Nonlinear Numerical Analysis of Thaw Consolidation of Ice Rich Frozen Soil

Xiaoliang Yao[1](✉), Boxiang Dang[2], and Jilin Qi[2]

[1] State Key Laboratory of Frozen Soil Engineering,
Northwest Institute of Eco-Environment and Resources,
Chinese Academy of Sciences, Lanzhou 730000, China
`yaoxl@lzb.ac.cn`
[2] School of Civil and Transportation Engineering,
Beijing University of Architecture and Civil Engineering, Beijing 100044, China

Abstract. A linear interpolation function fitting the nonlinear relationship between void ratio and compression modulus was proposed and corresponding strategy guaranteeing calculation accuracy and efficiency was developed for the nonlinear numerical analysis of 3-D large strain thaw consolidation of ice rich frozen soil. It was verified by a series of ice rich thaw consolidation tests that, with the proposed numerical implementation strategy, the calculated results match well with the tested results of pore water pressure and thaw displacement. Further analysis on the different stress-strain relationships shown that the prediction values and accuracies on pore water pressure and thaw displacement of nonlinear relationship is higher than that of linear relationship. This also leads to higher prediction accuracy on thaw consolidation degree and pore water pressure at thawing front of nonlinear relationship.

Keywords: Ice rich frozen soils · Thaw consolidation
Nonlinear stress-strain relationships

1 Governing Equations and Numerical Implementation

When ice rich frozen soil is involved, the large strain consolidation theory is usually employed to describe soil skeleton mechanical behavior and fluid flow in post-thawed domain [1]. Generally, the consolidation theory includes four parts (i.e., kinematic equation, constitutive equation, Darcy's law and fluid mass conservation equation), where the three dimensional linear constitutive theory us expressed as,

$$\dot{\sigma}_{ij} = \frac{E}{(1+v)}\dot{\varepsilon}_{ij} + \frac{vE}{(1-2v)(1+v)}\dot{\varepsilon}_{ij}\,\delta_{ij} - \delta_{ij}u \qquad (1)$$

In which, σ_{ij} is total stress tensor, E is Young's modulus, v is Poisson's ratio, δ_{ij} is the Kronecker symbol, and $\dot{\varepsilon}_{ij}$ is symmetric deformation tensor.

W. Wu and H.-S. Yu (Eds.): *Proceedings of China-Europe Conference on Geotechnical Engineering*, SSGG, pp. 1433–1437, 2018.
https://doi.org/10.1007/978-3-319-97115-5_117

The thermal conductive equations are implemented to detect the post-thawed domain as following,

$$\begin{cases} -h_{i'i} + h_v = \rho c \dfrac{\partial T}{\partial t} \\ h_i = -\xi T_{'i} \end{cases} \tag{2}$$

In Eq. (2), T is temperature (°C), h_v (W/m^3) is volumetric heat source intensity; c (J/kg·°C) and ξ (W/m·°C) are specific heat and thermal conductivity, respectively; ρ is the media density (kg/m^3). Both of the thermal parameters are temperature dependent, and the details can be referred to literatures [2, 3].

As it can be seen in Eq. (2), the linear stress-strain relationship was used in previous three dimensional analysis of frozen soil thaw consolidation [4, 5]. For ice rich frozen soil, the compressibility of which shows strong nonlinearity, and the nonlinear stress-strain relationship must be used to describe the soil skeleton mechanical behaviours. In the following, a nonlinear relationship between void ratio (e) and compression modulus (E_s) is used to modify the original linear thaw consolidation theory, i.e.,

$$E_s = \frac{1+e_0}{\lambda}\exp(\frac{e_0 - e}{\lambda}) \tag{3}$$

where, λ is the slope of K_0 compression *e-log* (stress) curve. The Young's modulus (E) can be further expressed as,

$$E = \left(1 - \frac{2v^2}{1-v}\right)E_s \tag{4}$$

In numerical analysis, a linear interpolation function Eq. (3) for fitting the nonlinear relationship between void ratio and compression modulus is proposed for guaranteeing calculation accuracy and efficiency as,

$$E_{s(t+t\Delta)} = (1 - \frac{e_{t+t\Delta} - e_m}{e_{m+1} - e_m})E_{s(m)} + \frac{e_{t+t\Delta} - e_m}{e_{m+1} - e_m}E_{s(m+1)}(e_m \le e_{t+t\Delta} \le e_{m+1}, \quad m = 0, 2, 3, \ldots, 4) \tag{5}$$

where, $Es_{(m)}$ and e_{m-} are the data points obtained from the K_0 compression *e-log* (stress) curve.

2 Verification and Results Discussion

To verify the applicability of the proposed linear interpolation function for fitting the nonlinear relationship between void ratio and compression modulus, a series of 1-D thaw consolidation tests under different dry unit weight and surcharge loads were conducted, and the corresponding parameters for consolidation calculation are obtained in Table 1.

Table 1. Parameters for consolidation calculation.

