Rittle-Johnson, Star, & Durkin, 2009), instructional explicitness (Gersten et al., 2009), and clear visual models to minimize cognitive load (Fuchs et al., 2009). Practice activities mix problems with more and less familiar fractions to develop fluency and improve retention (Carpenter, Fennema, & Romberg, 2012). Finally, fast-paced games help build both whole number and fraction fluency at the end of each lesson.

Although our intervention work is ongoing, preliminary findings have been positive. Participants (who were identified by their teachers as needing intervention or who performed below a predetermined cut-point on a reliable screener of fraction concepts) were randomly assigned to our intervention or a business-as-usual intervention contrast group. Children who received the intervention performed reliably better than controls, with large effect sizes on measures of fraction number line estimation, as well as more general fraction conceptual knowledge. For the most part, students maintained these gains on a delayed posttest administered 2 months after the conclusion of the intervention.

Overall, recent intervention work with fractions reveals that interventions that focus on fraction magnitude and that use the number line as a representational guide hold promise for helping all students learn fractions. The number line approach is likely to gain traction in US schools, including special education. In fact, the US benchmarks in math (i.e., CCSS) emphasize the use of the number line to teach fractions, starting in third grade. Future work is needed, however, to examine whether such interventions can help students succeed with respect to longer-term outcomes, such as algebra proficiency and using fractions in daily life.

Conclusion

In the early elementary years, the primary goal of mathematics instruction in the United States is to build children's number sense with whole numbers. Research shows that a good understanding of whole number magnitudes is critical for later facility with fractions, mastery of which is a key accomplishment in the intermediate grades. Failure to master fractions has severe long-term consequences for student success in mathematics, limiting eventual prospects for employment and leading to poor decisions in the increasingly number-rich environments of everyday life.

Acquiring both whole number and fraction knowledge is particularly challenging for students with MLD and thus a major educational concern in American schools, particularly in light of recent shifts in curriculum and standards (i.e., CCSS) toward deeper conceptual understanding of mathematics. One challenge that remains is how to balance the needs of students with MLD with these more rigorous standards; many US students with MLD have weak number sense and subsequent difficulty representing fractions as magnitudes on a number line, which prevents them from incorporating fractions and whole numbers into a coherent understanding of the rational number system. Fortunately, recent research suggests that both early difficulties with whole numbers and later difficulties with fractions can be remediated by helping students build solid magnitude representations, and interventions focused on representing fractions along with whole numbers on number lines lead to improved mathematics outcomes.

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