Dry Unit Weight (kN/m^3)	λ	e_0	Surcharge Load (kPa)	k (m/s)	E_{s0} (kPa)	Poisson's Ratio
16.1	0.029	0.68	50	5.00E−08	271	0.25
			100	3.40E−08	508	
13.0	0.052	1.08	50	7.00E−08	189	0.30
			100	4.40E−08	346	
11.5	0.080	1.35	50	9.55E−08	137	0.30
			100	6.55E−08	255	

As the two main indexes representing the thaw consolidation behavior of frozen soil, thaw consolidation degree (*TD*) and the pore water pressure at thawing front (*PPTF*) are closely related thaw consolidation ratio (*TCR*) [2, 6], which is defined as,

$$TCR = \frac{\alpha}{2(c_v)^{1/2}} \tag{6}$$

where, α is the thawing rate of soil sample, which is related to the thermal properties and boundaries; c_v is the consolidation coefficient and defined as,

$$c_v = \frac{kE_{s0}}{\rho_w} \tag{7}$$

where, the secant compression modulus (E_{s0}) is used for ease of analyzing the difference between linear and nonlinear relationships.

Figures 1 and 2 show the relationships between thaw consolidation degree (*TD*), normalized pore water pressure at thawing front (N_{PPTF}) and *TCR*, where *TD* and N_{PPTF} are defined as,

$$TD = \frac{h(\mathrm{t})}{h_{\max}(\mathrm{t})} \tag{8}$$

$$N_{PPTF} = \frac{u}{p_0} \tag{9}$$

where, $h(t)$ is the thaw displacement, $h_{\max}(t) = \frac{E_{s0}}{P_0}x(t)$ and $x(t)$ is the thaw depth at time t. It can be seen that for the calculated results of both stress-strain relationships, N_{PPTF} is proportionally related to *TCR*, while it is opposite for *TD*. This indicates that with increase of *TCR*, more post-thawed pore water is generated, while the rate of drainage (c_v) is relatively decreased. Subsequently, the *TCD* decreases and N_{PPTF} increases. By comparing the calculated results of both relationships (linear and nonlinear), it can be found that for both of the indexes, the linear results are lower than that of nonlinear results, which is due to the calculated differences of both stress-strain relationships on thaw displacement and pore water pressure. In addition, the nonlinear stress-strain relationship shows a higher accuracy on of the *TCD* and N_{PPTF} (Figs. 1 and 2) than the

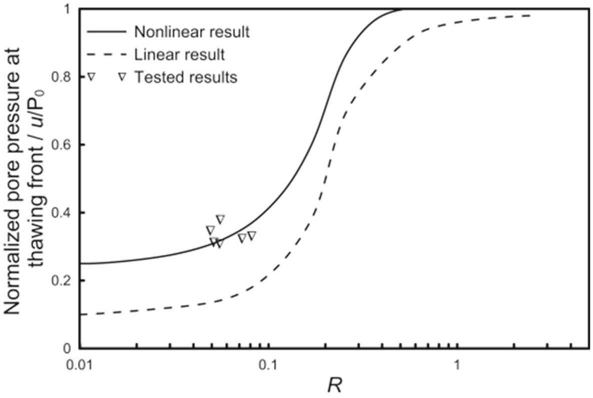

Fig. 1. Changes in N_{PPTF} vs. *TCR* (top)

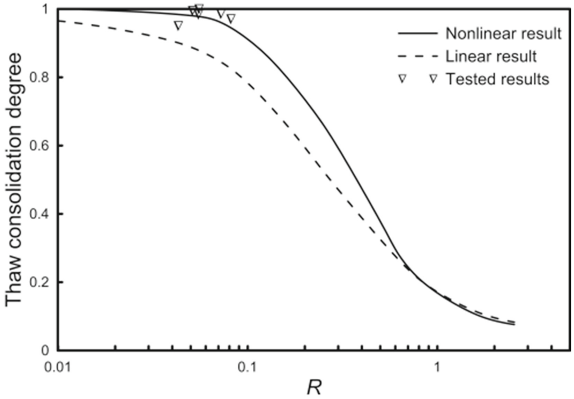

Fig. 2. Changes in *TD* with *TCR* (bottom).

linear relationship, which indicates the applicability of the proposed linear interpolation function in cold regions engineering when ice rich permafrost is involved.

Acknowledgements. This work was supported in part by the National Natural Science Foundation of China (No. 41671061) and the research Project of the State Key Laboratory of Frozen Soils Engineering granted to Dr. Xiaoliang Yao (SKLDSE-ZT-28).

References

1. Yao, X.L., Qi, J.L., Wu, W.: Three dimensional analysis of large strain thaw consolidation in permafrost. Acta Geotech. **7**(3), 193–202 (2012)
2. Qi, J.L., Yao, X.L., Yu, F.: Consolidation of thawing permafrost considering phase change. J. Civil Eng. KSCE **17**(6), 1293–1301 (2013)
3. Wang, S.H., Qi, J.L., Yu, F., Yao, X.L.: A novel method for estimating settlement of embankments in cold regions. Cold Reg. Sci. Technol. **88**(5), 50–88 (2013)
4. Sykes, J.F., Lennox, W.C., Charlwood, R.G.: Finite element permafrost thaw settlement model. J. Geotech. Eng. Div. **100**(11), 1185–1201 (1974)
5. Qi, J.L., Yao, X.L., Yu, F., Liu, Y.Z.: Study on thaw consolidation of permafrost under roadway embankment. Cold Reg. Sci. Technol. **81**(3), 48–54 (2012)
6. Morgenstern, N.R., Smith, L.B.: Thaw-consolidation tests on remoulded clays. Can. Geotech. J. **10**(1), 25–40 (1973)